



Solution and stability analysis of coupled nonlinear Schrodinger equations

M. Shahmansouri ^{a,*} B. Farokhi ^b

^a*Physics Department, Islamic Azad University, Arak Branch, Arak P.O. Box 38135-567, Iran.*

^b*Physics Department, Arak University, Arak P.O. Box 38156-879, Iran.*

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Abstract

We consider a new type of integrable coupled nonlinear Schrodinger (CNLS) equations proposed by our self [submitted to *Phys. Plasmas* (2011)]. The explicit form of soliton solutions are derived using the Hirota's bilinear method. We show that the parameters in the CNLS equations only determine the regions for the existence of bright and dark soliton solutions. Finally, through the linear stability analysis, the modulational instability condition is given.

Key words: Nonlinear equations; Nonlinear Schrödinger equation; Stability.

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* Corresponding author's E-mail: mshmansouri@gmail.com(M. Shahmansouri), Tel: +988613670017; Fax: +988613670017

1 Introduction

The nonlinear Schrodinger equation has appeared widely in study of dynamical behavior of dusty plasma crystals [1], in nonlinear plasma [2],[3], in laser plasma interaction [4], in many body systems [5], in nonlinear optics [6], and in optical communications [7]. The coefficients which appears in nonlinear Schrodinger equation determines the stability/instability region of propagating wave. When two waves interact nonlinearly with each other, the nonlinearity provides a coupling between waves. The coupling may leads to change of stability condition. The instability growth rate associated with a single unstable wave is increased by the presence of a second wave. A wave that is stable in isolation can be destabilized by the presence of a second unstable wave. The existence and uniqueness of global solutions for rough data of the nonlinear Schrodinger equation coupled with the nonlinear Klein-Gordon equation (NS-KG) with quadratic coupling and cubic auto-interactions have been proved recently in [8]. Recently modulational instability of the NS-KG system has investigated [9]. On the integrability of these coupled systems, Manakov [?],[11] showed first that if the coupling is only through cross-phase modulation (XPM), and the XPM coefficient is equal to the self-phase modulation (SPM) coefficient, then this system (now called the Manakov system) is integrable. Multisoliton solutions in the Manakov system have also been extensively investigated by the inverse scattering method and the Hirota method [10]-[14], and an interesting phenomenon of polarization rotation after collision has been found. Later studies revealed that when the XPM coefficient is opposite of the SPM coefficient, the system is still integrable [15]-[17]. The two- and three-soliton solutions in this model were obtained by the Hirota method in Ref. 18, and a phenomenon of energy redistribution between solitons after collision was reported. More general forms of integrable coupled NLS equations were also mentioned in Refs. 15 and 17, but multisoliton solutions in such systems have not been examined yet. Given the importance of the general coupled NLS equations for various physical problems, these equations deserve careful and detailed investigations. Generally speaking, there exist five principally different cases of the coupling between two modes in a Kerr medium and corresponding vectorial solitary waves (for simplicity, we assume the case of the tempo-

ral solitons and focusing nonlinearity):

* Bright solitons, each in the mode with the anomalous dispersion (vector bright solitons) [e.g., Manakov [11], Christodoulides and Joseph [19] and Menyuk [20]];

* Bright soliton in the mode with the anomalous dispersion coupled to a dark soliton in the mode with normal dispersion (normal dark-bright pair) [e.g., Afanasjev et al. [21], Hong et al. [22] and Kivshar [23]];

* Bright soliton in the mode with normal dispersion exists due to mutual coupling to a dark soliton in the mode with anomalous dispersion (the so-called inverted dark-bright pair) [e.g., Trillo et al. [24] and Afanasjev et al. [21]];

* Two dark solitons, each in the mode with the normal dispersion (vector dark solitons) [e.g., Kivshar and Turitsyn [25] and Sheppard and Kivshar [26]];

* Bright pulse supported by a dark soliton, both modes are with the normal dispersion (soliton induced waveguides, in the linear limit, or dark-bright pair, in a nonlinear regime) [e.g., Christodoulides [19] and Sheppard and Kivshar [26]].

