Advances in mathematical finance &applications, 4 (4), (2019), 83-97
DOI: 10.22034/amfa.2019.579510.1144

Published by IA University of Arak, Iran
Homepage: www.amfa.iau-arak.ac.ir

Portfolio Optimization by Means of Meta Heuristic Algorithms

Mahmoud Rahmani, Maryam Khalili Eraqi*, Hashem Nikoomaram

Department of Financial Management, Science and Research Branch, Islamic Azad University, Tehran, Iran

ARTICLE INFO

Article history:
Received 20 November 2018
Accepted 05 July 2019

Keywords:
Artificial Bee Colony,
Portfolio Optimization,
Genetic Algorithm,
Ant Colony Algorithm

ABSTRACT

Investment decision making is one of the key issues in financial management. Selecting the appropriate tools and techniques that can make optimal portfolio is one of the main objectives of the investment world. This study tries to optimize the decision making in stock selection or the optimization of the portfolio by means of the artificial colony of honey bee algorithm. To determine the effectiveness of the algorithm, its sharp criteria was calculated and compared with the portfolio made up of genes and ant colony algorithms. The sample consisted of active firms listed on the Tehran Stock Exchange from 2005 to 2015. The sample selected by the systematic removal method. The findings show that artificial bee colony algorithm functions better than the genetic and ant colony algorithms in terms of portfolio formation.

1 Introduction

Asset allocation for portfolio optimization is one of the fundamental issues in modern capital markets and financial management. Portfolio optimization involves multiple securities like stocks, debt and equity investment funds [1]. Optimization is an issue to find the best answer to the problem considering the objectives and constraints [2]. Markowitz, presented a quantitative framework for portfolio selection and the attractiveness of investment is determined in terms of both risk and return, the risk is defined based on the variance-covariance matrix [3]. This approach has several shortcomings, the most important of which include: First collecting enough information about the risks and returns. Second, the model is very simple for modelling in the real-world because all the features such as transaction fees, and management of prices are not covered. Third, the estimates of return and risk by using old data are measured with measurement errors [4]. To add a constraint to the original formulation and solving the basic problem, portfolio optimization will be more difficult, because, the traditional techniques and methods are not shown a satisfactory result, so, we develop the Meta-heuristic algorithms for solving optimization problems. Meta-heuristics are defined as an iterative process to deal with a set of solutions trying to improve them. The key point is that this algorithm metaheuristic does not guarantee the finding

*** Corresponding author Tel.:** *+989122172635*
E-mail address:*m.khaliliaraghi@gmail.com*

of optimal solution necessarily in spite of a satisfactory solution. Meta-heuristic algorithms based on population are used to solve portfolio optimization problem and mimic the behaviour of natural systems. They are divided into two groups based on collective intelligence algorithms, evolutionary algorithms. Of the most popular evolutionary algorithms, the genetic algorithm has been applied to the selection of portfolio optimization [5]. Arnone et al. [6] for solving the optimization issue by using meta-heuristic problem and artificial intelligence algorithms used a genetic algorithm to optimize the portfolio. Fernandez and Gomez [7] by using a genetic algorithm and neural network algorithms with a limited number of assets indicated a significant result. Chang et al. [8] Considered the limit cardinal through a mean - variance, meta-heuristic algorithms, including genetic algorithms, annealing analogy to carry out portfolio optimization. Eslamibigdeli and Tayebisani [9] by combining genetic and ant algorithms to optimize the portfolio optimization based on value at risk with regard to limits for a round number for the number of shares in portfolio optimization in the Stock Exchange, indicated that portfolio optimization algorithms simply provide better results than a genetic algorithm. Kiani Harchegani et al. [10] used a genetic algorithm for portfolio optimization based on minimum risk-taking and its components in Tehran Stock Exchange and concluded that efficient frontier obtained by the genetic algorithm equalled to the efficient frontier obtained from the exact solution and this indicates that genetic algorithm optimization had a high performance in portfolio optimization.

However, in each case, there are shortcomings such as slow convergence speed, entrapment in local optimization [11, 35]. To overcome these problems, this paper suggested artificial bee colony for portfolio optimization. Artificial bee colony algorithm used to mimic the simulation of the behaviour of bee groups searching for food [12]. In this algorithm, the variables are considered as a food source location. The amount of nectar and food source indicate a possible function or fitness solution. Each food source is derived only by a worker bee. In other words, the number of worker bees around a hive equals to the surrounding food sources. Artificial bee colony algorithm has been welcomed by scientific communities due to its special capabilities for solving optimization problems. Hence, the innovation of this research is to use it as an efficient model of speed and accuracy to increase investment model and help investment managers to make appropriate decisions. In this regard, to determine the effectiveness of the proposed algorithm, its performance has been compared with ant colony and genetic algorithm. Each of the above methods to solve the problem of portfolio optimization have the advantages of their own.

