A Nonlinear Optimal Control Model for a generator System

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Abstract

In this article, we apply this new method for solving an engineering system with initial and boundary conditions and integral criterion ,and we will using dynamic programming as iteration for solving model.

1 Introduction

Embedding method was introduced in 1986 by Rubio in his book([7]). In fact, by doing a deformation, the problems are converted into a measure one with some positive theoretical coefficient and by extending space and then applying discretization scheme, the optimal pair of trajectory and control is determined as a finite linear programming. In real life situations, continuous optimal control problems arise mostly in every aspect of human endeavour. Among these are the electric power systems, mainly the generation, transmission and distribution of electric energy.The industrial growth of any nation depends greatly on the reliability of large interconnected electric power system. Electric power system is a significant form of modern energy source, because of its application in nearly all spheres of human endeavour for economic development([4, 5, 6]). In an interconnected power system, the objective of an electric energy system engine is to generate electric energy in sufficient quantities at the most suitable generating locality, transmit it in bulk quantities to the load centres, and then distribute it to the individual customers in proper form and quality and at the lowest possible economic price. However, the factors influencing power generation at minimum cost are operating efficiencies of generators, fuel cost and transmission losses. The most efficient generator in the system may not guarantee minimum cost as it may be located in an area where fuel cost is high. In this article, we try to bring the attention to these two facts for an optimal control problem governed by a electric power generating system with initial and boundary conditions and an integral criterion ([4], [11], [3]). The problem present in a variational form and then, by doing a deformation, it is converted into a measure theoretical one with some positive coefficient. Next, by extending the underlying space, using some density properties and applying some discretization scheme,

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the optimal pair of trajectory and control is determined simultaneously as a result of a finite linear programming. The approach would be improved if the number of nods in discretization is exceeded.

2 Dynamic System Model

Consider the mathematical model of electric power generating system given below ([1]), where $x_1(t)$ is the amount of power generated by the ith generator at time t and $x_2(t)$ is the cost of production/ generation at a particular time. Thus we have,

$$\dot{x_1}(t) = (\alpha + \beta) - u_1(t)kx_1(t) + qx_1(t)x_2(t) - \gamma_1 x_1(t) \tag{2.1}$$

$$\dot{x_2}(t) = (a + b) - u_2(t)cx_1(t) + rx_1(t)x_2(t) + \gamma_2 x_1(t) \tag{2.2}$$

The problem to study is to find the controls u_1, u_2 that minimizes the cost functional

$$J(u_1, u_2) = \int_0^T [\delta x_2(t) + \xi_1 u_1^2 + \xi_2 u_2^2]dt \tag{2.3}$$

where $\alpha + \beta$ is the actual mechanical / electrical energy from the turbine, k is the rate of generation, q is the total running cost, γ_1 is the rate of energy loss during transmission, a is the labour cost at a particular time, b is the cost of maintenance, c is the capacity rate of generator, r is the fuel cost rate, γ_2 is the total cost of transmission. The control u_1 is the load shedding rate, u_2 is the generator actual capacity rate, δ is the unit of power generating station and η_1, η_2 are to balance the size of the control.
Now, we consider the following optimal control problem:

$$Minimize \quad \int_0^T [\delta x_2(t) + \eta_1 u_1^2 + \eta_2 u_2^2]dt$$

$$Subject\ to: \quad \dot{x_1}(t) = (\alpha + \beta) - u_1(t)kx_1(t) + qx_1(t)x_2(t) - \gamma_1 \tag{2.4}$$

$$\dot{x_2}(t) = (a + b) - u_2(t)cx_1(t) + rx_1(t)x_2(t) + \gamma_2 x_1(t)$$

$$x_1(0) = x_{10}, \quad x_2(0) = x_{20}.$$

We try to follow Rubio in [7]. This would guide us to introduce a new solution method for the problem with many advantages. In this manner, we will design an embedding method for solving such strong nonlinear problems in which determines the optimal solution by transferring the problem into a finite linear programming. The historical background of this method and its applications can be found in many literatures like [7, 8] and [10]. **Definition.** Let $X(t) = (x_1(t), x_2(t))$ be the trajectory vector and $U(t) = (u_1(t), u_2(t))$ be the control vector; the pair $\mathcal{P} = (X(t), U(t))$ is called admissible whenever it satisfies the equations (2.1-2.3) and its related initial and terminal conditions. Also for $J = [0, T]$, we suppose that the function $x_1 : J \to [a_{x_1}, b_{x_1}]$,$x_2 : J \to [a_{x_2}, b_{x_2}]$ be absolutely continuous and bounded on J; further, $u_1 : J \to [c_{u_1}, d_{u_1}]$ and $u_2 : J \to [c_{u_2}, d_{u_2}]$ are considered as bounded Lebesque measurable function on J. The set of all admissible pairs is shown by $\mathcal{W}$. Therefore, the purpose is to determine an admissible pair $\mathcal{P} \in \mathcal{W}$ so that it

$$I(\mathcal{P}) = \int_J F_0(t) = \int_J (\delta x_2(t) + \xi_1 u_1^2 + \xi_2 u_2^2)dt.$$

