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Measurement of Inefficiency Slacks in Network Data Envelopment Analysis

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Abstract

This article presents a set of additive DEA models (optimistic and pessimistic) to measure inefficiency slacks in which observations are shown with crisp numbers. In the concept of pessimistic efficiency, DMU with balanced input and output data can be scored as efficient. Hence, it is an inevitable necessity to integrate different performance sizes in order to achieve an overall performance assessment for each DMU. An example of resin manufacturer companies in Iran is presented to explain how to calculate the system and process inefficiency slacks.

Key words: Data envelopment analysis, inefficiency slacks, series systems, optimistic and pessimistic viewpoints, overall performance.

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1 Introduction

All organizations, whether profit, governmental, non-profit and other types of decision-making units (DMUs), are looking for producing more outputs and services using less resources. This is an efficiency problem with three stages, namely (i) efficiency measurement, (ii) target setting and (iii) goal achievement. This type of problems has been extensively studied by economists and management science scholars for many years. Since the pioneering work of Charnes et al. in 1978, data envelopment analysis (DEA) has become the most prominent nonparametric method for measuring efficiency of DMUs to produce multiple outputs by using multiple inputs [3]. In addition, to efficiency measurement, DEA is also able to show to what extent the output can be increased by maintaining the current input level. It is also capable of estimating the extent of input saving only by increasing efficiency while maintaining the current output level. In other words, DEA creates a target for non-efficient DMUs to be converted into efficient DMUs. Thus, DEA technique is able to respond questions arising in the first two stages of efficiency studies. Consequently, thousands of articles and books have been published since the advent of DEA in 1978.

A system usually consists of multiple subsystems that operate interdependently. Conventional DEA used for measuring efficiency only considers inputs to the system and outputs produced by the system and ignores its internal structure. Therefore, the overall system may be efficient, but its constituents may be non-efficient. More importantly, all components of a DMU show a lower performance than another DMU in some cases, but the first DMU has a higher system performance than the second DMU. Many ideas have been developed from conventional DEA to overcome these problems. The models developed for measuring the efficiency of production systems with different network structures are known as network DEA models.

Numerous studies have been recently conducted on DMUs with a network structure. Cook, et al. [4] pointed out several approaches for modeling DMUs with a two-stage network structure. Typically, these models are developed by efficiency decomposition using geometric or arithmetic means. Although the network DEA model proposed by Färe and Grosskopf [6] is able to operate with different network structures, it fails to decompose or score efficiency of sub-DMUs that forms the entire DMU network. Using slack-based models, Tone and Tsutsui [13] developed a network DEA model which measures both partial and overall efficiencies of DMUs. They assumed that (i) the network is consisted of several parts, (ii) partial efficiency is an index for a particular part relative to its counterparts in other networks and (iii) the overall efficiency of a network is weighted harmonic average of its partial scores that their weights are determined externally. Kao and Hwang [10] proposed a two-stage DEA model taking into account sequential relationship of two sub-processes in the overall production process in which the overall efficiency is obtained by multiplying the efficiencies of two sub-processes. For a unique efficiency decomposition, they used a method for finding the highest efficiency of sub-processes while maintaining maximum overall efficiency. Kao [8] introduced dummy processes to convert a general network structure to sequential stages consisting of several parallel processes. They used the approaches proposed by Kao and Hwang [10] and Kao [9] for decomposing sequential and parallel structures, respectively. On the other hand, Cook, et al. [4] proposed models based on additive efficiency decomposition of network DMUs using the centralized model of Liang, et al. [12]. This model assumes that the overall efficiency is the product or sum of partial efficiencies. For example, consider the approach used by Kao and Hwang [10] which assumes a set of insurance companies with a two-stage operation for earning premiums and generating profits. Therefore, the overall efficiency is the product of premium earning and profit generation efficiencies. Liang, et al. [12] called this modelling technique or efficiency decomposition as centralized or cooperative game approach, as all efficiency scores of all sub-DMUs or stages are optimized simultaneously.

In addition, Liang, et al. [12] examined modelling of two-stage network DMUs from a non-cooperative game perspective. Non-cooperative approach is considered in the form of leader-follower or Stackelberg game. For example, assuming that the first stage involving premium earning is leader, the performance of the first stage would be more important and the efficiency of the second stage (profit generation) will be calculated provided that the efficiency of the first stage would remain constant. Similarly, one can assume that the second stage is leader and the first stage is follower.

The centralized model of Liang, et al. [12] can be used for DMUs with any type of network structure assuming that the overall efficiency is weighted average efficiency of each of the stages (or parts). However, the leader-follower model cannot be easily applied. The models proposed by Liang, et al. [12] or Kao and Hwang [10] have been developed with this assumption that all outputs of the first stage are the only inputs to the second stage.

An intermediate product is one of the main features of network systems. Unlike endogenous external inputs and final outputs which are produced for exterior, intermediate products are produced and consumed within the system and thus cannot be seen from outside. It is noteworthy that the relative model is able to measure partial efficiencies, while the envelopment model in which the efficiency measure is expressed as a function of distance, is able to display projections or targets of agents. This is of particular importance for network systems in which an intermediate product is produced by a sector and thus is expected to have a larger value to be more efficient. At the same time, it is expected to have a smaller value to have a more efficient production sector.

Therefore, intermediate product supply and demand sectors have conflicting targets regarding the amount of intermediate product. This study aims at developing additive DEA models (optimistic and pessimistic) to measure inefficiency slacks of network systems. The proposed models measure the inefficiency slacks of intermediate product to obtain an efficient system (showing product increase or decrease only for one stage). Iranian resin producing companies are studied to explain how to calculate inefficiency slacks of the system and processes with crisp data.

