Shiraz Journal of System Management
Vol 4, No. 2, Ser. 14 (2016), 001-010

Offering a New Algorithm to Improve the Answer-Search Algorithm in Quadratic Assignment Problem

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Abstract. Layout design problem is one of the useful field of study used to increase the efficiency of sources in organizations. In order to achieve an appropriate layout design, it is necessary to define and solve the related nonlinear programming problems. Therefore, using computer in solving the related problems is important in the view of the researchers of this area of study. However, the designs produced by a computer to solve big problems require more time, so, this paper suggests an algorithm that can be useful in better performance of the known algorithms such as Branch and Bound.

Keywords. Facility Layout, Computer Algorithms, Exact Methods, Branch and Bound, Feasible Answer.

1. Introduction

Facility layout design is a position layout of the equipment of good production or service offering. Koopmans and Beckmann were the pioneers defining facility layout design problem as a common industrial problem which aims at configuring facility so that the cost of the transportable materials will be minimized(Koopmans and Beckmann,1957).

Azadivar and Wang defined layout design problem as a problem determining relative displacement and allocating space to the existed facilities.It is often hypothesized that material flow among departments is fixed and the designed layout will be applicable for a long time. But due to competitive atmosphere of the market and change in customers' taste, dynamism is considered as an inevitable element in industry today, that as a result of the production companies, they have to be able to answer it (Tompkins et al,1996). Due to this point, it can be inferred that facility layout for short time needs can inconsiderably increase the costs resulted from primary facility for a long time. Thus, it seems that considering a dynamic factor is necessary and important (Baykasoglu et al, 2006).

2. Quadratic Assignment Problem (QAP)

In 1957, Beckmann and Koopmans defined and formulated Quadratic Assignment Problem to be used in economic activities. Because of its quadratic nature, this problem is known as Quadratic AssignmentProblem, which has attracted researchers' attention working in several areas of study. Lots of researchers and scientists used it in areas of mathematics, computer, operations research and economics to model optimization problems. Assignment means that each facility should be conformed into one position and vice versa. In QAP,the number of facilities has to be equal to the number of positions. Mathematic form of this problem is as follows(Azadivar and Wang,2010):

$$Min\ C = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{s=1}^{n} d_{i,k} * w_{j,s} * x_{i,j} * x_{k,s}$$

$Subject\ to$:

$$\sum_{j=1}^{n} x_{i,j} = 1\ ;\ i = 1,2,\dots,n \qquad (1)$$

$$\sum_{i=1}^{n} x_{i,j} = 1\ ;\ j = 1,2,\dots,n$$

$$x_{i,j} = 0\ or\ 1\ ;\ \ i = 1,2,\dots,m\ \&\ j = 1,2,\dots,n$$

In the event that facility of j is located in cell of i and facility of s is located in cell of k $(x_{i,j} = 1 \& x_{k,s} = 1)$, by calculating the previous condition, the cost of displacement in this route is $c_{i,j,k,s} = d_{i,k} * x_{i,j} * x_{k,s}$. In the event that importance of this route is considered, average displacement cost in this route is $\bar{c}_{i,j,k,s} = d_{i,k} * w_{j,s} * x_{i,j} * x_{k,s}$.

3. Statement of Problem

Quadratic Assignment Problem(QAP) is one of the most complex optimization problem of nonlinear integer (Loon Lim et al,2016). In general, (in wide dimension problems) QAP does not include an exact solution because it is located in the group of NP-Hard problems and to solve it, Metaheuristic algorithm and Invasive Weed Optimization are often used.

To solve Quadratic Assignment Problem, some exact algorithms such as Dynamic programming, cut page method, Branch and Bound method can be used (Christo and Benavent,1989)&(Bazara and Sherali,1980). Branch and Bound method proves better function than the previous two methods does (Mautor and Roucairol,1994). One of the problems of the mentioned three methods is their incapability in solving wide dimension problems. In other words, using the mentioned algorithms are not possible for the problems with size more than 15 (Al-Hakim,2000).

In real world, all of facilities cannot be settled in all Locations. Thus, by making search space small, we can reach an optimum answer faster in problems having wide dimensions and such limitations.

4. Literature Review

Stützle(2006) offered a new method called Iterated Local search (ILS) to solve Quadratic Assignment Problem. Iterated Local Search is a simple random search method. First, some random points are created in search space, then based on the competence of the mentioned points, searching around them is started. One of the biggest challenge in Stützle's method is the radius in local search.

Hicks(2006) in a paper developed Genetic algorithm to be used in facility layout in a set of productive cells. The results showed that the approach of redesigning facilities determines intracellular layout, then it localizes the cells among empty departments.

Mc et al(1988) in a paper used Genetic algorithm as a general method to solve layout design problems. They developed a mathematical model to study layout of the devices and material flow pattern for workshop and product manufacturing environment. The suggested Genetic algorithm with the aim of minimizing material displacement cost, extracts an optimum machinery layout.

