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Coordinating the Two-Echelon Supply Chain of Perishable Products with Uncertain Demand: A Game-Theoretic Approach

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Abstract **This study examines the ability of contracts as one of the supply chain coordination mechanisms under competitive conditions. In this study, a two-tier supply chain model with two manufacturers and two retailers is considered to develop a competitive structure when demands are uncertain. The supply chain demand is random and depends on the price of the products. Moreover, the products manufactured by market manufacturers are replaceable. Therefore, the main competitive factor is the order decisions, and due to the nature of the demand, deciding on pricing and ordering is necessary. Each retailer is faced with the issue of determining the prices of goods from two manufacturers, which consequently forms a competitive ground between retailers. Therefore, the two-tiered supply chain model is based on the contract price and is optimized, followed by coordinated analysis. The result of the study shows that to maintain the structure of the supply chain, the manufacturer can increase its wholesale price to the extent that the retailer has zero profit. The lowest price of wholesalers is equal to the cost of production, and the wholesale price can be increased to the point where the retailer's profit is zero.**

Keywords *Pricing, Wholesale Price, Contract, Supply Chain, Demand*

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Introduction

The supply chain of perishable goods has always been one of the most important and challenging issues in supply chain management. Proper supply chain designs have optimal effects on system performance, while improper designs can cause many problems. In addition to economic problems, the deterioration of goods also causes environmental damage. Thus, resulting in increased waste and environmental pollution. One of the problems in the food supply chain is waste (Doan 2020). In recent years, many researchers in the food supply chain have focused on the importance of supply chain management (Zarei, Fakhrzad et al. 2011). Providing healthy and quality food, especially perishable products, is a concern for the companies involved in the supply chain (Morganti and Gonzalez-Feliu 2015). The supply chain for perishable goods has always been one of the most important and challenging management issues in different conditions. Perishable goods, such as foodstuffs, are a challenge for the supply chain management. These challenges are mainly due to the diversity in the number of goods, tracking the flow of goods during the supply chain, the short lifespan of products, and the need to control the temperature in the supply chain. Food hazards can occur at any stage of the food supply chain, and proper management can be used to determine critical control points for information about products, manufacturing and expiration dates, etc. And provide it in a transparent manner is essential for supply chain participants and consumers In the food supply chain, certain conditions, such as climate, raw products, deterioration, and special storage conditions, lead to greater uncertainty According to research, the supply chain of complex food industries is constantly changing. Changing diet preferences, shopping habits, and lifestyle increases the necessity to make products easier to prepare. On the other hand, population growth has led to higher demand for fresh produce as well as higher valueadded products ((Fournier and Jacobs 2020). Given the diversity of products in the market, the challenge of anticipating and responding to customer demand becomes more important for manufacturers and retailers. Moreover, to satisfy customers, manufacturers and retailers need to focus more on time management. Therefore, the food supply chain is faced with several challenges such as changes in customer demand, delivery time, and shortage of inventory. Since perishable foods have more specific conditions than non-perishable foods, managing the perishable food supply chain becomes very important. If products don't reach the customer and get spoiled on the way, high costs will be imposed on the entire supply chain. Moreover, previous research has shown that most of the methods used do not correspond to the variables in the real world, but the leading research tries to provide a model that can be used with the highest efficiency considering the variables close to the real world. In this study, by modeling a three-tiered supply chain with the help of game theory, the behavior of variables affecting the performance of the perishable goods are examined (Rasti-Barzoki and Moon 2020; Jafari, Hejazi et al. 2020). The variables affecting the supply chain performance have been considered in this study and include delivery time, uncertainty in demands, average consumer demand, and demand diversity (demand diversity refers to the level of variation in demand). With the help of game theory, the role of each member of the supply chain will be identified, and by creating different scenarios, the effect of these factors will be determined on the supply chain. In the second stage of the proposed model, effective indexes and variables in the form of mathematical models are made to maximize the level of customer service and to minimize costs. In this research, a supply chain is considered in which the producer offers a new product, then the distributor buys the goods and sells them to the final customers. Since the product is perishable, both its quantity and quality may be reduced during the transportation process. The products produced by manufacturers have different prices, and the retailers sell the products at a different price to the final customer. The demand function indicates the selling price of the competitor's retail goods for the current product. Due to factors such as transportation, product freshness, and market demand, the decisions of the three tiers involved in this supply chain are very complex and may damage the overall performance of the chain.

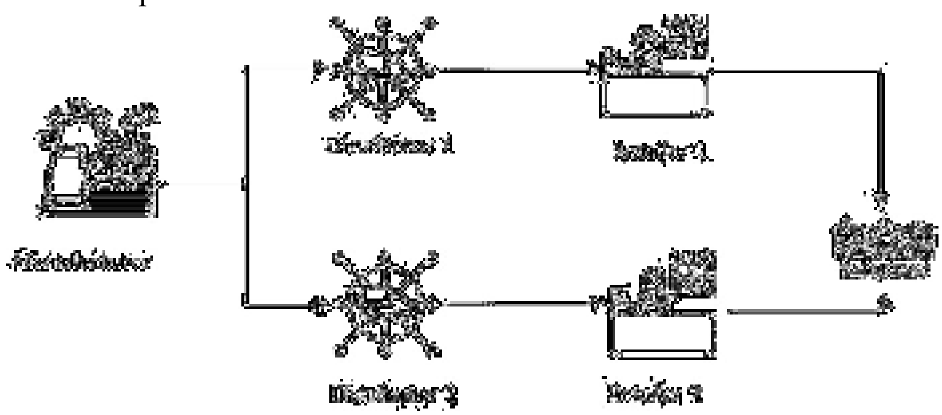

Figure 1.
Supply Chain of the Case Study

As shown in Figure 1, the three-tiered chain supply chain includes one manufacturer, two distributors, and two retailers. The manufacturer receives the required raw materials from a qualified supplier. The manufacturer then sells the final product to the distributor (two distributors) for the shortest time to transfer to the market after the production process. Once the product reaches the distributors, they deliver it to two retailers. At this stage, the manufacturer, distributor, and retailer are initially examined based on four competitive strategies (competitive scenarios and pricing) in the form of a game theory model (income and options contract and discount) in different periods.

