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Nonlinear disjunctive kriging for the estimating and modeling of a vein copper deposit

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Abstract

Estimation of mineral resources and reserves with low values of error is essential in mineral exploration. The aim of this study is to estimate and model a vein type deposit using disjunctive kriging method. Disjunctive Kriging (DK) as an appropriate nonlinear estimation method has been used for estimation of Cu values. For estimation of Cu values and modeling of the distribution of Cu in Chelkureh, samples have been taken from 48 drill holes in Chelkureh, and the values of Cu have been analyzed. Resulting data from analyzing Cu values were converted to standard normal values using Hermite polynomials. Variography has been done in the Chelkureh deposit. After studying variograms in different directions, it was found out that the ore deposit has a mild anisotropy. The best-fitted variogram model was considered for disjunctive kriging estimation. The model consists of a pure nugget effect with 0.46 amplitude plus a spherical scheme with sill 1.20 and range 140 m. Consequently, a three-dimensional model of estimated value and error estimated value was provided by disjunctive kriging to divide the ore into an economic and uneconomic part. The models based on the estimate of the DK method in the study area exhibited an increasing trend of concentration from the center to North. Finally, validation between the disjunctive kriging carried out by using cross-validation. The result showed that the correlation of estimated values and real values was strong (63.4%).
Keywords: *Disjunctive kriging, Vein type, Chelkureh, Zahedan*

1. Introduction

Disjunctive Kriging (DK) has been available for spatial estimation for more than 40 years. However, the seemingly complex theory makes it unappealing for most practitioners (Ortiz et al. 2004). DK is a technique that provides advantages in many applications. It can be used to estimate the value of any function of the variable of interest (Ortiz et al. 2004). DK provides a solution space larger than the conventional kriging techniques that only rely on linear combinations of the data. DK is more practical than the conditional expectation, since it only requires knowledge of the bivariate law, instead of the full multivariate probability law of the data locations and location being estimated (Ortiz et al. 2004). In linear estimators, the weights are optimized to ensure the minimal estimation variance, when one needs have a local expected value, so the primitive assumption is that a normal (frequency) distribution function is available. But when is needed to calculate the probability of values above a threshold, these probabilities are not a simple linear form of data, and there should be adapted techniques to calculate these probabilities (Webster and Oliver 2001). One of these techniques is Disjunctive Kriging (DK). Other geostatistical methods used to calculate the probability that the true value exceeds the threshold are conditional simulations (Chiles and Delfiner 1999), multi-Gaussian kriging (Verly 1983; Emery 2006) or nonparametric estimators such as indicator kriging (Deutsch and Journel 1998). Considering this capability, DK has been used in earth science; the works of some of the researchers can be mentioned. A lot of researchers have used DK in different cases such as Mining, Environment issues, Petroleum, Geotechnique and so on, to perform the estimations (Journel and Huijbregts 1978; Matheron 1984; Webster and Oliver 2001; Daya 2014). For instance, in Emery (2006), this method is used for describing the distribution of pollution in the earth and some other parameters in agriculture (Emery 2005). In this study, Emery has found more realistic results from DK versus Ordinary Kriging (OK). This paper aims to estimate and model the Cu values in Chelkureh using DK method as a non-linear geostatistical estimator, based on iso-factorial model and Hermit polynomials.

2. Theory of disjunctive kriging (DK)

Basically, in DK, the random field Z to be estimated is decomposed into a sum of disjoint (uncorrelated) components of sample values. When kriging of the separate components is possible, the procedure is onwards: i.e., when the joint probability density functions of Z (or a transform of it, Y) and each sample za (or Ya) is of isofactorial type (Matheron 1984). In practice, a continuous variable like the grade of an ore deposit can always be transformed by anamorphosis into a Gaussian equivalent Y, and then only a joint Gaussian hypothesis for the probability density function (PDF) of samples is required, for which there

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exists an isofactorial representation based on Hermite polynomials (Rivoirard 1994). The first step is to transform observations on the random field, Z(x), to observations on Y(x) such that:

$$Z(x) = \Phi[Y(x)] (1)$$

Where Y(x) is N (0, 1). The function Φ is expressed in terms of Hermite polynomials, which are related to the normal distribution by Rodrigue's formula:

$$H_k(y) = \frac{1}{\sqrt{k!}g(y)} \frac{d^k g(y)}{dy^k} (2)$$

In which g(y) is the normal probability density function, k is the degree of the polynomial taking values 0, 1, 2,… and $\frac{1}{\sqrt{k!}}$ is a standardizing factor. The first two Hermite polynomials are:

$$H_o(y) = 1 \ ,$$
$$H_1(y) = -y \ ;$$

Thereafter the higher order polynomials are obtained using the recurrence relation:

$$H_k(y) = -\frac{1}{\sqrt{k}} y H_{k-1}(y) - \sqrt{\frac{k-1}{k}} H_{k-2}(y)$$

The Hermite polynomials are orthogonal with respect to the weighting function exp (-y2/2) on the interval from $-\infty$ to $+\infty$. Many functions of a Gaussian field Y(x) can be represented as the sum of Hermite polynomials (Rivoirard 1994):

$$f\{Y(x)\} = f_o H_o\{Y(x)\} + f_1 H_1\{Y(x)\} + f_2 H_2\{Y(x)\} + \cdots (3)$$

Because the polynomials are orthogonal we can calculate the coefficients required for Eq. (1) as:

$$Z(x) = \Phi[Y(x)] = \Phi_0 H_0\{Y(x)\} + \cdots = \sum_{k=0}^{\infty} \Phi_k H_k\{Y(x)\} (4)$$

