

Biaxial Buckling Analysis of Symmetric Functionally Graded Metal Cored Plates Resting on Elastic Foundation under Various Edge Conditions Using Galerkin Method

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ABSTRACT

In this paper, buckling behavior of symmetric functionally graded plates resting on elastic foundation is investigated and their critical buckling load in different conditions is calculated and compared. Plate governing equations are derived using the principle of minimum potential energy. Afterwards, displacement field is solved using Galerkin method and the proposed process is examined through numerical examples. Effect of FGM power law index, plate aspect ratio, elastic foundation stiffness and metal core thickness on critical buckling load is investigated. The accuracy of this approach is verified by comparing its results to those obtained in another work, which is performed using Fourier series expansion.

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Keywords : Functionally graded material; Plate; Buckling analysis; Galerkin method; Elastic foundation.

1 INTRODUCTION

SINCE the introduction of Functionally Graded Materials (FGMs) in 1972, these materials are being used in aerospace, biomechanics, petrochemical, marine industry, also in civil engineering and mechanical engineering wherever it is needed to eliminate stress concentration or to reach higher strength and endurance [1, 2, 3, 4, 5]. FGMs are composite materials with non-homogenous micro-structures, that their mechanical characteristics changes continually through thickness [6, 7, 8] and obeys specific distribution functions such as power law index [9, 10, 11]. Metallic and ceramic material are widely used together in FGMs to improve the strength and temperature endurance. This combination also terminates contact surface, corrosion, cracking and de-bond problems which are usual in conventional composites [7, 10, 12, 13]. Because of these interesting characteristics, researchers have investigated the behavior of these materials from structural, vibrational and stress point of views. Pan [14] has derived an exact solution to stress analysis for rectangular FGM plates with simply supported edge conditions. Li et al. [3, 4], Cheng and Batra [13], Kashtalyan [15], Zenkour [16], Zheng and Zhong [17], Vel and Batra [18] and Nguyen et al. [19] analyzed these materials to investigate their stress-strain behavior. Hopkiins and Chamis [20] presented a unique set of micromechanics equations for high temperature. Birman [21] solved the buckling problem of FGM hybrid composite plates based on these equations. Feldman and Aboudi [8] investigated elastic bifurcation buckling of FGM plates with non-uniformly distributed fibers under uniaxial compressive loading by combining micromechanical and structural approach. Chen and Liew [22] investigated buckling behavior of a two-dimensional

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elastic plane stress problem of FGM plates subjected to nonlinearly distributed in-plane edge loads using Mindlin's plate assumption. Saidi et al. [23] investigated axisymmetric bending and buckling of FGM circular plates by means of Unconstrained Third-Order Shear Deformation Theory (UTST). Mohammadi et al. [24] presented Levy solution for the buckling analysis of thin rectangular FGM plates subjected to different mechanical loads under different boundary conditions using the principle of minimum potential energy based on the CPT (Kirchhoff Theory).

Another important issue is the behavior of FGM plate, when it is resting on elastic foundation. Dung DV, Thiem HT [25] investigated stability of eccentrically stiffened functionally graded imperfect plates resting on elastic foundation. Sobhy [26] investigated buckling and free vibration of exponentially graded sandwich plates resting on elastic foundation for different boundary conditions using shear deformation plate theory. Naderi and Saidi [27] extracted an exact solution for stability analysis of moderately thick functionally graded sector plates resting on elastic foundation using first order shear deformation plate theory. Yaghoobi and Fereidoon [28] investigated buckling behavior of functionally graded plates resting on elastic foundation using refined n th-order shear deformation theory. Thai and Kim [29] offered a closed-form solution for buckling analysis of thick functionally graded plates on elastic foundation. Surveying the literature, it is clear that the effect of elastic foundation, especially its effect on buckling of FGM plates, has not been in proper attention.

In this paper, governing equations for FGM plate resting on elastic foundation is extracted using Classic Plate Theory (CPT). Using Galerkin method, these equations are solved for eight different boundary conditions and critical buckling load are calculated for them. Then, effect of FGM power law index, plate aspect ratio, elastic foundation stiffness and metal core thickness on critical buckling load is investigated through numerical examples. Finally, in order to confirm the validity of the results a comparison between present work and a previous work is presented and it is shown that the difference of results is under 1 percent in almost all cases.

