Adv. Math. Fin. App., 2024, **9**(3), P.900-915

Advances in Mathematical Finance & Applications

www.amfa.iau-arak.ac.ir
Print ISSN: 2538-5569
Online ISSN: 2645-4610
Doi:10.71716/amfa.2024.23061895

Original Research

The Co-movement Between Bitcoin, Gold, USD and Oil: DCC-GARCH and Smooth Transition Regression (STR) Model

Yazdan Gudarzi Farahani [a] * , Ehsan Asghari Ghara [b], Mansour Haghtalab [c]

[a] *Assistant Professor ,Department of Economic and Administrative Sciences, Qom university ,Qom, Iran*
[b] *Department of Economic, Arizona University, Arizona, United States*
[c] *Department of Economics, Tehran University, Tehran, Iran*

ARTICLE INFO
Article history:
Received 2023-06-24
Accepted 2024-05-06

Keywords:
BTC
Gold Price
USD
Oil Price
DCC-GARCH
STR Model

ABSTRACT

This study investigates the relationships between Bitcoin (BTC) prices and fluctuations in relation to gold, USD, and Iran's oil prices from 2019 to 2022. We employed the dynamic conditional correlation generalized autoregressive conditional heteroscedasticity (DCC-GARCH) method to model the fluctuations of financial variables. Additionally, the smooth transition regression (STR) method was applied to explore the relationships between the variables. The results reveal significant positive correlations between BTC prices and gold, as well as oil, and a negative correlation with USD prices. We observed volatility persistence, causality, and phase differences between BTC and other financial instruments and indicators. Notably, a negative relationship was identified between Bitcoin and the USD in both linear and non-linear aspects, with a larger coefficient in the second regime. Furthermore, a positive relationship was found between Bitcoin and the variables of gold and oil prices, with coefficients being larger in the second regime compared to the first.

1 Introduction

The Spillover and connectedness between various financial assets are essential for risk management and forecasting aspects of financial markets. In the past few decades, the literature has extensively concentrated on developing methods to measure the aggregate connectedness between financial assets. However, analyzing only aggregate connectedness between assets is not adequate since different shocks to one asset may have different effects at different frequencies. On the one hand, some of the shocks may only affect the short-term; on the other hand, others may affect investor expectations and have more permanent, long-term effects. Furthermore, the effects of shocks may also be different on returns and volatilities. Therefore, investors must examine the effects of shocks on return/volatility structures and at different frequencies since they may affect investors' diversification decisions. During the last years, the value of many financial assets decreased rapidly. This general risky economic environment and market decline were accompanied by some assets considered safe havens. Calling an investment a safe haven depends on whether it is uncorrelated to stocks and against stocks, maintains its price level, or exhibits upward movements [8]. The price movements of Bitcoin (BTC) since its launch in 2009

* Corresponding author. Tel.: *09127535246*
E-mail address: *y.gudarzi@qom.ac.ir*

have raised the issue of whether BTC exhibits safe-haven properties as an alternative to stocks [41] [38]. BTC is seen as a safe haven for several reasons, including the independence from monetary policies in a country, its role in accumulating value, and its limited relationship with traditional assets.The term cryptocurrency is used to refer to digital currencies or assets based on blockchain technology. With the rapid development of blockchain technology, cryptocurrencies have received massive publicity in the financial markets due to some views that they can be considered a new category of investment assets. Today, the cryptocurrency market has become one of the fastest growing markets in the world regarding the trading volume and market cap [15]. Compared to traditional asset markets, cryptocurrency is an emerging market with a large market cap [33]. However, despite the decrease in trade volume, the enthusiasm for digital currencies continues at full speed. Crypto money offers a kind of freedom to people. In a research report published on 26/03/2023, Morgan Stanley said that expectations for increased USD liquidity to support the banking sector after some mandatory shutdowns helped the BTC (BTC) rally, but other factors were also active. Cryptocurrencies and BTC are classified as speculative investments [40]. The BTC market is the most volatile [16]. The explosive movements experienced in BTC recently show that it is not yet seen as a stable investment tool. BTC acts more like an investment tool open to speculative activities rather than a currency. Dwyer [17] emphasized that the return volatility observed in BTC is higher than in other investment instruments, revealing this fact. Likewise, Baur and Dimpfl (2017) emphasize that BTC's highly volatile nature distracts it from the fact that it can be seen as any currency. Rapid technology development has contributed to the recent dramatic growth in the cryptocurrency market, enabling users to more easily access digital currencies and transfer money globally at a much lower cost and time than traditional money transfers methods. However, it has also led to high speculation among network users. Although rapid technology updates have brought positive effects in many ways, this fast update has caused more speculators to join the market. As a result, the cryptocurrency market has become more volatile than the stock market or other commodity markets [23]. Chaim and Laurini [12] highlight that cryptocurrency volatility is higher than in traditional assets, indicating higher returns and risks. Therefore, an emerging and high-profile market with high recognition and income is desirable to owners, investors and risk managers. Yu et al [43] discovered that the market efficiency of the BTC market is higher than that of the overall financial market due to the asymmetry of volatility.While many assets subject to trading in financial markets were negatively affected by the hostile atmosphere and uncertainty created by the pandemic, there were also shining financial instruments in this period. At the beginning of these are cryptocurrencies and especially BTC.

