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The Iterative Method for Solving Non-Linear Equations

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Abstract

In this paper, we have combined the ideas of the False Position (FP) and Artificial Bee Colony (ABC) algorithms to find a fast and novel method for solving nonlinear equations. Additionally, to illustrate the efficiency of the proposed method, several benchmark functions are solved and compared with other methods such as ABC, PSO and GA.

Key words: Root-finding method, False position method, Artificial bee colony algorithm.

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1 Introduction

Solving nonlinear equations, which is very important in engineering and science fields, is usually a difficult task. Najmeddin Ahmad at.all introduced an iterative method to solve nonlinear equations using Simpson method[1].

Numerous modern heuristic algorithms have been introduced to solve combinatorial and numerical optimization problems [2]. These algorithms can be classified into several categories depending on the criteria being considered, such as population-based, iterative based, stochastic and deterministic.

Population-based algorithms can be classified by the nature of phenomenon simulated into two groups: evolutionary algorithms (EA) and swarm intelligence based algorithms[3,4].

Artificial bee colony (ABC) is a member of swarm intelligence and defined algorithms by Dervis Karaboga in 2005[5] and motivated by the intelligent behavior of honey bees [6,7]. It is as simple as Particle Swarm Optimization (PSO) and Differential evolution (DE) algorithms[9], Genetic algorithm (GA)[11], biogeography based optimization (BBO), and uses only common control parameters such as colony size and maximum cycle number.

Development of the ABC algorithm for solving generalized assignment problem, which is known as NP-hard problem, is presented in detail along with some comparisons[10,12,13]. Sources, depending on their own experience and their nest mates, adjust their positions. Moreover, FP has become a root-finding algorithms which combines features from the Bisection and the secant method. It is a very simple and robust method.

In this paper, we propose an iterative method which uses the ideas of the FP and ABC algorithms. We, then, show its performance and compare it with other methods. In Section 2, the ABC and False position algorithms are described. In Section 3, we explain and discuss about our method. In Section 4, we compare accuracy and complexity of proposed method with other algorithms on four benchmark functions, followed by conclusions in section 5.

2 Preliminaries

In this section, we present the ABC algorithm and False position method.

2.1 Artificial bee colony algorithm

Several approaches have been proposed to model the specific intelligent behaviors of honey bee swarms, but Real-world problems usually have many design parameters that should be considered in the design process. Thus, Algorithms that are not robust to large-scale problems cannot preserve their effectiveness against high dimensionality.

In the ABC algorithm, the colony of artificial bees contains three groups of bees: employed bees, onlookers and scouts. Short pseudo-code of the ABC algorithm is given in figure 1.

There are three main control parameters that are used in the ABC: the number of food sources which is equal to the number of employed or onlooker bees (S_N), the value of limit, the maximum cycle number (for more explaination see [7,5,2]).

2.2 False position algorithm

The False position algorithm is a numerical method for estimating the roots of the real-valued problem $f(x) = 0, x \in [a, b]$. Assume that $f(x)$ is continuous and $f(a)$ and $f(b)$ have opposite signs. We find the point $(c, 0)$ where the secant line L joining the points $(a, f(a))$ and $(b, f(b))$ crosses the x-axis, that is the better approximation (the bisection method used

(1) Initialize the population of solutions,

(2) Evaluate the population,

(3) Produce new solutions for the employed bees,

(4) Apply the greedy selection process,

(5) Calculate the probability values,

(6) Produce the new solutions for the onlookers,

(7) Apply the greedy selection process,

(8) Determine the abandoned solution for the scout,
and replace it with a new randomly produced solution,

(9) Memorize the best solution achieved so far.

Fig. 1. Artificial Bee Colony Algorithm.

the midpoint of the interval $[a, b]$)as the next iterate. Short pseudo-code of the False position algorithm is given in figure 3:

3 The proposed method

In the ABC algorithm, while onlookers and employed bees carry out the exploitation process in the search space, the scouts control the exploration process. An important difference between ABC and other swarm intelligence algorithms is that in the ABC algorithm the possible solutions represent food sources (flowers), not individuals (honeybees)[14]. In other algorithms, like PSO, each possible solution represents an individual of the swarm.