All these cases are described by two NLS equations, coupled due to crossphase modulation. These coupled equations become asymmetric for the interaction between envelopes of different carrier frequencies or some additional coupling terms, e.g. due to four-wave mixing effect, may appear. Park and Shin proved integrability of CNLS, include one coupling term due to four-wave mixing effect [18]. In this work, we consider CNLS equations include additional coupling terms in comparison with the known investigations, which appear as result of study of dynamical behavior of dusty plasma crystal. Then we investigate the stability condition of CNLS equations. A model to describe the interaction of two dust lattice modes is the CNLS equations. Nonlinearity is manifested via a slow modulation of the wave amplitudes, in time and space. The amplitude evolution is described by these equations[Wang [27]]

$$\frac{i\partial u_{11}}{\partial t} + P_1 \frac{\partial^2 u_{11}}{\partial x^2} + Q_{11} u_{11} |u_{11}|^2 + Q_{12} u_{11} |v_{11}|^2 + Q_{13} u_{11}^* v_{11}^2 + Q_{14} v_{11}^* u_{11}^2 = 0 \quad (1.1)$$

$$i\frac{\partial v_{11}}{\partial t} + P_1\frac{\partial^2 v_{11}}{\partial x^2} + Q_{21}v_{11}|v_{11}|^2 + Q_{22}v_{11}|u_{11}|^2 + Q_{23}v_{11}^*u_{11}^2 + Q_{24}u_{11}^*v_{11}^2 = 0 \quad (1.2)$$

where t and x represent the time and distance, respectively. Also P_i and Q_{ij} are the real constants, and indicates on dispersion and nonlinearity respectively. Integrability of this system can be investigated via Hirota's method, or existence of Lax pair.

2 Exact solution

The Hirota's method provides an efficient and straightforward procedure to obtain the soliton solutions of the NLEEs [27],[28]. When the bilinear is derived, one may get the soliton solutions, especially the multi-soliton solution directly through the truncated formal perturbation expansion at different levels [29]-[32]. In the following part, we will employ this method to construct the soliton solutions by means of symbolic computation. In order to construct Hirota's bilinear form of system (1-2), we consider the bilinear transformations

$$u_{11} = g/f, v_{11} = h/f \quad (2.1)$$

where g , h and f are functions of x and t , as g and h are complex functions and f is real function. The bilinear form of System (1-2) is obtained as follows:

$$(iD_t + P_1D_x^2)(g \cdot f) = 0, \quad (2.2)$$

$$(iD_t + P_2D_x^2)(h \cdot f) = 0, \quad (2.3)$$

$$(D_x^2(f \cdot f)) = (Q_{11}|g|^2 + Q_{12}|h|^2 + Q_{13}g^*h^2/g + Q_{14}gh^*)/P_1, \quad (2.4)$$

$$(D_x^2(f \cdot f)) = (Q_{21}|h|^2 + Q_{22}|g|^2 + Q_{23}h^*g^2/h + Q_{24}hg^*)/P_2, \quad (2.5)$$

So the left hand sides of Eqs.(3c) and (3d) become equal. Hence the right hand sides of these equations should also be equal which is true only when $Q_{11}/P_1 = Q_{22}/P_2 = q_1$, $Q_{12}/P_1 = Q_{21}/P_2 = q_2$,

$$Q_{13}/P_1 = Q_{24}/P_2 = q_3, Q_{14}/P_1 = Q_{23}/P_2 = q_4,$$

The above conditions can be obtained by equating the coefficients of $|g|^2$,

$|h|^2$, g^*h and gh^* respectively in Eqs.(3c) and (3d). One can easily check that Eq.(4) admits the already known integrability conditions. We believe that these conditions may be very useful for the experimental generation of solitons in nonlinear couplers. In order to obtain the soliton solutions, we are applying a perturbative technique by writing the variables g , h and f as a series in an arbitrary parameter ε

$$g = \varepsilon g_1 + \varepsilon^3 g_3 + \dots, h = \varepsilon h_1 + \varepsilon^3 h_3 + \dots, f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \dots, \quad (2.6)$$

So we can obtain the single and multiple- soliton solutions for system (1-2).