Metaheuristics algorithms, which are inspired by various phenomena of nature, are founded the base on two concepts of centralization and variation. These two concepts define the behaviour function of Metaheuristics. In general, it is a higher-level procedure or heuristic designed to find, generate, or select a heuristic by delimitation of the search through different methods including searching for the best configuration for a collection of variables to achieve the optimum results. Using this algorithm for the portfolio optimization, the investors can analyze the mass data of the different companies, summarize them and use them to select the optimized portfolio. Metaheuristics algorithm has some characteristics as follows:

- It is highly flexible.
- It can increase the capacity of optimization.
- It can increase the productivity of investment.

2 Research Theoretical Background

When an investor faces with a variety of options and choices while considering investment decisions, he needs to make a decision on the number of asset selection, as well as the amount of investment. So, making a proper decision on the amount of money that should be invested in each asset is of great importance [13]. The major role which the portfolio selection plays is to allocate the cash to the selection of stocks so that the investor faces a risk-return trade-off while making his decision to invest [6] Therefore, the portfolio selection problem is equivalent to the investor selecting the optimal portfolio from a set of possible portfolios by considering the return, risk, and many other factors. In other words, the problem is that the investor can't select a portfolio which is optimum in all aspect. Accordingly, portfolio optimization is followed by the financial decision-maker's choice to reconcile with the purposes and consideration of the factors mentioned above. That is, they have to choose one and forget about the others. This problem can be solved by using different approaches as Randomized Control Models, Multi-Purpose Planning. Neural Networks, and Metaheuristics algorithms which can help the investors to select an optimized portfolio, as suggested by financial scholars [14]. Modern portfolio theory (MPT) suggested by Markowitz [3] to select the portfolio by mean-variance analysis is a framework for assembling a portfolio of assets such that the expected return is maximized for a given level of risk. Its key insight is that an asset's risk and return should not be assessed by itself. Rather, an investor had to consider how each security co-moved with all other securities but by how it contributes to a portfolio's overall risk and return. Wang et al. [11] suggested the genetic algorithm to select the portfolio which can help the investors to overcome the constraint of the functions used in the portfolio optimization such as configuration, a time-consuming non-liner problem-solving method, procedures, complex and intangible calculation parameters, and inflexibility of the variables.

Karaboga and Busturk [15] proposed the bee colony algorithm to optimize the numerical functions and believed that this algorithm is an efficient means to optimize the portfolio. Gang et al. [16] used the genetic algorithm to solve the problems of the portfolio optimization based on mean-variance analysis and found out that the portfolio with fewer assets has a better performance. Tollo et al. [17] defined a multi-criteria optimization problem in which the two types of approaches were combined, and a hybrid meta heuristic that combines local search and quadratic programming to obtain an approximation of the Pareto set was introduced. They found that this Meta heuristic can be effectively used to solve multi-criteria portfolio selection problems and showed that the results obtained by the Meta heuristic are robust with respect to the return representation used. Mamanis [19] asserts that the resulting portfolio optimization problem becomes very hard to be tackled with exact techniques as it displays nonlinearities, discontinuities and high dimensional efficient frontiers. In his work, he provides a brief note on the field of portfolio optimization with meta heuristics and concludes that especially Multi objective meta heuristics(MOMHs) provide a natural background for dealing with portfolio selection problems with complex measures of risk (which define non-convex, non-differential objective functions), discrete constraints and multiple objectives. Jarraya [18] discussed Markowitz model in terms of minimizing risk and maximizing returns of expected portfolio and asserts that the proposed models in this issue are resolved basing on quadratic programming; while, the real state of financial markets makes these problems too complex. He suggests various Meta heuristics approaches which are proposed to solve asset allocation and portfolio optimization problems, and to do so, some approaches are surveyed by categorizing them, describing results and involved techniques. Next he aims to provide a good guide to the application of Meta heuristics to portfolio optimization and asset allocation problems. Chen [14] used the artificial bee colony algorithm to optimize the portfolio by considering the budget constraints and the findings showed that non-linear planning models and traditional approaches have failed to solve such combinations. However, the artificial bee colony algorithm is an efficient way to solve the portfolio optimization problems.

According to the study conducted by Miryekemam et al. [20], decision making has always been affected by two factors: risk and returns. Considering risk, the investor expects an acceptable return on the investment decision horizon. Accordingly, defining goals and constraints for each investor can have unique prioritization. They developed several approaches to multi criteria portfolio optimization. The maximization of stock returns, the power of liquidity of selected stocks and the acceptance of risk to market risk are set as objectives of the problem. In order to solve the problem of information in the Tehran Stock Exchange in 2017, 45 sample stocks have been identified and, with the assumption of normalization of goals, a genetic algorithm has been used. The results show that the selected model provides investors whit a good performance for selecting the optimal portfolio with specific goals and constraints. In a study on Portfolio-Optimization conducted by Darabi and Baghban [21] has been the Clayton-copula along with copula theory measures, Portfolio-Optimization is one of the activities in investment funds. They used copula as an alternative measure to model the dependency structure in research. In this regard, given the weekly data pertaining to the early 2002 until the late 2013, They used Clayton-copula to generate an optimized portfolio for both copper and gold. Finally, the Sharpe ratio obtained through this method has been compared with the one obtained through Markowitz mean-variance analysis to ascertain that Clayton-copula is more efficient in portfolio-optimization.