Literature Review

In this section, the background of the research is reviewed in two sections. Pricing in the supply chain, revenue sharing, and options contracts. At the end of these two sections, the research gap will be explained in more detail. Eghbali et al.(2019) , studied the theoretical and empirical aspects of supply chain and operational concepts and focused on optimal pricing decisions in a centralized three-tiered supply chain consisting of several suppliers, a manufacturer, and a retailer. The supplier offers a specific raw material, and the manufacturer requires it to produce two other specific products. Each product requires a different mix of raw materials, and as soon as the price of the product is below its actual value, the customer purchases the product. This determines the impact of raw material suppliers, production operations, and marketing conditions on choosing the best pricing strategy. Nowadays, sustainability is becoming more popular among industrial companies due to environmental issues. Hence, governments are following the Energy Efficiency Program (EEP) to control and reduce energy consumption worldwide. In Safarzadeh and Rasti Barzuki's , (2019) paper, a new sustainable supply chain with the hypothesis of maximum efficiency is equal to the increase in technical efficiency which can be used by manufacturers. Energy-saving costs are considered as a quadratic function, and the retail price of the product, produced by an inefficient and an energy-efficient manufacturer is greater than the costs of the finished products (excluding the cost of energy services). The retail price of the home and industrial energy services is greater than the cost of producing energy services for the supplier. Hence, the results examine an energy-efficient manufacturer, an inefficient manufacturer, and an energy-efficient supplier. In this regard, a pricing model for the energy product is presented in a two-way efficiency program. Parsai Fard et al. (2019) proposed that increasing the green growth index will not only increase demand but also increases the profits of all parts of the supply chain. Researchers have used a market hypothesis where all the parameters of the supply chain are definitive and known. A complete competition between retailers and suppliers exists, such that for each product and raw material, there must be at least two retailers and two suppliers. To maximize profits in the supply chain, many strong coordination tools are involved such as pricing, advertising, inventory management. Karimabadi et al. (2019) considered pricing in a fuzzy two-tiered supply chain that involves a retailer and a manufacturer that produces a product (a completely new product directly from the raw materials or applying operations on other finished products) and consequently sells it to the end customer through a retailer. This was considered while assuming all fuzzy variables that are nonlinear and independent. All members of the supply chain behave as if they have complete knowledge of the demand and cost structure of other chains and try to maximize their profits under uncertain conditions. Ranjan and Jha (2019) examine pricing strategies and coordination mechanisms among members in a dual-channel supply chain (DCSC) while assuming demand as a linear function of sales prices (online/offline), green quality level and sales effort level. The manufacturer does not want to spend extra variable costs for producing a green product. If both products have the same specifications (price, quality, and sales effort level), the demand for both products will be equal between the two channels. Zhang et al. (2019), shows a supply chain that includes an online retailer that can influence prices, and a carrier that expands its capacity to a high cost while assuming that the cost of purchasing goods is zero. Online retailers act as a leader and determine the selling price. The demand is a linear function when it is definitive, and the quadratic capacity expansion cost function is measured. In another study by Tao et al. (2019), which was based on pricing strategies, and inventory policies, RFID technology was adopted in the supply chain to deal with inventory inaccuracies. The incorrect assumption of inventory occurs as soon as the order is placed, and focusses on the demand of the product which is price-dependent and random. He et al. (2019) presented a closed-loop supply chain (CLSC) in which the manufacturer can distribute new products through an independent retailer, and sell re-manufactured products through a third-party company or platform only if government subsidies are available. Ma et al. (2018) showed the pricing strategies in a two-tier supply chain with two manufacturers and a retailer. M1 and M2 are two manufacturers where M1 is considered as a green manufacturer and M2 is considered as a traditional manufacturer. Customers not only influences the purchase decision but also poses a great challenge for the retail supply chain management. The two-tier supply chain consists of one manufacturer and a retailer in which the manufacturer produces two alternative products that belong to two different generations. The retailer determines product selection decisions and pricing policies, taking into account different customer demands based on different customer values. Moreover, Lowe et al.(2018) reviewed that the retailer purchases from a manufacturer that produces two generations of old and new products. Alamdar et al. (2018), considered a fuzzy closed-loop supply chain (CLSC) with a manufacturer, a retailer, and a collector, along with a nonlinear and linear market demand function hypothesis. In this model, all used products are collected from consumers and successfully remanufactured. The remanufacturing costs such as destruction costs, inspection, quality assurance, and other activities have been investigated. Based on the results, the collaboration between the manufacturer and retailer is beneficial for both the customer and the chain and the most effective model for collecting the used products is the collaboration between the manufacturer and the collector. Noori Daryan et al. (2017) examined optimal pricing and the decision to replenish a supply chain with a single supplier and multiple retailers and taking into consideration that scarcity is not permissible. Additionally, the demand depends on the sale price, which includes a combined contract, quantity and discount, and a free shipping contract. Sinayi and Rasti Barzoki (2018) concluded that there are three major models of sustainability in a supply chain that are at high levels, with the assumption of low demand at retail prices while being at a green level. Moreover, the governments impose taxes on the final price of the product. The production of a green product does not affect the traditional marginal costs of the manufacturer, and the manufacturer views the consumer surplus as a sign of social welfare in its profit performance. Jamali and Rasti Barzoki (2018) considered the environmental and economic aspects of sustainability. The research focused on pricing and determining the degree of the greenness of a product compared to a non-green product while considering two manufacturers, two retailers, and the internet. The pricing of two alternative products is a linear cost function for the manufacturers and retailers such that explicit results and parameters can be easily estimated. The retailers' prices should be more than wholesalers, and the price factor itself is more important. Retailers should be more than wholesalers, and the self-price coefficient is of high importance.Nazanin Pilevari and Mahsa Alahyari (2020) proposed an assessment model for evaluating the supply chain sustainability. Hajmohamad et al.(2020) introduced a new model of combining variables affecting the classification of customers which is based on a distribution system of goods and services. Chen et al (2017) dealt with pricing decisions and supply chain efforts with a manufacturer and retailer while assuming that consumer demand is not only sensitive to retail prices but also depends on sales of the retailers. The cost of producing unit C cannot exceed the unit retail price and the unit wholesale price. The manufacturer and the retailer decide to maximize their profits, to do this the retail price should be higher than the wholesale price. Both the manufacturer and the retailer have the same information about the demands and costs and analyze. Biswas & Avittathur (2019) assumed that there is no time constraint regarding product delivery, assuming that the definite demand and its random components are different for the buyer, and the buyer usually saves less of the product at a high retail price to save cost. This shows that the deals with a two-tiered supply chain network consisting of one supplier and several buyers can eliminate the channel conflict caused by concurrent costing and inventory competition. A loss in the fresh agriculture products (FAP) exists and the development of FAP e-commerce is hindered. Song & He's (2019) paper proposed to achieve a three-tier FAP supply chain coordination and profit through contracts between the supply chain members. The paper presented a three-tiered supply chain involving an e-commerce company, a third-party logistics service provider (TPLSP), and an e-commerce store while assuming that members of the chain take risks and are rational. The expected profit for the members of the chain is non-negative and an uncertain situation is examined. Wan & Chen's paper (2019) examines a supply chain involving a supplier while assuming that the time range is assured (T, 0), the retail price (due to the positive impact of inflation) increases over time leading to inflation, customer demand can be divided into definite and random, and the initial inventory of the retailer at the beginning of the sales period is assumed to be zero. It also examines the uncertainty situation where the manufacturer produces one type of perishable product and a retailer buys the product from the manufacturer and sells it to the end customers. Hu et al. (2018) considered two types of contracts. A two-tier supply chain, where the retail price and order quantity of the products are optimized while assuming that the market demand is random and sensitive to the products' retail prices. If retailers face a shortage of inventory, they will only have lost sales and will not be punished. The supplier and the retailer negotiate the product and the retail price while the leading supplier is Stackelberg, and taking uncertain conditions into account. Fu et al. (2018) , analyzed the Stockholm game model while assuming that information about price and demand is a shared knowledge between two players. All the parameters are known, and the result of bargaining does not affect the profit. The wholesale price is a linear function and the uncertainty situation has been developed to answer the question of how decentralized supply chains regulate profit-sharing using minimal information on demand and sales prices? The framework uses only the first and second moments of price and demand changes, and can, therefore, be implemented using a set of parsimonious parameters. Zhao et al. (2018) proposed a two-stage model to explore options contracts in a two-tier supply chain, given that the market is random and the retailer can buy from two alternatives. In addition, manufacturers or retailers alone cannot influence the price and demand of the market, since the retailer can buy as much as it wants from the market which creates uncertainty. Options contract plays an important role in the distribution of benefits between upstream and downstream members of the green supply chain and improving the overall performance. However, there are few studies of options contracts in green supply chains. Song and Gao (2018) presented a green supply chain game model with two types of options contracts with the assumption that the real market demand and retail prices would. Investment in R&D is a quadratic equation, and the green product research and development costs are fully funded by manufacturers and the retailer has a certain percentage of options contract with the manufacturer. The results of the conventional centralized control game model and the results of the decentralized decision game models are finally compared. Yang et al. (2017) proposed an option contract in a retail supply chain in which the market demand depends on sales effort. Moreover, the demand is a linear function and both suppliers and retailers ignore the risks.