In our study, the methodologies and relationships used in previous studies [8, 39, 18, 26, 21] are included. In these studies, the hedging features of BTC are discussed in general. Dyhrberg [18] investigated BTC's position in stock and currency price fluctuations with the GARCH model and found that BTC's gold has some hedging behaviors. According to Bouri et al. [8], while BTC exhibited distinct hedging properties for investment-grade energy commodity portfolios in the pre-crisis period, post-crisis BTC only functioned as diversifiers. Bouri et al. [8] propose the critical roles of BTC in diversifying and hedging the risk of equity markets. Kang et al. [26] examined the hedging and diversification properties of gold futures against BTC prices using dynamic conditional correlations (DCCs) and wavelet coherence. They find evidence of volatility persistence, causality and phase differences between BTC and gold futures prices. The wavelet consistency results show high co-movement between BTC and gold futures prices. The innovation of the present paper is that the fluctuations in the financial markets were modeled using the GARCH model, then the co-movement of these variables was evaluated using non-linear models, which was not used in previous studies of this approach. Also, our study examines the co-movements of BTC with gold, oil, and USD using the dynamic conditional correlation generalized autoregressive conditional heteroscedasticity (DCC-GARCH) method. This model, an extension of the CCC-GARCH model, permits the correlation matrix to vary over time. The DCC-GARCH model offers computational advantages by ensuring that the number of parameters for estimating correlation is independent of the number of series. Subsequently, we delve into the time-frequency structure of the correlation and co-movements between BTC prices and other financial assets and indicators. This analysis aims to contribute to the limited literature on the subject, providing valuable insights. Notably, our study stands out as the first to leverage the smooth transition regression method, shedding light on asymmetric relations between BTC and gold, foreign exchange, and oil markets.The study discusses BTC in the context of hedging features and volatility spillover. The gold, oil and USD is included in the scope of the study because there is no study in this context against BTC prices in the literature. Generally, BTC is considered together with one or two financial instruments and analyzes are carried out. At the same time, the BTC market is subject to intense speculation and expectations. This situation is observed intensively in Iran as well. Especially during the last years, speculative movements have intensified, and the combined movements of USD gold oil prices and BTC have become very interesting. For this reason, the motivation behind the period of the selected data sample is to examine the movements at a local scale, to make inferences about possible similar crises in the future, and to enable functional evaluations to be made in terms of understanding the consequences of similar problems in the past. The main goal of this study is to examine BTC's movements and volatility spillovers with traditional investment instruments and international financial indicators in Iran. The reason for taking the variables in Iran is to explore the movements in the last years locally and to obtain helpful results for local decision-makers and policymakers. In addition, it ensures that inferences are made to represent developing countries with similar economic, financial and social structures internationally. To this purpose, employing non-linear STR model, the current study evaluates the co-movement between bitcoin, gold, USD and oil in Iran.The remainder of this paper is organized as follows. Section 2 describes the theoretical foundations, followed by a review of relevant studies in Section 3. Section 4 estimates and analyzes the research model, and finally Section 5 concludes the paper and gives some concluding remarks.

2 Literature Review

BTC was introduced to the financial markets for the first time by Nakamoto, and although it has been about fifteen years since then, how it should be defined has not yet been fully answered. Whether BTC is a currency, a commodity, or an investment asset is still debatable—demand shock, significant price movements, etc., of cryptocurrencies, especially BTC. Exhibiting most of the commodity properties supports the idea that they are commodities [8, 11]. The fact that BTC is a mining reward and its supply is limited causes BTC to be considered a digital version of commodities used for savings. When cryptocurrencies are accepted as commodities, examining price volatility and co-movement with other entities is essential. The connection between cryptocurrencies, commodities, and other traditional assets is interesting.BTC, the first of the cryptocurrencies and the largest in total market capitalization (approx. $330.76B) and trading volumes, continues to be at the centre of discussions regarding its potential role in the global financial system. In a more analytical approach, BTC's underlying technology, the blockchain, holds great promise for financial institutions. However, some other studies question the future of BTC and its prospects for mediation. In some countries, financial regulators are trying to regulate or even ban the use of BTC in their country's economies, making the financial inclusion of BTC even more challenging [39]. The relationship between BTC and traditional assets (e.g., stocks, bonds) and commodities (e.g., gold, crude oil) has been gaining traction in academia for some time due to its significant implications for investors, academics, and policymakers [24,38]. Despite extreme price fluctuations [8, 22], market manipulations [12], and stock market security flaws, interest in BTC investment continues to grow.Bouri et al. [8] find no consistent evidence that BTC acts as a haven for global assets, while Selmi et al. It has been determined that it acts as a protection, safe haven, and diversifier. However, this feature seems sensitive to the different market conditions of BTC and Gold and whether the oil price is in a down, regular, or upside regime. In addition, Baek and Elbeck [4] argues that BTC is merely a speculative commodity rather than a currency.The view that gold can be considered a safe-haven asset is widely accepted, especially in the depressed market environment [32]. The traditional safe haven feature of gold emerges in short intervals, especially in crisis periods [9]. For example, Bulut & Rizvanoghlu [11] emphasize that while gold is generally considered a hedging tool, it is a strong safe haven in only 9 countries in their sample. BTC, the most popular and valuable among existing cryptocurrencies, has limited stock and short-term elasticity of supply [17]. BTC is also called synthetic commodity money due to its scarcity and lack of fiat money [35]. BTC and gold have many similar features, such as being apolitical, safe-haven, and inflation-free [38]. For this reason, BTC is also called digital Gold [5-6]. BTC also has advantages differentiating it from gold, such as being independent of a country's politics and economy and relying on suitable algorithms and sophisticated protocols. Therefore, it is stated that BTC will not be affected by the co-movement and financialization of commodities such as gold. Such features make it meaningful to compare the safe haven features between BTC and Gold. Gold and BTC are similar regarding being a value protection tool and not being controlled by states. The fact that BTC can be used as a general payment method, such as cash or gold, due to its convertibility advantage makes it attractive to investigate hedge properties [18,8, 36, 21, 3]. However, Wu [42] investigated the relationship between BTC and traditional financial instruments regarding the asset quality and hedge effect of BTC and found that BTC has a unique risk-return feature and volatility clustering performance, and its high volatility persistence is similar to gold. At the same time, it was argued that while BTC exhibits a significant one-way spillover effect with other variables, BTC is much more affected by different market shocks than other markets are affected by BTC shocks. Therefore BTC cannot be a safe haven. Since crude oil occupies a dominant position in the global energy market [44], the interaction of oil and BTC markets is another essential issue for policymakers and investors. This is because, according to the risk premium channel [10], a crude oil shock can significantly affect investors' willingness to take the risk of BTC. Therefore, it is crucial to uncover the link between crude oil and BTC to more effectively assess the potential risks of cryptocurrencies and thus increase earnings [28]. Selmi et al. [36] claim that BTC plays a diversified role in hedging from oil price changes and is seen as a private safe haven. However, this relationship is variable in different market conditions. Kurka (2019) pointed out that the unconditional link between cryptocurrency and crude oil can be ignored. However, recent studies have empirically confirmed the severe impact of financial shocks from extreme events (e.g., terrorist attacks, political events, and economic crises) on crude oil and BTC prices [29,45 ,28]. In particular, studies examine the relationship between BTC and strategic commodities such as gold and crude oil and suggest that BTC is a hybrid commodity and will be affected by crude oil prices are noteworthy [8,24]. Kwon [27] examines whether BTC can be classified as a currency, commodity, or investment asset. The author found a similarity between BTC and USD. In addition, he discovered that the tail of the stock market return is associated with the risk premium in BTC's return. The bottom line shows that BTC is traded as an alternative to a medium of exchange and investment rather than a commodity. On the other hand, supply-demand factors dominate the price behavior in the BTC market [13]. Thus, unlike standard currencies in circulation, BTC's liquidity and volatility are not influenced by a centralized system of financial institutions (e.g. central banks) or other major macroeconomic factors [13,6]. Therefore, the price of BTC could potentially be separate from the economic and trade cycles that result from monetary policy and the central bank's money supply management [26]. This latter feature suggests that BTC can serve as a dynamic diversification and hedging tool, thus managing volatility risks in the markets [26,19]. On the other hand, Baur et al. (2018) suggest that BTC's extreme returns and volatility are more like a highly speculative asset than gold or the USD.Studies investigating the relationships of cryptocurrencies with other investment alternatives state that they can provide hedging in crude oil [36] and Gold [20] prices. Another topic frequently emphasized in the literature is the co-movement of cryptocurrencies [33,1, 16]. However, it is noteworthy that the relationship between cryptocurrencies and energy commodities is also included in the literature [34]. Mensi et al. [31] focused on BTC's relationship with Islamic financial assets and stock markets and its co-movement and risk spillover. In another study by Mensi et al. [30], the effects of structural breaks (SB) on BTC and Ethereum price returns on long binary memory levels were investigated. Since its emergence in 2009, BTC has been intensively studied in the academic field, especially after its rise in 2015. Dyhrberg [18] investigated the economic asset properties of BTC with GARCH models. The author has determined that BTC exhibits hedging properties and is similar to Gold and the USD because of its advantages. The author has also shown that BTC can be helpful in risk management and is ideal for risk-averse investors regarding negative expectations about the market's future. The author also emphasized that it can be classified between Gold and the USD. Baur, Dimpfl & Kuck [6] stated that BTC displays distinctly different returns, volatility, and correlation characteristics than other assets, including Gold and the USD. Oad Rajput et al. [32] found that BTC price has an asymmetrical and negative relationship with USD in the short and long run.In addition to various financial and economic risks, studies have been conducted on how political risks affect the role of BTC, as revealed by Bouoiyour et al. [7]. The authors explored the role of different assets (especially oil, precious metals, and BTC) as a safe haven against US equities at times of heightened uncertainty about the outcome of the 2016 US presidential election. Its results show that oil is an effective safe haven against political risks. Similarly, gold and silver are a safe haven against US stock losses in the medium and long term and BTC. Li et al. [28] examined excessive risk transmission between BTC and the crude oil market under extreme and non-extreme shocks. They found strong evidence of excessive risk transfer between BTC and crude oil and explored the time-varying nature of the BTC-oil relationship. They found time-varying interactions in the oil-BTC relationship. Their study also shows stronger causal links during large movements in oil returns. Al-Nassar et al. [3] explore the potential hedging and safe-haven properties of various alternative investment assets, including Gold, BTC, oil, and the oil price volatility index (OVX), against the risks of the Saudi stock market and its constituent sectors at different stages of the COVID-19 pandemic. Their findings show that all researched alternative investment assets have a time-varying hedging role in the Saudi stock market, which has become expensive in the early stages of the COVID-19 pandemic. DCCs between Saudi indices and oil and, to a lesser extent, BTC peaked during the COVID-19 crisis, highlighting oil's role in transmitting financial contagion to the Saudi stock market.