3 Portfolio Optimization

Portfolio selection demonstrates how an investor allocates his liquidity with respect to efficiency targets and risk returns to different assets to achieve a satisfactory portfolio of assets [22]. The composition of the portfolio in question can be the result of accidental and irrelevant investment decisions, or the result of deliberate planning [22]. Selecting tools and techniques that can form the portfolio optimization is coveted by the world of investment [13]. The main objective in portfolio model helps investors to choose the optimal portfolio according to his preferences and environmental conditions. The best known and most common approach regarding portfolio optimization model is the choice of mean-variance by Harry Coetzee model in which investment risk model is not only based on the standard deviation of a stock, but also the risk of the investment [24]. Markowitz model aimed at minimizing the risk and returns of the stock portfolio is intended as a limitation. In this study, the model it is defined as follows:

$$\begin{aligned}
&minz=\sum_{i=1}^{n}\sum_{j=1}^{n}X_iX_jCOV_{(R_i,R_j)}-\sum_{i=1}^{n}X_iR_i\\
&S.T\\
&\sum_{i=1}^{n}X_i=1\\
&X_i\geq 0\ ,\forall_i\in(1,2,3,\dots,N)
\end{aligned}\tag{1}$$

N: Number of existing assets
R_i: The average return on assets
X_i(weight variable) investment ratio in asset to total investment
Covariance between stock returns i and j$COV_{(Ri,Rj)}$:
:
limit$\sum_{i=1}^{n} X_i = 1$
It ensures that all capital is available and the goal is to achieve minimizing portfolio to obtain expected returns.

3.1 Artificial Bee Colony (ABC)

The artificial bee colony is a search algorithm which simulates the behavior of bee groups in nature-inspired search of food [12]. The algorithms have been used by different researchers and its performance has been confirmed. Karaboga and Basturk [15] used this method to optimize numerical functions and juxtaposed it with genetic, and particle cumulative algorithms and concluded that the ABC algorithm has a better performance. Scientists in [25-30] used ABC algorithm to solve optimization problems and concluded that this algorithm has a better performance. In this algorithm, the variables considered as a food source location. The amount of nectar food source indicates the probability or fitness solution. Every food source is extracted only by a worker bee. In other words, the number of worker bees around the hive is equal to the number of food sources. The algorithm includes the three groups of worker bees, spectators, and the vanguard. At first, a set of food sources were randomly selected. Worker bees refer to them and calculate the nectar. Then, the bees return to the hive and share the information with others, i.e. audience bees. In the second phase, after exchanging the information, each worker bees travels to a source and based on visual information from the environment, they may select a new source adjacent to the previously selected source. This means that depending on the color and type of flower, the bee decides to go to the same source or select a new source. In the third stage, in regard to the information from the worker bees dancing in the site, the audience bees prefer a range of food sources based on nectar. When a source is finished or abandoned, a new source has found accidentally by scouts, is re-placed to meet the needs. This cycle will be repeated until requirements are fulfilled. In this model, there is a maximum number of one scout bee in every cycle and the number of worker and audience bees is equal. ABC algorithm starts with the first population of random answers. Repeating the process, random answers are tried to improve and the first population is created based on equation (2).

$$X_{ij} = X_j^{min} + rand(0,1)\left(X_j^{max} - X_j^{min}\right) \tag{2}$$

Where i = 1, 2, ..., Sn, Sn is the size of the initial population and j = 1, 2, ..., D, where D is the number of parameters;$X_j^{min}$is the lower limit and$X_j^{max}$is the upper limit of the problem parameters. The function of each bee is as follows:

3.1.1 Worker bee phase

In the worker bee phase, each worker bee is in search for solutions in the vicinity of the existing solution in its memory, namely in this phase for each Xi, j (solutions in the memory), a new periphery (Vj) is produced according to equation (3):

$$V_{i,j} = X_{i,j} + \phi_{(i,j)}\left(X_{ij} - X_{kj}\right) \tag{3}$$

Where, k = 1, 2, ..., Sn, is a periphery to the crowd and k is chosen randomly, and$\phi_{(i,j)}$is a random number in the interval (1 V1-), finally, by selecting greedily based on the priority between $X_{i,j}$ ‘ $V_{i,j}$,the more qualified is elected, and the reasoning of priority is calculated from equation (4):

$$fit_i = \begin{cases} \dfrac{1}{1+f(x_i)} & if \quad f(x) \geq 0, \\ 1+f(x_i) & if \quad f(x) < 0 \end{cases} \tag{4}$$

Where f (xi) is the objective function value for the solution. Having completed the search process, the audience bees evaluate the information and with a probability proportional to the quality nectar source (solution), they choose one of the food sources. The possibility of relation is obtained from equation (5).

$$P_i = \frac{fit_i}{\sum_{i=1}^{n} fit_n} \tag{5}$$

In this relation, $fit_i$is the priority for food source corresponding to each bee; SN is the number of available solutions. If a source is ended, which lacks appropriate quality, the worker bees abandons it and becomes a scout.