Method

In this study, game theory based on the grey number is used to analyze the performance of the supply chain of perishable materials and a dynamic cooperative game called Stackelberg is used to investigate this. It uses the full knowledge of how the manufacturer is leading the supply chain and achieving greater profitability. Consequently, the focus is on an inventory system managed by the manufacturer. The manufacturer is aware of the retailer's demands and begins to produce the products based on the demands of the retailers while considering the perishability of the products. In this study, while investigating four strategies in different periods, the authors seek to achieve optimal transition pricing policies in the form of a Stackelberg game in a competitive environment and a decentralized supply chain under different conditions. In the next section, the authors introduce a grey number theory and its application in pricing models and its parameters in contractual economic conditions. The supply chain model under review shows a competition between the two levels of manufacturer and distributor. The structure of this model is a two by two structure similar to that of Krishnan and Winter (2012) Manufacturers produce one alternative product, and retailers face two alternatives, which increases competition among manufacturers as the number of alternative goods in the market increases. Retailers, on the other hand, compete in the alternative pricing phase of the market, thus forming a two-tiered competitive structure at both the manufacturer and retail levels. The model under study is similar to the study of Li et al. (2017). However, in this model, the demand of the retailer is considered random and by adding contractual conditions to the wholesale price, an innovative approach is made in this research. The demand function is defined in both definitive and the random functions, the definitive part being the descending function of the retailer's sales price, and the random part is represented by a random variable of the density function and the specified distribution function. The competitive effect of the manufacturer and the retailer is determined by multiplying their competitiveness by the competing price in demand function. Due to the randomness of retailers' demand in the sales market, facing leftover inventory or lost sales at the end of the period involves costs that must be included in the model. Manufacturers sell the same product to two retailers at the same price, however, retailers sell the same product at different prices. Accordingly, the variables of the model are as follows.

$D_{ij}(\otimes)$: Demand of products produced by manufacturer i for retailer j

$a_{ij}(\otimes)$: Demand independent of product price produced by manufacturer i for retailer j

$v_{ij}(\otimes)$: Auction value of leftover goods at the end of the sales season for the product i and retailer j

$g_{ij}(\otimes)$: Shortage cost for the product i and retailer j

$c_i(\otimes)$: Cost of production of product t

$b_{ij}(\otimes)$: Price sensitivity of demand for the product i for retailer j

$x_i(\otimes)$: The level of competition between manufacturers $(0 \le x_i \le 1)$

$t_i(\otimes)$: The level of competition between retailers $(0 \le t_i \le 1)$

$q_{ij}(\otimes)$: Order quantity of product i by retailer j

$p_{ij}(\otimes)$: Sales price of product i

$w_i(\otimes)$: Wholesale price of goods i

$\varepsilon_{ij}(\otimes)$: Random demand for the product i by retailer j with density distribution $f(0)$ and cumulative distribution $F(0)$

$z_{ij}(\otimes)$: Storage level of product i by retailer j (random variable)

Since demand is random and there exists a competition between the manufacturers and retailers, the demand for the product of manufacturer *i* and retailer *j* is based on uncertain collective demand and is as follows.

$$D_{ij}(\otimes) = \begin{bmatrix} a_{ij}(\otimes) - b_{ij}(\otimes)p_{ij}(\otimes) + \left(1 - t_i(\otimes)\right)x_i(\otimes)p_{i,3-j}(\otimes) \\ + t_i(\otimes)(1 - x_i(\otimes))p_{3-i,j}(\otimes) + t_i(\otimes)x_i(\otimes)p_{3-i,3-j}(\otimes) + \varepsilon_{ij}(\otimes) \end{bmatrix} \quad (1)$$

- The price of the product offered by each manufacturer varies.
- Retailers sell the products in the market at different prices.
- The third part of this demand function is the selling price of the product that is intended by the competing retailers.
- The price offered by a competing manufacturer for their product in the current market.
- The price offered by a competing manufacturer for the current product in a competing retail market.

There are three types of competition in the market, competition between manufacturers, competition between retailers, and competition between retailers and manufacturers. Based on the impact of options and risk contracts, supply chain efficiency and profitability must be carefully considered when selecting the type of supply chain coordination contract. Since the leaders of the chain are the manufacturers, they choose the type of contract and offer it to the retailers. The costs involved in implementing each contract in a supply chain are as follows.

- Specifying the terms and conditions acceptable to both parties and making a contract between them

- The cost of accepting the contract by the other party

According to the aforementioned text, the authors can conclude that this research examines three major types of pricing contracts namely wholesale, options, and discount contracts.In this type of contract, the manufacturer supplies the product to the retailer at a fixed wholesale price with the intention of profit that must be greater than the cost, excluding the order quantity. The profit function of the retailer y is defined as. If the retailer's order exceeds the market demand, then at the end of the period there will be salvage value for the leftover inventories. On the other hand, the shortage of the retailer's inventory occurs when market demand exceeds the order quantity, which would result in lost sales. Therefore, according to the retailer's profit formula, which is derived from the subtraction of purchase and auction costs and the lack of sales revenue, the retailer's profit function is defined as follows.

$$\pi_{Rj}^{wpc}\left(q_{ij}(\otimes), p_{ij}(\otimes)\right) = \begin{cases} \sum_{i}^{2} = \left[p_{ij}(\otimes)D_{ij}(\otimes) - w_i(\otimes)Q_{ij}(\otimes) + v_{ij}(\otimes)\left(Q_{ij}(\otimes) - D_{ij}(\otimes)\right)\right] & Q_{ij}(\otimes) > D_{ij}(\otimes) \\ \sum_{i=1}^{2} \left[p_{ij}(\otimes)Q_{ij}(\otimes) - w_{ij}(\otimes)Q_{ij}(\otimes) - g_{ij}(\otimes)\left(D_{ij}(\otimes) - Q_{ij}(\otimes)\right)\right] & Q_{ij}(\otimes) < D_{ij}(\otimes) \end{cases}$$

According to the definitive demand structure of the retailer and based on (Pettruzi and Dada 1999), to simplify the problem the authors consider.

$$D_{ij}(\otimes) = y_{ij}(\otimes)(p(\otimes)) + \varepsilon_{ij}(\otimes) \quad (2)$$

The variable ε_{ij} is defined by the density distribution function $f(0)$ and the cumulative distribution function $F(0)$ of a random variable in the interval $[A, B]$, which is the stocking level of the product.

$$z_{ij}(\otimes) = q_{ij}(\otimes) - y_{ij}(\otimes)9\,p(\otimes) \quad (3)$$

Consequently, the average profit function of the retailer j is defined by the following equation.

$$E\left(\pi_{Rj}^{wpc}(z_{ij}(\otimes), p_{ij}(\otimes))\right) = \sum_{i=1}^{2}\left\{\begin{array}{l}\left[\begin{array}{l}(p_{ij}(\otimes)(y_{ij}(\otimes)(p(\otimes)) + u_{ij}(\otimes)) - w_i(\otimes)(z_{ij}(\otimes) \\ + y_{ij}(\otimes)(p(\otimes))) + v_{ij}(\otimes)(z_{ij}(\otimes) - u_{ij}(\otimes))\end{array}\right] f\left(u_{ij}\right) du_{ij}(\otimes) \\ + \int_{z_{ij}}^{B_{ij}}\left[\begin{array}{l}p_{ij}(\otimes)\left(z_{ij}(\otimes) + y_{ij}(\otimes)9p(\otimes))\right) - w_i(\otimes)(z_{ij}(\otimes) \\ + y_{ij}(\otimes)(p(\otimes))) - g_{ij}(\otimes)\left(u_{ij}(\otimes) - z_{ij}(\otimes)\right)\end{array}\right] f(u_{ij}) du_{ij}(\otimes)\end{array}\right\} \tag{4}$$

$$= \sum_i \begin{array}{l} p_{ij}(\otimes)\left[y_{ij}(\otimes)9p + \mu_{ij}(\otimes)\right] + v_{ij}(\otimes) \wedge_{Rj}(\otimes)\left(z_{ij}(\otimes)\right) \\ - \left(p_{ij}(\otimes) + g_{ij}(\otimes)\right)\Theta_{Rj}(\otimes)\left(z_{ij}(\otimes)\right) - w_i(\otimes)(z_{ij}(\otimes) + y_{ij}(\otimes)(p(\otimes)))\end{array}$$

Since the retailer buys from two manufacturers, the sum of its average profit function is i. In the profit function of the retailer the leftover inventory and lost sales are identified by two functions.