5. Branch and Bound Algorithm

Branch and Bound is a public algorithm used to find the optimum solutions of different problems, especially in discrete optimization and combinational optimization. This algorithm counts all the solutions of a problem, meanwhile, there are lots of useless solutions that, by deleting them through estimating upper and lower boundaries, can be optimized. This method was first introduced by Land and Doig for discrete programming in 1960. In this algorithm all the states preparing the probability of reaching better answers, will be studied and finally, the best answer will be chosen out of all the studied answers (Land and Doig,1960).

6. Introducing Feasible Search Algorithm

The new algorithm is explained in the following order:

1.start
2. put K=1
3.Put n=N
4.Put MaxCost= $+\infty$
5. Put NE=O
6. Put i=1
7. Put j=1
8. Choose a possible state (feasible) for $X_{i,j}$ from the set $S_{i,j}$ as if solution X isnot repetitive.
9. Put NE=NE+1
10. Calculate objective function for the present layout and copy it in variable Cost (NE)
11. If Cost (NE) <MaxCost, put Best Cost=Cost (NE)and Bestsolution=x.
12. If $j \leq$ n-1, add one unit to j and go to step 8, otherwise go to step 10.
13. If $i \leq$n-1, add one unit to i, and go to step 7, otherwise go to step 14.
14. Print NE
15. Put Best solution.

16. Print Best cost.
17. The end.

N means the number of facilities, $\mathbf{S}_{i,j}$ means all the possible (feasible) states for $\mathbf{x}_{i,j}$, also **NE** depicts the number of evaluations or the measured solutions.

7. Case Study

The case study in this paper includes an industrial workshop producing different kinds of wooden and metal products. This workshop includes 17 facilities and 17 Locations. The aim of this paper is to reach an optimized settlement of the facilities in the Locations based on the distance among the Locations and the transportation flow among machines.

8. Distance of the Locations

The distance among the Locations is shown in Table 1.

Table 1. Distance of Locations (meter)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	25	38	50	60	77	22	35	49	63	77	28	43	55	67	79	101
2	25	0	16	28	40	55	5.5	17.5	31.5	44.5	58.5	23	38	50	62	74	98
3	38	16	0	17	29	41	17.5	6.5	19.5	33.5	47.5	23	24	36	48	60	68
4	50	28	17	0	16	31	29.5	15.5	7.5	21.5	35.5	35	20	28	32	44	56
5	60	40	29	16	0	19	41.5	27.5	13.5	9.5	23.5	47	32	20	24	36	44
6	77	55	41	31	19	0	56.5	42.5	28.5	14.5	8.5	62	47	35	23	23	25
7	22	5.5	17.5	29.5	41.5	56.5	0	7	31	45	59	8	23	35	47	59	81
8	35	17.5	6.5	15.5	27.5	42.5	7	0	7	31	45	14	9	21	32	44	68
9	49	31.5	19.5	7.5	13.5	28.5	31	7	0	7	31	28	13	7	19	31	55
10	63	44.5	33.5	21.5	9.5	14.5	45	31	7	0	7	44	27	15	5	17	41
11	77	58.5	47.5	35.5	23.5	8.5	59	45	31	7	0	56	41	29	17	5	26
12	28	23	23	35	47	62	8	14	28	44	56	0	20	32	44	56	80
13	43	38	24	20	32	47	23	9	13	27	41	20	0	16	27	39	63
14	55	50	36	28	20	35	35	21	7	15	29	32	16	0	16	28	52
15	67	62	48	32	24	23	47	32	19	5	17	44	27	16	0	16	40
16	79	74	60	44	36	23	59	44	31	17	5	56	39	28	16	0	28
17	101	98	68	56	44	25	81	68	55	41	26	80	63	52	40	28	0

9. Percentage of Transportation Flows

Percentage of displacements among machines is shown in Table2.

Table 2. Matrix of the transportation percentage among facilities

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	0	0	0	0	0	0	0	16.9	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	12.15	0	0	0	0	0	0	0	4.97
3	0	0	0	0	0	0	0	0	1.83	0	10.35	0	0	0	0	0	3.65
4	0	0	0	0	0	12.27	2.79	2.79	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	12.61	0	0	2.23	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0.01	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0	0	0	0.01
9	0	0	0	0	0	0	0	0	0	0	0.03	0.03	0	0	0	0	0.09
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0.37	0	0	0	0	0	0.41	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	3.67	1.2	5.27	2.39	3.99	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

10. Creating a Mathematical Model of Facility Layout Problem

In the following discussion, the required model is defined generally and parametrically.