The paper is organized as follows. Additive DEA models for a two-stage process are proposed in Section 2. To illustrate the practical utility of the results, several Iranian resin producing companies are assessed in Section 3. Concluding remarks are presented in Section 5.

2 Additive DEA models for a two-stage process

2.1 Optimistic additive DEA model for a two-stage process

The flexibility of intermediate products should be increased as output of the first stage, and reduced as inputs to the second stage to maximize the overall efficiency [1]. This has led to some difficulties in modelling. To overcome existing problems, the following additive DEA model is proposed for identifying efficient and non-efficient DMUs of a network system:

$$
\begin{aligned}
&\max\ S_o^{\text{overall}*} = \sum_{i=1}^{m} s_i^- + \sum_{d=1}^{D} |s_d^{\text{intermediate}}| + \sum_{r=1}^{s} s_r^+ \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \quad i = 1, \ldots, m, \\
&\sum_{j=1}^{n} \lambda_j z_{dj} + s_d^{\text{intermediate}} = z_{do}, \quad d = 1, \ldots, D, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, \ldots, s, \\
&- z_{do} \le s_d^{\text{intermediate}} \le z_{do}, \quad d = 1, \ldots, D, \\
&s_i^-, s_r^+, \lambda_j \ge 0, \quad i = 1, \ldots, m; r = 1, \ldots, s; j = 1, \ldots, n, \\
&s_d^{\text{intermediate}} \text{ free in sign.}
\end{aligned}
\tag{2.1}
$$

Note that bounds are defined for the slacks of intermediate products to act freely and take negative values. Using these bounds, intermediate products will select realistic values to increase outputs or reduce inputs. Model 2.1 is a nonlinear model due to the presence of an absolute value function in its objective function. However, the absolute value function can be eliminated from the objective function by the following change of variable:

$$
\begin{cases}
s_d^- = \frac{1}{2}(|s_d^{\text{intermediate}}| + s_d^{\text{intermediate}}), & d = 1, \ldots, D, \\
s_d^+ = \frac{1}{2}(|s_d^{\text{intermediate}}| - s_d^{\text{intermediate}}), & d = 1, \ldots, D
\end{cases}
\tag{2.2}
$$

Therefore,

$$
\begin{aligned}
&\max \ S_o^{\text{overall}} = \sum_{i=1}^{m} s_i^- + \sum_{d=1}^{D} s_d^- + s_d^+ + \sum_{r=1}^{s} s_r^+ \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \quad i = 1, \ldots, m, \\
&\sum_{j=1}^{n} \lambda_j z_{dj} + s_d^- - s_d^+ = z_{do}, \quad d = 1, \ldots, D, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, \ldots, s, \\
&- z_{do} \leq s_d^- - s_d^+ \leq z_{do}, \quad d = 1, \ldots, D, \\
&s_d^- \times s_d^+ = 0, \quad d = 1, \ldots, D, \\
&s_d^-, s_d^+, s_i^-, s_r^+, \lambda_j \geq 0, \quad d = 1, \ldots, D; i = 1, \ldots, m; r = 1, \ldots, s; \\
&\qquad\qquad\qquad\qquad\qquad j = 1, \ldots, n.
\end{aligned}
\tag{2.3}
$$

Model 2.3 is nonlinear due to the constraint set $s_d^- \times s_d^+ = 0$ ($d = 1, \ldots, D$). Unfortunately, this model cannot be linearized by change of variable. Since only D constraints of $(m + s + 3D)$ constraints include insignificant nonlinear terms, it is not difficult to solve this problem. However, Model (2.3) can be converted into a mix integer linear programming model. The constraint set $s_d^- \times s_d^+ = 0$ ($d = 1, \ldots, D$) indicates that at least one of the variables s_d^- or s_d^+ ($d = 1, \ldots, D$) in each constraint should be equal to zero. Therefore, the following conditions are added to the problem instead of the constraint set $s_d^- \times s_d^+ = 0$ ($d = 1, \ldots, D$).

$$
\begin{cases}
s_d^- \leq M\beta_d, & d = 1, \ldots, D, \\
s_d^+ \leq M(1 - \beta_d), & d = 1, \ldots, D, \\
\beta_d \in \{0, 1\}, & d = 1, \ldots, D,
\end{cases}
\tag{2.4}
$$

where M is a large positive number. Therefore, the inefficiency caused by auxiliary variables can be measured using the following model:

$$
\begin{aligned}
&\max\ S_o^{\text{overall}} = \sum_{i=1}^{m} s_i^- + \sum_{d=1}^{D} s_d^- + s_d^+ + \sum_{r=1}^{s} s_r^+ \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \quad i = 1, \ldots, m, \\
&\sum_{j=1}^{n} \lambda_j z_{dj} + s_d^- - s_d^+ = z_{do}, \quad d = 1, \ldots, D, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, \ldots, s, \\
&-z_{do} \le s_d^- - s_d^+ \le z_{do}, \quad d = 1, \ldots, D, \\
&s_d^- \le M\beta_d, \quad d = 1, \ldots, D, \\
&s_d^+ \le M(1-\beta_d), \quad d = 1, \ldots, D, \\
&\beta_d \in \{0, 1\}, \quad d = 1, \ldots, D, \\
&s_d^-, s_d^+, s_i^-, s_r^+, \lambda_j \ge 0, \quad d = 1, \ldots, D; i = 1, \ldots, m; r = 1, \ldots, s; \\
&\qquad\qquad j = 1, \ldots, n.
\end{aligned}
\tag{2.5}
$$

If the slack set s_r^{+*} $(r = 1, \ldots, s)$, s_i^{-*} $(i = 1, \ldots, m)$, s_d^{-*} $(d = 1, \ldots, D)$ and s_d^{+*} $(d = 1, \ldots, D)$ equal to zero, DMU_o is optimistic efficient, otherwise is an optimistic non-efficient DMU.