3 Research Methodology

3.1 Smooth Transition Regression Model (STR)

This study assesses the relationship between Bitcoin, gold, USD, and oil in Iran using the Smooth Transition Regression (STR) method. Following the approach outlined by Becha et al. (2023), we define a STR model characterized by two limiting regimes and a transition function to capture the dynamics of these financial variables:

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_t F(q_t; \gamma, c) + u_t, \quad t = 1,2,\dots,T \tag{1}$$

Where Y_t denotes the dependent variable, representing the price of Bitcoin. X_t is a vector of exogenous variables, including USD, gold, and oil prices? The u_t represents the error term. The transition function, $F(q_t;\, \gamma, c)$, is unbounded and bounded, logistically specified according to the approach outlined by Becha et al. [20]:

$$F(q_t; \gamma, c) = \left[1 + exp\left(-\gamma \prod_{j=1}^{m}(q_t - c_j)\right)\right]^{-1},\ \gamma > 0, c_1 \leq c_2 \leq \cdots \leq c_m \tag{2}$$

Where c_j is an m-dimensional vector representing the value of the threshold limits and γ is the slope parameter, which represents the pace of transition from one regime to another and has a clear constraint. q_t represents the transition variable, which, according to the research of Colletaz & Hurlin [14], can be selected as one of the explanatory variables, the dependent variable lag, or any other variable outside the model that is theoretically related to the studied model and establishes a non-linear relationship [37].Becha et al. [20] argue that in practice, it is sufficient to consider one or two threshold values, m=1 or m=2, to account for the parameter variability. For m = 1, the STR model predicts two limiting regimes associated with (q_t) transition variable values less than and greater than the threshold (C_1) and with a uniform transition function of coefficients β_0 to $\beta_0 + \beta_1$. If the slope parameter γ approaches infinity, the STR model is replaced with the two-regime threshold regression (TR) model proposed by Hansen (1999). That is, if $q_t > C_1$, the transition function is the numeric value to be one, and else it is zero. For m = 2, the transition function at the point $(C_1 + c_2)$ / 2 is minimized and takes on the value 1 for lower and higher values of the transition variable (q_{it}). In this situation, when the slope parameter γ tends to 0, the STR model is simplified to a linear or homogeneous regression model with fixed effects, regardless of the value of m. Accordingly, in the PSTR model, the predicted coefficients for the transition variable and slope parameter vary constantly between the two limit states F = 0 and F = 1, which are defined as follows:

$$Y_t = \begin{cases} \mu_i + \beta_0' x_t + u_t & F = 0 \\ \mu_i + (\beta_0' + \beta_1')x_t + u_t & F = 1 \end{cases} \tag{3}$$

As mentioned earlier, the estimation of coefficients of different explanatory variables for different and variable sections throughout time is another notable aspect of the STR model, which totally resolves the problem of conventional heterogeneity in the consolidated data. The STR model will be estimated by removing the fixed effects by subtracting the individual averages and then applying the nonlinear least squares (NLS) approach, which corresponds to the maximum likelihood (ML) estimator. For the estimating stages in accordance with Becha et al. [20], the following are the estimating steps of a STR model: first, the linearity test used by Wald Lagrangian coefficient statistics (LM_W), Fisher's Lagrangian coefficient (LM_F), and likelihood ratio (LR) are applied following Colletaz & Hurlin [14], and if the null hypothesis that the relationship between the variables is linear is rejected, the number of transition functions must be assigned to fully describe the nonlinear behavior between the variables. The null hypothesis of the presence of a transition function is compared to the hypothesis of the existence of at least two transition functions for this purpose.

