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Elliptic Function solutions of (2+1)-Dimensional Breaking Soliton Equation by Sinh-Cosh Method and Sinh-Gordon Expansion Method

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Abstract

In this paper, based on sinh-cosh method and sinh-Gordon expansion method, families of solutions of (2+1)-dimensional breaking soliton equation are obtained. These solutions include Jacobi elliptic function solution, soliton solution, trigonometric function solution.

Key words: sinh-cosh method, soliton, Jacobi elliptic function, sinh-Gordon expansion method.

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1 Introduction

There exist many methods for obtaining solutions of the (2+1)-Dimensional breaking soliton equation, such as the Generalized Jacobi elliptic function method [2], (G'/G) Expansion method [3] and so on.
In this paper, by using the sinh-cosh method [1] and sinh-Gordon expansion method [4,5], we construct elliptic function solutions in the (2+1)-dimensional breaking soliton equation.

$$u_t - bu_{xxy} + 4b(uv)_x = 0, \tag{1.1}$$
$$v_x - u_y = 0, \tag{1.2}$$

Where b is an arbitrary constant, the system (1)-(2) was used to describes the (2+1)-dimensional interaction of Riemann was propagated along the y-axis with long wave propagated along the x-axis and it seems to have been investigated extensively where over lapping solutions have been derived.

2 Methods

Consider a given (2+1)-dimensional breaking soliton equation with independent variable $x = (t, x_1, x_2, ...)$ and dependent variables $u(x)$. The following formal solution of the given (2+1)-dimensional breaking soliton equation will be souqht by the following ansatz

$$u(x) = A_0 + \sum_{i=1}^{n} \cosh^{i-1}(w)\left[A_i \sinh(w) + B_i \cosh(w)\right], \tag{2.1}$$

Where n is an integer which is determined by balancing the highest order derivative term with the highest order nonlinear term in the given(1)-(2) [5], and $A_0 = A_0(x), ..., A_n = A_n(x),\ B_1 = B_1(x), ..., B_n = B_n(x), w = w(\mu), \mu = \alpha x + p + q$ Are all differentiable function.

satisfies ω

$$\left(\frac{dw}{dx}\right)^2 = \sinh^2(w(\mu)) + c, \tag{2.2}$$

Or in another form

$$\frac{d^2w}{dx^2} = \sinh(w)\cosh(w), \tag{2.3}$$

Where $c = 1 - m^2$ and m is the modulus of Jacobi elliptic function. Equation (2.2) has the following solution:

$$\sinh(w) = cs(\mu, m) = \frac{cn(\mu, m)}{sn(\mu, m)}, \tag{2.4}$$

$$\cosh(w) = ns(\mu, m) = \frac{1}{sn(\mu, m)}, \tag{2.5}$$

Where $sn(\mu, m)$, $cn(\mu, m)$ are jacobian elliptic sine function and the jacobian elliptic cosine function respectively.we can also seek (2+1)-dimensional breaking soliton equation s solution in the up form where $w = a(\xi)$, $\xi = k(x + \alpha y - \beta t)$ where ξ a real parameter and k, α, β are constant.

3 the application of methods

3.1 the application of sinh-cosh method

Inother to solve (1) and (2) by using our method , we first reduce (1) and (2) to a differential equations .we make transformations

$$u(x, y, t) = u(\mu), v(x, yt) = v(\mu), \tag{3.1}$$

$$\mu = \alpha x + p + q, \tag{3.2}$$

Where α is a nonzero constant and p is the function of, q is a function t. The substitutions of (8) and (9) into (1) and (2) yields

$$q'(t)u' - b\alpha^2 p'\beta(y)u''' + 4b\alpha u'v + 4b\alpha uv' = 0, \tag{3.3}$$
$$\alpha v' - p'(y)u' = 0, \tag{3.4}$$

And integrating yields, (10) and (11)

$$q'(t) - b\alpha^2 p'\beta(y)u'' + 4b\alpha uv = 0, \tag{3.5}$$
$$\alpha v - p'(y)u = 0, \tag{3.6}$$

The substitutions of $v = \dfrac{p'(y)}{\alpha}u$ into (12) yields

$$q'(t)u - b\alpha^2 p'\beta(y)u'' + 4bp'u^2 = 0. \tag{3.7}$$

Balancing u^2 with u'' then gives $n = 2$.
According to method we assume that (14) has the solution

$$u(x) = A_0 + A_1\sinh(w) + B_1\cosh(w) + A_2\sinh(w)\cosh(w) + B_2\cosh^2(w), \tag{3.8}$$

