American Option Pricing of Future Contracts in an Effort to Investigate Trading Strategies; Evidence from North Sea Oil Exchange

Ramin Ahmadi[*], Nahal Ariankia

Department of Financial Engineering, Faculty of Technical Engineering, Khatam University, Tehran, Iran.

ABSTRACT

In this paper, Black Scholes's pricing model was developed to study American option on future contracts of Brent oil. The practical tests of the model show that market priced option contracts as future contracts less than what model did, which mostly represent option contracts with price rather than without price. Moreover, it suggests call option rather than put option. Using t hypothesis test, price differences were obtained, which can serve as a useful strategy for traders interested in arbitrage practice and risk hedging. This research introduces an optimal strategy (both for call and put option states and buy and sell of future contract Δ) for all options of buy and sell future contracts with and without price. In this research, six-month data of the end of 2015 about oil option and option of future contracts of North Sea oil for three different maturities were used.

1 Introduction

In general, we can divide option right into two categories; call option and put option. The former actually gives its holder the right (rather than obligation) to buy the underlying asset with specified price on a specified date or prior to it. Likewise, put option gives its holder the right to sell the underlying asset with specified price on a specified date or prior to it. The price stated on the contract is called "strike price" or "exercise price", and specified date is "expiration date" or "option maturity".

Call or put options are divided into European and American option. The former can be just exercised on maturity date, while the latter can be exercised any time before maturity date or on maturity date. Option contracts on stock are seen as cash options, because buy or sell of asset with agreed price can be undertaken immediately when option is exercised. However, in option of future contracts the holder can have a trading situation in future contract when an option is exercised.

Each future contract option is associated with the name of a month in the year when asset maturity is the basis of future contract (as in oil in this paper) in that month. It should be noted that this month is not the time of option expiration. Typically, the precise maturity of future contract option is a few days before or at the maturity date of the underlying asset of future contract. There has been always a

question as to why traders prefer the option issued for future contract to the option issued for the underlying asset. The main reason for this is that future contract frequently has more liquidity power than underlying asset. Moreover, price of future contract is definite and certain any time on future contract exchange, but cash price of underlying asset may not be available at any moment. The most important advantage of future contract option is that exercising option does not always lead to transferring physical asset, yet underlying asset of option, future contract, is frozen before its transfer comes due; that is to say, it is settled in cash. This can appeal to retail investors to whom the procurement of necessary funds for buying underlying asset is problematic when an option is exercised. Another advantage of future contract option is that future contract and option of asset are both traded on exchange, which in turn facilitates risk hedging, arbitrage and speculation. Furthermore, it involves less transaction costs in most of the time as against cash option. The most important difference between option of future contract and stock option is that there is no need to pay for contract price in order to enter into transaction of future contracts.

Assuming that strike price and time remaining to maturity are the same in both cases, the profit from European cash call option with the strike price K is equal to Return=max(S_T-k,0) , and the profit from European call option of future contract with the strike price K is equal to Return=max(F_T-k,0).

If the maturity of European option of future contract and that of future contract are the same, then $F_T = S_T$, so these two options are equal. However, if the maturity of European call option of future contract is before the maturity date of future contract, then the value of the call option of future contract is greater than that of call option like that in normal market, where future prices are more than cash price; the reverse is true for European put option of future contract.

- Extremely out-of-the-money and in-the-money options have higher implied volatility compared with options at the money

- Under the assumption of the fixed state of interest rate, formula of future contract option price and its corresponding forward contract option become equal, because under this situation price of future contract and that of forward contract are the same. However, when interest rate is random, the price of forward contract and future contract is different due to the mechanism of deposit account in future contract.

In recent years, many articles were presented in the field of risk hedging of asset portfolio by risk hedging instrument. One of the main aspects of hedging risk of oil price with future oil contracts is that it allows company to insure itself against adverse movements of oil price without losing an opportunity for earning profit, which is not related to changes of oil exchange rate and its adverse movements. A number of researchers are facing problems with pricing of future oil contracts, which is believed to be the lack of a good model. The results show that Brent oil mispricing model follow a specific model. Such pricing is the result of a failure in the model. Shastri and Tandon [13] found a strategy for risk hedging, which yields excess profits in the market of oil option when there is price deviation from their corresponding model prices.