Then the optimistic non-efficient DMU_o can be transferred to the efficiency frontier using projection point 2.6:

$$
\begin{cases}
\hat{x}_{io} = x_{io} - s_i^{-*}, & i = 1, \ldots, m, \\
\hat{z}_{do} = z_{do} - (s_d^{-*} - s_d^{+*}), & d = 1, \ldots, D, \\
\hat{y}_{ro} = y_{ro} + s_r^{+*}, & r = 1, \ldots, s.
\end{cases}
\tag{2.6}
$$

Example 2.1 *Consider five DMUs that use the input x to produce the intermediate product z in the first stage and use this product in the second stage to produce the output y as shown in the columns* 2 *to* 4 *of Table 1. Figure 1 shows production frontiers of two stages where the right-hand side shows five DMUs with the superscript* (2.1) *that use x to produce z, while the left-hand side shows five DMUs with superscript* (2.2) *that* *use z to produce y. When the first stage is considered as an independent production process, the frontier is made of five DMUs under constant returns to scale with the radius $OC^{(1)}$.*

Table 1
Data sets and inefficiency slacks of the first and second stages

DMU	x	z	y	Inefficiency slacks of the first stage			Inefficiency slacks of the second stage		
				$s^{(1)-*}$	$s^{(1)+*}$	$S_o^{(1)*}$	$s^{(2)-*}$	$s^{(2)+*}$	$S_o^{(2)*}$
A	2	1	0.5	1.00	0.00	1.00	0.00	0.87	0.87
B	4	2	1	2.00	0.00	2.00	0.00	1.75	1.75
C	4	3	2	0.00	0.00	0.00	0.00	2.12	2.12
D	5	4	5.5	1.00	0.00	1.00	0.00	0.00	0.00
E	6	5	5.5	1.00	0.00	1.00	0.00	1.37	1.37

The fifth columns of Table 1 shows inefficiency slacks of five DMUs where only C is an optimistic efficient DMU. Similarly, when the second stage is considered as an independent production process, $\mathrm{OD}^{(2)}$ will be frontier and the inefficiency slacks of five DMUs are shown in the sixth column where only D is an optimistic efficient DMU. None of the DMUs is efficient in both stages. If the whole system is considered as a block box that consumes input x to produce output y, then D is an optimistic efficient DMU. Figure 2 shows the production frontier for five DMUs where OD shows the frontier. The inefficiency slacks measured by the Model (2.5) are shown in the third column of Table 2.

The black box model does not take into account the operation of each of the stages when is directly applied to measure the efficiency of the system. For example, consider the DMU_E to see the effect. This DMU makes use of 6 units of x to produce 5 units of z in the first stage with the projection point of $\hat{E}^{(1)}$ ($x_{\hat{E}^{(1)}} = 5$) from the first stage. This DMU uses 5 units of z to produce 5.5 units of y in the second stage with the projection point of $\hat{E}^{(2)}$ ($x_{\hat{E}^{(2)}} = 6.875$).

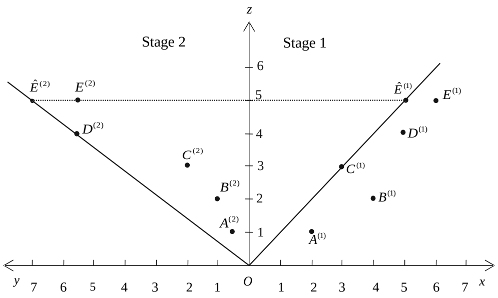

Fig. 1. Frontiers of the two stages of production

Table 2
Inefficiency slacks obtained from the black box model and Model 2.5

DMU	Inefficiency slacks from black box model			Inefficiency slacks from Model 2.5			
	s^{-*}	s^{+*}	S_o^*	s^{-*}	$s^{\text{intermediate}*}$	s^{+*}	$S_o^{\text{overall}*}$
A	0.00	1.70	1.70	0.00	-0.60	1.70	2.30
B	0.00	3.4000	3.40	0.00	-1.20	3.40	4.60
C	0.00	1.30	1.30	1.18	1.54	0.00	2.72
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E	0.00	1.10	1.10	1.00	1.00	0.00	2.00

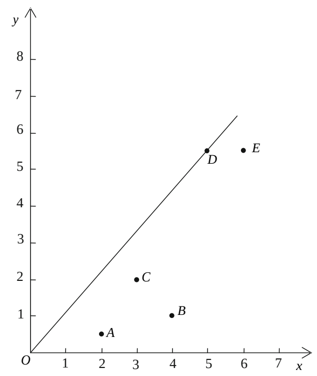

Fig. 2. The frontier of the black box

2.2 Pessimistic additive DEA model for a two-stage process

According to the pessimistic theory [2,5,7,14], intermediate products in the first stages should be reduced as outputs and should be increased as inputs to the second stage to calculate the efficiency measure from the inefficient production frontier. The following pessimistic additive DEA model is proposed for calculating inefficiency slacks of DMU_o:

$$
\begin{aligned}
&\max\ S_o^{\text{overall}*} = \sum_{i=1}^{m} s_i^+ + \sum_{d=1}^{D} |s_d^{\text{intermediate}}| + \sum_{r=1}^{s} s_r^- \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \lambda_j x_{ij} - s_i^+ = x_{io}, \quad i = 1, \ldots, m, \\
&\sum_{j=1}^{n} \lambda_j z_{dj} + s_d^{\text{intermediate}} = z_{do}, \quad d = 1, \ldots, D, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} + s_r^- = y_{ro}, \quad r = 1, \ldots, s, \\
&- z_{do} \leq s_d^{\text{intermediate}} \leq z_{do}, \quad d = 1, \ldots, D, \\
&s_i^+, s_r^-, \lambda_j \geq 0, \quad i = 1, \ldots, m; r = 1, \ldots, s; j = 1, \ldots, n, \\
&s_d^{\text{intermediate}} \ \text{ free in sign.}
\end{aligned}
\tag{2.7}
$$