The transform is invertible, and so we can express the results in the original units of measurement. To krige the variable of interest, Z(x), the Hermite polynomials are kriged separately and their estimates are summed to give the disjunctive kriging estimator:

$$\hat{Z}^{DK}(x) = \Phi_0 + \Phi_1 \hat{H}_1^k\{Y(x)\} + \Phi_2 \hat{H}_2^k\{Y(x)\} + \ldots (5)$$

So from n data points (x1,… xn) in the neighborhood of x0 where we want an estimate we estimate the Hermite polynomials by

$$\hat{H}_k^k\{Y(x_o)\} = \sum_{i=1}^{n} \lambda_{ik} H_k\{Y(x_i)\} \ (6)$$

and we insert them into Eq. (5). The λ_{ik} are the kriging weights, which are found by solving the simple kriging equations:

$$\sum_{i=1}^{n} \lambda_{ik} \operatorname{cov}[H_k\{Y(x_j)\}, H_k\{Y(x_i)\}] = \operatorname{cov}[H_k\{Y(x_j)\}, H_k\{Y(x_o)\}]$$

(7)

or alternatively

$$\sum_{i=1}^{n} \lambda_{ik} \rho^k (x_i - x_j) = \rho^k (x_j - x_0); for \ \ all \ \ j = 1..n$$

(8)

In particular, the procedure enables estimation of Z(x0) by

$$\hat{Z}(x_0) = \Phi\{\hat{Y}(x_0)\} = \Phi_0 + \Phi_1[\hat{H}_1^k\{y(x_0)\}] + \cdots$$

(9)

The kriging variance of $\hat{H}_k\{Y(x)\}$ is

$$\sigma_k^2 = 1 - \sum_{i=1}^{n} \lambda_{ik} \rho^k (x_i - x_0) (10)$$

and the disjunctive kriging variance of $\hat{f}[Y(x_0)]$ is

$$\sigma_{DK}^2(x_0) = \sum_{k=1}^{\infty} f_k^2 \sigma_k^2 (x_0) (11)$$

Once the Hermite polynomials have been estimated at x0 we can estimate the conditional probability that the true value there exceeds the critical value, Zc. The transformation $Z(x) = \Phi[Y(x)]$ means that Zc has an equivalent yc on the standard normal scale. Since the two scales are monotonically related their indicators are the same (Rivoirard 1994):

$$\Omega[Z(x) \le z_c] = \Omega[Y(x) \le y_c] (12)$$

where Ω is an indicator.

For $\Omega[Y(x) > y_c]$, which is the complement of $\Omega[Y(x) \le y_c]$, the kth Hermite coefficient is

$$f_k = \int_{-\infty}^{+\infty} \Omega[y \le y_c] H_k(y) g(y) dy = \int_{-\infty}^{y_c} H_k(y) g(y) dy$$

(13)

The coefficient for k=0 is the distribution function of Y(x) at yc , i.e.,

$$f_0 = G(y_c)$$

Where, G is the distribution function of the standard Normal variate. For larger k

$$f_k = \frac{1}{\sqrt{k}} H_{k-1}(y_c) g(y_c) (14)$$

The indicator can be expressed in terms of the cumulative distribution and the Hermite polynomials:

$$\Omega[Y(x) \le y_c] = G(y_c) + \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} H_{k-1}(y_c) g(y_c) H_k\{Y(x)\}$$

(15)

Its disjunctive kriging estimate is obtained by

$$\hat{\Omega}^{DK}[y(x_0) \le y_c] = G(y_c) + \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} H_{k-1}(y_c) g(y_c) \hat{H}_k^k\{y(x_0)\}$$

(16)

The kriged estimates $\hat{H}_k^k\{y(x_0)\}$ approach 0 rapidly with increasing k, and so summation need extend over only few terms. This is the same as $\hat{\Omega}^{DK}[z(x_0) \le z_c]$.

In this instance, we are interested in the probability of excess, and so we compute

$$\hat{\Omega}^{DK}[z(x_0)>z_c]=1-G(y_c)-\sum_{k=1}^{L}\frac{1}{\sqrt{k}}H_{k-1}(y_c)g(y_c)\hat{H}_k^k\{y(x_0)\} \quad (17)$$

3. Case study

The Chehelkureh deposit is located in the Nehbandan-Khash zone (eastern Iran) between the Afghan block to the east, the Neh Fault to the west, and the Bashagard Fault to the south (Stocklin 1977) (Fig 1). This zone, also known as the Sistan suture zone of eastern Iran (Tirrul et al. 1983), represents a narrow, short-lived strip of oceanic lithosphere that was consumed in the Sennonian and Paleogene and, in part, obducted during the Eocene continental collision (Tirrul et al. 1983).

Dikes and lavas from the Chehelkureh ophiolitic mélange are plagioclase-phyric basalts with chemical compositions that indicate that they were mid-ocean ridge and marginal basin tholeiites (Desmons and Beccaluva 1983).

The N-S–trending Lunka-Malusan Mountain Range is the highest in the region, with Kuh-e-Lunka (2,300-m elevation) comprising metaturbidites (Fig 2) and Kuh-e-Malusan (2,425-m elevation) comprising gabbro (Maanijou et al. (2012). The study area is divided into three lithotypes on the basis of rock components: igneous rocks (younger than ophiolites), sedimentary rocks, and the ophiolitic mélange (Fig 2). Each of these lithotypes is described below, relative to its age (i.e., from the oldest to the youngest unit). Sedimentary layers, which consist of greywacke, shale, and limestone, are tightly folded, steeply dipping, and faulted (Maanijou et al. 2012). Cretaceous turbidites have faulted contacts with the ophiolitic complex and are composed of phyllite and small lenses of marble (Maanijou et al. 2012). Paleocene turbidites are composed of shale and sandstone with rare limestone layers (Fig 2). Eocene turbidites are up to 1 km thick and widespread. In metamorphosed turbidites, the basal conglomerate is the oldest unit.