3.1.2 Observer Bee Phase

Observer bee randomly selects a solution, according to equation (3), it seeks a periphery for the selected solution. In accordance with formula (4), a more qualified solution is selected. To avoid being trapped in local optimum, a solution that the counter is less than a certain amount was abandoned. The worker bees are responsible for the solution to become the scout. Using a random search in accordance with the formula (2), a new solution is selected and replaces the alternative memory solutions. The counter of the removed solution becomes zero. In each algorithm, the best answer is maintained. The main stages of the implementation of artificial bee colony can be stated as follows

- Initialization
- Evaluation of population
- Cycle = 1
- Repeat
- Produced a new solution by worker bees using formula (3) and its evaluation
- Evaluation of new and old solutions using equation (4)
- Calculation of the probability value solutions using equation (5)
- To any observer bee, a new solution using equation (3) to be provided and the amount is likely to be calculated.
- Evaluation of old and new solutions using equation (4).

- Setting a limit for the scout bee, specify whether the solution should be abandoned or be pursued.
- Summarize the answers
- Cycle = Cycle +1
- Proceed until the cycle equals to MCN

3.2 Genetic Algorithm

Holland used a genetic algorithm for engineering optimization [32]. The idea was the transmission of inherited properties by genes. This algorithm is about Darwin's life concerning the most appropriate choice. With a series of structured information and random integration, a search algorithm is created [33]. The genetic algorithm should present a better and worse definition, which is done by the fitness function as a non-negative function which is usually the adaptation of a chromosome to target the measures. Genetic algorithm usually tries to maximize value fitness function and chromosomes are similar to an individual in the community [34] Genetic algorithm, with the random initial population it creates, began to act and often chooses to have better fitness as parents. The algorithm uses two operators of mutation and the crossover to produce offspring with features different from the two parents. Gene mutation is a change in a gene. Mutation genetic algorithm creates a small change in the nature of a solution. It causes a variety of causes to be preserved and a new genetic structure of the population form so as to prevent being trapped in local optimum [36]. The crossover of two or more chromosomes, the genetic composition is said to produce children. This practice prevents children from being a perfect resemblance to one of the parents [37]. Eben and Smith [38] have expressed the running of a genetic algorithm as follows:

1. Create an initial population of solutions as big as Pop size
2. Practice justification on the initial answers in case of necessity
3. Produce children (Pop size × Pc) by using the crossover (rate of crossover= Pc).
4. Produce children (Pop size × Pc) children by using crossover (rate of crossover= Pc)
5. Practice justification proceedings on the produced children using mutation a crossover method in case of necessity
6. Add a unit to the number of repetitions
7. If the number of iterations is a multiple, proceed to step (8); otherwise, go to the step (9). (A random number)
8. %N number of chromosomes is randomly selected, and do a local search (% random number)
9. Update the community
10. If the conditions for stopping the algorithm is set, stop and report the best answer; otherwise, you must be back to step (3).

3.3 Ant Colony Algorithm

The main idea of the behaviour of ants in nature to solve optimization problems introduced by Marco in 1991 for solving the wandering salesman problem [36]. Ant colony algorithm has features such as flexibility, robustness, and self-organization. The algorithm consists of three main processes of building answer by ants, updating pheromone, and auxiliary activities [39]. In making an answer, ant colony checks its status simultaneously and independently by moving to neighbouring nodes. The decision is

made with a possible by pathway pheromone and heuristic information. Therefore, they gradually provide solutions for the optimization problem. When an ant makes an answer, it evaluates the answer. Using pheromone updating process, it decides how the pheromones should be updated.
Two phases for answer construction, each ant selects consecutive decisions independently and randomly from the graph. The possibility that ant k is in node i (provided that it is in the vicinity of node i) as the next group is calculated from the

$$p_{ij}^{k} = \frac{(t_{ij})^{\alpha}(n_{ij})^{\beta}}{\sum_{iENK}(t_{ij})^{\alpha}(m_{ij})^{\beta}} \tag{6}$$

following equation:

Where : t_{ij} is the ant footprint
$n_{ij} = \frac{1}{dij}$ is a part of innovative information?