Another method which is proposed for calculation of option price is Monte Carlo simulation method. This method is based on a binomial tree built for derivatives. Certain paths are assumed on this tree and forward-motion is used instead of moving from the end to the back of the tree. is used instead of moving from the end to the back of the tree. Monte Carlo Simulation cannot be used for American

option, because there is no way to make sure whether early exercise is optimal or retention of an option in a node. Thus, this method is thus used for controlling valuation formula of European options.

Variations in oil price index are negatively correlated to variations of base oil exchange rate which is announced in the base country, USA. This makes option and future contract appealing alternatives to risk hedging. Numerous studies have been conducted into pricing of future oil and currency contracts. Many researchers have shown that future premium on pricing model is introduced in calculations by replacing oil future exchange rate with current exchange rate. By using future contracts of oil, risk hedging power depends on liquidity power in market. Roman [12] showed "Roll Geske Waley" for call option with a software programs, and compared it with Black and Sholes model. He found that RGW has less volatility calculation errors and eventually less pricing error compared with B-S model.

Jacobs and Clyman [6] demonstrated that market of oil future contract is more liquid than its proposed volume, which maybe because of risk arbitragers. We can turn pricing model into Black's pricing form of future contract option. Muzzioli and Reynaerts [11] studied an American option price by using a multi-period binomial model when there is uncertainty on the volatility of asset, showing that this model is superior to other models from three aspects; first, triangular and trapezoidal fuzzy numbers; second, the use of no-arbitrage condition in extracting risk-neutral probability; third, simple and fast computational algorithm for obtaining option price. Zhu and Chen [14] presented a correction to Merton's classical case of pricing American options. By using perturbation method, two analytic formulae were derived for option price and optimal exercise price. Agarwal et al [1] introduced an approximation of American option price under stochastic volatility models, by using the maturity randomization method known Canalization, and came to the result that the proposed method outperforms the least squares regression method.

Thus, we can practically test market pricing by using historical pricing data of option and oil future contract, which is accomplished by comparing market price and model price derived from MATLAB output. In what follows, some questions left unanswered in studies conducted until now are answered; what is the model suitable for option on oil future contract? How does this model work with respect to historical prices? Can traders earn profit by setting up risk-neutral portfolio on the basis of signals extracted from the model?

2 The Mathematical Background

When a put option is exercised on a future contract, option holder has a put option on future contract of an underlying asset, as well as cash flow which is equal to a difference between future asset price at the time of option exercise and future price at the present time; if future price of the asset increases, trader earn profit by exercising option with cheaper price.

In this paper, an attempt was made to obtain the value of differentiation equations for option on future based on Black-Scholes formulae. It is assumed that the rate of a fixed interest and price movement of the underlying asset follow a geometric Brownian process. It is because in Black-Scholes pricing model interest rate is assumed to be constant, so future price is a time deterministic function of the price of the underlying asset; therefore, volatility of future price should be the same as the price of the underlying asset.

Variations of future price f_t are as follows;

$$\frac{df_t}{f_t} = \mu_f \mathrm{dt} + \sigma \, dz_t \tag{1}$$

Where μ_f is efficiency expected for future and σ price volatility of asset, Thus, V (f_t,t) represents the value of option of future contract. Now, we consider a portfolio with α_t unit of future contract in call situation and one unit of option of future contract in put option, in which α_t is set dynamically to create a simultaneous risk hedging situation for portfolio in all moments.

Value of the portfolio $\Pi(f_t, t)$ is as follows;

$$\Pi(f_t, t) = -v(f_t, t) \tag{2}$$

Since there is no cost for entering into future contract, as we should note that the portfolio profits from future call option as much as …, by using Ito's Lemma we have:

$$\mathrm{dv}\,(f_t,\mathrm{t}) = \left(\frac{\partial v}{\partial t} + \frac{\sigma^2}{2} f_t^2 \frac{\partial^2 v}{\partial f_t^2} + \mu f_t \frac{\partial v}{\partial f_t}\right)\mathrm{dt} + \sigma \frac{dv}{df_t} dz_t \tag{3}$$

Total profit earned by portfolio;

$$\int_0^t -dv(f_u, u)du + \int_0^t \alpha_u \, df_u = \int_0^t \left(-\frac{\partial v}{\partial u} - \frac{\sigma^2}{2} f_u^2 \frac{\partial^2 v}{\partial f_u^2} + (\alpha_u - \frac{\partial v}{\partial f_u})\mu_f f_u\right) du + (\alpha_u - \frac{\partial v}{\partial f_u})\sigma f_u dz_u \tag{4}$$