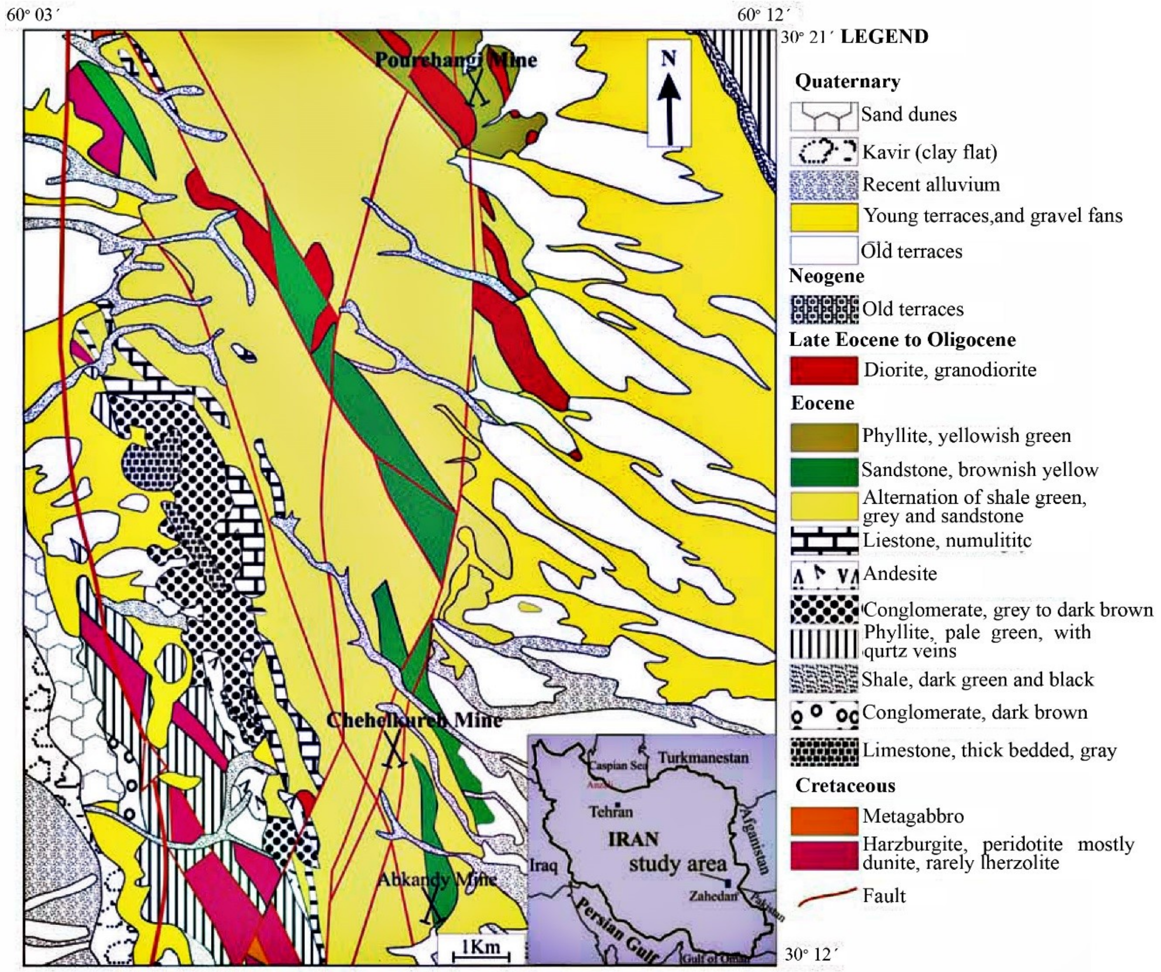

Fig 1. Geologic map of the Chehelkureh ore deposit (Maanijou et al. 2008).

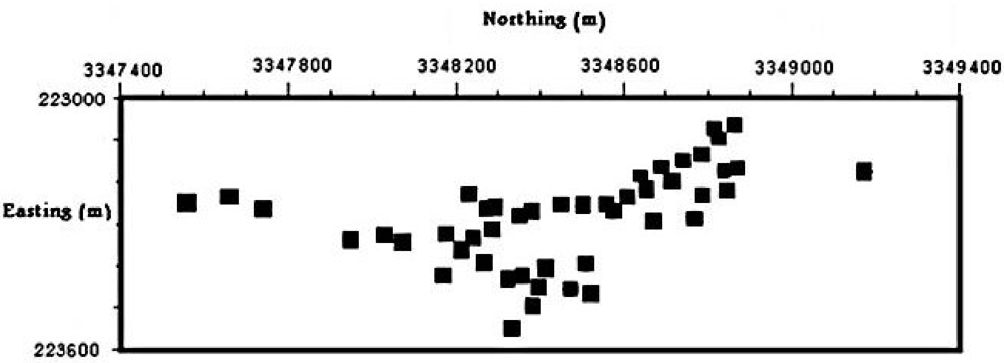

Fig 2. Borehole location map of Chelkureh deposit

The western turbidites, which are altered, host the Chehelkureh ore deposits (Maanijou et al. 2012). Several granitoid stocks and dikes intruded the sedimentary sequence where they are oriented parallel to the major NWSE– trending fault set (Fig 2). Plutonic rocks crop out mostly to the west of the Chehelkureh Fault in the Lunka-Malusan Mountain Range (Valeh and Saeedi 1989). Intrusive bodies consist of quartzmonzodiorite and granodiorite at the Chehelkureh deposit. Exposures of rock in the vicinity of the Chehelkureh deposit are controlled by major N-S– and NW-SE–trending faults, based on air photo lineaments, surface traces, and offsets of geologic features (Maanijou et al. 2012).