In general, the set $\mathcal{W}$ may be empty; even if $\mathcal{W}$, the infimum of $I(\mathcal{P})$ may not be in $\mathcal{W}$. Moreover, even the minimizing pair does exist in $\mathcal{W}$, it may be difficult to be characterized (necessary conditions are not always helpful because the information they give, may be impossible to interpret generally). Transforming the problem into another appropriate space can be helpful to conquest these difficulties; it is exactly our purpose to introduce the new approach. Hence, it is necessary at this stage to point out some characteristics of the admissible pairs in $\mathcal{W}$. For our purpose, we will find out some effects of these pairs on different type of functions.

3 Embedding Method Technique

Let $A = [a_{x_1}, b_{x_1}] \times [a_{x_2}, b_{x_2}]$, $[c_{u_1}, d_{u_1}] \times [c_{u_2}, d_{u_2}]$ and $\Omega = J \times A \times U$. Let $g = (g_1, g_2)$ such that

$$g_1 = (\alpha + \beta) - u_1(t)kx_1(t) + qx_1(t)x_2(t) - \gamma_1 x_1(t)$$
$$g_2 = a + b) - u_2(t)cx_1(t) + rx_1(t)x_2(t) + \gamma_2 x_1(t).$$

Assume that $\mathcal{P} = [X(t), U(t)]$ be an admissible pair, and B be an open ball in R^3 containing $J \times A$; the space of real-valued continuously differentiable functions on B denote by $\acute{C}(B)$ such that they and their first derivatives are bounded on B. Let $\phi \in \acute{C}(B)$, we define

$$\phi^g(t, X, U) = \phi_x(t, X)g(t, X, U) + \phi_t(t, X).$$

Since $\mathcal{P}$ is admissible,

$$\int_J \phi^g(t, X, U))dt = \phi(0, X_\phi) - \phi(T, X_T). \tag{3.1}$$

If J^0 be the interior points of the time interval, we denote $D(J^0)$ as the space of infinitely differentiable real-valued functions with compact support in J^0. For all $\psi \in D(J^0)$ define:

$$\psi_1(t, X, U) = X_1\psi(t) + g_1(t, X, U)\psi(t);$$
$$\psi_2(t, X, U) = X_2\psi(t) + g_2(t, X, U)\psi(t);$$

one can easily show that:

$$\int_J \psi_j(t, X(t), U(t))dt = 0, \quad \forall \psi \in D(J^0). \tag{3.2}$$

Let $C_1(\Omega)$ be the set of all functions which depend only on time; in other words, $f \in C_1(\Omega)$ if $f(t, X(t), U(t)) = \theta(t)$. Thus we have :

$$\int_J f(t)dt = a_f, \quad \forall f \in C_1(\Omega) \tag{3.3}$$

where a_f is the Lebesque integral of $f(t)$ over J.
For each $\mathcal{P}$ we introduce the functional Λ_P that:

$$\Lambda_P : F \to \int_J F(t, X(t), U(t))dt \quad F \in C(\Omega)$$

One can easily show that the transformation $\mathcal{P} \mapsto \Lambda_{\mathcal{P}}$ is injection and then the problem of electric power generate can be equally represented as the following one but on the space of positive linear functionals on Ω:
$Min: \quad \Lambda_{\mathcal{P}}(F_0)$

$$S.to: \quad \Lambda_{\mathcal{P}}(\phi^g) = \delta\phi \qquad \forall \phi \in \acute{C}(B), \tag{3.4}$$

$$\Lambda_{\mathcal{P}}(\psi_j) = 0, \qquad \forall \psi \in D(J^0), \;\; j = 1, 2$$
$$\Lambda_{\mathcal{P}}(f) = a_f, \qquad \forall f \in C_1(\Omega)$$

According to the Riesz representation theorem ([7]), for each linear positive functional $\Lambda_{\mathcal{P}}$ there exist a unique positive regular Borel measure is called Radon measure on Ω such that :

$$\Lambda_{\mathcal{P}}(F) = \int_\Omega F(t, X(t), U(t))dt = \mu_{\mathcal{P}}(F). \tag{3.5}$$