3.2 Model Specification and Data Analysis

In this study, the movement of daily BTC (BTC) price, spot Gold price, USD in and Brent Crude Oil Future in terms of USD (Real exchange rate) in the period 12/31/2019-12/31/2022 are examined. In the analysis, the logarithmic values of the variables were used.Descriptive statistics regarding the variables used in the study are given in Table 1. Table 1 presents the summary statistics of the BTC and the other related variables. The numbers in the table are statistics calculated over logarithmic values. In the whole period from December 31, 2019, to December 31, 2022, the mean daily logarithmic price of BTC is 12.29. Skewness, Kurtosis, Jarque-Bera, and Probability values indicate that the data are typically not normally distributed. The number of observations included in the analysis is 662. Other statistics that can be used in the context of the structure of the data set in the table are mean, median, maximum, minimum, and standard deviation statistics. For example, standard deviation *(sd)* can evaluate every variable's volatility. BTC and oil series are skewed to the left because the skewness value is negative, while other variables are skewed to the right. Given the daily frequency of observations in our dataset, we can leverage the Central Limit Theorem, which allows us to infer that our estimators follow a normal distribution due to the sufficiently large sample size.

Table 1: Descriptive Statistics

	LNBTC	LNGOLD	LNUSD	LNOIL
Mean	12.28887	6.249286	2.191029	4.131062
Median	12.68362	6.176066	2.102694	4.193964
Maximum	13.57571	6.940316	2.858015	4.860742
Minimum	10.37841	5.670453	1.767245	2.978077
Std. Dev.	0.945311	0.341647	0.310753	0.389820
Skewness	-0.429817	0.547431	0.801836	-0.338378
Kurtosis	1.599817	2.323615	2.323144	2.565442
Jarque-Bera	74.46074	45.68405	83.57472	17.84196
Probability	0.000000	0.000000	0.000000	0.000134
Sum	8135.229	4137.028	1450.461	2734.763
Sum Sq. Dev.	590.6787	77.15375	63.83124	100.4454

Source: Research finding

The graphs in Figure 1 were also created in the study. In Figure 1, BTC, GOLD, OIL and USD prices show a severe upward trend with the pandemic. Although BTC prices exhibit similar movements with other variables in the chart, it is seen that the movements differ in some periods.

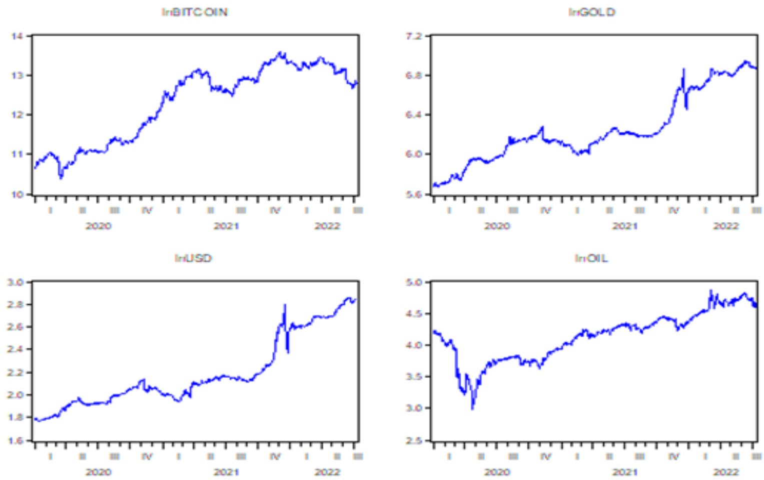

Fig. 1: Time Series of Variables
Source: Research Finding

4 Model Estimation

According to the introduction of the variables and the model used in this paper, the modeling steps are described in figure (2).

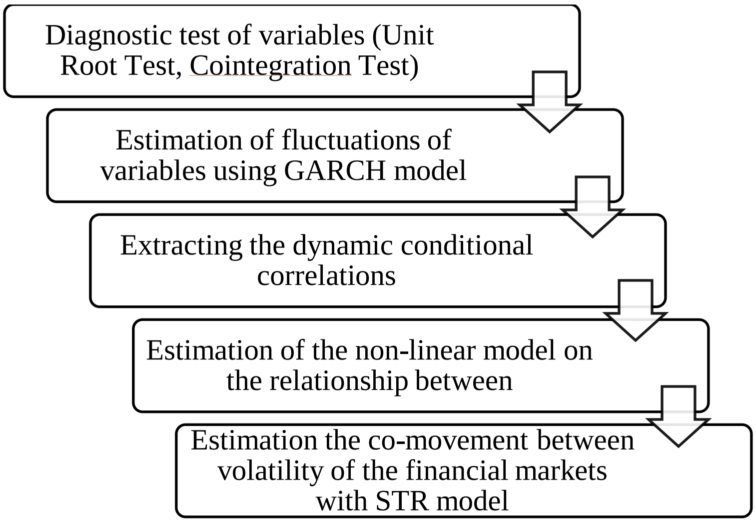

Fig. 2: The structure of model steps
Source: Research Finding

4.1 Static Variable Testing

According to the econometric literature, before any estimation and in order to prevent the emergence of false regressions, the variables must be static. If the model's variables are static, estimates will not be subject to the issue of false regression. Using the HEGY test, the variables were examined in terms of stationarity. The null hypothesis in these tests is that there is a unit root. The summary of test results is shown in Table 2. According to the results, all variables are within 5 percentage points of the y-intercept. The graphs depicted in Figure 1 illustrate raw variables at the level. However, the variables utilized in the statistical models are transformed using logarithms. This transformation leads to the conclusion that all variables are stationary at the level.

Table 2: Results of the unit root test (Level of Variables)

Variables	**HEGY**	
LNBTC	Tstatistic	-1.213
	P-value	0.523
LNGOLD	Tstatistic	-1.532
	P-value	0.896
LNUSD	Tstatistic	-0.943
	P-value	0.715
LNOIL	Tstatistic	-1.357
	P-value	0.687

Source: Research findings

The results are seen in Table 2. In general, that all series has a unit roots in their level values. Next, the HEGY test is performed first at the level and then with the seasonal difference of the first order.