2.1.Single soliton solution

To obtain the single soliton solution (SSS), we assume solutions in a series form in ε such that:

$$g = \varepsilon g_1, h = \varepsilon h_1, f = 1 + \varepsilon^2 f_2, \quad (2.7)$$

We shall now substitute these expressions into Eqs. (3) and collect the contributions appearing in each power in ε . At first order we have:

$$(iD_t + P_1 D_x^2)(g_1 \cdot 1) = 0(iD_t + P_2 D_x^2)(h_1 \cdot 1) = 0 \quad (2.8)$$

In the second order:

$$2D_x^2(f_2 \cdot 1) = (q_1|g_1|^2 + q_2|h_1|^2 + q_3g_1^*h_1^2/g_1 + q_4g_1h_1^*) \quad (2.9)$$

In the third order:

$$(iD_t + P_1 D_x^2)(g_1 \cdot f_2) = 0(iD_t + P_2 D_x^2)(h_1 \cdot f_2) = 0 \quad (2.10)$$

In this case, the solutions are found to be

$$g_1 = ae^{\alpha_1}, h_1 = be^{\alpha_2 + \varphi}, \alpha_1 = k_1/\sqrt{-P_1x - ik_1^2t}, \alpha_2 = k_1/\sqrt{-P_2x - ik_1^2t} \quad (2.11)$$

If we suppose $P_1 = P_2 = P$, then we have $\alpha_1 = \alpha_2 = \alpha$, so for simplicity we apply this suppose in the following calculations. Substituting from Eq.(10) into Eq.(8), we can obtain

$$f_2 = \frac{1}{2(k_1 + k_1^*)^2} (q_1|a|^2 e^{\alpha + \alpha^*} + q_2|b|^2 e^{\alpha + \alpha^* + \varphi + \varphi^*} + q_3a^*b^2 e^{\alpha + \alpha^* + \varphi}/a + q_4ab^* e^{\alpha + \alpha^* + \varphi^*}) \quad (2.12)$$

Substituting Eqs. (10) and (11) into Eq.(2), through the Eq.(6) and after absorbing ε , the single soliton solution to be

$$u_{11} = \frac{(k + k^*)a}{\sqrt{2}(q_1|a|^2 + q_2|b|^2e^{\varphi+\varphi^*} + q_3a^*b^2e^\varphi \div a + q_4ab^*) \cosh(kx)} \exp(-ik^2t) \quad (2.13)$$

$$v_{11} = \frac{(k + k^*)b}{\sqrt{2}(q_1|a|^2 + q_2|b|^2e^{\varphi+\varphi^*} + q_3a^*b^2e^\varphi \div a + q_4ab^*) \cosh(kx)} \exp(-ik^2t) \quad (2.14)$$

In order to ensure a real dispersion relation and a continuation of the dark soliton regime, we must require that the dispersion parameter P be a negative constant.

2.2. Multiple soliton solution

To obtain the two soliton solution (TSS), we assume solutions in a series form in ε such that:

$$g = \varepsilon g_1 + \varepsilon^3 g_3, h = \varepsilon h_1 + \varepsilon^3 h_3, f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4, \quad (2.15)$$

Then following the procedure of previous section, two soliton solution of System (1) is presented as

$$u_{11} = \varepsilon g_1 + \varepsilon^3 g_3 \div 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4, v_{11} = \varepsilon h_1 + \varepsilon^3 h_3 \div 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 \quad (2.16)$$

where

$$\begin{aligned} g_1 &= e^{\alpha_1} + e^{\alpha_2}, h_1 = g_1, \alpha_j = k_j/\sqrt{-P}x - ik_j^2t \\ f_2 &= q_1 + q_2 + q_3 + q_4 \div 8[1 \div k_1^2 e^{2\alpha_1} + 1 \div k_2^2 e^{2\alpha_2} + 4 \div (k_1 + k_2)^2 e^{\alpha_1 + \alpha_2}] \\ g_3 &= -g_1 f_2, h_3 = -h_1 f_2 \end{aligned}$$

Through the asymptotic analysis of Solutions (14), the collisions between two solitons, including two dark solitons, and two bright solitons have been found to be elastic (as seen in Figures. 1-2).