In the ABC algorithm the fitness of a food source is given by the value of the objective function of the problem. In this section, by combining the ideas of the FP and ABC algorithms (FP-ABC), we propose a method to solve the non-linear equation $f(x) = 0, x \in [a, b]$ where $f(a)f(b) < 0$. To this end, we consider following steps:

(1) Let $f(x), x \in [a_0, b_0]$ is continues function,

(2) $k = 0$, if $f(a_0).f(b_0) < 0$, then

   (a) Find the point $(c_0, 0)$ where the secant line L joining the points $(a_0, f(a_0))$ and $(b_0, f(b_0))$ crosses the x-axis, that is the better approximation, as follows:

      (i) $m = \frac{f(b_0)-f(a_0)}{b_0-a_0}$,

      (ii) where the points $(a_0, f(a_0))$ and $(b_0, f(b_0))$ are used, then $m = \frac{0-f(b_0)}{c_0-b_0}$,

      (iii) where the points $(c_0, 0)$ and $(b_0, f(b_0))$ are used, we have $\frac{f(b_0)-f(a_0)}{b_0-a_0} = \frac{0-f(b_0)}{c_0-b_0}$,

(3) if $f(a_0)$ and $f(c_0)$ have opposite signs, a zero lies in $[a_0, c_0]$
$k = k + 1, [a_k, b_k] = [a_0, c_0]$.

(4) if $f(c_0)$ and $f(b_0)$ have opposite signs, a zero lies in $[c_0, b_0]$
$k = k + 1, [a_k, b_k] = [c_0, b_0]$.

(5) if $f(c_0) = 0$, then the zero is c_0.

(6) Convergence condition: At each step the approximation of the zero r is $c_k = b_k - \frac{f(b_k)(b_k-a_k)}{f(b_k)-f(a_k)}$ and it can be proved that the sequence c_k will converge to r.

   (a) if $(f(c_{k-1}).f(a_{k-1})) < 0)$,then $b_k = c_{k-1}$.

   (b) else $a_k = c_{k-1}$,

   (c) end.

   (d) If $abs(b_k - a_k) \leq \varepsilon$, then $r = c_k$, Stop.

Fig. 2. The False Position Algorithm.

4 Numerical Examples

In this section, we solve the following benchmark functions with our proposed method. (FP-ABC), where at least one root of the function lies in the given domain.

$$f_1(x) = x^2/4000 - cos(x) + 1 - x = 0,\ x \in [-20, 20]$$

$$f_2(x) = 10 + x^2 - x - 10cos(2\pi x) = 0,\ x \in [-20, 1)$$

(1) Initial population of possible solutions $x_i \in [a, b], i = 1, 2, ..., n$, randomly generated by ABC algorithm,
(2) chooses a food source depending on the probability value associated with that food source
(3) By using False position method(FP),
if $sign(f(x_i)f(a)) < 0, i = 1, 2, ..., n$, then

$$\alpha_i = x_i - \frac{f(x_i)(x_i - a)}{f(x_i) - f(a)}, (else \quad \alpha_i = x_i - \frac{f(x_i)(b - x_i)}{f(b) - f(x_i)})$$

and choose α such that $|f(\alpha)| = min|f(\alpha_i)|, i = 1, 2, ..., n(f(\alpha)$ is nearest to zero or we can say α is better approximation of the root and update interval $[a, b]$,
(4) ABC algorithm searches around the best approximation of the root in step 2, construct some neighbors and with comparison value of f in these nodes and $f(\alpha)$, better approximation of the root will be achieved,
(5) If the best solution determines with arbitrary accuracy, algorithm will be stopped, otherwise, with ABC algorithm new population will be generated and algorithm will go to step 2.