$\wedge_{Rj}(\otimes)\left(z_{ij}(\otimes)\right)$ And $\Theta_{Rj}(\otimes)\left(z_{ij}(\otimes)\right)$

$$\wedge_{Rj}\left(z_{ij}(\otimes)\right) = \int_{A_{ij}}^{z_{ij}}\left(z_{ij}(\otimes) - u_{ij}(\otimes)\right) f(u_{ij}(\otimes)) du_{ij} \tag{5}$$

$$\Theta_{Rj}\left(z_{ij}(\otimes)\right) = \int_{z_{ij}}^{B_{ij}}\left(u_{ij}(\otimes) - z_{ij}(\otimes)\right) f(u_{ij}(\otimes)) du_{ij} \tag{6}$$

Furthermore, by subtracting income from the cost of production, the profit function i is derived. Following is the optimized profit function of the manufacturer and retailer in a competitive environment.

$$\pi_{Mi}^{wpc}\left(q_{ij}(\otimes)\right) = \sum_{j=1}^{2}(w_i(\otimes) - c_i(\otimes))Q_{IJ}(\otimes) \tag{7}$$

The retailer j seeks to maximize its profit by obtaining the optimal order quantity using the selling price and the optimal storage quantity. Therefore, the authors equate the retailer's profit function to zero to obtain the optimal order quantity.

$$Maximize_{st.p_{ij(\otimes)}, z_{ij(\otimes)}}^{E\left(\pi_{R_j}^{wpc}(z_{ij}(\otimes), p_{ij}(\otimes)))\right)} \tag{8}$$

Proposition 1. To determine the optimal price of product i is purchased by retailer j.

Proposition 1. The optimal selling price of product i for retailer j, under uncertain conditions, for the two produces and retailers, is as follows.

$$p_{ij}^{wsp}(\otimes)=\frac{\begin{bmatrix}a_{ij}(\otimes)+\mu_{ij}+(1-t_i(\otimes))x_i(\otimes)p_{i,3-j}(\otimes)+t_i(\otimes)(1-x_i(\otimes))p_{3-i,j}(\otimes)\\+t_i(\otimes)x_i(\otimes)p_{3-i,3-j}(\otimes)+b_{ij}(\otimes)w_i(\otimes)-\Theta_{Rj}(\otimes)\left(z_{ij}(\otimes)\right)\end{bmatrix}}{2b_{ij}(\otimes)} \tag{9}$$

Proof. The profit function of retailer j for the selling price of product i is optimized using the following equation.

$$\frac{\partial E\left(\pi_{Rj}^{WPC}(z_{ij}(\otimes),p_{ij}(\otimes))\right)}{\partial p_{ij}}=\sum_{i=1}^{2}\left\{\begin{matrix}\left[a_{ij}(\otimes)-2b_{ij}(\otimes)p_{ij}(\otimes)+(1-t_i(\otimes))x_i(\otimes)p_{i,3-j}(\otimes)\right]\\+\begin{bmatrix}t_i(\otimes)(1-x_i(\otimes))p_{3-i,j}(\otimes)t_i(\otimes)x_i(\otimes)p_{3-i,3-j}(\otimes)\\+\mu_{ij}(\otimes)+b_{ij}(\otimes)w_i(\otimes)-\Theta_{Rj}(\otimes)\left(z_{ij}(\otimes)\right)\end{bmatrix}\end{matrix}\right\}=0 \tag{10}$$

Equation (10) shows this optimal condition. On the other hand, if the second-order derivative is negative, the profit function will be a concave function based on the selling price, where the optimal price is the only point where the function is maximum. This allows us to determine whether the acquired point is the only optimal point or not. The second-order derivative is examined in the following way.

$$\frac{\partial E\left(\pi_{Rj}^{WPC}(z_{ij}(\otimes),p_{ij}(\otimes))\right)}{\partial p_{ij}(\otimes)}=\sum_{i}-2b_{ij}(\otimes)<0 \tag{11}$$

Based on the above equation, the optimal pricing policy can be introduced in equation (10) as a response to the selling price of retailer j for the product i. Proposition 2. In the uncertainty of collective demand, the optimal storage response of product i for customer j can be shown as follows.

$$1-F(z_{ij}^{wsp}(\otimes))=\frac{(w_i(\otimes)-v_{ij}(\otimes))}{p_{ij}(\otimes)-v_{ij}(\otimes)+g_{ij}(\otimes)} \tag{12}$$

Proof. The steps described above are repeated for this case. The derivative of the retailer profit function is determined in terms of storage level such that the optimal point is obtained. Hence, the second-order derivative of the function is also obtained.

$$\frac{\partial E\left(\pi_{Rj}^{WPC}(z_{ij}(\otimes), p_{ij}(\otimes))\right)}{\partial p_{ij}(\otimes)} = \sum_{i=1}^{2}\begin{bmatrix} -\left(w_i(\otimes) - v_{ij}(\otimes)\right) + \\ (p_{ij}(\otimes) - v_{ij}(\otimes) + g_{ij}(\otimes))(1 - F\left(Z_{ij}(\otimes)\right) \end{bmatrix} \quad (13)$$

The second-order derivative is as follows.

$$\frac{\partial E\left(\pi_{Rj}^{WPC}(z_{ij}(\otimes), p_{ij}(\otimes))\right)}{\partial p_{ij}(\otimes)} = \sum_{i=1}^{2} -(p_{ij}(\otimes) - v_{ij}(\otimes) + g_{ij}(\otimes)) f\left(z_{ij}(\otimes)\right) < 0 \quad (14)$$

Therefore, the maximum amount of retailer profit is achieved at the optimal storage level. Since the derivative of the retailer profit function generates two random retailer variables, to maximize the profit function at both points, the determinant of the Hessian matrix must be used. This value must be positive, and the calculation of the determinant determines the positive value.

$$\begin{vmatrix} \dfrac{\partial^2 E\left(\pi_{Rj}^{WPC}((z_{ij}(\otimes), p_{ij}(\otimes)))\right)}{\partial^2 p_{ij}(\otimes)} & \dfrac{\partial^2 E\left(\pi_{Rj}^{WPC}((z_{ij}(\otimes), p_{ij}(\otimes)))\right)}{\partial p_{ij}(\otimes)\partial Z_{ij}(\otimes)} \\ \dfrac{\partial^2 E\left(\pi_{Rj}^{WPC}((z_{ij}(\otimes), p_{ij}(\otimes)))\right)}{\partial Z_{ij}(\otimes)\partial p_{ij}(\otimes)} & \dfrac{\partial^2 E\left(\pi_{Rj}^{WPC}((z_{ij}(\otimes), p_{ij}(\otimes)))\right)}{\partial^2 z_{ij}(\otimes)} \end{vmatrix} \quad (15)$$

As a result, both points of the retailer's profit function is maximized. However, the wholesale price is introduced as a random variable. Based on the linear relationship of the manufacturer's profit function with the wholesale price, these two variables are directly related to each other and the higher wholesale price leads to more profit. Moreover, the profitability of the retailer is exposed by the manufacturer's high price so it must be determined using the wholesale supply chain coordination policy. Collectively, the supply chain optimization points and both members of the supply chain can be checked for supply chain coordination. This is done by comparing the optimal points of the retailer with the order quantity and optimal storage level. The relationships

$$\pi_{ST}(\otimes)=\sum_{j=1}^{2}\left[E(\pi_{Mi}(q_{ij}(\otimes))+E(\pi_{Rj}(z_{ij}(\otimes),p_{ij}(\otimes))\right]$$

$$\pi_{ST}(\otimes)=\sum_{j=1}^{2}\begin{bmatrix}(p_{ij}(\otimes)-c_i(\otimes))(y_{ij}(\otimes)(p(\otimes))+\mu_{ij}(\otimes))+(v_{ij}(\otimes)\\ -c_i(\otimes))\wedge_R(z_{ij}(\otimes))\\ -\Theta_R(\otimes)(z_{ij}(\otimes))(p_{ij}(\otimes)+g_{ij}(\otimes)-c_i(\otimes))\end{bmatrix} \qquad (17)$$

related to supply chain optimization are discussed in the next section. For the supply chain considered, two structures can be defined.

