

Separation of Geochemical Anomalies Using Factor Analysis and Concentration-Number (C-N) Fractal Modeling Based on Stream Sediments Data in Esfordi 1:100000 Sheet, Central Iran

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Abstract

The aim of this study is separation of Fe_2O_3 , TiO_2 and V_2O_5 anomalies in Esfordi 1:100,000 sheet which is located in Bafq district, Central Iran. The analyzed elements of stream sediment samples taken in the area can be classified into 5 groups (factors) by factor analysis. The Concentration–Number (C-N) fractal model was used for delineation of the Fe2O3, TiO2 and V2O5 thresholds. According to the thresholds, the distribution of elemental concentration for Fe_2O_3 and TiO_2 were divided to four classifications and V2O5 has five geochemical populations in the area. Based on correlation between obtained results with geological and remote sensing data, the results show that the major anomalies of Fe_2O_3 , TiO_2 and V_2O_5 and related factor are mostly situated around granitic/rhyolitic rocks, iron alterations and along faults.

Keywords: Factor Analysis, Concentration-Number (C-N) Fractal Model, Esfordi, Stream Sediment.

1. Introduction

The definition of geochemical anomalies from backgrounds is important for interpretation of geological evolution and the ore-forming processes [1, 2, 3, 4]. Mineral exploration based on stream sediment data has been widely utilized for various types of ore deposits" and separation of geochemical anomalies based on stream sediment data is an essential stage to outline discoveries and prospects for mineral exploration [5, 6, 7]. There have been some classical statistics methods for definition of geochemical anomalies from background such as histogram analysis, box plot, summation of mean and standard deviation coefficients and median [8, 9, 10, 11, 12, 13, 14]. The statistical methods consider only the frequency distribution of the elemental concentration paying no attention to spatial variability; the information about the spatial correlation is not always available. Additionally, these methods are only appropriate to cases where geochemical data follows a normal distribution [3, 15, 16, 17, 18].

Fractal/multifractal modeling, established by Mandelbrot (1983), have been widely applied in different branches of geosciences since 1980s [19, 20, 21, 22]. Bolviken et al. (1992) and Cheng et al. (1994) proved geochemical patterns of various elements have fractal dimensions [22, 23].

Several fractal models have been proposed in geochemical data analysis including Concentration-Area (C-A) by Cheng et al. (1994), Number-Size (N-S) by Mandelbrot (1983), Concentration-Distance (C-D) by Li et al. (2003), Concentration-Volume (C-V) by Afzal et al. (2011) and Concentration-Number (C-N) by Hassanpour and Afzal (2013) [4, 22, 24, 25, 26]. Moreover, Studies on many cases revealed that geochemical data could have a multifractal nature [5, 27, 28, 29, 30, 31, 32, 33]. Geochemical data including lithogeochemical, stream sediments and in-situ soil have a multifractal nature which represents differences in geological factors such as alterations, rock units, geochemical and mineralization processes [22, 31, 34, 35, 36, 37, 38, 39].

Multivariate statistical analysis specifically factor analysis is proper techniques to classify and reduce the number of geochemical variables. Factor analysis, as one of the methods of multivariate analysis, has been widely used for interpretation of stream sediment geochemical data. The principal aim of factor analysis is to explain the variations in a multivariate data set by a few factors as possible and to detect hidden multivariate data structures. Factor analysis is suitable for analysis of the variability inherent in a geochemical data set with many analyzed elements. Consequently, factor analysis is often applied as a tool for exploratory data analysis [14, 37, 40, 41].

The purpose of this study was to use the application of C-N fractal modeling and factor analysis to distinguish factors based on Fe2O3, TiO2 and V2O5

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geochemical anomalies for further mineral exploration in Esfordi 1:100000 sheet, Central Iran.

2. Methodology

2.1. C-N fractal model

The N-S model, which was introduced by Mandelbrot (1983), can be employed to demonstrate the distribution of geochemical populations without pre-processing of data [22]. The model shows that there is a relationship between desirable attributes (e.g., ore element) and their cumulative numbers of samples of the elemental concentrations in this studied area. Agterberg (1995) proposed a multifractal model titled determination of the Size-Grade for spatial distributions of giant and super-giant mineral deposits [21]. Monecke et al. (2005) utilized the N-S fractal model to describe element enrichments accumulated with metasomatic processes during the formation of hydrothermal ores in the Waterloo Australia massive sulfide deposit [42]. A power-law frequency model has been intended to describe the N-S relationship according to the frequency distribution of elemental concentrations and cumulative number of samples with those attributes [5, 43, 44, 45, 46, 47, 48, 49]. Hassanpour and Afzal (2013) developed the N-S model and proposed concentration-number (C-N) model. The model is expressed by the following equation [4, 5, 221:

 $N(\geq \rho) = F\rho - D \tag{1}$

where ρ indicates element concentration, $N(\geq \rho)$ denotes cumulative number of samples with elemental concentration values greater than or equal to ρ , F is a constant and D is the scaling exponent or fractal dimension of the distribution of elemental concentrations. Log-log plots of $N(\geq \rho)$ versus ρ show straight line segments with different slopes –D corresponding to different concentration intervals [4, 5].