Model (2.7) is a nonlinear model due to the existence of an absolute value function. However, the absolute function value can be eliminated from the objective function of the model by the following change of variable:

$$
\begin{cases}
s_d^- = \frac{1}{2}(|s_d^{\text{intermediate}}| + s_d^{\text{intermediate}}), \quad d = 1, \ldots, D, \\
s_d^+ = \frac{1}{2}(|s_d^{\text{intermediate}}| - s_d^{\text{intermediate}}), \quad d = 1, \ldots, D
\end{cases}
\tag{2.8}
$$

Substituting in Model (2.7), we have:

$$
\begin{aligned}
&\max\ S_0^{\text{overall}} = \sum_{i=1}^{m} s_i^+ + \sum_{d=1}^{D} s_d^- + s_d^+ + \sum_{r=1}^{s} s_r^- \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \lambda_j x_{ij} - s_i^+ = x_{io}, \quad i = 1, \ldots, m, \\
&\sum_{j=1}^{n} \lambda_j z_{dj} + s_d^- - s_d^+ = z_{do}, \quad d = 1, \ldots, D, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} + s_r^- = y_{ro}, \quad r = 1, \ldots, s, \\
&s_d^- \times s_d^+ = 0, \quad d = 1, \ldots, D, \\
&- z_{do} \leq s_d^- - s_d^+ \leq z_{do}, \quad d = 1, \ldots, D, \\
&s_d^-, s_d^+, s_i^+, s_r^-, \lambda_j \geq 0, \quad d = 1, \ldots, D; i = 1, \ldots, m; r = 1, \ldots, s; \\
&\qquad\qquad j = 1, \ldots, n.
\end{aligned}
\tag{2.9}
$$

To obtain a linear model, the following conditions replace the constraint set $s_d^- \times s_d^+ = 0$ $(d = 1, \ldots, D)$:

$$
\begin{cases}
s_d^- \leq M\beta_d, & d = 1, \ldots, D, \\
s_d^+ \leq M(1 - \beta_d), & d = 1, \ldots, D, \\
\beta_d \in \{0, 1\}, & d = 1, \ldots, D,
\end{cases}
\tag{2.10}
$$

Finally, the following model can be used to measure inefficiency slacks from a pessimistic point of view:

$$
\begin{aligned}
&\max \ S_o^{\text{overall}*} = \sum_{i=1}^{m} s_i^+ + \sum_{d=1}^{D} s_d^- + s_d^+ + \sum_{r=1}^{s} s_r^- \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \lambda_j x_{ij} - s_i^+ = x_{io}, \quad i = 1, \ldots, m, \\
&\sum_{j=1}^{n} \lambda_j z_{dj} + s_d^- - s_d^+ = z_{do}, \quad d = 1, \ldots, D, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} + s_r^- = y_{ro}, \quad r = 1, \ldots, s, \\
&- z_{do} \le s_d^- - s_d^+ \le z_{do}, \quad d = 1, \ldots, D, \\
&s_d^- \le M\beta_d, \quad d = 1, \ldots, D, \\
&s_d^+ \le M(1-\beta_d), \quad d = 1, \ldots, D, \\
&\beta_d \in \{0,1\}, \quad d = 1, \ldots, D, \\
&s_d^-, s_d^+, s_i^+, s_r^-, \lambda_j \ge 0, \quad d = 1, \ldots, D; i = 1, \ldots, m; r = 1, \ldots, s; \\
&\qquad\qquad\qquad\qquad\qquad j = 1, \ldots, n.
\end{aligned} \tag{2.11}
$$

If the slack sets s_r^{-*} $(r = 1, \ldots, s)$, s_i^{+*} $(i = 1, \ldots, m)$, s_d^{-*} $(d = 1, \ldots, D)$ and s_d^{+*} $(d = 1, \ldots, D)$ equal to zero, DMU_o is pessimistic inefficient, otherwise is a pessimistic non-inefficient DMU.
Then the pessimistic non-inefficient DMU_o can be transferred to the inefficiency frontier using projection point (2.12):

$$
\begin{cases}
\hat{x}_{io} = x_{io} + s_i^{+*}, & i = 1, \ldots, m, \\
\hat{z}_{do} = z_{do} - (s_d^{-*} - s_d^{+*}), & d = 1, \ldots, D, \\
\hat{y}_{ro} = y_{ro} - s_r^{-*}, & r = 1, \ldots, s.
\end{cases} \tag{2.12}
$$

3 Explanatory Example

In this section, a performance scoring problem is examined by using the additive DEA models developed in this study.