So that, Pseudo-code of the proposed algorithm is given in figure **??**.
(1) Initialize the population of optimal solution $X_i = (x_{i1}, x_{i2}, ..., x_{in}) \in R^n$. Let X_i represent the i^{th} food source in the population, and each food source is generated by ABC algorithm as follows:

$$x_{ij} = x_{minj} + rand(0, 1)(x_{maxj} - x_{minj}), \ j = 1, 2, ..., n$$

(2) Repeat UNTIL (requirements are met)
(3) Construct initial employed bee colony solutions by using False position method as follows:

$$X_{i+1}^{(k+1)} = X_{i+1}^{(k)} - \frac{f(X_{i+1}^{(k)})(X_{i+1}^{(k)} - X_i^{(k)})}{f(X_{i+1}^{(k)}) - f(X_i^{(k)})},$$

$$X_i = [x_{i1}, x_{i2}, ...x_{in}], i = 1, 2, ..., k = 1, 2, ...$$

(4) Initial food sources are produced for all employed.
(5) Repeat UNTIL (requirements are met)
(6) Each employed bee goes to a food source in her memory and determines a neighbor source, then evaluates its nectar amount and dances in the hive.
(7) Each onlooker watches the dance of employed bees and chooses one of their sources depending on the dances, and then goes to that source. After choosing a neighbor around that, she evaluates its nectar amount as follows.
   (a) $ObjVallb = f(X_i)$,
   (b) $ObjValub = f(X_j), i \neq j$.
   (c) if $((ObjVallb).(ObjValub) < 0)$, then
   $ObjValGlPar = f(c)$(compute from step 3),
   (d) if $(sign(ObjValGlPar) \neq sign(ObjVallb))$, then
   $ABCOpts.ub = GlobalParams$,
   (e) $ObjValub = f(X_j)$,
   (f) $GlobalParams = X_j - (ObjValub \times (X_j - X_i)/(ObjValub - ObjVallb))$,
(8) Abandoned food sources are determined and are replaced with the new food sources discovered by scouts.
(9) The best food source found so far is registered.

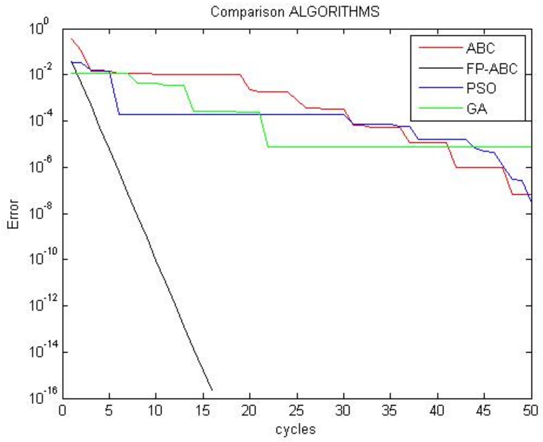
Fig. 3. The Proposed algorithm(FP-ABC)

$$f_3 = 20 + e - 20e^{-0.2\sqrt{1/x^2}} - e^{1/cos(2\pi x)} - x = 0, \ x \in [1, 21]$$

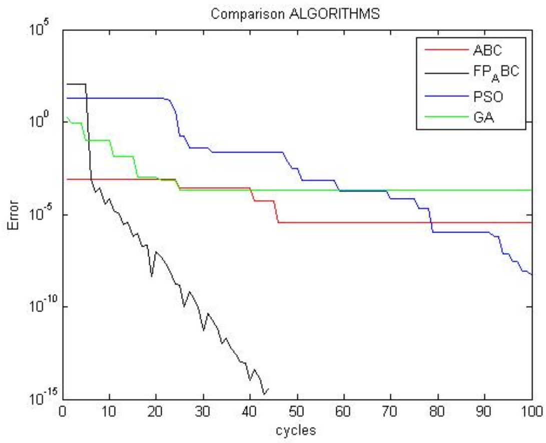
$$f_4 = 418.9829 - xsin\sqrt{x} - x = 0, \ x \in [400, 500]$$