- Two vertical supply chains with one manufacturer and two retailers
- A supply chain with two manufacturers and two retailers

In the basic structure, the supply chain profit function k is derived from the sum of the manufacturer's and retailer's profits.

$$\pi_{sk}(\otimes)=\pi_{mk}(\otimes)+\sum_{j=1}^{2}E(\pi_{Rj}(\otimes)(z_{ij}(\otimes),p_{ij}(\otimes)) \qquad (fori=1) \qquad (16)$$

Using the first-order derivative, the optimal supply chain storage level and sales price can be achieved by coordinating the chain and performing the optimality test using the second-order derivative. By comparing this optimal point with the optimal points of the retailer, one can determine the wholesale price of the manufacturer and the conditions of coordination. The profit function of the secondary structure of the supply chain, with two manufacturers and two retailers, is as follows.

$$F(z^0(\otimes))=\frac{(p_{ij}(\otimes)+g_{ij}(\otimes)-c_i(\otimes))}{(p_{ij}(\otimes)-v_{ij}(\otimes)+g_{ij}(\otimes))} \qquad (18)$$

$$p_{ij}^0(\otimes)=\frac{\begin{bmatrix}\mu_{ij}(\otimes)+a_{ij}(\otimes)+(1-t_i(\otimes))x_i(\otimes)p_{i,3-j}(\otimes)+t_i(\otimes)(1-x_i(\otimes))p_{3-i,j}(\otimes)\\ +t_i(\otimes)x_i(\otimes)p_{3-i,3-j}(\otimes)+b_{ij}(\otimes)c_i(\otimes)-\Theta_R(\otimes)(z_{ij}(\otimes))\end{bmatrix}}{2b_{ij}(\otimes)} \qquad (19)$$

Since the secondary structure of the supply chain represents the general design of the primary structure, the secondary structure is considered in this research. Proposition 3. The wholesale price contract and the concurrent pricing and storage decisions in a two-tiered supply chain, is as follows.

Proof. The optimal wholesale price and the storage level are obtained using the previous steps and they are also checked using a second-order derivative. Thus, the optimal storage point is first determined by the first-order derivative by the following equation.

$$\frac{\partial \pi_{st}(\otimes)}{\partial z_{ij}(\otimes)} = \sum_{j1}^{2}\sum_{i=1}^{2}\begin{bmatrix} -\left(c_i(\otimes) - v_{ij}(\otimes)\right)F\left(z_{ij}(\otimes)\right) + \\ (p_{ij}(\otimes) - c_i(\otimes) + g_{ij}(\otimes))(1 - F(z_{ij}(\otimes))) \end{bmatrix} \tag{20}$$

The optimal selling point for the first-order derivative of the profit function is obtained as follows.

$$\frac{\partial \pi_{st}(\otimes)}{\partial p_{ij}(\otimes)} = \sum_i \sum_j \begin{bmatrix} \mu_{ij}(\otimes)2b_{ij}(\otimes)p_{ij}(\otimes) + a_{ij}(\otimes) + (1 - t_i(\otimes))x_i(\otimes)p_{i,3-j}(\otimes) \\ + t_i(\otimes)(1 - x_i(\otimes))p_{3-i,j} \\ (\otimes) + t_i(\otimes)x_i(\otimes)p_{3-i,3-j}(\otimes) - \Theta_R(\otimes)(z_{ij}(\otimes)) + c_i(\otimes)b_{ij}(\otimes) \end{bmatrix} \tag{21}$$

According to the second-order derivative of the supply chain profit function, the optimization of the retailer price and the storage level is determined to maximize the chain profit function. The optimal points are obtained by calculating the determinants of the matrix. In this model, the wholesale price offered by each manufacturer is the same as the retailer, but a small amount is offered to wholesalers so that they become partners with the retailers at the end of the retail period. The manufacturers' income and options contracts have the same parameters as the retailers. The profit function of retailer j where the retailer is assumed to share its income to manufacturer i will be as follows:

$$E\left(\pi_{Rj}^{RS}(z_{ij}(\otimes), p_{ij}(\otimes))\right) = \sum_i \begin{bmatrix} f_i(\otimes)p_{ij}\left[y_{ij}(\otimes)(p(\otimes)) + \mu_{ij}(\otimes)\right] \\ + f_i(\otimes)v_{ij}(\otimes) \wedge_{Rj}(\otimes)(z_{ij}(\otimes)) \\ -(f_i(\otimes)p_{ij}(\otimes)) + g_{ij}(\otimes))\Theta_{Rj}(\otimes)(z_{ij}(\otimes)) \\ - w_i(\otimes)\left[y_{ij}(\otimes)(p(\otimes)) + z_{ij}(\otimes)\right] \end{bmatrix} \tag{22}$$

The manufacturers' profit function is based on the residual profit of the retailer, which is equal to f_i

$$\pi_{mi}^{RS}(\otimes)=\sum_{j=1}^{2}\begin{bmatrix}(w_i(\otimes)-c_i(\otimes))(y_{ij}(\otimes)+z_{ij}(\otimes))\\+(1-f_i(\otimes))p_{ij}(\otimes)(y_{ij}(\otimes)+\mu_{ij}(\otimes))\\+(1-f_i(\otimes))v_{ij}(\otimes)\wedge_{Rj}(\otimes)(z_{ij}(\otimes))\\.(1-f_i(\otimes))p_{ij}(\otimes)\Theta_{Rj}(\otimes)(z_{ij}(\otimes))\end{bmatrix} \tag{23}$$

To obtain the optimal points of sale and the stocking level while using high profit functions, the profit of each member is optimized. In the income and options contract, the price and the optimal level of retail sales for coordination analysis have to be determined.

Proposition 4: For an options contract, pricing decisions for retailers must be made as follows (Etebari 2020);

$$p_{ij}^{RS}(\otimes)=\frac{\begin{bmatrix}f_i(\otimes)(a_{ij}(\otimes)+(1-t_i(\otimes))x_i(\otimes)p_{i,3-j}(\otimes)+t_i(\otimes)(1-x_i(\otimes))p_{3-i,j}(\otimes)\\+t_i(\otimes)x_i(\otimes)p_{3-i,3-j}(\otimes)+\mu_{ij}(\otimes))+b_{ij}(\otimes)w_i(\otimes)-f_i(\otimes)\Theta_{Rj}(\otimes)(z_{ij}(\otimes))\end{bmatrix}}{2b_{ij}(\otimes)f_i(\otimes)} \tag{24}$$

Proof: As mentioned earlier, by deriving the first order profit function in terms of the selling price of the retailer, the desired variable is obtained as follows:

$$\frac{\partial E\left(\pi_{Rj}^{RS}(z_{ij}(\otimes),p_{ij}(\otimes))\right)}{\partial p_{ij}(\otimes)}=\sum_i\begin{bmatrix}f_i(\otimes)\left(a_{ij}(\otimes)+(1-t_i(\otimes)\right)x_i(\otimes)p_{i,3-j}(\otimes)\\+t_i(\otimes)(1-x_i(\otimes))p_{3-i,j}(\otimes)\\+t_i(\otimes)x_i(\otimes)p_{3-i,3-j}(\otimes)+\mu_{ij}(\otimes))+b_{ij}(\otimes)w_i(\otimes)\\-f_i(\otimes)\Theta_{Rj}(\otimes)(z_{ij}(\otimes))-2b_{ij}(\otimes)f_i(\otimes)p_{ij}(\otimes)\end{bmatrix} \tag{25}$$

Consequently, a second-order derivative is required to maximize the optimal selling point in the profit function:

$$\frac{\partial^2 E\left(\pi_{Rj}^{RS}(z_{ij}(\otimes), p_{ij}(\otimes))\right)}{\partial p_{ij}(\otimes)} = \sum_i -2b_{ij}(\otimes) f_i(\otimes) \qquad (26)$$

Since the second derivative of the profit function is negative, a concurrent profit function is seen, in which the optimal selling point is an absolute maximum point.Proposition 5: Based on the equation below, the optimal stocking level in which the profit function is maximum is obtained.