Briefly, sustainable supply chains consider social and environmental impacts of activities and products of the supply chain along with economic benefits of businesses with the aim of optimizing the supply chain management from economic, social and environmental aspects. Due to current global environmental challenges such as climate changes and greenhouse gas emissions as well as social concerns such as the use of children in the industry, industries pursue a broader range of objectives than just achieving profitable economic performance.
We are witnessing attitude changes in management theories and political economy with a special attention to social responsibilities of enterprises in the social and environmental spheres. This attitude change is taking place both in the macroeconomic and microeconomic levels so that economic enterprises assume themselves as a whole along with the government and public institutions and take responsibilities regarding social welfare and damages to water, land and air.
In this section, the additive DEA models proposed in this study are used to analyze the performance of 27 resin producing companies in Iran. Data in Table 3.1 were obtained from Khodakarami, et al. [11]. The inputs, intermediate products and outputs of DMUs are as follows:
x_1: Annual cost,
x_2: Annual personnel turnover,
x_3: Environmental cost,
z_1: Number of products from supplier to manufacturer,
z_2: Partnership cost in green production plans,
y_1: Number of trained personnel in the fields of job, safety, and health,
y_2: Number of green products,
y_3: Revenue.
Table 3

DMU	Inputs				Intermediate products			Outputs		
	x_1 (1000)$	x_2	x_3 (1000)$		z_1	z_2 (1000)$		y_1	y_2	y_3 (1000)$
Aria Resin Co.	2982	0.2	117		8	145		158	5	4760
Azar Resin Co	2684	0.5	101		6	135		191	5	3240

Peka Chemie Co.	3753	0.15	84		11	213		217	9	4850
Bonyan Kala Chemie Co.	2961	0.1	121		9	152		295	13	4190
Pars Pamchal Chemical Co.	2789	0.35	116		5	139		337	7	4710
Paint Sahar Co.	2951	0.6	135		14	91		263	8	4510
Taba Coatings	2856	0.2	174		8	153		338	13	4930
Paksan Co.	2654	0.45	132		11	175		194	11	4350
Chemical Carbon Acid Co.	2921	0.2	110		7	97		172	4	4130
Alborz Chelic Co.	2723	0.7	98		10	64		387	3	3860
Mobin Petrochemical Co.	3975	0.5	164		11	142		419	6	5157
Marun Petrochemical Co.	1855	0.65	135		7	118		476	9	4230
Fajr Petrochemical Co.	4186	0.3	139		13	164		117	10	5970
Laleh Petrochemical Co.	2774	0.2	112		7	143		218	6	3370
Khosh & Kcc Co.	2657	0.45	176		9	115		176	5	4670
Rang Afarin Co.	3852	0.5	161		12	178		197	12	5110
Dorsa Chemie Co.	3758	0.1	95		8	126		423	9	4840
Bushehr Chemical Industries Co.	3984	0.3	153		15	114		259	12	5710
Rang Avar Paint & Chemical Co.	3656	0.55	76		11	89		110	9	4380
Rangsazi Iran Co.	2814	0.6	241		7	135		73	6	3850
Petromad Kimia Co.	3881	0.4	135		9	84		198	5	5650
Pars Zinc Dust Co.	3175	0.1	92		6	124		331	6	4140
Peik Chimie Co.	746	0.5	168		7	97		578	8	4470
Resin Fam Co.	2667	0.2	114		8	119		114	5	3750
Doreen Chimie Co.	2894	0.65	139		11	142		135	9	4180
Pars Eshen Co.	3651	0.5	175		9	136		238	7	4460
Nikoo Resin Co.	1956	0.1	131		13	157		194	12	4290

4 Analysis of resin producing companies from an optimistic perspective

Optimistic efficient and non-efficient DMUs can be identified by applying the Model (2.5) on data in Table 3.1. It is clear from Table 4 that

eight DMUs including DMUs 3, 4, 10. 12. 17, 19, 23 and 27 are identified by the Model (2.5) as optimistic DMUs. According to the results of Model (2.5), the slack of the intermediate product z_1 is negative for eight DMUs. Moreover, the slack of the intermediate product z_2 is negative for ten DMUs. Positive slacks of intermediate products are identified for DMUs 2, 25, and 26. The value of M is equal to 10^5 in this numerical example.
Optimistic efficient and non-efficient DMUs in the first and second stages can be identified by applying the optimistic additive DEA model of traditional black box under CRS assumption on data in Table 3.1. The results are listed in Tables 4 and 4. According to the slacks in Table 4, DMUs 3, 19, 23 and 27 are identified as optimistic efficient DMUs in the first stage. Among the inefficiency slacks in Table 4, the DMUs 5, 7, 10, 18, 21 and 23 are identified as optimistic efficient DMUs in the second stage. It should be noted that DMU_{23} which was identified as an optimistic efficient DMU in the first stage, is also identified as an optimistic efficient DMU and system in the second stage.
Table 4

DMU	s_1^{-*}	s_2^{-*}	s_3^{-*}	$s_1^{intermediate*}$	$s_2^{intermediate*}$	s_1^{+*}	s_2^{+*}	s_3^{+*}	$s_o^{overall*}$
1	223.66	0.00	0.0000	-3.32	-36.27	122.03	5.10	0.00	390.40
2	1521.15	0.22	0.00	0.20	41.91	141.27	0.86	0.00	1705.63
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.0000	0.00	0.00	0.00	0.00	0.00
5	175.36	0.06	0.00	-4.48	-30.10	17.39	1.62	0.00	229.05
6	866.36	0.16	0.00	5.22	0.00	103.26	0.00	0.00	975.01
7	376.97	0.00	0.00	-7.4620	-38.28	0.00	1.67	727.52	1151.91
8	637.44	0.28	0.00	0.00	33.07	99.44	0.00	0.00	770.24
9	772.3028	0.0000	0.0000	-2.6780	-51.8602	99.6687	4.86	0.00	931.37
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	13.9841	0.06	0.00	0.00	-44.07	105.23	4.58	659.34	2182.30
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	508.19	0.00	0.00	0.00	-66.55	274.91	1.49	72.23	923.39
14	1417.50	0.00	0.00	0.00	45.09	67.24	1.79	0.00	1531.64
15	1525.55	0.00	0.00	-0.25	-7.33	364.16	4.84	217.68	2119.84
16	1786.90	0.23	0.00	0.00	23.31	207.92	0.00	0.00	2018.38
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
18	610.22	0.07	0.00	0.00	-114	76.02	1.46	292.99	1094.78