It can be seen that $f_1(0) = 0, f_2(0) = 0$. By plotting the graphs of f_3 and f_4, it is observed that f_3 has a root very close to 20 and f_4 has a root very close to 490. we show details of solving function $f_1(x) = 0$ as follows:
It is clear that $f_1(-20).f_1(20) < 0$ then

$$\exists\alpha \in [-20, 20] : \ f_1(\alpha) = 0.$$

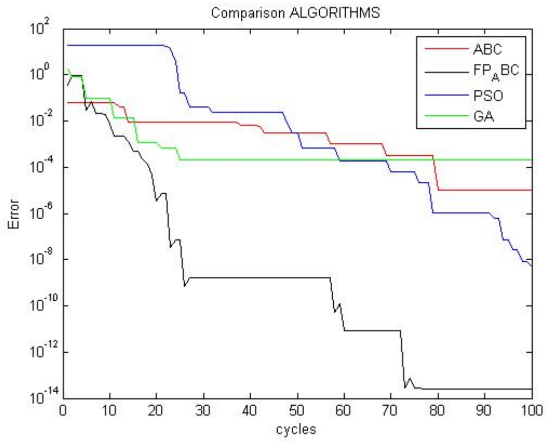


(a) result of proposed method and ABC, GA and PSO algorithms to solve the problem of function f_1

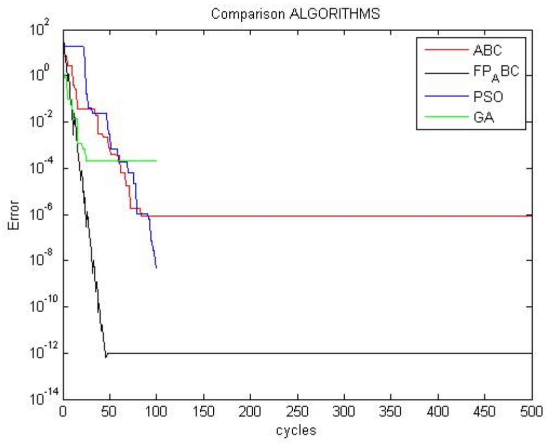


(b) The result of proposed method and ABC, GA and PSO algorithms to solve the problem of function f_2

At first, initial value of α generated randomly with ABC algorithm (x_0, $I_0 = [-20, 20]$), and then with proposed algorithm, we obtained new approximate value of the root($c_1 \in I_1 = [a_1, b_1] \subseteq [-20, 20]$). We will reach to best value of root with arbitrary accuracy $|b_k - a_k| = 1.e^{-t}$, $t >> 1$ after some iteration of our algorithm. Results for the four functions are shown in Table 4 and in Figures: **??**,5,6 and 7. The table and the figures



(a) The result of proposed method and ABC, GA and PSO algorithms to solve the problem of function f_3



(b) The result of proposed method and ABC, GA and PSO algorithms to solve the problem of function f_4

show that proposed method is too much faster than other algorithms and accuracy is very good respect to other methods.

Method		f_1	f_2	f_3	f_4
GA	Root	-7.3765e-6	0	20.09812	490.0312407
	Mean	0.0021087	1.2478e-005	0.04974	0.3874
	SD	0.003990	1.639e-005	0.2264	3.1252
PSO	Root	2.64455e-008	0	20	490.031
	Mean	0.002341	0.0023	4.725e-009	1.23482e-007
	SD	0.00735	0.0152	0	0
ABC	Root	-5.25832e-017	1.7034e-005	20.098	490.031
	Mean	0	3.54996e-006	1.038e-005	8.33846e-007
	SD	0	0	0	0
FP-ABC	Root	-1.4478e-015	8.55087e-016	19.92	490.031
	Mean	1.44329e-015	0	7.0658e-014	1.02318e-012
	SD	0	0	0	0

Table 4 : The result of proposed method and ABC, GA and PSO algorithms to solve the problem of functions f_1, f_2, f_3 and f_4

5 Conclusions and Suggestions

In this paper, we presented a novel approximation method to find the real single root α of a function $f(x) = 0$, by using Artificial Bee Colony Algorithm and False Position method. As, with the help of examples including the benchmark functions, we illustrate that our method outperforms most of the existing algorithms like the classical GA, PSO and ABC as well most of the time.