Table 3: Results of the unit root test (First Diffrence of Variables)

Variables	**HEGY**	
Δ(LNBTC)	Tstatistic	-4.694
	P-value	0.000
Δ(LNGOLD)	Tstatistic	-8.165
	P-value	0.000
Δ(LNUSD)	Tstatistic	-8.559
	P-value	0.000
Δ(LNOIL)	Tstatistic	-5.785
	P-value	0.000

Source: Research findings

The analysis of unit root tests confirms that the estimated model contains first-difference stationary, that is, I (1) variables. As such, the proper cointegration test in this case is the bounds tests.The null hypothesis of the bound test states that the dependent variable does not have a cointegrating relationship with the independent variables. The cointegration test performed in this case has been extracted to Table 4. Looking at the result, we find that dependent variable is significantly cointegrated with the independent variables in the model.

Table 4: Results of the Cointegration Test

t_{φ}	t_{λ_1}	t_{λ_2}	LM(4)	Prob
-4.25	-2.74	-1.89	9/36	0.000

Source: Research findings

The obtained results have confirmed the existence of cointegration and long-term relationship between the variables, so there is no need to differentiate the variables to solve the unit root of the variables.

4.2 Model Estimation

DCC-GARCH Results

The DCC-GARCH analysis is used in this study as a second method. With *DCC-GARCH* analysis, the conditional correlations between the variables were determined. As a result of the *DCC-GARCH* analysis, we can say that the positive correlation between *BTC* prices and other variables is characteristic for the entire period. Higher values of parameter α marked theta (1) in tables make our models more dynamic. Therefore, *DCC-GARCH* models can respond flexibly to changes in measured correlations.

Estimations of the DCC-GARCH models meet the requirement that the sum of dynamic parameters *theta (1) + theta (2) <1*. It means that it fulfilled the positive definiteness of matrix Q_t. In addition, the estimated parameters of both *DCC-GARCH* models are statistically significant because of the high values of the sum of the dynamic parameters achieved; high persistence in conditional volatility can be observed. All parameters for conditional variances and correlations were also statistically significant. The estimate of the v parameter shows that the *t* distribution is correctly adjusted to the data. The symbols Θ_1 and Θ_2, which explain the dynamic correlation relationship between *BTC* and *GOLD* prices in Table 3, are statistically significant at the 5% significance level. Therefore, a positive and influential relationship exists between prices.

Table 5: BTC and GOLD DCC-GARCH Dynamic Correlations

	Coefficient	Std. Error	z-Statistic	Prob.
Θ_1	0.045212	0.015426	2.930908	0.003380
Θ_2	0.932843	0.024963	37.36861	0.0000
	t-Distribution (Degree of Freedom)			
v	4.315128	0.330931	13.03938	0.000000
Log-likelihood	3.279443	Schwarz criterion		-13.04418
Avg. log-likelihood	-13.07844	Hannan-Quinn criteria.		-13.03439
Akaike info criterion	-13.06073			
* Stability condition: theta(1) + theta(2) < 1 is met.				

Source: Research finding

Based on these parameters, it is possible to build a model for BTC and Gold series as referred to below:

$$Q_{i,j,t} = \omega_{i,j} + 0.045212\varepsilon_{i,t-1}\varepsilon_{j,t-1} + 0.932843Q_{i,j,t-1},$$

Where i represents the first financial market (BTC) and J represents the second financial market (GOLD). The symbol Θ_1, which explains the dynamic correlation relationship between *BTC* and *OIL* in Table 6, is statistically significant at the 10% significance level. Therefore, a negative and weak relationship exists between prices.

Table 6: BTC and OIL DCC GARCH Dynamic Correlations

	Coefficient	Std. Error	z-Statistic	Prob.
Θ_1	-0.014650	0.007886	-1.857782	0.063200
Θ_2	0.386593	0.880788	0.438918	0.6607
	t-Distribution (Degree of Freedom)			
v	4.122509	0.292315	14.10295	0.000000
Log-likelihood	2.930274	Schwarz criterion		-11.64751
Avg. log-likelihood	-11.68176	Hannan-Quinn criteria.		-11.66634
Akaike info criterion	-11.69269			
* Stability condition: theta(1) + theta(2) < 1 is met.				

Source: Research finding

Based on these parameters, it is possible to build a model for *BTC* and *OIL* series as referred to below:

$$Q_{i,j,t} = \omega_{i,j} - 0.014650\varepsilon_{i,t-1}\varepsilon_{j,t-1}$$

Where i represents the first financial market (BTC) and J represents the second financial market (OIL). The symbols Θ1 and Θ2, which explain the dynamic correlation between BTC and USD in Table

5, are statistically significant at the 10% and 5% significance levels, respectively. Therefore, a positive and robust relationship exists between prices.

Table 7: BTC and USD DCC-GARCH Dynamic Correlations

	Coefficient	Std. Error	z-Statistic	Prob.
Θ_1	0.124915	0.073874	1.690912	0.090854
Θ_2	0.683981	0.212479	3.219045	0.0013
	t-Distribution (Degree of Freedom)			
v	3.310168	0.155514	21.28529	0.000000
Log-likelihood	3.469103	Schwarz criterion		-13.80282
Avg. log-likelihood	-13.83708	Hannan-Quinn criteria.		-13.81988
Akaike info criterion	-13.84622			
* Stability condition: theta(1) + theta(2) < 1 is met.				

Source: Research finding

Based on these parameters, it is possible to build a model for BTC and USD series as referred to below:

$$Q_{i,j,t} = \omega_{i,j} + 0.124915\varepsilon_{i,t-1}\varepsilon_{j,t-1} + 0.683981Q_{i,j,t-1},$$

Where i represents the first financial market (BTC) and J represents the second financial market (USD). Figure 3 shows estimated dynamic correlations. As of December 31, 2019, it is seen that the correlation coefficients created by the DCC-GARCH models have reached positive and negative values for the examined bilateral relations. When BTC-Gold movements are concerned, positive and negative trends are observed between July and October and October-December, respectively, in 2020. Between 2021 November-2022 and February-2022 April, positive and negative movements were observed. Especially in the November-February 2022 period, significant positive and negative correlation trends were observed between February 2022 and April 2022. On the oil side, the first thing to notice is the profound negative correlation in March 2020.