19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	1187.91	0.00	0.00	-6.09	-37.06	652.31	7.80	2887.74	4778.93
21	437.20	0.1269	0.00	-1.15	-84	294.55	5.39	0.00	822.43
22	144.64	0.00	0.00	-2.22	0.00	0.00	2.58	118.67	268.12
23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
24	1005.51	0.00	0.00	0.00	11.40	205.33	3.30	0.00	1225.55
25	1538.64	0.34	0.00	2.01	20.81	248.88	0.00	0.00	1810.70
26	2775.14	0.00	0.00	1.16	29.11	345.63	1.74	254.06	3406.87
27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 5

DMU	s_1^{-*}	s_2^{-*}	s_3^{-*}	$s_1^{\text{output}*}$	$s_2^{\text{output}*}$	S_o^{1*}	Peer group
1	1128.73	0.10	0.00	3.72	0.00	1132.56	3,27
2	865.52	0.41	0.00	4.35	0.00	870.29	3,27
3	0.00	0.00	0.00	0.00	0.00	0.00	NA
4	998.93	0.00	0.00	3.17	0.00	1002.11	3,27
5	1057.35	0.26	0.00	6.50	0.00	1064.12	23,27
6	495.59	0.41	0.00	0.00	70.43	566.43	19,27
7	1058.06	0.00	25.96	4.42	0.00	1088.45	23,27
8	309.31	0.33	0.00	2.49	0.00	312.14	3,27
9	1858.38	0.00	1.60	0.70	0.00	1860.69	23,27
10	1059.37	0.58	0.00	0.00	53.28	1113.24	19,27
11	2447.66	0.1805	0.0000	0.2185	0.0000	2448.0595	23,27
12	578.99	0.3909	0.0000	2.3376	0.0000	581.7225	3,27
13	2154.25	0.18	0.00	0.5540	0.00	2154.99	23,27
14	906.58	0.10	0.00	4.3225	0.00	911.01	3,27
15	1565.05	0.00	2.12	0.00	4.3276	1571.50	23,27
16	1700.65	0.32	0.00	2.5910	0.00	1703.56	23,27
17	2069.35	0.01	0.00	1.71	0.00	2071.09	3,27
18	1732.89	0.17	0.00	0.00	67.38	1800.46	23,27
19	0.00	0.00	0.00	0.00	0.00	0.00	NA
20	1674.57	0.00	26.23	2.96	0.00	1703.76	23,27
21	2666.51	0.12	0.00	0.00	30.35	2697.00	3,23
22	1495.66	0.01	0.00	3.45	0.00	1499.13	3,27

23	0.00	0.00	0.00	0.00	0.00	0.00	NA
24	1262.55	0.05	0.00	1.67	0.00	1264.28	23,27
25	1233.86	0.4563	0.00	0.51	0.00	1234.83	23,27
26	2283.43	0.10	0.00	1.53	0.00	2285.07	23,27
27	0.00	0.00	0.00	0.00	0.00	0.00	NA

Table 6

DMU	$s_1^{\text{input}*}$	$s_2^{\text{input}*}$	s_1^{+*}	s_2^{+*}	s_3^{+*}	S_o^{2*}	Peer group
1	0.00	0.00	190.91	2.66	1458.98	1652.56	5,21
2	0.00	0.00	135.22	1.94	1870.90	2008.06	5,21
3	0.00	0.00	296.25	2.15	3932.94	4231.35	5,21
4	0.00	0.00	132.43	0.00	1173.04	1305.48	7,18,23
5	0.00	0.00	0.00	0.00	0.00	0.00	NA
6	2.05	0.00	0.00	0.00	470.72	472.78	10,18,21
7	0.00	0.00	0.0000	0.00	0.00	0.00	NA
8	0.00	0.00	431.36	0.00	3511.27	3942.63	5,21,27
9	0.00	0.00	59.85	1.33	803.27	864.46	5,21
10	0.00	0.00	0.00	0.00	0.00	0.00	NA
11	0.00	0.00	0.0000	2.52	2310.36	2312.89	5,21,23
12	0.00	0.00	24.1011	0.00	479.0353	503.13	5,7,23
13	0.00	0.00	381.3529	0.00	2773.29	3154.64	5,21,23
14	0.00	0.00	126.94	1.44	2345.99	2474.37	5,21
15	0.00	0.00	98.21	1.41	1507.48	1607.11	5,21
16	0.00	0.00	530.20	0.00	3143.59	3673.79	5,21,23
17	0.00	0.00	166.22	0.00	671.54	837.77	5,21,23
18	0.00	0.00	0.00	0.00	0.00	0.00	NA
19	0.00	0.00	132.51	0.00	79.53	212.05	18,21,23
20	0.00	0.00	252.27	1.07	1729.86	1983.21	5,21
21	0.00	0.00	0.00	0.00	0.00	0.00	NA
22	0.00	0.00	0.00	0.68	741.13	741.82	5,21,23
23	0.00	0.00	0.00	0.00	0.00	0.00	NA
24	0.00	0.00	170.99	1.47	2026.58	2199.04	57,21

25	0.00	0.00	345.41	0.00	3205.17	3550.58	5,21,23
26	0.00	0.00	87.84	0.37	2074.81	2163.03	5,21
27	0.00	0.00	575.35	0.00	3879.94	4455.29	18,21,23