3 Stability analysis

For analyze of stability condition, we apply a small perturbation on the system. Then we consider perturbation as an additional term to the solution of equilibrium state. Substituting perturbation solution in the system

of equations, one can find that the perturbation term satisfy a set of equations from which can leads to the nonlinear dispersion relation (NDR). The stability condition can extract from behavior of NDR. First, we shall seek an equilibrium state in the form

$$u_{11} = U_0 \exp[i\Omega_1\tau] \text{ and } v_{11} = V_0 \exp[i\Omega_2\tau] \quad (3.1)$$

into equations (1), where U_0, V_0 are (constant real) amplitude and Ω_1, Ω_2 are (real) phase. We can find a solution of the form

$$\Omega_1 = [Q_{11}U_0|U_0|^2 + Q_{12}U_0|V_0|^2 + Q_{13}U_0V_0^2 + Q_{14}V_0U_0^2]/U_0 \quad (3.2)$$

$$\Omega_2 = [Q_{21}V_0|V_0|^2 + Q_{22}V_0|U_0|^2 + Q_{23}V_0U_0^2 + Q_{24}U_0V_0^2]/V_0 \quad (3.3)$$

Then we consider a small perturbation around equilibrium situation, and substitute

$$u_{11} = [U_0 + \varepsilon(U_{1R} + iU_{1I})] \exp(i\Omega_1\tau) \text{ and } v_{11} = [V_0 + \varepsilon(V_{1R} + iV_{1I})] \exp(i\Omega_2\tau) \quad (3.4)$$

into (1). Separating real and imaginary parts of equations, the first order terms in ε , leads to

$$\frac{\partial M_{1I}}{\partial \tau} = F_1 M_{1R} + F_2 M_{2R} \quad (3.5)$$

$$\frac{\partial M_{2I}}{\partial \tau} = F_3 M_{1R} + F_4 M_{2R} \quad (3.6)$$

$$\frac{\partial M_{1R}}{\partial \tau} + F_5 M_{1I} + F_6 M_{2I} = 0 \quad (3.7)$$

$$\frac{\partial M_{2R}}{\partial \tau} + F_7 M_{1I} + F_8 M_{2I} = 0 \quad (3.8)$$

where coefficients has defined in appendix-A.

Eliminating M_{1I} and M_{2I} , these equations yield

$$\frac{\partial^2 M_{1R}}{\partial \tau^2} + (F_5 F_1 + F_6 F_3) M_{1R} + (F_5 F_2 + F_6 F_4) M_{2R} = 0 \quad (3.9)$$

$$\frac{\partial^2 M_{2R}}{\partial \tau^2} + (F_7 F_1 + F_8 F_3) M_{1R} + (F_7 F_2 + F_8 F_4) M_{2R} = 0 \quad (3.10)$$

We consider a harmonic perturbation in the form

$$M_{1R} = M_{1R0} \exp[i(K_\xi \xi + K_\eta \eta - \Omega \tau)], \quad M_{2R} = M_{2R0} \exp[i(K_\xi \xi + K_\eta \eta - \Omega \tau)] \quad (3.11)$$

Eqs. (22) and (23) can be reduced to

$$\begin{bmatrix} A_{ST} - \Omega^2 & B_{ST} \\ C_{ST} & D_{ST} - \Omega^2 \end{bmatrix} \begin{bmatrix} M_{1R} \\ M_{2R} \end{bmatrix} = 0$$

where the matrix elements are complicated expressions of the coefficients in (1), which has defined in appendix-A. The determinant in the system of equations (25) must vanish, for consistency, leading to a dispersion relation (for the perturbation) in the form

$$\Omega^4 - \Omega^2 T + D = 0 \quad (3.12)$$

where

$$T = A_{ST} + D_{ST} \text{ and } D = A_{ST} D_{ST} - B_{ST} C_{ST} \quad (3.13)$$