2.2. Factor analysis

Factor analysis is one of the most popular multivariate analysis which is determined as a powerful implement to visualize high dimensional data in lower dimensional spaces based on variance and covariance matrix. It is a useful tool for combining several correlated variables into a single variable, and hence for reducing the dimensionality of datasets into uncorrelated principal components based on covariance or correlations of variables which represents the interrelationships among the multi-dimensional variables [14, 50, 51]. A large dataset of geochemical variables could be combined in a few factors by this method. Factors could be illustrative of the geological and mineralization processes that generate the correlations among these variables (elemental concentrations) [52, 53].

3. Geological Setting

The Central Iran structural zone includes the Anarak-Bafq-Kerman metallogenic belt, parts of the Uremia-Dokhtar volcanic belt and Sanandaj-Sirjan structural-metamorphic zone. There are mineralization of different types of iron ores (> 2 Gt) which are located in the Bafq district region. The largest reserve of iron ore ever discovered in Bafq district is the Chadormalu mine which contains 400 Mt of iron ore [54, 55, 56].

The Esfordi 1:100000 sheet is one of the most important of geological maps in the Bafq district. The Bafq region is one of the most essential mineralized zones of central Iran with the upper Precambrian metamorphic-sedimentary rocks and Rift series of Precambrian - Paleozoic [54, 55]. Central Iran is a portion of Gondwana land with a Precambrian basement. Within a Pan-African rift zone a huge Infracambrian volcanic field was formed on top of a silicic diapir, with ignimbritic cauldrons, ring fracture intrusions, and resurgent granites (Figs 1) [56, 57].

Intrusive rocks host of magnetite; apatite-magnetite and sometimes they are also rich with Rare Earth Elements (REEs) mineralization. The mineralized zones are commonly associated with calc-alkaline volcanic rocks and metasomatic alteration. Iron oxideapatite deposits and occurrences occur within felsic volcanic tuffs and volcano-sedimentary sequences of the Early Cambrian age [58, 59]. The magnetiteapatite deposits comprise several orebodies with largescale replacement and brecciation textures, and a sodic-calcic alteration envelope [58]. The previous studies show that the genesis of the iron ores are similar to Kiruna-type deposits [56, 57]. There are many large and rich iron and iron-apatite deposits and occurrences around the Bafq region, e.g., Chadormalu, Choghart, Seh-Chahoon and Chah-Gaz iron ores and the Esfordi iron-apatite ore deposits (Fig 1).

4. Discussion

In this study, 843 collected stream sediment samples were analyzed by ICP-MS for 34 elements and oxides which correspond to iron mineralization (Fig. 2). Statistical results indicate that Fe2O3, TiO2 and V2O5 mean values are 5.66%, 0.796% and 86.2 ppm, respectively (Table 1). Their histograms have not normal distribution, as depicted in Fig. 3. Based on the elemental distribution, median is assumed to be equal to threshold values [9, 15]. The obtained thresholds are 5%, 0.78% and 77 ppm for Fe2O3, TiO2 and V2O5 respectively. The elemental distributions were built up by IDW estimation method in the area using RockWorks software package.



Fig. 1. The metallogenic province of Bafq showing a N–S striking section of the Kashmar–Kerman Tectonic Zone and the location map of the Bafq magnetic occurrences and deposits and the Esfordi deposit [60, 61] and geological map of Esfordi 1:100000 sheet [59].

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Fig. 2. Samples' location map in the Esfordi 1:100000 sheet

Table 1. Statistical parameters for $Fe_2O_3,\ TiO_2$ and V_2O_5 in the Esfordi 1:100000 sheet

	Mean	Median	Maximum	Minimum	Standard Deviation
Fe ₂ O ₃ (%)	5.66	5.00	30.05	3.00	1.74
TiO ₂ (%)	0.796	0.78	0.01	1.89	0.17
V ₂ O ₅ (ppm)	86.2	77	705.10	7.00	74.26

This procedure is suggested because it eliminates the undesired smoothing effects caused by usage of Kriging method. Since the IDW method clarifies the ore grade boundaries and ore concentration values, it is more desirable to use IDW method instead of Kriging which inherently has rather high amounts of truncation errors for the upper and lower boundaries of ore grades. The studied area was gridded by 100×100 m2 cells. The cell sizes dimensions were calculated based on geometry of sample collection gridding.

4.1. Factor analysis application

For reduction of variables, factor analysis was performed in the stream sediments geochemical data. The factor analysis was applied and 17 elements and oxides were classified in six factors by using SPSS software to the following groups (Table 2):

(1) Fe_2O_3 , TiO_2 and V_2O_5 , (2) SiO_2 , CaO and B, (3) Ni and Cu, (4) MgO and (5) Li. The first group including Fe_2O_3 , TiO_2 and V_2O_5 are important in the area for iron mineralization in the area. For better indication of extracted factors, the factor plot in rotated space is illustrated in Fig. 4.