The Chehelkureh deposit comprises numerous lenses and veins. There were two stages of mineralization, the first of which consists of metallic mineralization concentrated along the brittle, finely fractured parts of the beds of sandstone, siltstone, and shale. The second stage of mineralization formed along fractures that crosscut sandstone, siltstone, and shale, displacing them by several millimeters. The first stage includes quartz, calcite, dolomite, ankerite, siderite, ilmenite, rutile, molybdenite, pyrrhotite, arsenopyrite, pyrite, and chalcopyrite. The second stage consists of quartz, dolomite, ankerite, siderite, chalcopyrite, sphalerite, pyrite, galena, selenian galena, marcasite, nevskite, and paraguanajuatite. The gangue minerals are dominated by quartz and various carbonates, locally associated with chlorite. Hypogene alteration consists of silicification, carbonatization (ankerite, magnesite, siderite, and dolomite), chloritization, kaolinitization, sulfidation, and, less commonly, sericitization (Maanijou 2007, Maanijou et al. 2012).

4. Statistical analysis on data

This deposit was explored principally by 48 boreholes (Fig 2) with total 2976 m of drilling. In general, the drilling grid is irregular; the distance between the two boreholes varies from 50m to 100m (Fig 2). Borehole samples were analyzed by ICPMS method. They were of unequal length. It is very important in estimation to work with equal support samples and therefore the data were composited to equal lengths (Daya 2012; Afzal et al 2013; Daya 2015a and b; Daya and Bejari 2015; Daya and Hosseininasab 2019).

Statistical studies were performed on the raw data, and the results are shown in Figure 3 for Cu (%) concentration.

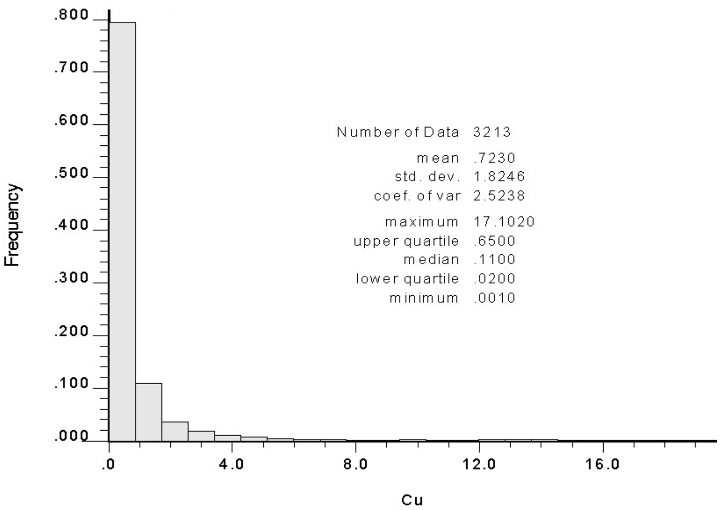

Fig 3. Histogram of the data for Chelkureh deposit

Figure 4 also shows the histogram of normal transformed data. Logarithmic transformation was used for normalizing the raw data. Statistical analysis was done using the GSLIB Software (Deutsch and Journel 1998).

5. Variogram Modelling

After computing and drawing the experimental variogram, a theoretical model was fitted on resulted experimental variogram. This was done by a series of codes in MATLAB software. The experimental variogram, γ(h) was fitted to a theoretical model to find three parameters, such as the nugget (c0), the sill (c) and the range (a). After the experimental variogram was calculated from the regularized data, a nested structure with a nugget and a spherical variogram was used. It consists of a spherical model with sill 1.20 and range 140m plus a nugget of 0.46 (Fig 5). This model was required since DK estimation will be based on it.

6. Estimation by disjunctive kriging

To estimate the Cu concentration, the disjunctive kriging method was used to receive estimates at points on a grid 20m x 20m x 12.5m. These points may be taken as the center-points of cubes of dimension 20m x 20mx 12.5m. The estimation and 3-D modeling process commenced from the elevation of 650m above the sea level to 1250m above the sea level in the mine. For the application of DK, MATLAB Software has been used. Figure 6 and 7 show estimates and kriging errors of Cu concentration in different elevations computed by DK.