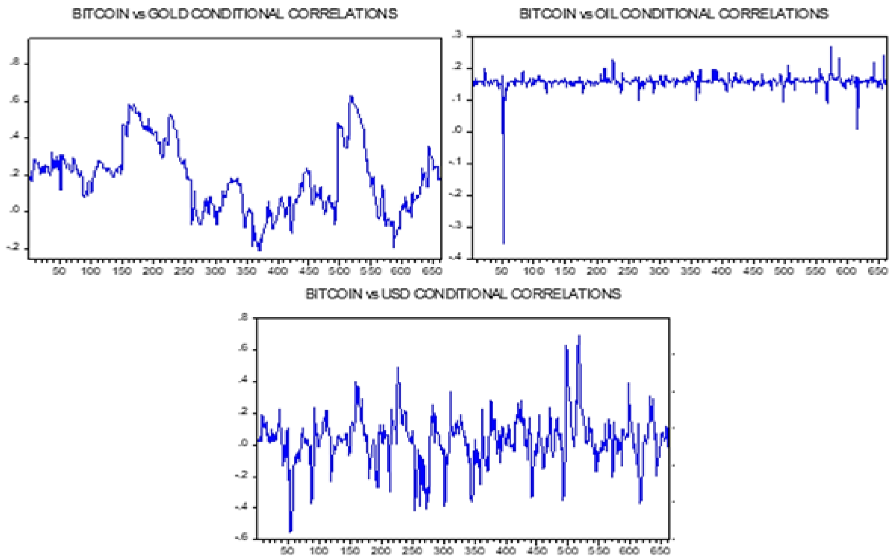

Fig. 3: Dynamic Conditional Correlations

Source: Research finding

4.3 STR Results

Following the discussions in the methodology section, the null hypothesis of linearity vs the hypothesis of the presence of the STR model are evaluated. The test program output is shown in Table 8. At a significance level of $\alpha = 0.05$, the statistics of Wald Lagrangian coefficient, Fisher Lagrangian coefficient, and likelihood ratio for one and two thresholds m = 1 and m = 2 confirm the existence of STR model.

Table 8: Nonlinear Relation Test

One threshold (m=1)			**Two thresholds (m=2)**		
LM_W	LM_F	LR	LM_W	LM_F	**LR**
53.354 (0.000)	17.764 (0.000)	55.769 (0.000)	85.572 (0.000)	15.017 (0.000)	**92.027 (0.000)**
H0: r = 0 vs H1: r = 1					

Note: r denotes the number of transition functions. The probabilities associated with each statistic are denoted by the values in parentheses.

Source: Research findings

Next, the number of transition functions must be determined by examining the existence of a residual nonlinear relationship. The results imply that the null hypothesis that considering a transition function is adequate has not been rejected at either the one or two criteria.

Table 9: Test for Residual Nonlinear Relationship

One threshold (m=1)			**Two thresholds (m=2)**		
LM_W	LM_F	LR	LM_W	LM_F	**LR**
1.400 (0.706)	0.423 (0.737)	1.402 (0.705)	11.938 (0.063)	1.824 (0.092)	**12.054 (0.061)**
H0: r =1 vs H1: r = 2					

Note: r denotes the number of transition functions. The probabilities associated with each statistic are denoted by the values in parentheses.

Source: Research findings

After verifying nonlinearity and identifying the number of transition functions required to accurately characterize the model, the optimal state of the threshold limit number must be estimated and the optimal model will be chosen by comparing Schwarz and Akaike criteria according to Jude's [25] algorithm. The results of Table 6 indicate that, according to the Schwarz and Akaik criteria, the PSTR model will be selected with a threshold if it is chosen based on the minimum value.

Table 10: Determining the Number of Threshold Points in a Transition Function

	Residual sum of squares	**Schwarz information criterion (SIC)**	**Akaike information criterion (AIC)**
m=1	0.6634	-6.969	**-7.01**
m=2	0.6601	-6.964	**-7.01**

Source: Research findings

After assigning the number of transition functions and the optimal threshold limit, a two-regime model is estimated, and the results are displayed in Table 8.

According to the findings of the model estimation, the slope parameter indicating the speed of adjustment between regimes is equal to 5.35. The location of regime switching and threshold crossing was determined to be 1.38, and the amount of antilogarithm is 4.28. Therefore, as long as the financial inclusion index is less than 4.28, the variables will behave according to the first regime, and if it is greater than 4.28, they will behave according to the second regime. The first limiting regime relates to a situation in which the slope parameter tends to infinity and the value of the transition variable is smaller than the threshold (where the regime switches). In this state, the transition function has a numerical value of zero and the model is linear. The obtained results indicated that there was a negative relationship between Bitcoin and the USD in the linear and non-linear part, and the second regime of this coefficient was larger.

Table 11: STR Model Estimation Results

Nonlinear part of the model		**Linear part of the model**	
LNGOLD	0.272 (4.813)	**LNGOLD**	**0.516 (4.626)**
LNOIL	0.085 (3.490)	**LNOIL**	**0.117 (3.200)**
LNUSD	-0.016 (-0.294)	**LNUSD**	**-0.287 (2.864)**
Regime switching location C= 1.38 Antilogarithm C= 4.28 **Slope parameter γ= 5.34**			

Source: Research findings

In addition, there has been a positive relationship between the variables of gold and oil prices with bitcoin, these coefficients were larger in the second regime than in the first regime.