dij is the distance between i and j nodes.
αβare two parameters that determine the relative influence of pheromone trail and innovative information and are located at the distance of (1, 0), respectively. After each ant have created an answer, algorithm will be through an updated phase. In the first pheromone update phase, regarding equation (7), the amount of pheromone of edges is evaporated, then from equation (8), the pheromone of each edge is updated:

$$t_{ij} \leftarrow (1-p)\, t_{ij} \tag{7}$$

*p*is parameter evaporation rate and are is located at (1, 0).

$$t_{ij} \leftarrow t_{ij} + \sum_{k-1}^{m} \triangle t_{ij}^{k} \tag{8}$$

$\triangle t_{ij}^{k}$is the amount of pheromone on each edge that puts k-th ant passes through. It is calculated through equation (9):

$$\Delta t_{ij}^{k} = \begin{cases} \frac{1}{c_k} \\ 0 \end{cases} \tag{9}$$

J is the part of ant k
9) if the edge is i)
ck is the length of a track by ant

By using the mechanism, it would be expected that when an ant finds a better answer, more pheromone is added to its edge over the other edges. The comparison between artificial bee colony and genetic and ant colony algorithms was indicated in no particular study, but the juxtaposition of two algorithms have been made, especially in the field of engineering and mathematical optimization problems. Karaboga and Basturk [10], in an article, compared the optimization of numerical functions of bee artificial colony and genetic algorithm and concluded that artificial bee colony provides better performance. Mala et al. [27] used ant colony algorithm and artificial bee colony algorithm and concluded that ABC algorithm has several advantages over ant colony algorithm. Wu et al. [31] in a study compared artificial bee colony with five meta-heuristic techniques such as genetic algorithm, PSO and concluded that the ABC algorithm optimization has better performance.

4 The Proposed Methodology

This study aims to select portfolio optimization in Tehran Stock Exchange using artificial bee colony optimization algorithm and compares the performance of this portfolio optimization with genetic and ant algorithms so, as to identify better algorithms with more appropriate performance more suitable for more effective decision making to invest in stocks listed on the Tehran Stock Exchange. To achieve the mentioned research objective, the following hypothesis was presented:

Selected portfolio optimization using artificial bee colony outperforms the portfolio optimization using ant and genetic colony algorithms.

The research in terms of data collection, since the study is to describe the conditions, is put in the descriptive research category because it can be useful for deciding on an investment. Also, the research is developmental as it explains mathematical models to select investment portfolio based on the Markowitz theory in Iran capital market. This study is also an applied research because it discusses the relationships between variables using data from Tehran Stock Exchange. In this research, to determine portfolio optimization, monthly returns from the companies listed on Tehran Stock Exchange from 2005 to 2016 were used. Thus, for each year, 12 rates of return were achieved and their total annual return was calculated. Risk per share is calculated based on the variance of monthly returns. To calculate the covariance, monthly stock returns are used. Using MATLAB, each year's stock optimization basket in which weights are calculated per share using three artificial bee colony, ant colony, and genetic algorithms. To determine the performance of portfolios, the three mentioned algorithms was used through equation (10).

$$\mathrm{SR_P} = \frac{\mathrm{R_p - R_f}}{\sigma_\mathrm{p}} \tag{10}$$

$\mathrm{SR_P}$:Portfolio's sharp benchmark

R_P: Portfolio returns calculated through equation (11)

σ_P : Portfolio risk calculated from equation (12)

$$R_P = \sum_{i=1}^{n} x_{i\,R_i} \tag{11}$$

$$\sigma_\mathrm{p} = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j cov_{(Ri,Rj)}\right]^{\frac{1}{2}} \tag{12}$$

x_i : Percentage of investment on asset *i*

R_i : Return of asset *i*

$COV_{(Ri,Rj)}$: Covariance between stock returns of i and j

R_f : Risk-free rate of return. In this study, it equals to the rate of return on deposits of one year as announced by the central bank. Statistically, there is significant difference between the performance of the portfolio optimization algorithms derived from the analysis of variance (ANOVA) and Tukey test. The study population consisted of all companies listed on Tehran Stock Exchange, and the systematic elimination method is used to select samples. For this purpose, from the stock listed on the Tehran Stock Exchange each year, the stocks with the desired features were selected as a statistical sample of the intended year. The features to select the sample include:

1) The Company's shares experience interruption not more than three months a year
2) Their stock is traded on the stock exchange the following year

5 Analysis and Findings

The Excel software used to calculate the input data. To determine the portfolio for each year, genetic, artificial ant colony and colony of bee algorithms were analysed using MATLAB. For the analysis of algorithms, SPSS software was used. After performing the steps listed in the previous parts and collecting the required data from Tehran Stock Exchange for the years between 2005 and 2015, and solving Markowitz model through artificial bee colony, ant algorithm and genetic environment using MATLAB software for each selected year. Each basket was included in the basket of stocks and weighing per share and for each portfolio returns, risk and sharp measure calculated. Table 1 presents descriptive statistics of the baskets:

Table 1: Descriptive statistics for selected baskets in each algorithm

Algorithms		**Baskets**	**Min**	**Max**	**Aver.**	**Standard Deviation**	**Skewness**	**Elongation**
Ant Algorithm	Sharp measure	10	-7.2	19.64	4.8	8.248	0.933	1.334
	Risk	10	2.3	9.36	4.29	2.265	2.507	1.597
	Efficiency	10	-8.74	169.56	37.5	50.015	2.397	6/485
ABC Algorithm	Sharp measure	10	9.5	75.78	28.427	28.044	0.109	-0.901
	Risk	10	0.45	5.14	1.774	1.746	0.279	1.197
	Efficiency	10	1.96	18643	49.26	52.296	2.303	6.251
Genetic Algorithm	Sharp measure	10	-25.87	55.99	3.88	22.35	1.434	2.928
	Risk	10	0.47	9.36	2.797	2.898	1.932	1.522
	Efficiency	10	-6.42	154.93	30.79	50.97	2.018	3.792

As Table 1 shows, the average sharp measure and efficiency of artificial bee colony was more and its risk was less than other algorithms. After the sharp measure, risk and efficiency of each basket was found, to determine the type of test, using the Kolmogorov-Smirnov, statistical distribution was determined. And the results from this test was summarized in Table 2. As Table 2 indicates, the level of significance for all algorithms is higher than the level of significance (0.05), so the distribution is normal and ANOVA and Tukey used to compare the performance of selected baskets in the mentioned algorithms.

Table 2: Results from Kolmogorov-Smirnov (for Sharp measure)

Algorithms	genetic algorithm	artificial bee colony	Ant colony
Number	10	10	10
average	3.88	28.427	4.08
Standard deviation	22.35	28.04	8.24
Z	0.772	0.453	0.596
Sig	0.590	0.986	0.87

To determine whether there is a significant difference between the performances of selected portfolio optimization algorithm through the three algorithms, the analysis of variance used and the results of this test summarized in Table 3.

Table 3: Results of analysis of variance from different algorithms

Algorithm	number	average	SD	test
Bee colony	10	4.08	8.24	4.412F =
Artificial Bee colony	10	28.42	28.04	Sig=0.022
Genetic Algorithm	10	3.88	22.35	

As Table 3 shows, the average sharp measure of artificial bee colony, genetic algorithm and ant algorithm equals to 28.42,3.88and 4.08, respectively. This significant difference in the mean is significant by F test because the test confidence level is 0.05 less than research confidence level. With 95% of confidence, there is a significant difference between the performances of different algorithms. And to determine which algorithm is better, Tukey test was used and the results were summarized in Table 4:

Table 4: Results from Toki test (Sharp measure in various algorithms)

Test	Algorithm	Ant colony	Artificial Bee Colony	Genetic Algorithm
Toky	Ant colony	-	24.347m = -	1.00 = Sig
	Artificial bee test	0.042 =Sig	-	0.040 = Sig
	genetic algorithm	0.19 =-[1]m	24.54 = -[1]m	-

As Table 4 shows, the performance of selected portfolio optimization through artificial bee colony significantly better than the performance of selected baskets using ant colony and genetic algorithms. However, there is no significant difference between the performance of selected portfolio optimization through genetic algorithms and ant colony.

8 Conclusion and Suggestions

In the present study, to select the appropriate portfolio optimization, Markowitz model through artificial bee colony in the MATLAB environment was solved and the results were compared with genetic and ant's algorithms. Sharp measures, efficiency, and risk were calculated for each portfolio. These parameters were compared with the three algorithms as Table 3 shows, there is a significant difference between the portfolio optimization by the artificial bee colony algorithm, ant colony and genetic algorithm at a statistically significant level of 0.05. Portfolio optimization selected by the artificial Bee colony algorithms, according to Tavakoli's test at 90% confidence interval, shows better performance as indicated at Table 4, so that sharp ratios average is 28.43, 4.08, and 3.88 for the artificial bee colony algorithms, ant colony, and genetic algorithm, respectively. However, there is no significant difference among the artificial bee colony, ant colony and genetic algorithms in term of performance of selected portfolios. On the other hand, the selected portfolio by the artificial bee colony algorithms compared to the other algorithms shows a lower a risk. These results are consistent with the findings of Karabago and Akai [26], Cho et al [8], Karabago [15] that compared the artificial bee colony algorithm, with the genetic algorithm. These results are also consistent with the findings of Mala et al. [27] that compared the artificial bee colony algorithm, ant algorithm. Accordingly, the findings suggest that investors pursuing the investigation at the Tehran Stock Exchange, particularly the professional managers of investment enterprises like funds and credit companies, can use the artificial bee colony algorithm to achieve better results. It should also be noted that the model used in this study is a modified version of the Markowitz stock basket model, in which the "rationality" of investors is one of the main foundations. From the viewpoint of behavioural financial knowledge, this assumption is not capable of explaining the behaviour of investors due to its lack of realization. For this reason, optimization models that are based on this premise can barely reflect the real-world conditions. In case the investors intend to apply a low-risk strategy for investment, the findings suggest that using the artificial bee colony algorithm can help them to achieve the optimum results as this algorithm detects the portfolio with lower risk compared to other algorithms. Furthermore, according to the findings, it is recommended that this title can be added to the curriculum of the postgraduate financial courses since metaheuristic algorithms have a wide range of applications.