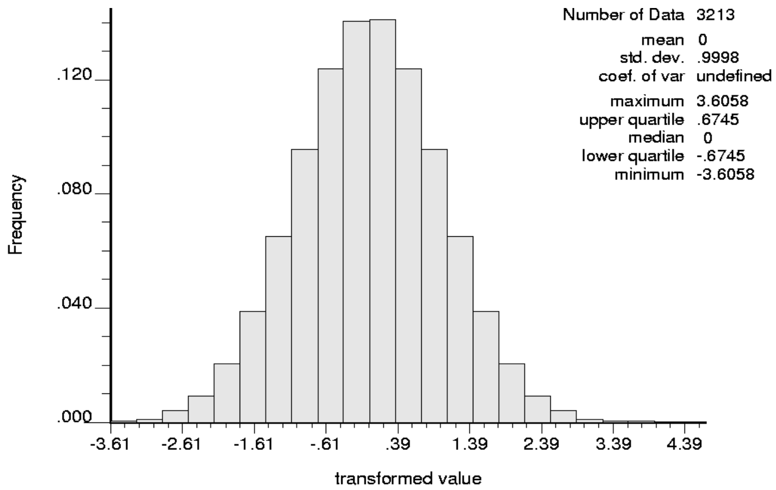

Fig 4. Transformed histogram of the data for Chelkureh deposit

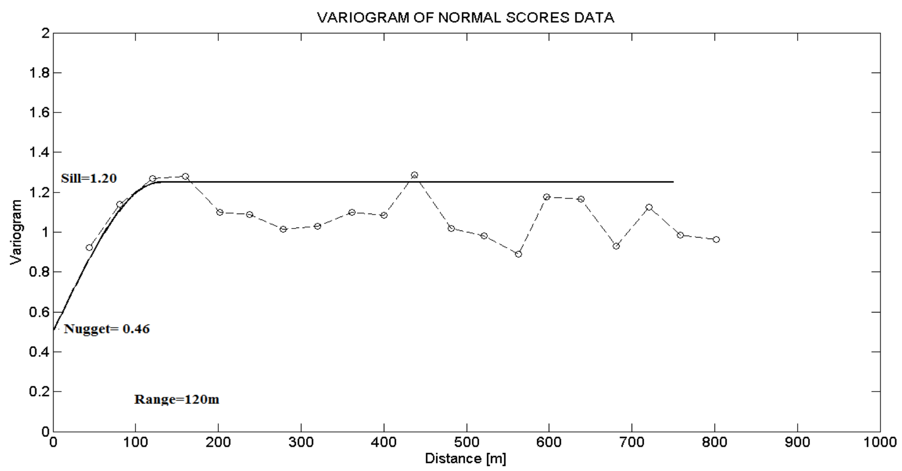

Fig 5. Experimental omnidirectional variogram and its fitted model in Chelkureh deposit

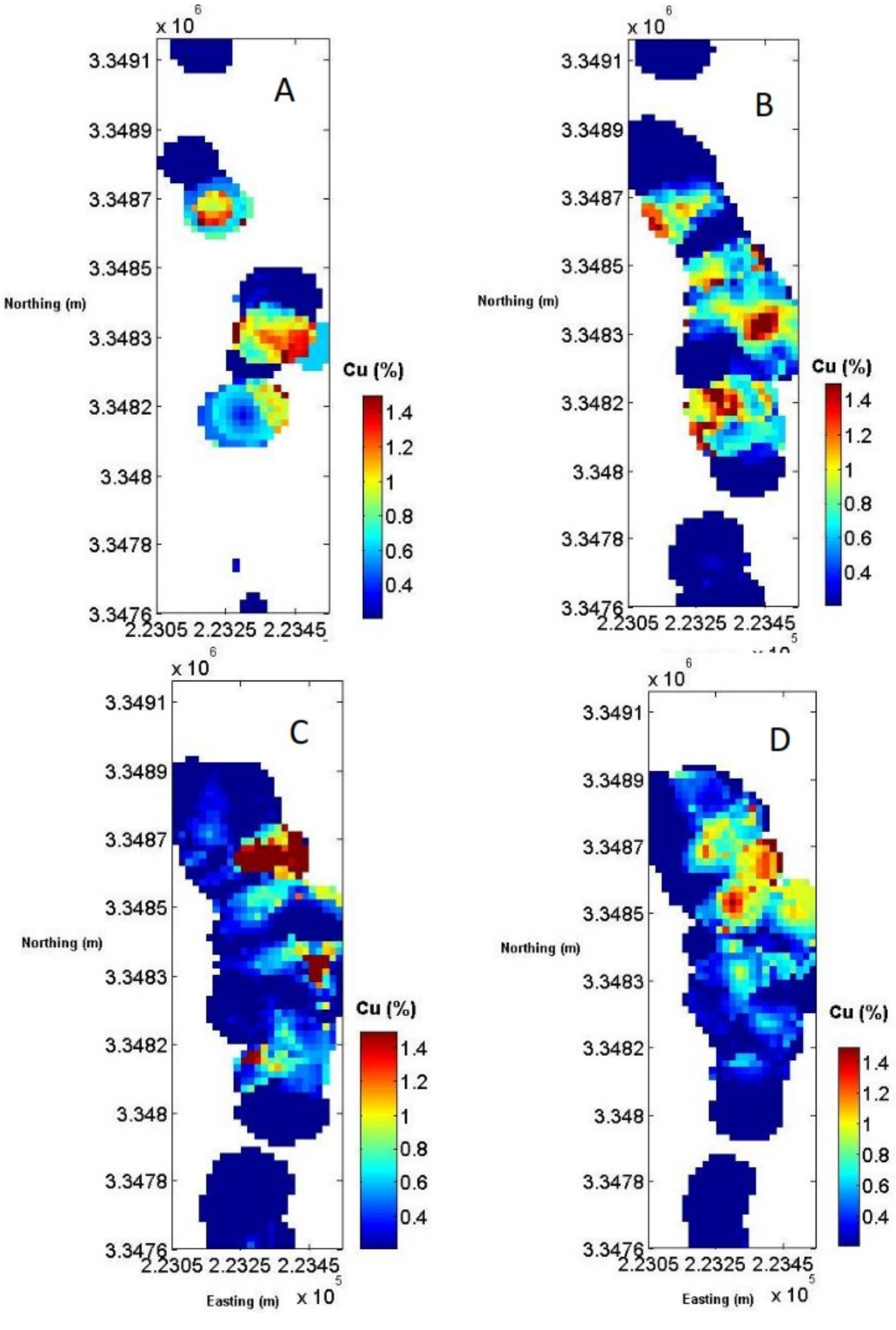

Fig 6. Estimates of Cu concentration by disjunctive kriging in different elevations (A=1300m, B=1400m, C=1500m, D=1600m above the sea level) in Chehelkureh deposit

Figures 8 and 9 show the three-dimensional model of DK estimates and DK errors of Cu concentration in Chelkureh deposit. Based on the disjunctive kriging method, the extremely and highly Cu concentration happened in the northern and central parts of the Chelkureh deposit (Fig 8).