Substituting (15) into (14) along with (4) and (5), yields a differential equation about setting the coefficients of $\sinh^i(w)\cosh^j(w)(\sinh^2(w) + c)^{\frac{k}{2}}, i = 1, 2, ...; j = 0, 1; k = 0, 1$.
$\sinh^i(w)\cosh^j(w)(\sinh^2(w) + c)^{\frac{k}{2}}, i = 1, 2, ...; j = 0, 1; k = 0, 1$ tozero, we

get the overdetermined equations:

$$
\begin{aligned}
&q'(t)A_0 + q'(t)B_2 + 2b\alpha^2 p'(y)B_2 c + 4bp'(y)A_0^2 + 4bp'(y)B_1^2 + 4bp'(y)B_2^2 \\
&+ 8bp'(y)A_0B_2 = 0, \\
&q'(t)A_1 - b\alpha^2 p'(y)A_1 - b\alpha^2 p'(y)A_1 c + 8bp'(y)A_0A_1 + 8bp'(y)A_1B_2 \\
&+ 8bp'(y)B_1A_2 = 0, \\
&q'(t)A_1 - b\alpha^2 p'(y)B_1 c + 8bp'(y)A_0B_1 \\
&+ 8bp'(y)B_1B_2 = 0, \\
&q'(t)A_2 - 4\alpha^2 bp'(y)A_2 c - b\alpha^2 p'(y)A_2 + 8bp'(y)A_1B_1 + 8bp'(y)A_2B_2 \\
&+ 8bp'(y)A_0A_2 = 0, \\
&q'(t)B_2 - 4\alpha^2 bp'(y)A_2 c - 4\alpha^2 bp'(y)B_2 c + 4bp'(y)A_1^2 + 4bp'(y)A_2^2 \\
&+ 8bp'(y)B_2^2 \\
&+ 8bp'(y)A_0B_2 + 4bp'(y)B_1^2 = 0, \\
&8bp'(y)B_1A_2 - 2b\alpha^2 p'(y)A_1 + 8bp'(y)A_1B_2 = 0, \\
&8bp'(y)A_1A_2 + 8bp'(y)B_1B_2 - 2b\alpha^2 p'(y)B_1 = 0, \\
&- 6b\alpha^2 p'(y)A_2 + 8bp'(y)A_2B_2 = 0, \\
&- 6b\alpha^2 p'(y)B_2 + 4bp'(y)A_2^2 + 4bp'(y)B_2^2 = 0.
\end{aligned}
$$

Solving equations with Maple, we derive the solutions of the partial differential equations.

$$
\begin{aligned}
&A_0 = \frac{1}{2}\alpha^2\sqrt{\frac{1}{16} - c + c^2} - \frac{5}{8}\alpha^2 + \frac{1}{2}\alpha^2 c, A_1 = 0, B_1 = 0, \\
&A_2 = -\frac{3}{4}\alpha^2, B_2 = \frac{3}{4}\alpha^2, p = \frac{1}{\alpha^2}y, q = \left[-4b\sqrt{\frac{1}{16} - c + c^2}\right] t
\end{aligned}
\tag{3.9}
$$

We have obtained solutions of (12) and (13) if $v = \frac{1}{\alpha}p'(y)$, these solutions are

$$\begin{cases} u_{11} = \frac{1}{2}\alpha^2\sqrt{\frac{1}{16} - c + c^2} - \frac{5}{8}\alpha^2 + \frac{1}{2}\alpha^2 c \\ \quad -\frac{3}{4}\alpha^2 cs\left(\alpha x + \frac{1}{\alpha^2}y + \left[-4b\sqrt{\frac{1}{16} - c + c^2}\right]t, m\right) \\ \quad +ns\left(\alpha x + \frac{1}{\alpha^2}y + \left[-4b\sqrt{\frac{1}{16} - c + c^2}\right]t, m\right) \\ \quad +\frac{3}{4}\alpha^2 ns^2\left(\alpha x + \frac{1}{\alpha^2}y + \left[-4b\sqrt{\frac{1}{16} - c + c^2}\right]t, m\right) \\ v_{11} = \frac{1}{2\alpha}\sqrt{\frac{1}{16} - c + c^2} - \frac{5}{8\alpha} + \frac{1}{2\alpha}c \\ \quad -\frac{3}{4\alpha}cs\left(\alpha x + \frac{1}{\alpha^2}y + \left[-4b\sqrt{\frac{1}{16} - c + c^2}\right]t, m\right) \\ \quad +ns\left(\alpha x + \frac{1}{\alpha^2}y + \left[-4b\sqrt{\frac{1}{16} - c + c^2}\right]t, m\right) \\ \quad +\frac{3}{4\alpha}ns^2\left(\alpha x + \frac{1}{\alpha^2}y + \left[-4b\sqrt{\frac{1}{16} - c + c^2}\right]t, m\right) \end{cases} \tag{3.10}$$