References

[1] Fabozzi, F., Kolm, P., Pachamanova, D., Focardi, S., *Robust portfolio optimization and management*, John Wiley and Sons, 2007.

[2] Karaboga, D., GorkemLi, B., *A quick Artificial Bee colony (qABC) algorithm and its performance on optimization problems*, Appl. Soft Compact, 2014, **23**, P.227-238. Doi: 10.1016/j.asoc.2014.06.035.

[3] Markowitz, H., *Portfolio Selection*, Journal of Finance, 1952, **7**(1), P.77-91, Doi:10.1002/nav.3800030110.

[4] Tollo, G., Roli, A., *Meta heuristics for the Portfolio Selection Problem*, International Journal of Operation Research, 2008, **15**, P.13-35 , Doi:10.1.1.401.8706.

[5] Lin, C.C., Liu, T.Y., *Genetic algorithms for portfolio selection problems with minimum transaction lost,* European Journal of Operational Research and Applications, 2008, **185**(1), P.393-404, Doi.org/10.1016/j.ejor. 2006.12.024.

[6] Arnone, S., Loraschi, A., Tettamanzi, A., *A genetic approach to portfolio selection*, Neural Network World, 1993, **6**(93), P.597- 604.

[7] Fernandez, A., Gomez, S., *Portfolio selection using neural networks*, Computers and Operations Research, 2007, **34**(4), P.1177-1191, Doi:10.1016/j.cor.2005.06.017

[8] Chan, T. J., Beasley, J. E., Medds, N., Sharaiha Y. M., *Heuristic for cardinality constrained portfolio optimization*, Computers and Operation Research, 2000, **27**, P.1271-1302 , Doi:10.1016/S0305-0548(99)00074-X

[9] Eslami Bidgoli Gh., Tayebi Sani E., *A novel Meta-Heuristic method for solving an extended Markowitz Mean–Variance portfolio selection model,* Journal of Investment Knowledge, 2014, **3**, P.122-101(In Persian)

[10] Kiani Harchegani, M., Nabavi Chashmi, S., Memarian, E., *Optimizing Stock Portfolio with regard to Minimum Level of Total Risk using Genetic Algorithm*, Journal of Investment Knowledge, 2013, **3**, P.125-164.

[11] Wang, Z., Ouyang, R., Kong, X., *A hybrid artificial bee colony algorithm for portfolio for portfolio optimization problem*, Journal of Theoretical and Applied Information Technology, 2013, **4**(4), P.8-16. Doi: 10.4156/ ijact.vol4.issue 4.2.

[12] Karaboga, D., An idea based on honey bee swarm for numerical optimization Technical report-tr06, Erciyes university, Engineering Faculty, Computer Engineering Department, 2005, **200**, P.1-10.

[13] Raei, R., Bahrani Jahromi, M*., Portfolio optimization using a hybrid of fuzzy ANP, VIKOR and TOPSIS*, Management Science Letters, 2012, **2**, P. 2473–2484. Doi: 10.5267/j.msl.2012.07.019.

[14] Chen, W., *Artificial bee colony algorithm for constrained possibilistic portfolio optimization problem*, Physca, 2015, **12**, P.122- 139. Doi:10.1016/j.physa.2015.02.060.

[15] Karaboga, D., Basturk B., *Artificial bee colony(ABC) optimization algorithm for solving constrained optimization problems.* In: proceeding of the 12th international fuzzy systems association world congress on foundations of fuzzy logic and soft computing. Springer, Berlin, IFSA, 2007, **7**, P.789-798, Doi:10.1007/978-3-540-72950-1_77

[16] Chang, T. J., Yang, S. C., Chang, K.J., *Portfolio optimization Problems in different risk measures using Gentic algorithm*, Expert, systems with Application, 2009, **36**(1), P.1-52. Doi: 10.1016/j.eswa.2009.02.062.

[17] Tollo, G.D., Thomas, S., Birattari, M., *A metaheuristic multi-criteria optimisation approach to portfolio selection,* Journal of Applied Operational Research, 2014, **6**(4), P. 222-242.

[18] Jarraya, B., Asset Allocation and Portfolio Optimization Problems with Meta heuristics: A Literature Survey, Business Excellence and Management, 2013, **3**, P.38-56.

[19] Mamanis, G., *Portfolio Optimization with Metaheuristics*, Finance and Market, 2017, **2**(2), Doi: http://dx.doi.org/10.18686/fm.v2i2.1048.