References

[1] N. Ahmad and V. P. Singh. A New Iterative Method for Solving Non Linear Equations Using Simpson Method, International Journal of Mathematics And Its Applications. Vol. 5, Issue 4-B, pp. 189-193, 2017.

[2] D. T. Pham and D. Karaboga. Intelligent Optimisation Techniques. Springer, London, 2000.

[3] Xin-She Yang. Nature-Inspired Metaheuristic Algorithms, Luniver Press, 2008

[4] J. Dreo and P. Siarry. Alain Petrowski and Eric Taillard, Metaheuristics for Hard Optimization, Springer Berlin Heidelberg, pp. 1-19, 2006.

[5] D. Karaboga. An Idea Based On Honey Bee Swarm for Numerical Optimization. Technical Report-TR06, Erciyes University, Engineering Faculty, Computer Engineering Department, 2005.

[6] D. Karaboga and B. Basturk. A powerful and efficient algorithm for numerical function optimization: Artificial Bee Colony (ABC) algorithm. Journal of Global Optimization,Vol. 39 , pp.459-471, 2007.

[7] D. Karaboga and B. Basturk. On the performance of Artificial Bee Colony (ABC) algorithm. Applied Soft Computing. Vol.8, pp. 687-697, 2008.

[8] A. Baykaso. L. zbakr and P.Tapkan. Artificial Bee Colony Algorithm and Its Application to Generalized Assignment Problem. Swarm Intelligence, Focus on Ant and Particle Swarm Optimization, Book edited by: Felix T. S. Chan and Manoj Kumar Tiwari, ISBN 978-3-902613-09-7, Itech Education and Publishing, Vienna, Austria, pp. 532, December 2007

[9] R. Storn. K. Price. Differential evolution A simple and efficient heuristic for global optimization over continuous spaces. Journal of Global Optimization, Vol.23, pp.689-694, 2010.

[10] P. Mansouri. B. Asady and N.Gupta. A Novel Iteration Method for solve Hard Problems(Nonlinear Equations) with Artificial Bee Colony algorithm. World Academy of Science, Engineering and Technology, Vol. 59, pp. 594-596, 2011.

[11] J.H.Holland. Adaptation in Natural and Artificial Systems. University of Michigan Press, Ann Arbor, MI,1975.

[12] B. Asady.P.Mansouri and N. Gupta. The modify version of artificial bee colony algorithm to solve real optimization problems. International Journal of Electrical and Computer Engineering (IJECE) Vol.2, No.4, pp. 473-480, 2012.

[13] N. Stanarevic. M.Tuba and N.Bacanin. Modified artificial bee colony algorithm for constrained problems optimization, International journal of mathematical model and methods in applied sciences, Issue 3, Vol. 5, 2011.

[14] D.Karaboga.B.Akay
and C.Ozturk. Artificial Bee Colony (ABC)Optimization Algorithm for Training Feed-Forward Neural Networks,Modeling Decisions for Artificial Intelligence, Vol. 4617, No.0302-9743, pp. 318-329, 2007.

[15] G. Roussy, J. A. Pearcy, *Foundations and industrial applications of microwaves and radio frequency fields*, John Wiley, New York, 1995.

[16] R. A. Van Gorder, K. Vajravelu, A variational formulation of the Nagumo reaction-diffusion equation and the Nagumo telegraph equation, *Nonlinear Analysis: Real World Applications 4*, pp.2957-2962, 2010.