When $m \to 1, cs(\mu, m) \to csch(\mu)$ and $ns(\mu, m) \to \coth(\mu), c \to 0$ so obtain the following soliton solutions of (1) and(2). (figure 1)

$$\begin{cases} u_{12} = -\frac{1}{2}\alpha^2 - \frac{3}{4}\alpha^2 csch\left(\alpha x + \frac{1}{\alpha^2}y - bt\right)\coth\left(\alpha x + \frac{1}{\alpha^2}y - bt\right) \\ \quad +\frac{3}{4}\alpha^2\coth^2\left(\alpha x + \frac{1}{\alpha^2}y - bt\right) \\ v_{12} = -\frac{1}{2\alpha} - \frac{3}{4\alpha}csch\left(\alpha x + \frac{1}{\alpha^2}y - bt\right)\coth\left(\alpha x + \frac{1}{\alpha^2}y - bt\right) \\ \quad +\frac{3}{4}\alpha^2\coth^2\left(\alpha x + \frac{1}{\alpha^2}y - bt\right) \end{cases} \tag{3.11}$$

When $m \to 0, cs(\mu, m) \to \coth(\mu)$ and $ns(\mu, m) \to \csc(\mu), c \to 1$ so obtain the following trigonometric function solutions of (1) and (2). (figure

2)

$$\begin{cases} u_{13} = -\dfrac{3}{4}\alpha^2 \coth\left(\alpha x + \dfrac{1}{\alpha^2}y - bt\right) \csc\left(\alpha x + \dfrac{1}{\alpha^2}y - bt\right) \\ \qquad +\dfrac{3}{4}\alpha^2 \csc^2\left(\alpha x + \dfrac{1}{\alpha^2}y - bt\right), \\ v_{13} = -\dfrac{3}{4\alpha} \coth\left(\alpha x + \dfrac{1}{\alpha^2}y - bt\right) \csc\left(\alpha x + \dfrac{1}{\alpha^2}y - bt\right) \\ \qquad +\dfrac{3}{4\alpha} \csc^2\left(\alpha x + \dfrac{1}{\alpha^2}y - bt\right) \end{cases} \tag{3.12}$$

3.2 the application of sinh-Gordon expansion method

In other to solve (1) and (2) by using our method, we first reduce (1) and (2) to differential equations. we make transformations

$$u(x, y, t) = u(\xi), v(x, y, t) = v(\xi) \tag{3.13}$$

$$\xi = k(x + \alpha y - \beta t) \tag{3.14}$$

Where ξ is real parameters and k, α, β are constant. The substitutions of (20) and (21) into (1) and (2) yields

$$-k\beta u' - bk^3\alpha u''' + 4bku'v + 4bkuv' = 0, \tag{3.15}$$

$$kv' - k\alpha u' = 0, \tag{3.16}$$

And integrating yields, (22) and (23)

$$-k\beta u - bk^3\alpha u''' + 4bkuv = 0, \tag{3.17}$$

$$kv - k\alpha u = 0, \tag{3.18}$$

The substitutions of $v = \alpha u$ into (24) yields

$$-k\beta u - bk^3\alpha u'' + 4bk\alpha u^2 = 0. \tag{3.19}$$

Balancing u^2 with u'' the gives $n = 2$.
According to method we assume that (26) has the solution

$$u(\xi) = A_0 + A_1 \sinh(w) + B_1 \cosh(w) + A_2 \sinh(w)\cosh(w) + B_2 \cosh^2(w), \tag{3.20}$$

Subtituting (27) and (26) along with (4) and (5), yields a hyperbolic polynomial about

$$w'^s \sinh^i(w)\cosh^j(w) \quad (i = 0, 1; s = 0, 1; j = 0, 1, 2, ...). \tag{3.21}$$