Equation (26) is a quadratic polynomial equation in Ω , possessing four (complex, in general) roots. Thanks to its (bi-quadratic) structure, it can be viewed as a quadratic polynomial equation in Ω^2 . Therefore, stability is ensured if both solutions (for Ω^2), say Ω_\pm^2 , are positive real (hence all four solutions $\pm\sqrt{\Omega_\pm^2}$ are real). Since the roots satisfy $T = \Omega_+^2 + \Omega_-^2$ and $D = \Omega_+^2 \Omega_-^2$, stability will be ensured if the following three conditions are satisfied simultaneously: $T \geq 0, D \geq 0$ and $\Delta = T^2 - 4D \geq 0$. Figures 3 and 4 depicts the quantities T, D and Δ defined above, for two modes of a wave propagating in the x-direction in a dusty plasma crystal. We see that stability is always ensured for one mode but no for another in this case.

4 Conclusion

In this work we consider CNLS equations, which describe the interaction between dust lattice modes in dusty plasma crystal, and investigated it via system (1), mathematically. The bilinear form, Eqs.(3), has been derived via the Hirota method, and then proved condition of the integrability of this system. The soliton solutions in have been obtained through bilinear form. Through the asymptotic analysis of Solutions (14), the collisions between two solitons, including two Bright solitons, have been found to be elastic (as seen in Figures. 1-2). Finally, we have made the linear stability analysis and obtained condition for the stability. Through suitable choices of the parameters, we will graphically analyze stability conditions (Figures 3-4).

Acknowledgment.

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Appendix A.

$$\begin{aligned}
F_1 &= P_1 \frac{\partial^2}{\partial x^2} - 3Q_{11}\psi_{10}^2 - (Q_{12}\psi_{20}\psi_{10} + 2Q_{14})\psi_{10}\psi_{20} - Q_{13}\psi_{20}^2 + \Omega_1 \\
F_2 &= -Q_{14}\psi_{10}^2 - (2Q_{12} + 2Q_{13})\psi_{10}\psi_{20} \\
F_3 &= -2Q_{23}\psi_{10}\psi_{20} - Q_{24}\psi_{20}^2 \\
F_4 &= P_2 \frac{\partial^2}{\partial x^2} - 3Q_{21}\psi_{20}^2 - (Q_{22} + 2Q_{24})\psi_{10}\psi_{20} - Q_{23}\psi_{10}^2 + \Omega_2 \\
F_5 &= P_1 \frac{\partial^2}{\partial x^2} - Q_{11}\psi_{10}^2 - (Q_{12}\psi_{20}\psi_{10} + 2Q_{14})\psi_{10}\psi_{20} - Q_{13}\psi_{20}^2 + \Omega_1 \\
F_6 &= -2Q_{13}\psi_{10}\psi_{20} + Q_{14}\psi_{10}^2 \\
F_7 &= -2Q_{23}\psi_{10}\psi_{20} + 2Q_{24}\psi_{20}^2 \\
F_8 &= P_2 \frac{\partial^2}{\partial x^2} - Q_{21}\psi_{20}^2 + Q_{23}\psi_{20}^2 - (Q_{22} + 2Q_{24})\psi_{10}\psi_{20} + \Omega_2 \\
A_{ST} &= [-P_1 K^2 - 2Q_{13}\psi_{20}^2 + Q_{14}\psi_{10}\psi_{20}][-P_1 K^2 + 2Q_{11}\psi_{10}^2 + Q_{14}\psi_{10}\psi_{20}] \\
&+ [Q_{24}\psi_{20}^2 + 2(Q_{22} + Q_{23})\psi_{10}\psi_{20}][-Q_{14}\psi_{10}^2 + 2Q_{13}\psi_{10}\psi_{20}] \\
B_{ST} &= [-P_1 K^2 - 2Q_{13}\psi_{20}^2 + Q_{14}\psi_{10}\psi_{20}][Q_{14}\psi_{10}^2 + 2(Q_{12} + Q_{13})\psi_{10}\psi_{20}] \\
&+ [-Q_{14}\psi_{10}^2 + 2Q_{13}\psi_{10}\psi_{20}][-P_2 K^2 + 2Q_{21}\psi_{20}^2 + Q_{24}\psi_{10}\psi_{20}] \\
C_{ST} &= [Q_{24}\psi_{20}^2 + 2Q_{23}\psi_{10}\psi_{20}][-P_1 K^2 + 2Q_{11}\psi_{10}^2 + Q_{14}\psi_{10}\psi_{20}] \\
&+ [Q_{24}\psi_{20}^2 + 2(Q_{22} + Q_{23})\psi_{10}\psi_{20}][-P_2 K^2 - Q_{23}\psi_{10}^2 + Q_{24}\psi_{10}\psi_{20}] \\
D_{ST} &= [-Q_{24}\psi_{20}^2 + 2Q_{23}\psi_{10}\psi_{20}][Q_{14}\psi_{10}^2 + 2(Q_{12} + Q_{13})\psi_{10}\psi_{20}] \\
&+ [-P_2 K^2 + 2Q_{21}\psi_{20}^2 + Q_{24}\psi_{10}\psi_{20}][-P_2 K^2 - Q_{23}\psi_{10}^2 + Q_{24}\psi_{10}\psi_{20}]
\end{aligned}$$