Fig. 4. Component plot in rotated space in the factor analysis

	Component						
	1	2	3	4	5		
SiO ₂	.044	.813	.026	130	.133		
Fe_2O_3	.822	.034	.134	048	.148		
MgO	075	123	.009	.834	163		
CaO	248	828	.096	.127	.055		
Ni	.028	022	.779	.187	100		
Cu	011	.168	.775	049	.113		
V_2O_5	.643	.066	169	327	147		
в	.229	.617	.288	.149	036		
Li	119	.156	.079	.099	.754		
Al_2O_3	019	.333	.107	.008	.282		
Zn	.190	001	.297	.541	.286		
Cr	.539	506	.433	116	.239		
Co	.160	.019	114	020	.383		
Ba	.520	.273	055	.287	079		
Sr	.025	422	.106	140	.556		
TiO ₂	.714	.106	.114	.122	.057		
MnO	.588	.217	361	.449	.137		

Table 2. Elemental factor analysis in the stream sediment samples from the Esfordi 1:100000 sheet Rotated Component Matrix^a

4.2. C-N fractal modeling

Based on C-N log-log plots of the elements, there are four geochemical populations for Fe₂O₃ and TiO₂ and six geochemical populations for V₂O₅ respectively (Fig. 5). High intensive anomalies of Fe₂O₃, TiO₂ and V₂O₅ commence from 9.12%, 1.17% and 200 ppm, respectively. However, Fe₂O₃, TiO₂ and V₂O₅ thresholds are 4.46% and 0.64% and 56 ppm, respectively (Table 3). Moreover, C-N log-log plot of first factor was generated which shows five populations (Fig. 5). Main anomalous parts of the Fe₂O₃, TiO₂ and V₂O₅ and related factor were located in the northern, central and western parts of the sheet, as depicted in Fig. 6 which could be prospects for iron ore mineralization. However, high intensive anomalies of V2O5 was indicated in the NW part of the area which associated with moderate intensive Fe₂O₃ and TiO₂ anomalies (Fig. 6).

Table 3. Thresholds of Fe_2O_3 , TiO_2 and V_2O_5 in Esfordi 1:100000 sheet based on C-N fractal model.

High intensity threshold	Moderate intensity threshold	Low intensity threshold	
9.12	5.75	4.46	Fe ₂ O ₃ (%)
1.17	0.89	0.64	TiO ₂ (%)
200	113	56	V ₂ O ₅ (ppm)
3.99	0.80	0.16	F1 (Fe ₂ O ₃ , TiO ₂ , V ₂ O ₅)



Fig. 5. C-N log-log plots of Fe_2O_3 , TiO_2 , V_2O_5 and related factor (F1) in the studied area



Fig. 6. Geochemical maps obtained by the C-N fractal model of Fe₂O₃, TiO₂, V₂O₅ and first factor (F1) in the studied area

6. Correlation between C-N fractal modeling results and Geological particulars

For validation of the results obtained by C-N fractal modeling, the Fe_2O_3 , TiO_2 and V_2O_5 anomalies were correlated with faults and structures, iron alterations and lithological maps. The most parts of these anomalous areas have a good overlap with the structures and faults which determined by remote sensing and geological map (Fig. 7). Faults intersect the anomalies situated near those structures, as depicted in Fig. 7. Main anomalies specifically in the western and central parts of the area associated with iron oxides and alterations which is show in Fig. 8. The high intensive anomalies are near to Esfordi and Choghart iron ores (Fig. 8). Main anomalies in central, western and NW parts of the area accumulated with intrusive and volcanic rocks especially granitic/ryolitic rocks which host main iron ores in the Bafq district (Fig. 9).

Conclusions

The results obtained by this study show that the C-N fractal model is a proper method for separation of different anomalies from background. The classical statistics methods are able to separate only two geochemical populations by median because elemental distribution in most cases are not normal. This process will reduce the accuracy of such studies so utilizing the fractal models could be useful. Moreover, factor analysis could be helped for reduction of variables and separation of multi-elemental anomalies. The results derived via C-N fractal model exhibit Fe₂O₃, TiO₂ and V_2O_5 anomalies in the western, central, NW and northern of the Esfordi area.

Additionally, Main anomalies of Fe_2O_3 , $TiO_2 V_2O_5$ and related factor (F1) obtained by C-N fractal modeling were validated with geological particulars consisting of faults, rock types and iron alterations. The anomalies are correlated with faults and structures especially their intersections. Furthermore, there are granitic/ryolitic intrusive and volcanic rocks which are major host rocks of iron ores in the Bafq region. Moreover, iron alterations and iron minerals were determined in the anomalous parts based on remote sensing operation. Results of this study indicate that major iron ore prospects illustrate in the central and western parts of the Esfordi area.

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Fig. 7. Correlation between the geochemical anomalies obtained by the C-N fractal model (Yellow ellipsoids) and faults derived via remote sensing and geological map [57] in the Esfordi area.



Fig. 8. Correlation between iron alterations resulted from remote sensing methods with main iron ores of the sheet [57] and the geochemical anomalies obtained by the C-N fractal model (Blue ellipsoids).



Fig. 9. Correlation between rock types of the sheet [57] and the geochemical anomalies obtained by the C-N fractal model (Blue ellipsoids)

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