Setting the coefficients of (28) to zero, we get the following of equations:

$$\begin{aligned}
&-k\beta A_0 - 2bk^3\alpha B_2 + 2bk^3\alpha B_2 c + 4bk\alpha A_0^2 - 4bk\alpha A_1^2 = 0,\\
&bk^3\alpha A_1 - bk^3\alpha A_1 c + 8bk\alpha A_0 A_1 - k\beta A_1 = 0,\\
&-k\beta B_1 + 2bk^3\alpha B_1 - bk^3\alpha B_1 c + 8bk\alpha A_0 B_1 - 8bk\alpha A_1 A_2 = 0,\\
&-k\beta A_1 + 5bk^3\alpha A_2 - 4bk^3\alpha A_2 c + 8bk\alpha A_1 B_1 - 8bk\alpha A_0 A_2 = 0,\\
&-K\beta B_2 + 8bk^3\alpha B_2 - 4bk^3\alpha B_2 c + 4bk\alpha A_1^2 - 4bk\alpha A_2^2\\
&+ 8bk\alpha A_0 B_2 + 4bk\alpha B_1^2 = 0,\\
&-2bk^3\alpha A_1 + 8bk\alpha A_1 B_2 + 8bk\alpha B_1 A_2 = 0,\\
&-2bk^3\alpha B_1 + 8bk\alpha A_1 A_2 + 8bk\alpha B_1 B_2 = 0,\\
&-6bk^3\alpha A_2 + 8bk\alpha A_2 B_2 = 0,\\
&-6bk^3\alpha B_2 + 4bk\alpha A_2^2 + 4bk\alpha B_2^2 = 0.
\end{aligned}$$

Solving equations with Maple, we derive the following solutions :

$$\begin{aligned}
&A_0 = \frac{\beta}{8b\alpha} - \frac{5}{8}k^2 + \frac{1}{2}k^2 c, A_1 = 0, B_1 = 0,\\
&A_2 = -\frac{3}{4}\alpha k^2, B_2 = \frac{3}{4}k^2, \beta = -4b\alpha k^2\sqrt{\frac{1}{16} + c^2 - c}
\end{aligned} \tag{3.22}$$

We have obtained solutions of (24) and (25) if $v = \alpha u$, these solutions are

$$\begin{cases} u_{21} = \left(\frac{\beta}{8b\alpha} - \frac{5}{8}k^2 + \frac{1}{2}k^2\right) - \frac{3}{4}k^2 cs\left(k(x+\alpha y-\beta t), m\right) \\ \qquad ns\left(k(x+\alpha y-\beta t), m\right) + \frac{3}{4}k^2 ns^2\left(k(x+\alpha y-\beta t), m\right), \\ v_{21} = \left(\frac{\beta}{8b\alpha} - \frac{5}{8}k^2 + \frac{1}{2}k^2\right) - \frac{3}{4}k^2 \alpha cs\left(k(x+\alpha y-\beta t), m\right) \\ \qquad ns\left(k(x+\alpha y-\beta t), m\right) + \frac{3}{4}k^2 \alpha ns^2\left(k(x+\alpha y-\beta t), m\right) \end{cases} \tag{3.23}$$

When $m \to 1, cs(\xi, m) \to \csc h(\xi)$ and $ns(\xi, m) \to \coth(\xi), c \to 0.$ so we obtain the following soliton solutions of (1) and (2). (figure 3)

$$\begin{cases} u_{22} = -\frac{3}{4}k^2 - \frac{3}{4}k^2 csch\left(k(x+\alpha y+b\alpha k^2 t)\right)\coth\left(k(x+\alpha y+b\alpha k^2 t)\right) \\ \qquad +\frac{3}{4}k^2 \coth^2\left(k(x+\alpha y+b\alpha k^2 t)\right), \\ v_{22} = -\frac{3}{4}k^2\alpha - \frac{3}{4}k^2 \alpha csch\left(k(x+\alpha y+b\alpha k^2 t)\right)\coth\left(k(x+\alpha y+b\alpha k^2 t)\right) \\ \qquad +\frac{3}{4}k^2\alpha \coth^2\left(k(x+\alpha y+b\alpha k^2 t)\right) \end{cases} \tag{3.24}$$

when $m \to 0, cs(\xi, m) \to \coth(\xi)$ and $ns(\xi, m) \to \csc(\xi), c \to 1$ so we obtain the following trigonometric function solutions of (1) and (2)

$$\begin{cases} u_{23} = -\frac{1}{4}k^2 - \frac{3}{4}k^2 \coth\left(k(x+\alpha y+b\alpha k^2 t)\right)\csc\left(k(x+\alpha y+b\alpha k^2 t)\right) \\ \qquad +\frac{3}{4}k^2 \csc^2\left(k(x+\alpha y+b\alpha k^2 t)\right), \\ v_{23} = -\frac{1}{4}k^2\alpha - \frac{3}{4}k^2 \alpha \coth\left(k(x+\alpha y+b\alpha k^2 t)\right)\csc\left(k(x+\alpha y+b\alpha k^2 t)\right) \\ \qquad +\frac{3}{4}k^2\alpha \csc^2\left(k(x+\alpha y+b\alpha k^2 t)\right) \end{cases} \tag{3.25}$$