The number of units of future contract, which is kept at any u time, is balanced dynamically, so that profit from portfolio is definite, can be obtained by a logical choice;

$$\alpha_u = \frac{\partial v}{\partial f_u} \tag{5}$$

In this case, deterministic financial gain from risk hedging is as follows;

$$\int_0^t \left(-\frac{\partial v}{\partial u} - \frac{\sigma^2}{2} f_u^2 \frac{\partial^2 v}{\partial f_u^2}\right) du \tag{6}$$

To avoid arbitrage, high deterministic gain should be the same as risk-neutral asset profit with a dynamic situation which is equal to $-v$. These two deterministic gains are equal to $V = (f, t)$ at any time;

$$\frac{\partial v}{\partial t} + \frac{\sigma^2}{2} f^2 \frac{\partial^2 v}{\partial f^2} - rv = 0 \tag{7}$$

When equation [7] is compared with a corresponding equation for value of American option on asset that pay dividend q, we reach the same [7] equation by putting $q = r$ g in the following equation;

$$\frac{\partial c}{\partial t} + \frac{\sigma^2}{2} s^2 \frac{\partial^2 c}{\partial s^2} + (r - q)s \frac{\partial c}{\partial s} - \mathrm{rc} = 0 \tag{8}$$

It should be noted that the growth rate expected from payable profit of the asset under risk-neutral conditions is equal to … and then … represents the rate of zero motion of future price. Thus, it can be said that f_t is Martingale and the following equation is used;

$$f_t = e^{r(T-r)} s_t \tag{9}$$

We know that risk-neutral motion is r.

According to the above equations, we can obtain American put and call option price of future contract by inserting $q = r$ in corresponding price equation of call and put option on asset which yields no profit. Thus, American call and put option price of future contract can be obtained as follows;

$$c(f, \tau; X) = e^{-r\tau}[fN(\widetilde{d_1}) - XN(\widetilde{d_2})] \tag{10}$$

$$p(f, \tau; X) = e^{-r\tau}[XN(-\widetilde{d_2}) - fN(-\widetilde{d_1})] \tag{11}$$

$$\widetilde{d_1} = \frac{\ln\frac{f}{X} + \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}} \tag{12}$$

$$\widetilde{d_2} = \widetilde{d_1} - \sigma\sqrt{\tau} \tag{13}$$

$$\tau = T - t \tag{14}$$

Where f, x are future price and price of option exercise respectively and τ is the time until maturity.

$$(f, \tau; x) + fe^{-r\tau} = C(f, \tau; x) + xe^{-r\tau} \tag{15}$$

As future price of every asset approximates to its cash price in the maturity of future contract, American future contract option should be more valuable than option of underlying asset. If maturity date of option and future contract is the same, it should be noted that τ is the maturity date of future option. When option of future contract and that of the underlying asset have the same maturity, future price is equal to $f = se^{r\tau}$. If this equation is put in equation [15], B-S formula is then obtained.

3   Proposed Methodology

The resulting prices are compared with the historical market prices of each option to assess whether the market is mispricing these options. The accomplish this, a paired t-test is used to determine whether the difference between the market price and the model price is significantly different than zero. We use a paired t-test instead of a regular t-test with equal or unequal variances to avoid running into the econometric problem of overlapping data. The difference between the market and model price can be viewed as independent variables.

The hypotheses for this test are as follows:

Testing hypothesis 1

$H_0 : \mu_d = C_{MKT} - C_{MOD} = 0$

$H_1 : \mu_d = C_{MKT} - C_{MOD} \neq 0$

where

C_{MOT}: price of option calculated by the model

C_{MKT}: market price of option

The t statistic can be found using the following equation:

$$t = \frac{\bar{d}}{S_d / \sqrt{n}}$$

(16)

Where $\bar{d}$ is the difference between model price and market price, and S_d is standard deviation of this difference.

Rejecting of null hypothesis will lead to the conclusion that the market value is significantly different than the value obtained from the model. T-tests are performed for all the options in the dataset. To check for trends in mispricing, the data are divided into in-the-money and out-of-the-money options. In order to determine the size of mispricing and effect of expiration time and moneyless on the degree of mispricing, root mean square error is used;

$$RMSE = \sqrt{\frac{\sum(C_{MKT} - C_{MOD})^2}{N}}$$

(17)

The above equation determines whether option is overvalued or undervalued, but it do not answer the question of whether the number of contracts that are underpriced. To answer this question, the proportion of contracts that are overpriced is determined and the following hypothesis 2 test is used.