Therefore problem (3.3) can be transferred into the space of measures by an one-to-one mapping. Note that the above mentioned difficulties are still exist, because the map $\mathcal{P} \mapsto \Lambda_{\mathcal{P}}$ is injection and the induced measure is unique. Hence we develop the solution space and consider the set of all positive Radon measures that just satisfy the conditions of (3.4) . Indeed, the minimization in (3.4), takes place on $M^+(\Omega)$, the set of all positive radon measures on Ω, as follows:

$Min: \quad \mu(F_0)$

$$S.to: \quad \mu(\phi^g) = \delta\phi \qquad \forall \phi \in \acute{C}(B), \tag{3.6}$$

$$\mu(\psi_j) = 0, \qquad \forall \psi \in D(J^0), \;\; j = 1, 2$$
$$\mu(f) = a_f, \qquad \forall f \in C_1(\Omega).$$

Indeed, we have extended the solution space and moreover, we will show soon that the optimal solution is also existed; thus the optimal solution of (3.6) is global. Assume that Q be the set of all positive radon measures in $M^+(\Omega)$ that satisfies in the equations of system (3.6). By equipping Q with weak*-topology , according to the following proposition, the existence of the optimum measure ,μ^*, for (3.6) is guaranteed (see [7, 8]).

Theorem 3.1. *The set of measures Q is compact in the topology induced by the topology on $M^+(\Omega)$.*

Proof. see [7]. □

Theorem 3.2. *The function $\mu \to \mu(F_0)$, mapping Q into the real line, is continuous.*

Proof. see [7]. □

Since each continuous function on a compact set, takes its infimum on this set, then the function $\mu \to \mu(F_0)$ takes its infimum on Q. Now we reached to a very important point; problem (3.6) is linear, since all the functions are linear with respect to the variable. Furthermore, the measure is required to be positive; thus (3.6) is a linear programming problem. But the number of constraints are infinite , while the dimension of space is infinite too. It is the most desirable if we could obtain the optimal solution just by solving a finite linear programming one. This process can be done by applying two steps of approximation. First, we choose suitable countable subsets of functions whose linear combinations are dense in the appropriate spaces of constraints, and then selecting a finite number of their elements.

For the first set of equalities in (3.6), let the set

$$\{\phi_i \in \acute{C}(B),\ i = 1, 2, ..., M\}$$

be such that the linear combinations of these functions are uniformly dense (dense in the topology of uniform convergence in the space $\acute{C}(B)$). For instance, these functions can be the polynomials in terms of x_1, x_2 and t. In the other hand, We choose M_2 number of these functions for the second set of equalities in (3.6) as follows:

$$\psi_j(t) = \begin{cases} sin[2\pi j \left(\frac{t-0}{T-0}\right)] & j = 1, 2, ..., M_2, \\ 1 - cos[2\pi j \left(\frac{t-0}{T-0}\right)] & j = M_2 + 1, M_2 + 2, ..., 2M_2. \end{cases} \tag{3.7}$$

Obviously, these functions are real-valued infinitely differentiable functions with compact support in J^0.
By dividing the time interval into L subintervals $J_1, J_2, ..., J_L$ we introduce the third set of functions in (3.6) as the following characteristic functions

$$f_s(t) = \begin{cases} 1 & t \in J_s, \\ 0 & otherwise, \end{cases} \tag{3.8}$$

Note that although these functions are not continuous , but they have two remarkable properties which are very helpful for our purpose. Each function $f_s,\ s = 1, 2, ..., L$ is the limit of an increasing sequence of positive continuous functions, say $\{f_{sk}\}$; then, if μ is any positive Radon measure on Ω, we have $\mu(f_s) = Lim_{k\to\infty}\mu(f_{sk})$. Also consider now the set of all such functions, for all positive L. The linear combinations of these functions can approximate a function in $C_1(\Omega)$ arbitrarily well.
Remark. It must be noted that by applying this step, problem (3.6) is changed into a semi-infinite linear programming ($SILP$) ; hence one may use some of the $SILP$ solution methods to find μ^* (see [7] and [10]).