$$F(z_{ij}^{RS}(\otimes)) = \frac{f_i(\otimes) p_{ij}(\otimes) - w_i(\otimes) + g_{ij}(\otimes)}{f_i(\otimes)(p_{ij}(\otimes) - v_{ij}(\otimes)) + g_{ij}(\otimes)} \qquad (27)$$

Proof: By deriving the profit function from the stocking level and equating it to zero, the maximum profit function j can be obtained and the second-order derivative is used to determine the type of stocking level function. Based on the following equation, the first order derivative is obtained:

$$\frac{\partial^2 E\left(\pi_{Rj}^{RS}(z_{ij}(\otimes), p_{ij}(\otimes))\right)}{\partial z_{ij}} = \sum_i \begin{bmatrix} -(f_i(\otimes) p_{ij}(\otimes) - v_{ij}(\otimes) + g_{ij}(\otimes))(1 - F(z_{ij}^{RS}(\otimes))) \\ -w_i(\otimes) f_i(\otimes) v_{ij}(\otimes) \end{bmatrix} \qquad (28)$$

And based on the following equation, the second order derivative is obtained:

$$\frac{\partial^2 E\left(\pi_{Rj}^{RS}(z_{ij}(\otimes), p_{ij}(\otimes))\right)}{\partial z_{ij}(\otimes)} = \sum_i -(f_i(\otimes) p_{ij}(\otimes) - v_{ij}(\otimes) + g_{ij}(\otimes)) f(z_{ij}(\otimes)) \qquad (29)$$

Findings

In this study, according to Chakroborty, (2015), it is assumed that the parameters of the two manufacturers and the retailers are identical. The purpose of this study is to first establish optimal pricing and storage levels and quantities of retailer, manufacturer and supply chain profits. Next, the sensitivity of the variables is analyzed. Finally, the variables are compared for the retailers and manufacturers and prioritize their sections for selecting the type of contract. Now, on simplifying the demand function the authors have.

$$D(\otimes) = a(\otimes) + Lp(\otimes) + \varepsilon(\otimes) \tag{30}$$

$$L(\otimes) = -B(\otimes) + x(\otimes) + t(\otimes) - tx(\otimes) \tag{31}$$

As a result, a Random demand with a uniform distribution is defined. The leftover storage and lost sales can be excluded by considering the random demand distribution.

$$\wedge_R(\otimes)(z) = \frac{z^2(\otimes)}{200} \tag{32}$$

$$\Theta_R(\otimes)(z) = 50 - z(\otimes) + \frac{z^2(\otimes)}{200} \tag{33}$$

The following value was considered for the parameters of the model.

$$a(\otimes) = 5000 \quad , c(\otimes) = 500 \quad , v(\otimes) = 450 \quad , g(\otimes) = 700$$

$$\varepsilon \in U(0{,}100) \quad , x(\otimes) = 0.5 \quad , t(\otimes) = 0.5 \quad , b(\otimes) = 0.8$$

The profit function of the retailer with respect to the simplification of the relationships is.

$$\pi_R^{WPC}(\otimes) = 2\begin{bmatrix} p(\otimes)(a(\otimes) + LP(\otimes) + \mu(\otimes)) + V(\otimes)\wedge_R \\ -(p(\otimes) + g(\otimes))\Theta_R(\otimes) - w(\otimes)(a(\otimes) + LP(\otimes) = z(\otimes)) \end{bmatrix} \tag{34}$$

Using the first-order derivative in the wholesale price contract, the authors can reach the optimal point of retail price.

$$p^{WPC}(\otimes) = \frac{a(\otimes) + \mu(\otimes) - LW(\otimes) - 50 + z(\otimes) - \dfrac{z^2(\otimes)}{200}}{-2L(\otimes)} \tag{35}$$

In this respect, Lemma 1 is defined by considering the negative relation of demand to the sales price and where the coefficient $L(\otimes)$ should be less than zero. Considering the optimal price relationship, the Lemma value $L(\otimes)$

is negative when the price is positive. As a result, the following constraint is considered.

$$b(\otimes) > x(\otimes) + t(\otimes) - t(\otimes)x(\otimes) \tag{36}$$

The optimal point of storage is as follows.

$$z^{WPC}(\otimes) = \frac{100 * (p(\otimes) - w(\otimes) + g(\otimes))}{(p(\otimes) - v(\otimes) + g(\otimes))} \tag{37}$$

The optimal storage level along with the optimal retailer price is a function of the optimal storage level. Due to the need to simplify the relationships of the decision variables, it is substituted in the optimal price and is therefore converted into a 3rd-degree equation.

$$Ap^3(\otimes) + Bp^2(\otimes) + Cp(\otimes) = D(\otimes) \tag{38}$$

And the parameter value of the coefficients of the equation is as follows.

$$A(\otimes) = -2L(\otimes) \tag{39}$$

$$B(\otimes) = -4L(\otimes)(g(\otimes) - v(\otimes)) - I(\otimes) - 50 \tag{40}$$

$$I(\otimes) = a(\otimes) + \mu(\otimes) - LW(\otimes) - 50 \tag{41}$$

$$C(\otimes) = -2L(\otimes)(g(\otimes) - v(\otimes))^2 - 2(g(\otimes) - v(\otimes))(I(\otimes) + 50) \tag{42}$$

$$K(\otimes) = (I(\otimes) + 50)(g(\otimes) - v(\otimes))^2 - 50(w(\otimes) - v(\otimes))^2 \tag{43}$$

Furthermore, the optimal solution for-profit functions and optimal points are obtained using the software. The results show that at the point w = 500, the supply chain is coordinated. The results are presented in Table 1.

Table 1.
Results of the Wholesale Price Contract

$x(\otimes)=0.5, t(\otimes)=0.5, b(\otimes)=0.8, w(\otimes)=500$	
Symbol	Value
sales price	50749.99
Storage level	99.9
Order amount	2562.4
Retailer Profit	252501254.9
Manufacturer Profit	0
Supply chain profit	505002509.9

The wholesale price contract is equal to the manufacturer's cost. Therefore, the profit margin of the manufacturer will be zero, which does not create good conditions for the manufacturer and they seek to increase the profit function by raising the wholesale price. Due to this, the parameters $T(\otimes)$ $X(\otimes)$ $T(\otimes)$ are changed to allowable intervals and are checked in such a way that at each level, the maximum wholesale price that the manufacturer can choose is determined. This can only happen if the retailer's profit is greater than zero. To achieve this, the effect of changing the sensitivity of demand to price $b(\otimes)$ at variable levels of competition between manufacturers and retailers is examined. The results vary at each level of competition and the values of $X(\otimes)$ and $T(\otimes)$ belong to the interval $b(\otimes)$ while considering the above constraint. The results of the study on the coordinated and uncoordinated supply chain in the wholesale price contract are shown in Table 2.

Table 2.

Impact of Parameter Changes on Coordinated and Non-Coordinated Supply Chains

Parameter change interval			**Uncoordinated supply chain**				**Coordinated supply cha**			
$X(\otimes)$	$T(\otimes)$	$b(\otimes)$	$p(\otimes)$	$z(\otimes)$	$w_{max}(\otimes)$	$\pi_{sc}(\otimes)$	$p^0(\otimes)$	$z^0(\otimes)$	$\pi_{sc}(\otimes)$	$Ef(\otimes$
0.25	0.25	0.5	79279.94	1.82	78531	14615459.11	40649.99	99.87	402995637.22	**3.**
		1	8666.32	10.54	8426	4303637.36	4738.88	98.99	40418503.0	**10.**
		1.5	4553.17	18.07	4385	2802751.57	2626.46	98.26	19208115.76	**14.**
	0.5	0.7	66011.10	2.08	65328	13291044.25	33916.66	99.85	334992097.96	**3.**
		1	13053.80	7.50	12755	5506809.99	6983.33	99.30	63040485.79	**8.**
		1.5	5542.23	15.33	5354	3221325.85	3135.70	98.52	24304611.96	**13.**
0.5	0.25	0.8	28149.99	4.02	27706	8464487.43	14678.56	99.66	140712354.92	**6.**
		1	13053.80	7.50	12755	5506809.99	6983.33	99.30	63040485.79	**8.**
		1.5	5542.23	15.33	5354	3221325.85	3135.70	98.52	23404611.96	**13.**
	0.5	0.9	32876.33	3.56	32396	9200622.20	17083.33	99.71	164994195.51	**5.**
		1	19651.45	5.37	19282	6590287.74	10349.99	99.52	97012547.16	**7.**
		1.5	6478.19	13.45	6273	3575120.49	3616.66	98.70	29130962.64	**12.**