Some of the properties of these solutions of (1) and (2) are shown by means of figures as follows: figure1 and figure 2 and figure 3 show the properties of u_{12}, v_{12} and u_{13}, v_{13} and u_{22}, v_{22}, respectively, where we select parameters as follows:

$$k = \frac{1}{2}, \alpha = \frac{1}{2}, b = 4$$

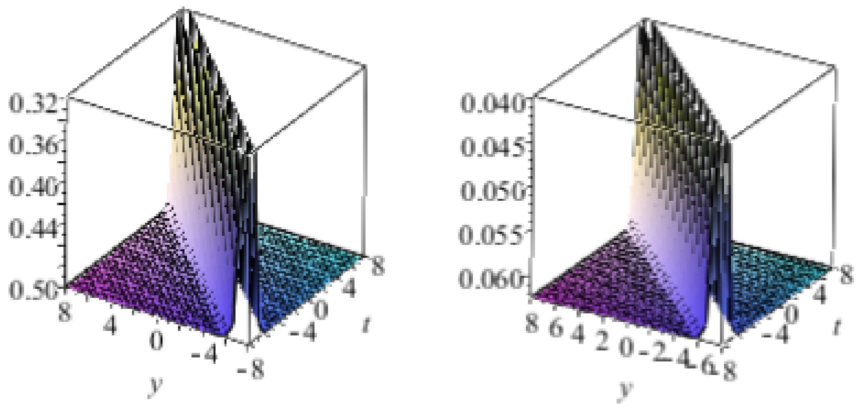

Fig. 1. the soliton solutions u_{12}, v_{12} of the (2+1)-dimensional breaking soliton equation are shown at $x = 0$.

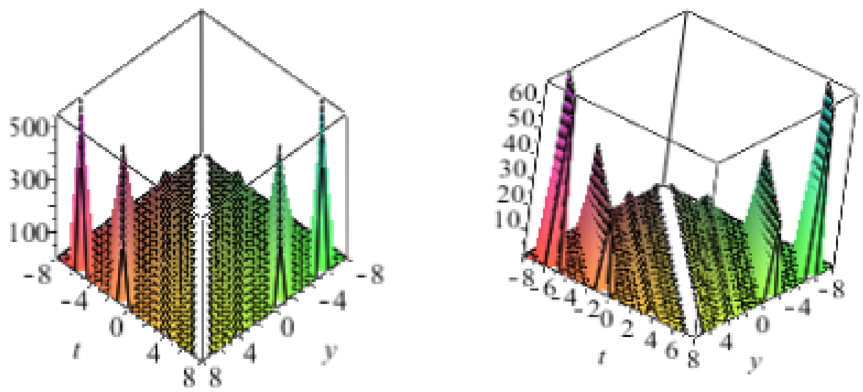

Fig. 2. trigonometric function solutions u_{13}, v_{13} of the (2+1)-dimensional breaking soliton equation are shown at $x = 0$.

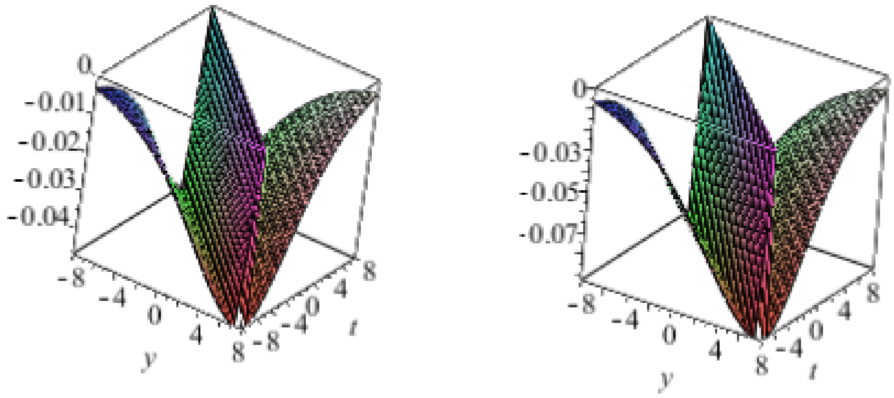

Fig. 3. the soliton solutions u_{22}, v_{22} of the (2+1)-dimensional breaking soliton equation are shown at $x = 0$.

In summary, we have the sinh-Gordon expansion method and sinh-cosh method to the (2+1)-dimensional breaking soliton equation. As a result, Jacobi elliptic function solutions are obtained. When $m \to 1$, we get the soliton solutions; while when $m \to 0$, we get the trigonometric function solutions.

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