5 Conclusion

This study examines the relationship between Bitcoin (BTC) prices and fluctuations with gold, USD, and oil. For this purpose, we used the Smooth Transition Regression (STR) model from 2019 to 2022. The estimation results revealed the existence of a nonlinear link between these variables, and the addition of a transition function with a threshold or regime-switching location is adequate to properly characterize nonlinear behaviors. The results indicate that the regime switches when the financial inclusion index, considered a transition variable, surpasses 1.38. The estimated slope parameter is 5.34, indicating the speed of adjustment from one regime to another. The results show that there is a negative relationship between Bitcoin and the USD in the linear and non-linear parts, with the coefficient being larger in the second regime. Additionally, there has been a positive relationship between the variables of gold and oil prices with Bitcoin, with these coefficients being larger in the second regime than in the first regime. The importance of cryptocurrencies is increasing in terms of the number of transactions concerning international markets due to their ease of use and digital support. The results indicate that BTC and gold are not seen as alternatives to each other in shorter maturities and that they are traded for speculative purposes. The findings obtained in this study show a positive and effective relationship between gold prices and oil prices with BTC. As a result of DCC-GARCH analysis, co-movements and significant relations between Bitcoin, gold, USD, and oil were determined. The findings of this study show that the BTC market should be constantly monitored, given its ability to transfer volatility risk to strategic commodities (such as crude oil) and even safe havens (such as gold), often seen as hedging instruments. The results indicate short-term co-movements of BTC and Gold, oil, USD are challenging to predict. The results also reflect the behavior of assets that appeal to speculators and uninformed noise investors, causing significant market fluctuations with their excessive transaction volumes during crisis periods that potentially affect the entire world economy and financial markets, such as the pandemic. Considering that before the pandemic, BTC was considered a relatively weak hedging tool or diversifier, the findings from this study become more remarkable. The results obtained from this paper are consistent with the results of Al-Nassar et al. [3], Li et al. [28], Oad Rajput et al [32], Abdul-Rahim et al [1], Disli et al [16], Qiao et al [33], Bouoiyour et al [7] studies. Additionally, policymakers should pay close attention to the tight interconnections between crude oil, especially during a crisis, if they want to implement optimal economic and energy policies to minimize the destabilizing effects of oil/BTC return shocks and avoid contagion risks. The results of this study also serve as a cautionary note for portfolio managers and investors who include BTC in their portfolios as a hedge against uncertainty. These results also show whether each asset/commodity can be used to manage and hedge the risk of the other asset/commodity due to the downward movement of the general market or sector. It would also be helpful to consider recent developments regarding the banking sector crisis and cryptocurrency exchange crashes in the United States in future studies. Once this is done, deciding whether BTC is a reliable option will be easier, enabling them to understand the issue better and make practical policy implications for investors and policymakers. Investors can gain new perspectives by using different cryptocurrencies or different country currencies. These research topics can be studied using other or new econometric methods. In the study, the relationship between BTC, USD, gold, and oil prices was analyzed only on the Iran economy, and it is thought that panel data analyses to be carried out on different countries or groups of countries will also make significant contributions to the literature.

References

[1] Abdul-Rahim, R., Khalid, A., Karim, Z. A., Rashid, M. Exploring the driving forces of stock-cryptocurrency comovements during COVID-19 pandemic: An analysis using wavelet coherence and seemingly unrelated regression. *Mathematics*, 2022; 10(12): 2116. Doi: 10.3390/math10122116

[2] Adebayo, T. S., Testing the EKC hypothesis in Indonesia: empirical evidence from the ARDL-based bounds and wavelet coherence approaches, *Applied Economic Research*, 2021; 28(1): 1-23. Doi: 10.1016/j.frl.2016.09.025

[3] Al-Nassar, N. S., Boubaker, S., Chaibi, A., Makram, B., In search of hedges and safe havens during the COVID─19 pandemic: Gold versus Bitcoin, oil, and oil uncertainty, *The Quarterly Review of Economics and Finance*, 2022; 12(5): 12-29. Doi: 10.1016/j.qref.2022.10.010

[4] Baek, C., Elbeck, M., Bitcoins as an investment or speculative vehicle? A first look, *Applied Economics Letters*, 2015; 22(1): 30-34. Doi: 10.1080/13504851.2014.916379

[5] Baur, D. G., Dimpfl, T., Realized bitcoin volatility. *SSRN*, 2017; 23(2): 1-26. Doi: 10.2139/ssrn.2949754

[6] Baur, D. G., Dimpfl, T., Kuck, K. Bitcoin, gold and the US dollar–A replication and extension,*Finance research letters*, 2018; 25(1): 103-110. Doi: 10.1016/j.frl.2017.10.012

[7] Bouoiyour, J., Selmi, R., Wohar, M. E., Safe havens in the face of Presidential election uncertainty: A comparison between Bitcoin, oil and precious metals, *Applied Economics*, 2019; 51(57): 6076-6088. Doi: 10.1080/00036846.2019.1645289

[8] Bouri, E., Molnár, P., Azzi, G., Roubaud, D., Hagfors, L. I., On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? *Finance Research Letters*, 2017; 20(2): 192-198. Doi:

[9] Bredin, D., Conlon, T., Potì, V., Does gold glitter in the long-run? Gold as a hedge and safe haven across time and investment horizon, *International Review of Financial Analysis*, 2015; 41(2): 320-328. Doi: 10.1016/j.frl.2016.09.025

[10] Bruno, V., Shin, H. S., Capital flows and the risk-taking channel of monetary policy, *Journal of monetary economics*, 2015; 71(2): 119-132. Doi: 10.1016/j.jmoneco.2014.11.011

[11] Bulut, L., Rizvanoghlu, I., Is gold a safe haven? The international evidence revisited, *Acta Oeconomica*, 2020; 70(4): 531-549. Doi: 10.1556/032.2020.00035

[12] Chaim, P., Laurini, M. P., Is Bitcoin a bubble? *Physica A: Statistical Mechanics and its Applications*, 2019; 517(1): 222-232. Doi: 10.1016/j.physa.2018.11.031

[13] Ciaian, P., Rajcaniova, M., Kancs, D. A., The economics of BTC price formation, *Applied economics*, 2016; 48(19): 1799-1815. Doi: 10.1080/00036846.2015.1109038

[14] Colletaz, G., Hurlin, Ch., Threshold Effects of the Public Capital Productivity: An International Panel Smooth Transition Approach, *Orleans Economics Laboratory*, 2021. Doi is not available

[15] Delfabbro, P., King, D. L., Williams, J., The psychology of cryptocurrency trading: Risk and protective factors, *Journal of behavioral addictions*, 2021; 10(2): 201-207. Doi: 10.1556/2006.2021.00037

[16] Disli, M., Abd Rabbo, F., Leneeuw, T., Nagayev, R., Cryptocurrency comovements and crypto exchange movement: The relocation of Binance, *Finance Research Letters*, 2022; 48(2): 29-42. Doi: 10.1016/j.frl.2022.102989

[17] Dwyer, G. P., The economics of Bitcoin and similar private digital currencies, *Journal of financial stability*, 2015; 17(1): 81-91. Doi: 10.1016/j.jfs.2014.11.006

[18] Dyhrberg, A. H., Bitcoin, gold and the dollar–A GARCH volatility analysis,*Finance Research Letters*, 2016; 16(3): 85-92. Doi: 10.1016/j.frl.2015.10.008

[19] Feng, W., Wang, Y., Zhang, Z., Can cryptocurrencies be a safe haven: a tail risk perspective analysis, *Applied Economics*, 2018; 50(44): 4745-4762. Doi: 10.1080/00036846.2018.1466993

[20] Becha, H., Kalai, M., Helali, K., Smooth transition regression model relating inflation to economic growth in Tunisia, *Economic Structures*, 2023; 12(2): 14-29. Doi: 10.1186/s40008-023-00308-9

[21] Goodell, J. W., Goutte, S., Co-movement of COVID-19 and Bitcoin: Evidence from wavelet coherence analysis, *Finance Research Letters*, 2021; 38(3): 74-89. Doi: 10.1016/j.eneco.2011.10.009