Testing hypothesis 2

$H_0 : \bar{p} = 0.5$

$H_a : \bar{p} \neq 0.5$

$$t = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

(18)

p is equal to 0.5 and $\bar{p}$ is proportion of options that are overvalued.

The use of implied volatility is not appropriate for evaluation of pricing model, because the purpose of this paper is to test model performance. Given that dataset are six-month period, so the results are not affected by short-time periods with unusual volatility.

The following equation is used for obtaining historical volatility:

$$HSD = \sqrt{\frac{(\sum_{j=1}^{m} R_J) - \bar{R}}{n-1}} \quad \begin{cases} R_J = Ln(\frac{F_J}{F_{J-1}}) \\ \bar{R} = \sum_{j=1}^{n-2} \frac{R_J}{n} \\ \text{Annualized } \sigma = HSD * \sqrt{250} \end{cases}$$

(19)

F: future contract price

n: the number of observations for consecutive future price

Having obtained the price difference, we can create risk-neutral portfolio and we need to test if we can yield unusual profit by creating the risk-neutral portfolio Δ for one-two-day holding periods. The portfolio consisting of call and put of a call option and entering into opposite situation namely put Δ of oil future contract is taken into consideration.

Hull [7] demonstrated that call delta is equal to $\Delta = N(d_1)e^{-rT}$ for option of future contract and put option is equal to $\Delta = (N(d_1)e^{-rT} - 1)$. Thus, for call options which are overvalued, a strategy

consisting of call of an option on oil purchase and for entering future contract of oil is appropriate. If option of call option is undervalued, the strategy is to purchase option and Δ of oil future contract is sold. For put options which are undervalued, the optimal strategy is to purchase put option and enter into Δ of call future contract, and if option of put options are overvalued, the optimal strategy is to sell option and sell Δ of put future contract.

A non-parametric sign test is then used to determine if the number of hedge portfolios with positive returns is significantly more than that of portfolio with negative return.

Testing hypothesis 3

In this test, the number of portfolio with positive return, critical value c and significance level α;

H_0: P=0.5

H_1: P >0.5

P(X≥C ǀ P=0.5)= α

To conduct this test, the critical value is computed as $c = \mu + Z_\alpha \sigma$, where n is the number of portfolio, p is equal to 0.5, $\mu = np$, and standard deviation is equal to $\sigma = \sqrt{np(1-p)}$.

If $x \geq c$, then the null hypothesis is rejected, and it is concluded that the number of portfolios with positive returns is significantly more than that of portfolio with negative return.

4 Implementation of the Methodology

Historical data, price of future contracts, and option of Brent oil are taken from Theice website. The maturity months of option for 6 months of the end of 2015 were available, but their corresponding future contracts were exercisable for three maturity dates namely 19 February, 18 March, and 15 April in 2016. Furthermore, it was just options with the exercise prices 67 and 92 $ that could be examined.

The research period started from July 2015 and lasted until the end of 2015. In this research, it was just call option that was used for pricing; pricing manner and presentation of put option follow the same procedure.

From 46985 call options, 81.4% are in the money and 18.6% are out of the money. Thus, trader can reap good profit easily by option on oil future contract and designing a good strategy which is elaborated in what follows, because the options are traded lower than their intrinsic value. The number of at-the-money options is here zero, which can be due to the fact that options are kind of future contract option and settlement prices are normally displayed in decimal form, while exercise price are shown in natural numbers.

Testing market pricing

For each of three maturities with both exercise prices, hypothesis 1 is applied. Hypothesis t test was conducted to compare values of T stat with T critical two-tale, so that we can judge the rejection or the acceptance of null hypothesis of this test. In Table 1, the results of t-pair test are presented.

Table 1: Output of t-pair test

Exercise price- maturity	$\bar{d}$	t statistic	Two-tail critical t statistic
Feb-67	0.812	10.27	1.98
Feb-92	0.189	5.93	1.98
Mar-67	1.45	15.43	1.98
Mar-92	0.389	8.14	1.98
Apr-67	1.062	13.12	1.98
Apr-92	0.25	7.071	1.98
Total	0.692	9.993	1.98

Given Table 1, it is observed that since the value of t statistic is more than the critical value 1 in all maturities, the null hypothesis is rejected; this means that price difference made by model and the price traded in market are significantly different from zero and it is positive, which is a good opportunity for arbitrage practice.