References

- [1] B. Farokhi, M. Shahmansouri, and I. Kourakis, Modulated transverse off-plane dust-lattice wave packets in hexagonal two-dimensional dusty plasma crystals,
- [2] V. I. Karpman, Nonlinear Waves in Dispersive Media, Pergamon, Oxford (1975).
- [3] C. Sulem and P. L. Sulem, The Nonlinear Schrödinger Equation: Self-Focusing and Wave Collapse, Springer, Berlin (1999).
- [4] M. Marklund and P. K. Shukla, Nonlinear collective effects in photon-photon and photon-plasma interactions,
- [5] J. M. Dixon, J. A. Tuszynski and P. J. Clarkson, From Nonlinearity to Coherence. Universal Features of Nonlinear Behavior in Many-Body Physics”, Cambridge university, Cambridge (1997).
- [6] Y. R. Shen, Principles of Nonlinear Optics, Wiley, New York (1984).
- [7] A. Hasegawa and Y. Kodama, Solitons in Optical Communications, Oxford University, New York (1995).
- [8] C. X. Miao and G. X. Xu, Global solutions of the Klein-Gordon-Schrödinger system with rough data in R^{2+1} ,
- [9] Xiao-yan Tang and P. K. Shukla, Modulational instability and exact solution of the nonlinear Schrödinger equation coupled with the nonlinear Klein-Gordon equation
- [10] S. V. Manakov, Zh. Eksp, On the theory of two-dimensional stationary self focussing of electromagnetic waves,
- [11] S. V. Manakov, Zh. Eksp, Analytical solution of the coupled nonlinear Schrödinger equations,
- [12] R. Radhakrishnan, M. Lakshmanan, and J. Hietarinta, Inelastic collision and switching of coupled bright solitons in optical fibers,
- [13] T. Tsuchida and M. Wadati, The Coupled Modified Korteweg-de Vries Equations,
- [14] T. Tsuchida, N-Soliton Collision in the Manakov Model,