4.1 Analysis of resin producing companies from a pessimistic perspective

More advanced technology in industries that require high capital costs such as high-tech or largescale manufacturing industries usually means a lot of money. In high-risk industries such as insurance or banking, high profits are usually associated with high risk. Investment risk assessment is of great importance for financial institutions or individuals investing in high-risk industries or those requiring high capital. In the case of bad investment, Type I risk and loss from a failed investment cannot be identified. Type I error indicates investment risk that should be minimized. Obviously, the cost of Type I error is much greater than that of Type II risk (the loss that a financial institution or an investor would suffer in the case of a successful investment). Therefore, it is very important to identify and measure the risk of investment. Financial institutions or individual investors should evaluate the performance of industrial companies before investing in such corporates.
Optimistic models identify companies that are potentially under pressure based on the extent of inefficiency in the optimal scenario which is not suitable for the real world. In the vulnerable competitive business space, potential companies that first exit business are usually those with the lowest competitive power in the worst scenario, especially when a recession or financial crisis occurs. Therefore, for investment risk assessment or bankruptcy prediction, a more meaningful method is to evaluate the worst performance of entities in the most unfavorable scenario.
For this real case study, the Model (2.11) is first applied to all DMUs to measure their inefficiency slacks from a pessimistic perspective. Then the pessimistic additive DEA model of the traditional black box is applied in both stages to measure their inefficiency slacks from a pessimistic point of view. The results are shown in Tables 4.1 to 4.1. It is clearly seen that

inefficiency slacks measured from two different DEA perspectives are significantly different.

According to Table 4.1, the DMUs 2, 9, 10, 11, 19, 20, 21, and 26 are identified as pessimistic inefficient DMUs by the Model (2.11). Based on the results obtained from the Model (2.11), the slack of the intermediate product z_1 is negative for 4 DMUs. The slack of the intermediate product z_2 is also negative for 3 DMUs. Both slacks of the intermediate products of DMUs 1, 4, 7, 8, 13, 15, 16, 24, and 27 are positive while both are identified negative for the intermediate products of DMUs 12 and 22.

Table 7

DMU	s_1^{-*}	s_2^{-*}	s_3^{-*}	$s_1^{\text{intermediate}*}$	$s_2^{\text{intermediate}*}$	s_1^{+*}	s_2^{+*}	s_3^{+*}	$s_o^{\text{overall}*}$
1	0.0000	0.1693	0.0000	0.6708	60.6076	0.0000	0.0000	768.7939	830.2416
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.4241	1.9582	0.0000	112.3963	87.9348	0.0000	348.5727	551.2861
4	0.0000	0.3159	0.0000	1.2818	51.6528	129.0398	6.9032	591.0501	780.2436
5	0.0000	0.1725	0.0000	-1.3032	0.0000	148.7125	1.7426	1307.3946	1459.3254
6	0.0000	0.0000	0.0000	4.9969	0.0000	60.2967	2.5836	565.0472	632.9243
7	0.0000	0.2662	0.0000	0.9376	36.0616	190.4403	7.3128	1296.9346	1531.9530
8	0.0000	0.0000	0.0000	3.8257	80.6059	97.1659	5.0986	997.2105	1183.9066
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	1621.6361	0.0000	0.0000	-0.7938	-56.6661	232.5242	2.5000	19.1736	1933.2938
13	0.0000	0.4652	87.9715	1.5301	11.1040	0.0000	0.4064	587.9171	689.3942
14	0.0000	0.2165	0.0000	0.0000	39.0148	52.8573	0.4155	2.7678	95.2719
15	0.0000	0.0146	0.0000	2.1137	1.8201	19.9988	0.0000	1133.0168	1156.9640
16	0.0000	0.0647	0.0000	1.8231	47.6780	0.0000	3.8808	394.5142	447.9608
17	0.0000	0.5130	5.5194	-2.2785	0.0000	255.3055	0.5453	325.6684	589.8301
18	0.0000	0.3005	0.0000	4.2104	0.0000	119.6756	3.7583	510.4407	638.3855
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	0.0000	0.4864	25.0768	-1.2078	-31.9983	110.9117	0.0000	308.5875	478.2684
23	1883.2780	0.0053	0.0000	0.0000	-8.3039	506.1369	2.1034	1022.723	3422.5503
24	0.0000	0.1804	0.0000	1.1588	39.1729	0.0000	0.0000	202.5974	243.1094
25	173.8689	0.0000	111.0764	3.1972	0.0000	44.6079	2.5078	0.0000	335.2581
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

27	0.0000	0.2432	0.0000	8.1561	73.5471	105.2537	8.0375	1755.8196	1951.0573

Pessimistic inefficient and non-inefficient DMUs in the first and second stages are identified by applying the pessimistic additive DEA model of traditional black box on data in Table 3.1. The results are listed in Table 4.1 and 4.1. According to the slacks in Table 4.1, the DMUs 2, 5, 12, 20, 21, and 22 are identified as pessimistic inefficient DMUs in the first stage. According to the slacks reported in Table 4.1, the DMUs 1, 2, 3, 6, 10, 13, 19, 20, 24, 25, and 27 are identified as pessimistic inefficient DMUs in the second stage. DMU_2 , DMU_{10} and DMU_{20} identified as pessimistic inefficient DMUs in the first stage are also identified as pessimistic inefficient DMUs and systems in the second stage.
Table 8