$$\text{Cost} = \sum_{i=1}^{17}\sum_{j=1}^{17}\sum_{k=1}^{17}\sum_{s=1}^{17} d_{i,k} w_{j,s} x_{i,j}\, x_{k,s}$$

Subject to:

$$\sum_{j=1}^{17} x_{i,j} = 1\ ;\ i = 1,2,\ldots,17 \tag{2}$$

$$\sum_{i=1}^{17} x_{i,j} = 1\ ;\ j = 1,2,\ldots,17$$

$$x_{12,j} = 0\ ; j \epsilon \{1,2,3,4,5,6,7,8,9,10,11,12,13,15,16\}$$

$$x_{6,j} = 0\ ; j \epsilon \{1,2,3,4,5,6,7,8,9,10,11,12,13,15,16\}$$
$$x_{1,j} = 0\ ; j \epsilon \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,17\}$$
$$x_{17,j} = 0\ ; j \epsilon \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,17\}$$
$$x_{7,j} = 0\ ; j \epsilon \{1,3,4,5,8,10,11,13,14,15,16,17\}$$
$$x_{8,j} = 0\ ; j \epsilon \{1,3,4,5,8,10,11,13,14,15,16,17\}$$

$x_{9,j} = 0 \; ; j \epsilon \{1,3,4,5,8,10,11,13,14,15,16,17\}$
$x_{10,j} = 0 \; ; j \epsilon \{1,3,4,5,8,10,11,13,14,15,16,17\}$
$x_{11,j} = 0 \; ; j \epsilon \{1,3,4,5,8,10,11,13,14,15,16,17\}$
$x_{2,j} = 0 \; ; j \epsilon \{2,6,7,9,12,14,15,16,17\}$
$x_{3,j} = 0 \; ; j \epsilon \{2,6,7,9,12,14,15,16,17\}$
$x_{4,j} = 0 \; ; j \epsilon \{2,6,7,9,12,14,15,16,17\}$
$x_{5,j} = 0 \; ; j \epsilon \{2,6,7,9,12,14,15,16,17\}$
$x_{13,j} = 0 \; ; j \epsilon \{2,6,7,9,12,14,15,16,17\}$
$x_{14,j} = 0 \; ; j \epsilon \{2,6,7,9,12,14,15,16,17\}$
$x_{15,j} = 0 \; ; j \epsilon \{2,6,7,9,12,14,15,16,17\}$
$x_{16,j} = 0 \; ; j \epsilon \{2,6,7,9,12,14,15,16,17\}$
$x_{12,14} = x_{1,16}$
$x_{12,17} = x_{1,15}$
$x_{6,14} = x_{17,16}$
$x_{6,17} = x_{17,15}$
$x_{i,j} = 0 \text{ or } 1 \; ; \; i = 1,2,\ldots,17 \;\& \; j = 1,2,\ldots,17$

Due to this point that in the current problem, based on real condition of the studied workshop, some new stipulations are added to that do not exist in base QAP, thus these stipulations are explained briefly in Table 3.

Table 3. Explanation of new stipulations

Limitation in reality	Mathematical stipulation
Location 12 can only accept facility 14 and 17	$x_{12,j} = 0 \; ; j \epsilon \{1,2,3,4,5,6,7,8,9,10,11,12,13,15,16\}$
Location 7 can only accept facilities {6,7,9,12,2}	$x_{7,j} = 0 \; ; j \epsilon \{1,3,4,5,8,10,11,13,14,15,16,17\}$
Location 2 can only accept facilities {1,11,10,8,4,3,2,5,13}	$x_{2,j} = 0 \; ; j \epsilon \{2,6,7,9,12,14,15,16,17\}$
If facility 14 is settled in Location 12, facility 16 shouldbe settled in Location 1	$x_{12,14} = x_{1,16}$

11. Comparison of the Results of the Proposed Algorithm and Branch and Bound Algorithm

Table 4 shows the results of performing the two algorithms by a common computer (CPU: 3.2GHz&RAM: 4096MB).

Table 4. Results of the two algorithms' performance

Branch and Bound algorithm			Proposed algorithm		
Time of performance (seconds)	Number of Evaluations made	Optimum amount	Time of performance (seconds)	Number of evaluations made	Optimum amount
2256.3	19353600	1400.845	1003.1	9676800	1400.845

As it is shown in Table 4, the new algorithm could reach the optimum answer by spending less time and making less evaluations. The optimized facility layout is inserted in Table 5. For example, facility 16 should be located in the first Location.

Table 5. Optimum facility layout

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

12. Conclusion

To reach an appropriate layout design, it is necessary to define and solve the related nonlinear programming problems. Thus, using computer to solve the related problems seems to be important to the researchers of this area of study. But, usually the designs produced by computers for solving big problems need more time. In this paper, an algorithm is proposed that can be useful for better performance of the known algorithms such as Branch and Bound and so it can identify the best answers by spending less time.

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