4 Metamorphosis

According to the Rosenbloom's theorem (see [7]), the optimum measure of (3.6), has the following presentation

$$\mu^* = \sum_{k=1}^{N} \alpha_k \delta(z_k), \quad \alpha_k \geq 0,\ k = 1, 2, ..., N \tag{4.1}$$

where $\delta(z_k) \in M^+(\Omega)$ is the unitary atomic measure with support the singleton set $\{z_k\} \subset \Omega$ which is characterized by

$$\delta(z_k)(F) = F(z_k), \quad F \in C(\Omega).$$

Replacing μ in (3.6) by (3.7), changes problem (3.6) into a nonlinear programming with unknown coefficients α_k and unknown supporting points z_k. If we can minimize the problem just with respect to coefficients α_k, then the problem is converted into a finite linear programming. This process is possible if we employ a discretization on the space Ω and just choose the nodes z_j which belong to a dense subset of Ω. As a result, we attain the following finite linear programming problem in which its solution is a very suitable approximation for (3.6):

$$Min: \quad \sum_{j=1}^{N} \alpha_j F_0(z_j)$$

$$S.to: \quad \sum_{j=1}^{N} \alpha_j \phi_i^g(z_j) = \Delta\phi_i; \quad i = 1, 2, ..., M_1,$$

$$\sum_{j=1}^{N} \alpha_j \psi_{i_j}(z_j) = 0; \qquad i = 1, 2, ..., M_2, \tag{4.2}$$

$$\sum_{j=1}^{N} \alpha_j f_s(t_j) = a_f; \qquad s = 1, 2, ..., L$$

where $\phi_i \in \acute{C}(B)$, $\psi_i \in D(J^0)$, $f_s \in C_1(\Omega)$ and $\alpha_j \geq 0$ for $j = 1, 2, ..., N$.
According to the density properties, it can be proved , that when $M_1, M_2, L, N \to \infty$, the solution of (3.8) will tend to the solution of the main problem (see [7]).

5 Numerical Example

Based on the explained approach, we incline to find the optimal pair of trajectory and control in the following numerical example.
Example. The parameters given in table below is used for find the optimal pair of trajectory and control for given electric power generating system

Parameter	Meaning	Value
$\alpha + \beta$	actual mechanical / electrical energy available	800 MW
q	total running cost	0.3217
r	fuel cost rate	0.347
x	actual capacity rate	0.0606
k	rate of generation	0.606
γ_1	rate of energy loss during transmission	0.002MW
a	labour cost	200
b	maintenance cost	100
γ_2	Cost of transmitting from generating station	0.3421
δ	unit of power generating station	1
ξ_1	no of hours for which the machines is on	16
ξ_2	the no of hours of operation	16

Interval	Partition
$x_1 = [a_{x_1}, b_{x_1}] = [600, 1800]$	10
$x_2 = [a_{x_2}, b_{x_2}] = [-1000, 3500]$	10
$J = [0, T] = [0, 1]$	10
$u_1 = [c_{u_1}, d_{u_1}] = [0, 1]$	10
$u_2 = [c_{u_2}, d_{u_2}] = [0, 1]$	10

Suppose that $M_1 = 4, M_2 = 16$ and $L = 10$, we set up a finite linear programming like (4.2) by 100000 nodes and 30 constraints , and then we used the revised simplex method from **Compaq Visual Fortran** software to solve it.As a result, we attained **1140.946524727022100000**.

The graph of the control and trajectory are shown in Figs. 1-2, respectively.

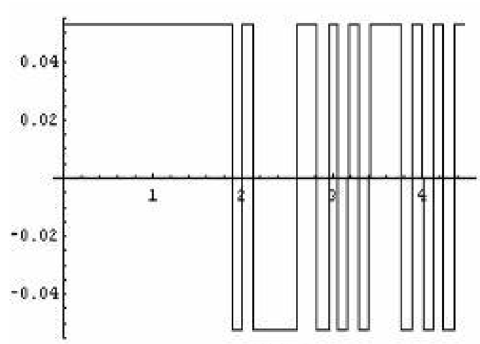

Fig 1

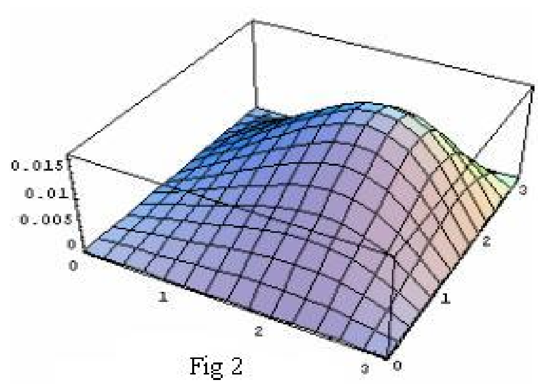

Fig 2

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