[20] Miryekemami, S., Sadeh, E., Sabegh, Z., *Using Genetic Algorithm in Solving Stochastic Programming for Multi-Objective Portfolio Selection in Tehran Stock Exchange*, Advances in Mathematical Finance and Applications, 2017, **2**(4), P.107-120.Doi: 10.22034/amfa.2017.536271.

[21] Darabi, R., Baghban, M., *Application of Clayton Copula in Portfolio Optimization and its Comparison with Markowitz Mean-Variance Analysis*, Advances in Mathematical Finance and Applications, 2018, **3**(1), pp. 33-51. doi: 10.22034/amfa.2018.539133.

[22] Aouni, B., *Multi-attribute portfolio selection: New Perspectives,* INFOR: Information Systems and Operational Research, 2009, **47**(1), P.1-4. Doi: https://doi.org/10.3138/infor.47.1.1.

[23] Abzari, M., Ketabi, S., Abbasi, A., *Using linear programming and providing a functional model for investment optimization,* Journal of Humanities and Social Sciences, Shiraz University, 2005, **22**(2), P.43-57. (In Persian)

[24] Markowitz, Harry M*., The Optimization of a Quadratic Function Subject to Linear Constraints*, Naval Research Logistics Quarterly, 1956, **3**, P.111–133.Doi: https://doi.org/10.1002/nav.3800030110 .

[25] Karaboga, D., Basturk, B., *Artificial bee colony(ABC) optimization algorithm for solving constrained optimization problems,* In: proceeding of the 12th international fuzzy systems association world congress on foundations of fuzzy logic and soft computing. Springer, Berlin, IFSA, 2007, **7**, P.789-798, Doi:10.1007/978-3-540-72950-1_77.

[26] Karaboga, D., Akay, B., *A comparative study of artificial bee colony algorithm*, Applied Mathematics and Computation, 2009 , **214**(1) , P.108-132, Doi:10.1016/j.amc.2009.03.090.

[27] Mala, D.J., Kamalapriya, M., Shobana, R., Mohan, V., *A non-pheromone based intelligent swarm optimization technique in software test suit optimization*. In :IAMA:2009 international conference on intelligent agent and multi – agent systems , IEEE Madras section ; IEEE computer society m Madras chapter ; Computer society of india Div II ; Council of science & industrial Research; Govtindia, Department of information Technology , 2009, **33**, P.188-192, Doi:10.1109/IAMA.2009.5228055

[28] Lin C.J., Lee C., *Efficient artificial bee colony algorithm for 3d protein folding simulation.* In: 17th national conference on fuzzy theory and its applications, 2009, P.705-710, Doi:10.1016/j.ins.2010.07.015 .

[29] Akay, B., Karaboga D., *A modified artificial bee colony algorithm for real- parameter optimization,* Information Sciences, 2012, **192**, P.120-142. Doi:10.1016/j.ins.2010.07.015 .

[30] Ruiz, V., Daz parra, O., *Similarities between meta-heuristics algorithms and the science of life,* Central European Journal in Operational Research, 2011, **19** ,445-466. Doi:10.1007/s10100-010-0135-x.

[31] Wu D., Yu W., Yin Z., Parameter estimation of rational models based on artificial bee colony algorithm. In: Proceedings of international conference on modeling, identification and control (ICMIC), 2011, **43**, P. 219-224. Doi: 10.1109/ICMIC.2011.5973704 .

[32] John, H., Holland, *Adaptation in natural and artificial systems*, MIT Press, Cambridge, MA, 1992.

[33] Renner, G., Ekart, A., *Genetic algorithms in computer aided design,* Computer-Aided Design, 2003, **35**, P.709-726. Doi:10.1016/S0010-4485(03)00003-4.

[34] Deng, Y., *Genetic Algorithm for Financial portfolio selection*, Master`s Thesis, University of Science & Technology Beijing, China, 2002.

[35] Izadikhah, M., Tavana, M., Di Caprio, D., Santos-Arteaga, FJ, *A novel two-stage DEA production model with freely distributed initial inputs and shared intermediate outputs*, Expert Systems with Applications, 2018, **99**, P. 213-230

[36] Coloni, A., Dorigo. M., Maniezzo, V., Distributed optimization by ant colony, in proceeding of European conference on Prtifical life (ecsla1), Elsevier publishing, Amsterdam ,1991.

[37] Marthins, P., Riberio, C., *Metaheuristics for optimization problems in computer communications*, Journal of Computer Communications, 2007, **30**(4), P.656-669.Doi:10.1016/j.comcom.2006.08.027 .

[38] Eiben, A., Smit, E., Interoduction to Evolutionary, Springer Verlag, 2003.

[39] Dorigo, M., Birattari, M., Stutzle, T., *Ant colony optimization-artificial ants as a computational intelligence technique*, Belgium: Universit Libre de Bruxelles. IRIDIA Technical report Series, 2006, Doi: 10.1109/MCI.2006.329691 **.**