[22] Hansen, B. E., Threshold effects in Non-Dynamic panels: Estimation, testing and inference, *Journal of Econometrics*, 1999; 93(2): 345-368. Doi: 10.1016/S0304-4076(99)00025-1

[23] Hashemi Joo, M., Nishikawa, Y., Dandapani, K. Cryptocurrency, a successful application of blockchain technology, *Managerial Finance*, 2020; 46(6): 715-733. Doi: 10.1016/j.ipm.2018.12.015

[24] Ji, Q., Zhang, D., Zhao, Y., Searching for safe-haven assets during the COVID-19 pandemic, *International Review of Financial Analysis*, 2020; 71(4): 56-73. Doi: 10.1016/j.irfa.2020.101526

[25] Jude, E., Financial development and growth: A panel smooth regression approach, *Journal of Economic Development*, 2010; 35(1): 15-33. Doi: 10.1016/j.frl.2020.101528

[26] Kang, S. H., McIver, R. P., Hernandez, J. A., Co-movements between Bitcoin and Gold: A wavelet coherence analysis, *Physica A: Statistical Mechanics and its Applications*, 2019; 536(3): 87-96. Doi: 10.1016/j.physa.2019.04.124

[27] Kwon, J. H., Tail behavior of Bitcoin, the dollar, gold and the stock market index, *Journal of International Financial Markets, Institutions and Money*, 2020; 67(2): 22-35. Doi: 10.1016/j.physa.2018.11.063

[28] Li, D., Hong, Y., Wang, L., Xu, P., Pan, Z. Extreme risk transmission among bitcoin and crude oil markets, *Resources Policy*, 2022; 77(3): 27-41. Doi: 10.1556/032.2020.00048

[29] Luo, J., Ji, Q., Klein, T., Todorova, N., Zhang, D., On realized volatility of crude oil futures markets: Forecasting with exogenous predictors under structural breaks, *Energy Economics*, 2020; 89(4): 78-99. Doi: 10.1080/00036846.2019.1645289

[30] Mensi, W., Al-Yahyaee, K. H., Kang, S. H., Structural breaks and double long memory of cryptocurrency prices: A comparative analysis from Bitcoin and Ethereum, *Finance Research Letters*, 2019; 29(2): 222-230. Doi:

10.1111/twec.13138

[31] Mensi, W., Rehman, M. U., Maitra, D., Al-Yahyaee, K. H., Sensoy, A.,Does bitcoin co-move and share risk with Sukuk and world and regional Islamic stock markets? Evidence using a time-frequency approach, *Research in International Business and Finance*, 2020; 53(2): 54-69. Doi: 10.1016/j.ribaf.2020.101230

[32] Oad Rajput, S. K., Soomro, I. A., Soomro, N. A., Bitcoin sentiment index, bitcoin performance and US dollar exchange rate, *Journal of Behavioral Finance*, 2022; 23(2): 150-165. Doi: 10.1556/2006.2021.00019

[33] Qiao, X., Zhu, H., Hau, L., Time-frequency co-movement of cryptocurrency return and volatility: Evidence from wavelet coherence analysis, *International Review of Financial Analysis*, 2020; 71(2): 67-82. Doi: 10.1016/j.jmoneco.2014.11.014

[34] Rehman, M. U., Kang, S. H., A time–frequency comovement and causality relationship between Bitcoin hashrate and energy commodity markets, *Global Finance Journal*, 2021; 49(3): 100-156. Doi:

[35] Selgin, G., Synthetic commodity money. *Journal of Financial Stability*, 2015 ,17, 92-99.

[36] Selmi, R., Mensi, W., Hammoudeh, S., Bouoiyour, J., Is Bitcoin a hedge, a safe haven or a diversifier for oil price movements? A comparison with gold, *Energy Economics*, 2018; 74(2): 787-801. Doi: 10.1016/j.eneco.2018.07.007

[37] Shahbazi, K. & Saeedpour, L., Threshold Effects of Financial Development on Economic Growth in D8 Countries, *Quarterly Journal of Economic Growth and Development Research*, 2013; 3(12): 38-21. Doi: 10.1016/j.frl.2016.09.025

[38] Shahzad, S. J. H., Bouri, E., Rehman, M. U., & Roubaud, D., The hedge asset for BRICS stock markets: Bitcoin, gold or VIX, *The World Economy*, 2019; 45(1): 292-316. Doi: 10.1111/twec.13138

[39] Vacha, L., Barunik, J., Co-movement of energy commodities revisited: Evidence from wavelet coherence analysis, *Energy Economics*, 2012; 34(1): 241-247. Doi: 10.1016/j.eneco.2011.10.007

[40] Vo, A., Chapman, T. A., Lee, Y. S., Examining Bitcoin and Economic Determinants: An Evolutionary Perspective, *Journal of Computer Information Systems*, 2022; 62(3): 572-586. Doi: 10.1080/08874417.2020.1865851

[41] Wang, G. J., Xie, C., Wen, D., Zhao, L., When Bitcoin meets economic policy uncertainty (EPU): Measuring risk spillover effect from EPU to Bitcoin, *Finance Research Letters*, 2019; 31(1): 24-39. Doi: 10.1016/j.frl.2015.10.012

[42] Wu, S. Co-movement and return spillover: evidence from Bitcoin and traditional assets. *SN Business & Economics*, 2021; 1(10): 122-145. Doi: 10.1007/s43546-021-00126-w

[43] Yu, J. H., Kang, J., Park, S., Information availability and return volatility in the bitcoin market: analyzing differences of user opinion and interest, *Information Processing & Management*, 2019; 56(3): 721-732. Doi: 10.1016/j.ipm.2018.12.002

[44] Zhang, D., Ji, Q., Energy finance: Frontiers and future development,*Energy Economics*, 2019; 83(2): 290-292.Doi: 10.1080/00036846.2019.1645289

[45] Zhang, D., Hu, M., Ji, Q., Financial markets under the global pandemic of COVID-19, *Finance research letters*, 2020; 36(2): 52-78. Doi: 10.1016/j.frl.2020.101528

[46] Zhang, L., Li, Z., Kirikkaleli, D., Adebayo, T. S., Adeshola, I., Akinsola, G. D., Modeling CO2 emissions in Malaysia: an application of Maki cointegration and wavelet coherence tests, *Environmental Science and Pollution Research*, 2021; 28(20): 26030-26044. Doi: 10.1007/s11356-021-12430-x