The estimate of the grade at an unsampled location is not enough. It is necessary to obtain an estimate of the probability that exceeds a given threshold, say 0.5 %. Figure 10 shows probabilities to exceed the chosen threshold value of 0.5% in different elevations computed by DK. Figure 11 also shows the three dimensional model of probabilities to exceed the chosen threshold value of 0.5%. Finally, validation of the disjunctive kriging is carried out by using cross-validation. Cross-validation uses all the data to estimate the trend and autocorrelation models. It removes each data location one at a time and predicts the associated data value. Results show that the correlation between the estimated values and the real values is 63.4% (Fig 12). If we wish to label the strength of the correlation, 0-0.19 is regarded as very weak, 0.2-0.39 as weak, 0.40-0.59 as moderate, 0.6-0.79 as strong and 0.8-1 as very strong correlation (Hassanipak and Sharafaddin 2001, Vural 2014). According to the above classification, the correlation between the real values and the estimated value in this study is strong.

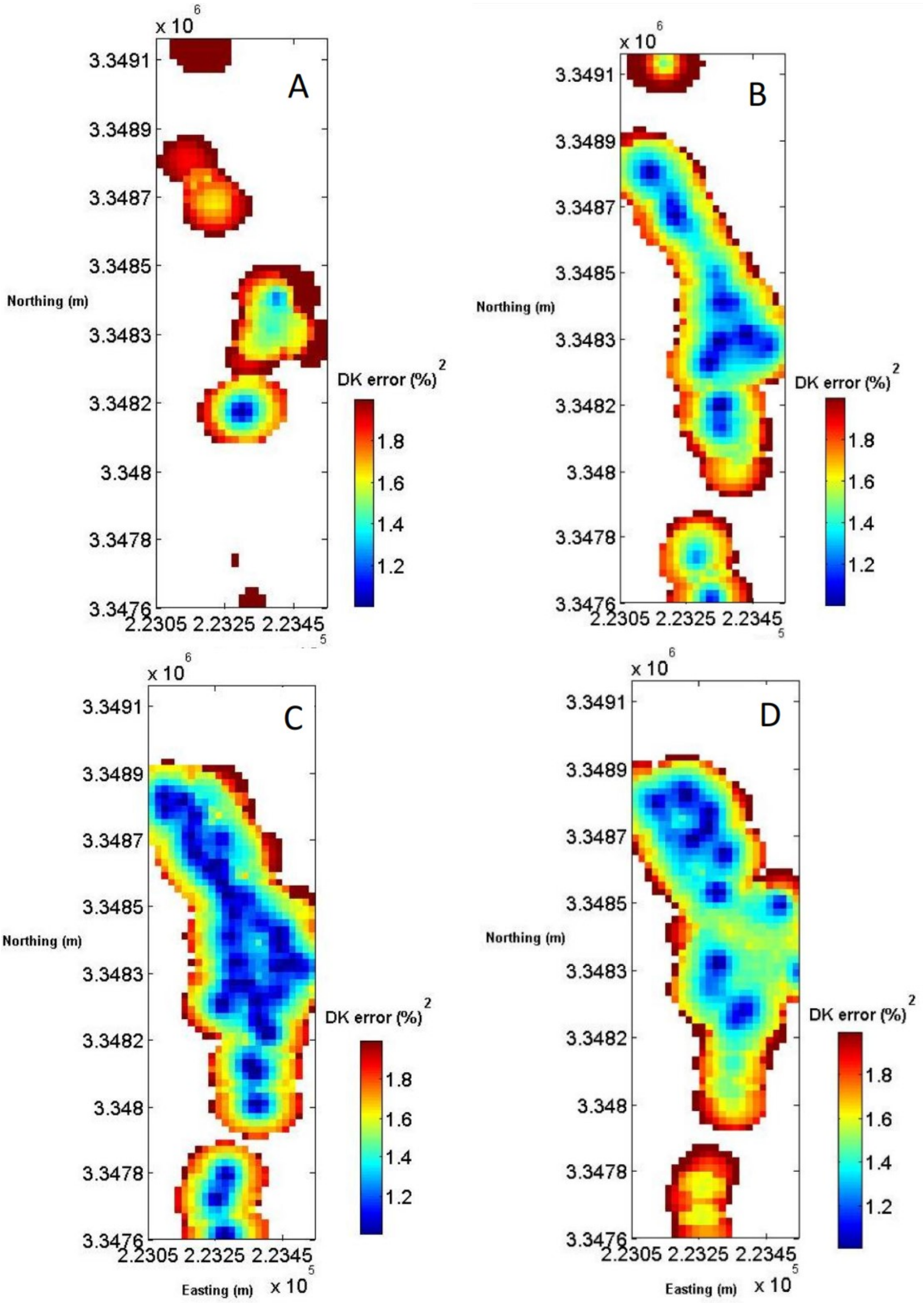

Fig 7. Disjunctive kriging errors of Cu concentration in different elevations (A=1300m, B=1400m, C=1500m, D=1600m above the sea level) in Chehelkureh deposit