Due to the lack of positive profits of the manufacturer in the coordinated supply chain and since the wholesale price is equal to the production costs, the manufacturer seeks to choose a price higher than the wholesale supply chain price. To maintain the supply chain structure and the collaboration of the retailers, the profits of the retailers should be positive. Table 2 shows the maximum wholesale price chosen by the manufacturer at each level. Moreover, the performance of the non-ideal supply chain has been calculated by comparing the ideal and non-ideal supply chains, which shows that the profitability of the supply chain decreases with distance from the supply chain coordination point. Finally, as the retailer seeks to achieve a positive profit, the authors can conclude that the increase in wholesale prices by the manufacturers leads to an increase in prices by the retailers. Table 2 also shows that since falling prices are necessary to increase profits and increase demand, lower retail prices are faced as demand-sensitive parameters increase. Moreover, due to the sensitivity of consumer demand, an increase in the stocking level is faced with an increase in demand sensitivity. Taking all of these into account, the authors can conclude that while the wholesale contract is easier to implement than other contracts. It is not a good contract in terms of profitability in the supply chain. This contract can be considered appropriate if there is complete certainty in the supply chain members to adapt to the problem. After reviewing the harmonized supply chain in the options contract, a survey was conducted using a numerical study. The profit function of each of the two retailers while taking the demand function into account, is as follows:

$$E\left(\pi_R^{RS}((z(\otimes), p(\otimes)))\right) = 2((fp(\otimes) - w(\otimes))[y(p(\otimes)) = \mu(\otimes)] \\ - (w(\otimes) - fv(\otimes)) \wedge_R (z(\otimes)) - (fp(\otimes) - w(\otimes) + g(\otimes))\Theta_R(\otimes)(z(\otimes))) \tag{44}$$

The optimal retail price is simplified and is as follows:

$$p^{RS}(\otimes) = \frac{f(a(\otimes) + \mu(\otimes) - 50 + z(\otimes) - \dfrac{z^2(\otimes)}{200}) - LW(\otimes)}{-2L(\otimes)f(\otimes)} \tag{45}$$

The optimal storage level is also obtained from the following equation:

$$z^{RS}(\otimes) = 100\,\frac{fp(\otimes) - w(\otimes) + g(\otimes)}{fp(\otimes) - fv(\otimes) + g(\otimes)} \tag{46}$$

To convert a retail price equation into a 3rd degree equation, z is substituted in p.

$$Ap^3(\otimes) + Bp^2(\otimes) + Cp(\otimes) + D(\otimes) = 0 \tag{47}$$

In this regard, the coefficients of the 3rd degree function are as follows:

$$Ap^3(\otimes) + Bp^2(\otimes) + Cp(\otimes) + D(\otimes) = 0 \tag{48}$$

$$A(\otimes) = -2L(\otimes)f^3(\otimes) \tag{49}$$

$$B(\otimes) = (-4) * L(\otimes) * f^2(\otimes) * (g(\otimes) - f(\otimes) * v(\otimes)) \\ - f^2(\otimes) * (f(\otimes) * (a(\otimes) + \mu(\otimes)) - L(\otimes) * w(\otimes)) \quad (50)$$

$$C(\otimes) = -2L(\otimes) f(\otimes)(g(\otimes) - fv(\otimes))^2 - 2f(g(\otimes) \\ - fv(\otimes))(f(a(\otimes) + \mu(\otimes)) - L(\otimes)w(\otimes)) \quad (51)$$

$$D(\otimes) = f(\otimes) * (a(\otimes) + \mu(\otimes)) - L(\otimes) * w(\otimes)) * (g(\otimes) \\ - f(\otimes) * v(\otimes)) - 50 * f(\otimes) * (w(\otimes) - f(\otimes) * v(\otimes))^2 \quad (52)$$

The numerical value of the parameters is assumed to be equal to or equal to the numerical study of the wholesale price contract. Given that $g(\otimes) = 0$, the value of $f(\otimes) = 0$ is determined. The equation was then solved and the results are presented in Table 3. At the moment when the price and the stocking level are in harmony i.e. when $w(\otimes) = f(\otimes)c(\otimes)$ and $g(\otimes) = 0$. These are the optimal points of the selling price and the stocking level. The profit functions of the members are calculated and listed in the table.

Table 3.
Optimal Values in the Coordinated Supply Chain

$x(\otimes) = 0.5 \quad, t(\otimes) = 0.5 \quad, b(\otimes) = 0.8 \quad, f(\otimes) = 0.5 \quad, g(\otimes) = 0$

sales price	50749.99
Storage level	99.9
Order amount	2562.4
Retail profit	126250627.48
Manufacturer's profit	126250627.48
Supply Chain Profit	505002509.94

When the wholesale price is equal to 500, the zero profit of the supplier is not suitable due to the equalization of the coordinated wholesale price and the cost of production of the producers. However, in the joint options contract, the producer's profit increases which in turn shows the benefit of this type of contract. Consequently, sensitivity analysis is performed by making changes in $X(\otimes)$, $b(\otimes)$ and $T(\otimes)$ affecting the parameters and profit functions. With values $x(\otimes) = (0,0,2,0,4,0,6,0,8,1)$ and $t(\otimes) = (0,0,2,0,4,0,6,0,8,1)$, the value of $b(\otimes)$ is calculated for the mentioned constraint and is given in Table 4.

Table 4.

Determining the Limits of C

$X(\otimes)$	$T(\otimes)$	$< b(\otimes)$
0	0	0
0.2	0.2	0.36
0.4	0.4	0.64
0.6	0.6	0.84
0.8	0.8	0.96
1	1	1

To create an acceptable state in all values $T(\otimes)$ and $X(\otimes)$ for the value of $b(\otimes)$, two values (1.5 and 1.25) are considered. Then, taking into account the change in parameters, and the change of $T(\otimes)$ and $X(\otimes)$ at $b(\otimes)=1.25$, the retail price is examined and presented in Section A of Figure2.

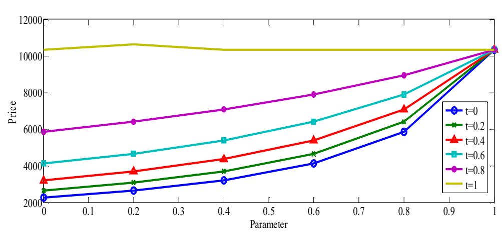

Figure A: Changes in the Retail price at $b(\otimes)=1.25$

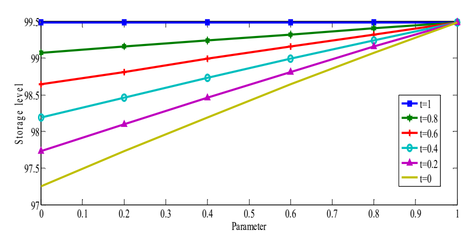

Figure B: Changes in the Stocking Level at $b(\otimes)=1.25$

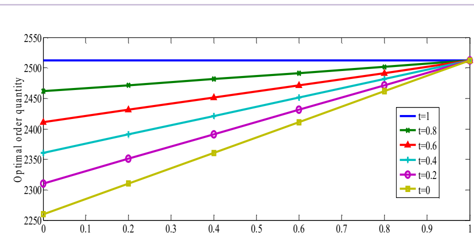

Figure C: Changes in the Optimal Order Value Relative to the Intensity of Competition at $b(\otimes)=1.25$