DMU	s_1^{-*}	s_2^{-*}	s_3^{-*}	$s_1^{\text{output}*}$	$s_2^{\text{output}*}$	S_o^{1*}	Peer group
1	1067.76	0.00	6.88	0.00	0.00	1074.65	5,21,22
2	0.00	0.00	0.00	0.00	0.00	0.00	NA
3	1943.96	0.06	84.24	0.00	0.00	2028.27	21,22
4	1507.66	0.133	16	0.00	0.00	1523.80	21,22
5	0.00	0.00	0.00	0.00	0.00	0.00	NA
6	1154.76	0.00	9.20	2.92	0.00	1166.88	10,21
7	1005.95	0.29	0.00	0.00	0.00	1006.25	5,20,21
8	2650.04	0.00	50.09	0.00	0.00	2700.13	5,21,22
9	353.68	0.07	0.00	0.00	0.00	353.75	5,21,22
10	0.00	0.00	0.00	0.00	0.00	0.00	NA
11	1058.24	0.00	13.78	0.00	0.00	1072.02	5,21,22
12	0.00	0.00	0.00	0.00	0.00	0.00	NA
13	1788.62	0.17	57.25	0.00	0.00	1846.04	21,22
14	878.81	0.00	3.85	0.00	0.00	882.67	5,21,22
15	1259.67	0.04	0.00	0.00	0.00	1259.72	5,20,21
16	1833.17	0.00	34.82	0.00	0.00	1868.00	5,21,22
17	135.40	0.12	26.50	0.00	0.00	162.04	21,22
18	1283.07	0.24	30.21	2.78	0.00	1316.31	21
19	381.31	0.00	65.48	0.46	0.00	447.26	10,21
20	0.00	0.00	0.00	0.00	0.00	0.00	NA

21	0.00	0.00	0.00	0.00	0.00	0.00	NA
22	0.00	0.00	0.00	0.00	0.00	0.00	NA
23	1868.33	0.00	0.00	0.11	0.00	1868.44	10,20,21
24	1165.91	0.0469	7.3039	0.00	0.00	1173.26	21,22
25	1741.01	0.0000	101.69	0.00	0.00	1842.71	5,20,22
26	476.65	0.0029	0.00	0.00	0.00	476.65	5,20,22
27	3958.12	0.39	65.04	0.00	0.00	4023.56	21,22

Table 9

DMU	$s_1^{\text{input}*}$	$s_2^{\text{input}*}$	s_1^{+*}	s_2^{+*}	s_3^{+*}	S_o^{2*}	Peer group
1	0.00	0.00	0.00	0.00	0.00	0.00	NA
2	0.00	0.00	0.00	0.00	0.00	0.00	NA
3	0.00	0.00	0.00	0.00	0.00	0.00	NA
4	0.00	0.00	132.13	5.31	560.82	698.28	3,27
5	2.18	0.00	195.38	1.12	1544.97	1743.67	3
6	0.00	0.00	0.00	0.00	0.00	0.00	NA
7	0.00	0.00	181.44	6.42	1431.78	1619.65	3,27
8	0.00	0.00	2.05	1.45	78.38	81.89	3,27
9	0.00	0.00	0.00	0.00	946.34	946.34	1,3,10,24
10	0.00	0.00	0.00	0.00	0.00	0.00	NA
11	0.00	0.00	73.54	0.00	481.61	555.16	2,3,10
12	0.00	0.00	349.48	3.02	1410.6910	1763.1881	3,27
13	0.00	0.00	0.00	0.00	0.00	0.00	NA
14	0.32	0.00	69.10	0.00	103.93	173.36	2,3
15	0.00	0.00	0.00	0.00	661.30	661.30	3,6,10,24
16	0.00	0.00	0.00	1.51	601.20	602.71	3,25,27
17	0.00	0.00	284.24	2.03	1752.74	2039.02	3,27
18	0.00	37.87	14.87	0.00	801.25	854.00	6,27
19	0.00	0.00	0.00	0.00	0.00	0.00	NA
20	0.00	0.00	0.00	0.00	0.00	0.00	NA
21	0.00	0.00	0.00	0.00	2297.85	2297.85	3,6,10,24
22	0.40	0.00	204.67	0.76	1316.52	1522.36	3
23	0.00	0.00	465.32	1.71	1970.32	2437.36	3,27
24	0.00	0.00	0.00	0.00	0.00	0.00	NA

25	0.00	0.00	0.00	0.00	0.00	0.00	NA
26	0.00	0.00	69.72	0.00	911.25	980.97	3,6,27
27	0.00	0.00	0.00	0.00	0.00	0.00	NA

5 Conclusion

There are two concepts for measuring the efficiency of a system including efficiency decomposition and aggregation. When efficiency decomposition is used, the system efficiency should be defined as objective function as the ratio of aggregate exogenous output to exogenous input taking into account operation of sectors. The partial efficiency is calculated by dividing the aggregate output by the aggregate input. The efficiency aggregation concept is based on the definition of system efficiency as the sum of partial efficiencies. Due to the nonlinear form, the weight used in the weighted average should be selected carefully to obtain an implicit linear model. The weight of a part is usually defined as the ratio of input aggregate efficiency used in this part to that consumed by the two parts.

Several additive DEA models were developed to measure the inefficiency of inputs and outputs of two-stage DEA from optimistic and pessimistic perspectives. Unlike efficiency decomposition and aggregation approaches, the inefficiency slacks of intermediate products are considered in the objective function. It was also shown that how the network DEA analysis of the worst performance with the aim of identifying inefficient firms can be used to identify DMUs with the worst performance, especially for bankruptcy assessment. In addition, it was found that the use of optimistic and pessimistic perspectives may lead to a much higher classification accuracy. Furthermore, optimistic and pessimistic perspectives provide more flexibility in choosing the best DMU, and thus the risk can be included in the calculations.

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