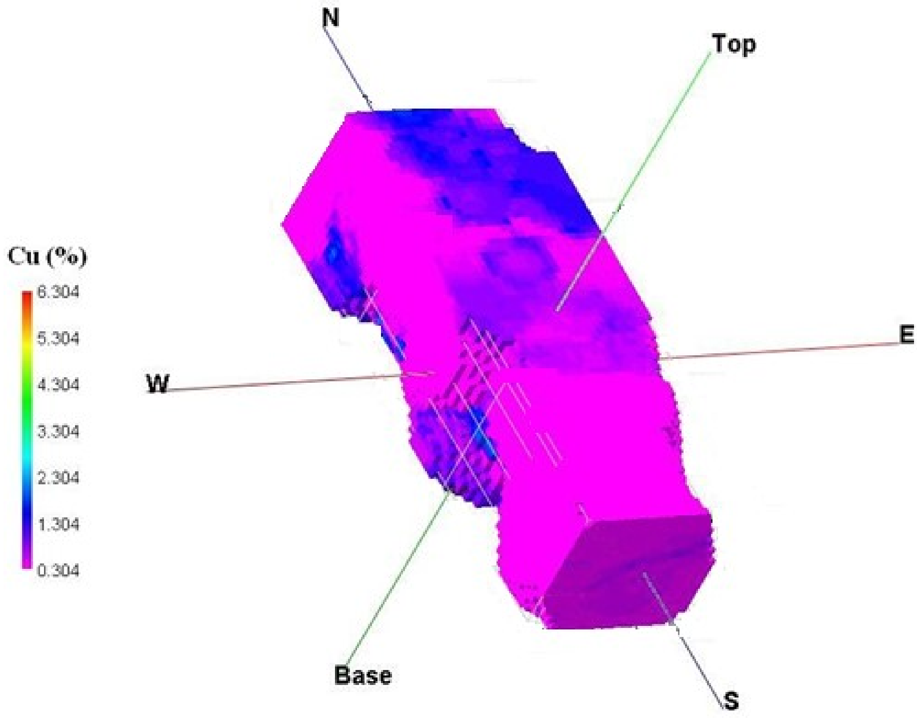

Fig 8. 3D model of Estimates of Cu concentration by disjunctive kriging

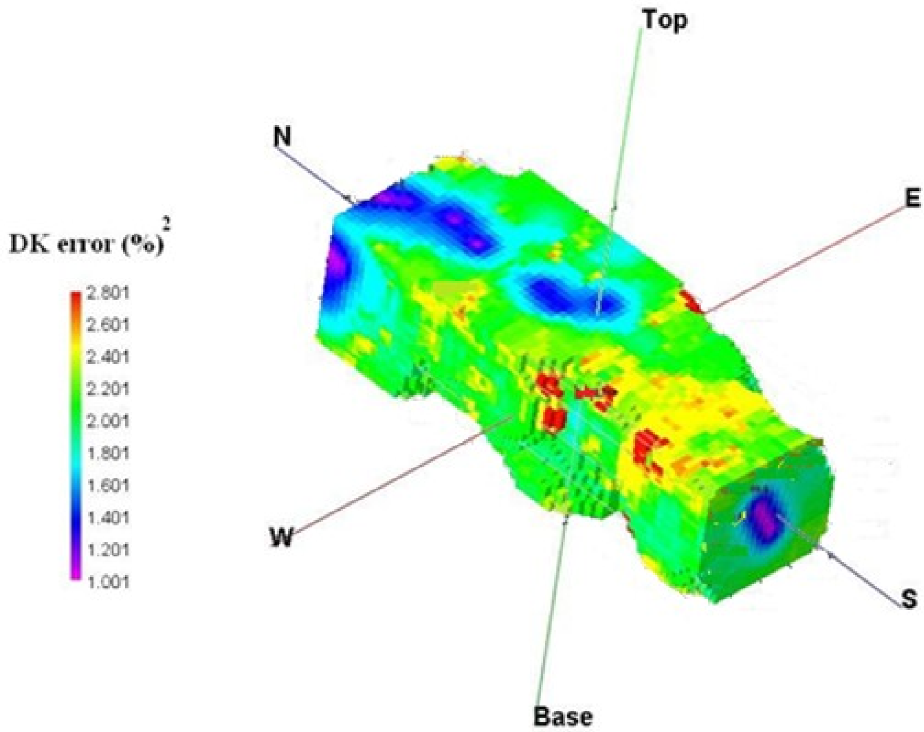

Fig 9. 3D model of disjunctive kriging variance of Cu concentration

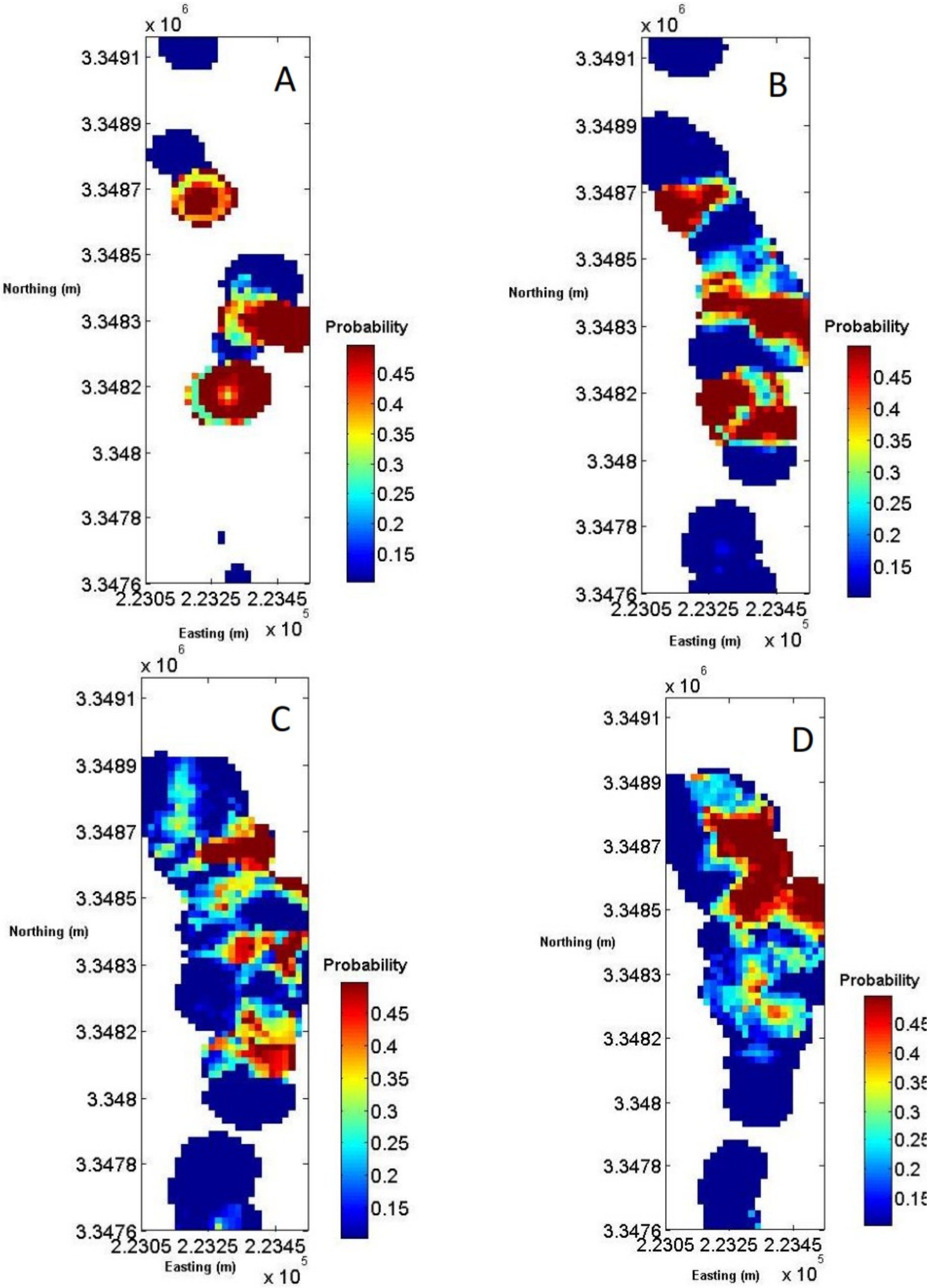

Fig 10. Probabilities to exceed the chosen threshold value of 0.5% in different elevations (A=1300m, B=1400m, C=1500m, D=1600m above the sea level) in Chehelkureh deposit

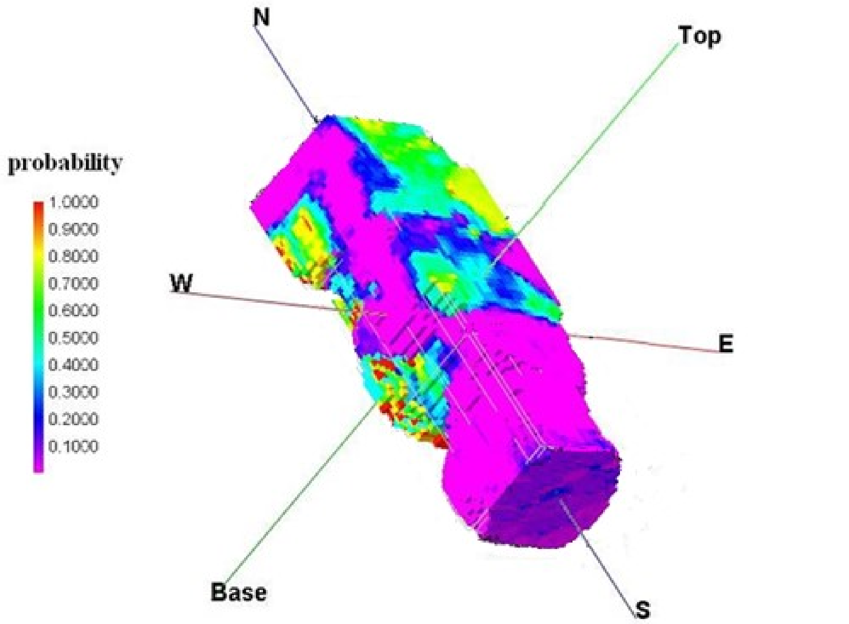

Fig 11. 3D model of Probabilities that the Cu concentration exceed the threshold value (0.5 %)

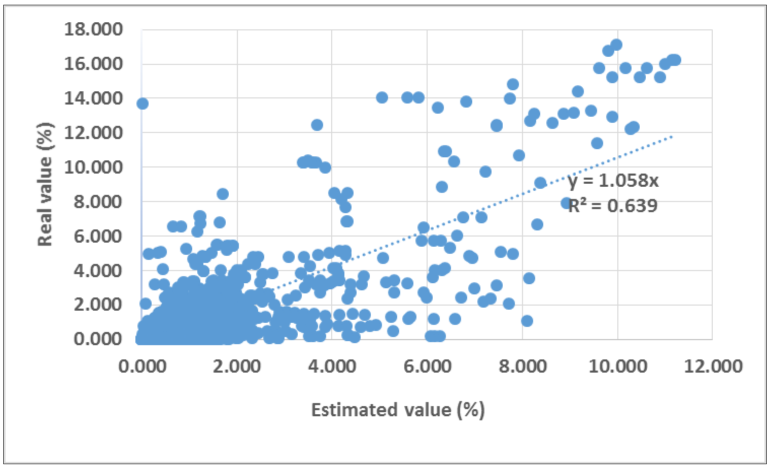

Fig 12. Scatterplot of real value versus estimated value for Cu concentration in Chelkureh Deposit (cross-validation)

7. Conclusions

Choosing the proper method for estimation of resource or reserve with a minimum error is very important in geostatistical operations in mining engineering. Because assessment of project economics (or another critical decision making) based on estimation with high error is risky. The case study presented in this paper show that disjunctive kriging is a useful method in the estimation of reserves or resources of vein type deposits, such as in Chelkureh copper deposit. After trial and error, a variogram with the best summary statistics was chosen. The model consists of a pure nugget effect with 0.46 plus a spherical scheme with sill 1.20 and range 140 m. From the case-study, the author concludes that disjunctive kriging can be used to model and estimate the grade of a Copper ore deposit (Cu concentration). Validation of the disjunctive kriging carried out by using cross-validation. The result showed that the correlation between estimated values and real values was strong (63.4%).

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