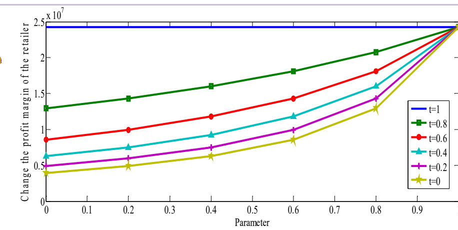

Figure D: Changes in the Retail Profits Relative to Changes in Competition Intensity at Level $b(\otimes)=1.25$

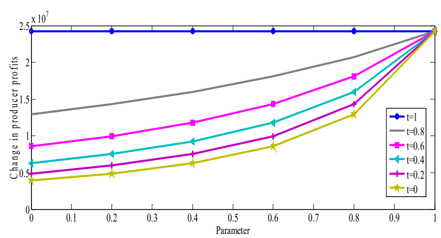

Figure E: Changes in the Producer Profits Relative to Changes in the Intensity of Competition at $b(\otimes)=1.25$

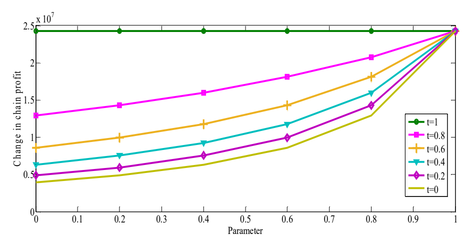

Figure F: Changes in the Supply Chain Profits Relative to Changes in Competition Intensity at Level $b(\otimes)=1.25$

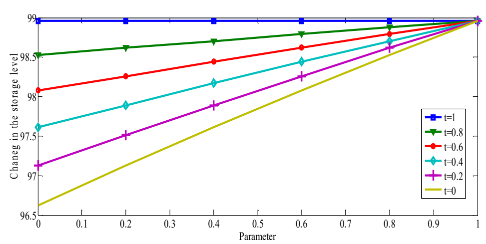

Figure G : Retail Price Change Compared to the Level of Competitiveness at $b(\otimes)=1.25$

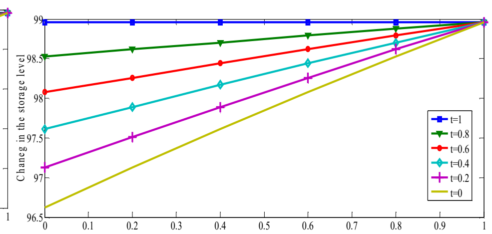

Figure H: Changes in the Stocking Level Relative to the Change in Competition at $b(\otimes)=1.25$

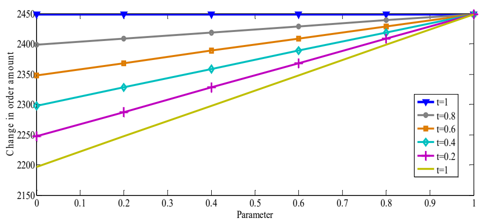

Figure I: Changes in the Order Relative to the Change in Competition at $b(\otimes)=1.25$

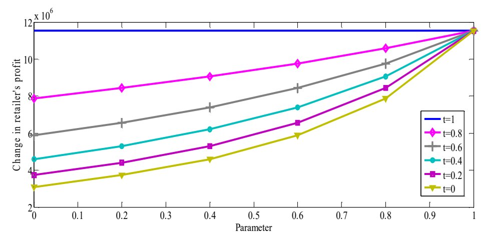

Figure J: Changes in the Retail Earnings Relative to Changes in Competition at $b(\otimes)=1.25$

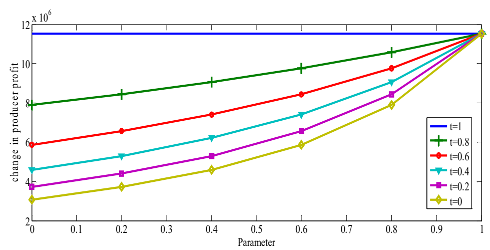

Figure K: Changes in Producer Profits Relative to Changes in Competition at $b(\otimes)=1.25$

Figure 2.
Sensitivity Analysis

According to Section A of Figure 2, competition increases between producers' at a steady level, the competition between retailers and price sensitivity also increases. When the intensity of competition between retailers increases. In the latter case, when the intensity of competition between retailers and producers is at a constant level, the price increases with the sensitivity of demand to price. This is due to the control of demand in a situation where the dependence of demand on price increases. Consequently, the sensitivity of the stocking level of goods to the sensitivity of demand-price has been analyzed and the results of the variations in the intensity of competition between producers and retailers are presented in Section B of

Figure2 Based on the results, the higher the competition, the higher the stocking level of goods, and the faster it will reach its peak when the competition reaches 100%. This is because the demand fluctuations have increased and retailers need to increase their inventory to meet this demand. In other words, as this sensitivity increases, the stocking level of goods decreases. The decrease in price and stocking level is due to the shortage of inventory costs and the predictability of demand by retailers. In Section C of Figure2 shows that due to competition among manufacturers for the amount of orders received and efforts to increase retail orders, the amount of retail orders also increases. Moreover, the competition from retailers to attract demand and pricing increases demand and ultimately increases retail orders. According to Section D of Figure2, the increase in competition between retailers will lead to higher profits as market share and sales increase, and this will eventually lead to an increase in supply chain profits if it does not conflict with increased producers' profits. As seen in in Section E of Figure2, increased competition leads to higher profits, and this continues until the market is balanced. It can be said that the increase in producers' profits that follow the increase in their competition is due to the increase in demand for retail orders, which itself seeks to respond to the increase in demand. In Section F of Figure2 show an increase in supply chain profits with the increase in competition between the manufacturers and retailers. This in turn leads to increased efficiency. By increasing competition between the members of the supply chain and increasing their efforts to attract more demand, increases the profitability of the supply chain. To reduce the selling price of goods, it is necessary to increase the sensitivity of demand-price while keeping the level of competition constant. This is seen in (Section G of Figure2). The significant effects of price variations on increasing consumer sensitivity to prices cause consumers to change course and move toward alternative goods on the market as prices rise. This will cause retailers to keep prices low to counter declining demand. The scenario of increasing competitiveness begins if the prices are raised. These events show that in terms of price sensitivity to demand, the price variation approach is not the right approach. If the intensity of competition is constant, the increase in the sensitivity of demand-price changes due to the alignment of price changes and the stocking level. Therefore, a decrease in the level of storage of goods is seen. In other words, increasing definite demand by lowering prices is something that leads to a decrease in random demand and thus a decrease in the stocking level of goods. The results can be seen in Section H of Figure2. According to Section I of Figure 2 Hence, the sensitivity of demand-price is inversely related to market demand. In other words, an increase in the former will reduce the latter. According to Section J of Figure 2. Sales are reduced at high levels by lower selling prices, caused by the sensitivity of demand to prices. This will lead to a decrease in retail orders and consequently decrease profits. However, with increasing competition, retail profits can be raised to a constant level of price sensitivity. According to Section K of Figure 2.This is the direct relationship between the producer's profit and the sales of the retailers, which will also decrease if demand decreases.

Conclusion

In this study, a competitive supply chain coordination with random demand was investigated. The coordination mechanism is discussed in the contracts where the wholesale price contract was applied in the supply chain model. The results show that the wholesale price contract is not suitable for supply chain coordination as it earns all profits from the retailer and the manufacturer is at a disadvantage in terms of costs and revenues. In other words, the manufacturer's profit is equal to zero. The manufacturer does not accept coordination and increases its wholesale price. To maintain the supply chain structure, the manufacturer can increase its wholesale price to the point where the retailer has zero profit. The lowest wholesale price of the manufacturer is equal to the cost of production, and to calculate the maximum wholesale price, it can be increased to the point where the retailer's profit is zero. For future studies, the authors can examine the supply chain structure in a way, where manufacturers provide different contracts to the retailers. For instance, one can choose a manufacturer for the wholesale price contract and another manufacturer for the options contract. For future studies, the authors can examine the supply chain structure in a way, where manufacturers provide different contracts to the retailers. For instance, one can choose a manufacturer for the wholesale price contract and another manufacturer for the options contract.

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