- [15] A. V. Mikhailov, The reduction problem and the inverse scattering method,
- [16] V. E. Zakharov and E. I. Schulman, To the integrability of the system of two coupled nonlinear Schrödinger equations
- [17] V. S. Gerdjikov, in Proceedings of the Sixth International Conference on Geometry, Integrability and Quantization, Varna, Bulgaria, 3-10 June 2004, edited by I. M. Mladenov and A. C. Hirshfeld, Sofia, Softex (2005).
- [18] Q. H. Park, and H. J. Shin, Painlevé analysis of the coupled nonlinear Schrödinger equation for polarized optical waves in an isotropic medium
- [19] D. N. Christodoulides, and R. I. Joseph, Vector solitons in birefringent nonlinear dispersive media
- [20] C.R. Menyuk, Pulse propagation in an elliptically birefringent Kerr medium.
- [21] V. V. Afanasjev, Yu. S. Kivshar, V. V. Konotop, V. N. Serkin, Dynamics of coupled dark and bright optical solitons
- [22] B. J. Hong, C. C. Yang, L. Wang, Generating dark solitons through cross-phase modulation in optical fibers
- [23] Yu. S. Kivshar, Stable vector solitons composed of bright and dark pulses
- [24] S. Trillo, S. Wabnitz, E. M. Wright, G. Stegeman, Optical solitary waves induced by cross-phase modulation
- [25] Yu. S. Kivshar, S. K. Turitsyn, Optical double layers
- [26] A. P. Sheppard, Yu. S. Kivshar, Polarized dark solitons in isotropic Kerr media
- [27] Deng-Shan Wang, Da-Jun Zhang, and Jianke Yang, Integrable properties of the general coupled nonlinear Schrödinger equations,
- [28] H. Q. Zhang, T. Xu, J. Li, B. Tian, Integrability of an N-coupled nonlinear Schrödinger system for polarized optical waves in an isotropic medium via symbolic computation,
- [29] T. Xu, B. Tian, L.L. Li, X. Lu, C. Zhang, Dynamics of Alfvén solitons in inhomogeneous plasmas,

- [30] R. Hirota, *The Direct Method in Soliton Theory*, Cambridge University, Cambridge (2004).
- [31] W. Hereman, W. Zhuang, *Symbolic Computation of Conserved Densities for Systems of Nonlinear Evolution Equations*,
- [32] E. M. E. Zayed, Khaled A. Gepreel, *A generalized (G'/G) expansion method for finding traveling wave solutions of coupled nonlinear evolution equations*,

List of Figures

Figure 1. Elastic collision between two dark solitons via solutions (14). Parameters are $k_1 = 2k_2 = 0.5$, $P = -1$, $q_1 = 0.5q_2 = -q_3 = -q_4 = 2$.

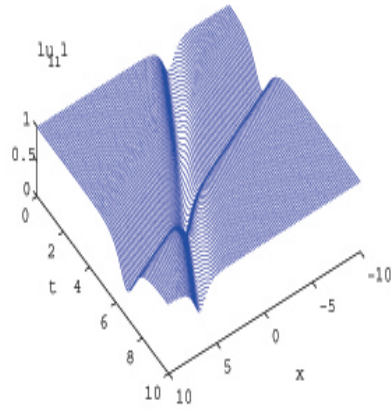


Figure 2. Elastic collision between two bright solitons via solutions (14). Parameters are $k_1 = 2k_2 = 0.5$, $P = 1$, $q_1 = 0.5q_2 = -q_3 = -q_4 = 2$.

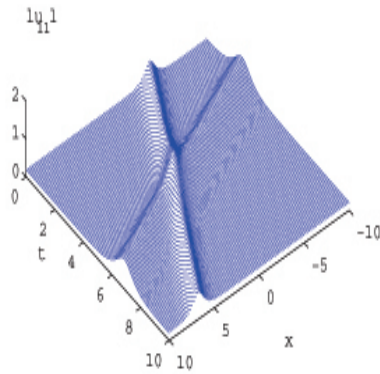


Figure 3. The behavior of (a) T , (b) D and (c) $T^2 - 4D$ versus K for $ka = 0.1$ and $\theta = 0$ (first mode).

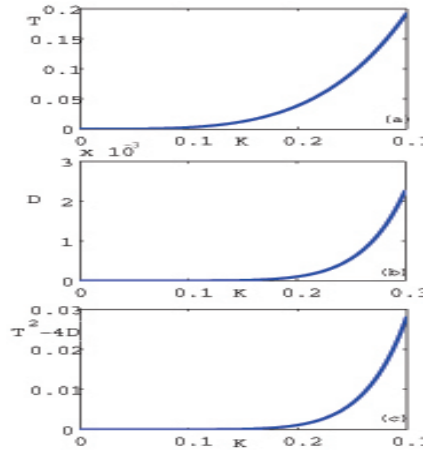


Figure 4. The behavior of (a) T , (b) D and (c) $T^2 - 4D$ versus K for $ka = 0.1$ and $\theta = 0$ (second mode).