Root mean square error (RMSE)

This test shows an additional reason for difference between market price and model price. In Table 2, the results of RMSE are presented. The mean of RMSE is 1.07 for all maturities of in-the-money call options, and 0.015 for option undergoing loss.

Table 2: Output of RMSE test

Feb 67*	Feb 67**	Feb 92*	Feb 92**	Mar 67*	Mar 67**	Mar 92*	Mar 92**	Apr 67*	Apr 67**	Apr 92*	Apr 92**
1.31	0.011	0.53	0.0132	1.84	0.004	0.73	0.009	1.45	0.045	0.58	0.011

* options which are undervalued
** options which are overvalued

Nonparametric test

This test, which is hypothesis 3, examines whether the number of hedged portfolio with positive return is more than the number of hedged portfolio with negative return. According to hypothesis test, if $X \geq C$ (X is the number of portfolio with positive return, C critical value), then it can be said that the number of portfolio with positive return is significantly more than the number of portfolio with negative return with 95% significance level.

Table 3: Strategy of hedged portfolio

Trading situation of future contract	Trading situation of future contract option	Nature of trading price
Δ_c put contract	Buying a contract	Undervalue call option
Δ_p put contract	Selling a contract	Overvalue call option
Δ_c call contract	Buying a contract	Undervalue put option
Δ_p call contract	Selling a contract	Overvalue pit option

X is the number of portfolio with positive return of the sum of positive values ds (difference between model price and market price) and is equal to 640 for all maturities and C is equal to 76.8, so the above unequal equation is true. In Table 3, the results of nonparametric test are presented, in which strategy of hedged portfolio is designed accordingly.

In this research, strategy of call option was just investigated. In this respect, corresponding deltas of options which are in the money and out of the money were computed for all maturities, so that a trader can practically use this research for designing a profitable strategy.

In figures 1 and 2, changes of difference between model price and market price (d) and Δ was shown as maturity time ratio (T) for both in-the-money and out-of-the-money options.
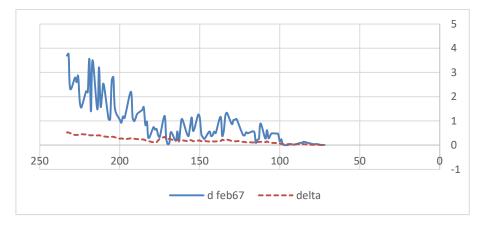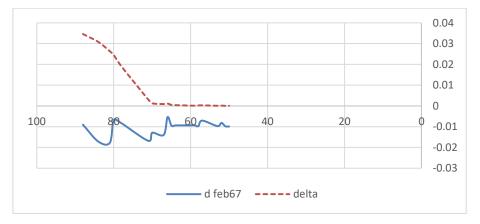

Fig. 1: In-the-money option

Fig. 2: Out-of-the-money option

As seen in the figures, as time difference between contract and maturity approaches zero, difference between price model and market model declines, so does Δ gradually.

The main difference that exists between figures of in-the-money option and loss is that d volatilities are much greater over time until maturity date in in-the-money options, which represents another sign

of taking strategy for the option. Moreover, Δ changes over time are roughly monotonous in regard to in-the-money options, as opposed to out-of-the-money options which fall down to zero.

5   Conclusion

This research offers a model of option pricing of oil future contract, and shows how this model can be developed into model of pricing of oil future contract.

Some tests were conducted to determine whether current market prices of oil future contract option are different from prices presented by the model. The results show that most of the options are under-valued in market less than their intrinsic value. It is expected that undervalue options are kind of in-the-money options, which proved to be true. The point is that overvalue options are expected to be types of call options more. In addition to this, the number of contracts of overvalue options is greater, which is a good sign of taking a profitable strategy for traders.

In the paper conducted by recent results concerning pricing of future contracts of US index, mis-pricing was not almost seen. The publication of this paper can offer a correction in the market and eliminate unnatural profits which accrued to certain parts of market traders. These results can be reasons for lack of efficiency on oil option market. Model tests suggests that positive arbitrage profits are yielded by exercising transactional strategy of delta hedged against risk in a day. Existence of such profits, which represents inefficiency of oil option market, can pave the way for future studies.

For market activists, this paper provides traders with arbitrage instruments and risk hedgers with fair prices in order to learn if they entered into a contract with fair price or not.

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