2 CLASSIC PLATE THEORY

Generally modeling plates is done based on categorizing them into thick, mid thick and thin. Since thin plates are considered in this paper, Kirchhoff classic theory is used. Thus, displacement field is given as:

$$u_x = u(x, y) - z \frac{\partial w(x, y)}{\partial x} \quad (1a)$$

$$u_y = v(x, y) - z \frac{\partial w(x, y)}{\partial y} \quad (1b)$$

$$u_z = w(x, y) \quad (1c)$$

In which, u_x, u_y and u_z are representing displacements in respective directions. Also, linear strain-displacement relations in elasticity are given as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2} \quad (2a)$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} - z \frac{\partial^2 w}{\partial y^2} \quad (2b)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z \frac{\partial^2 w}{\partial x \partial y} \quad (2c)$$

In which, u, v and w are displacements in mid-plane in respect to three directions. Developed form of stress-strain relationship in 2D theory of elasticity is given by:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E(z)}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \quad (3)$$

In which E , ν are poisson ratio and young modulus. Directly integrating both sides of Eq. (3) results in forces and multiplying it by Z and integrating results in moments:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = A \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} \quad (4)$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = -D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (5)$$

In which M_{ij} , N_{ij} , D and A are total moments, total forces, momentary and linear stiffnesses, respectively. These parameters are given by:

$$(N_{ij}, M_{ij}) = \int_{-h/2}^{h/2} \sigma_{ij}(1, z) dz \quad (6a)$$

$$(A, D) = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu^2} (1, z^2) dz \quad (6b)$$

3 FGM COMPOSITE MATERIALS

Based on composite material law, final product properties will be sum of primary materials properties multiplied by their volume fractions. In functionally graded materials where each material ratio is variable through thickness, as shown in Fig. 1, final properties are also varied through thickness.

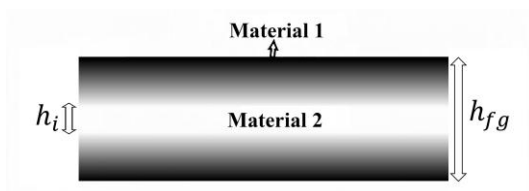


Fig.1
Schematic of symmetric FGM cored plate.

There are various offered functions describing volume fraction of FGM components, one of them is power law index. In this case, volume fraction of both materials is varied through thickness and dependent on the power law index (Fig. 2).

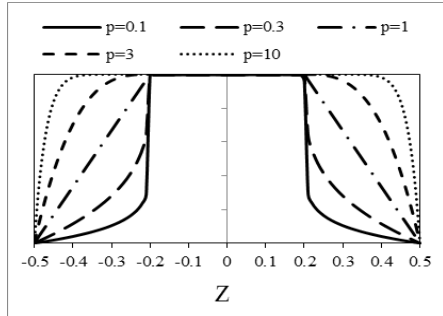


Fig.2
Effect of power law index on symmetric FGM cored plate properties through thickness.

For symmetric FGM cored plates, shown in Fig. 1, mechanical properties are given by [7, 9]:

$$E(z) = E_i + (E_0 - E_i) \left(\frac{\text{real} \left(\sqrt{|z| - \frac{h_i}{2}} \right)^2}{h_{fg}} \right)^p \tag{7a}$$

$$\rho(z) = \rho_i + (\rho_0 - \rho_i) \left(\frac{\text{real} \left(\sqrt{|z| - \frac{h_i}{2}} \right)^2}{h_{fg}} \right)^p \tag{7b}$$

In which $E_i, \rho_i, E_0,$ and ρ_0 are elasticity modulus and density for internal and external material, h_{fg} is thickness of functionally graded part, h_i is thickness of the plate core and p is plate power law index.

There are several methods such as newton law, virtual work, energy methods, minimizing total potential energy, etc., for obtaining displacement field of elastic materials. In this paper minimum potential energy method is used. This method is based on minimizing the sum of external and internal work:

$$\delta(W_I + W_E) = \delta W_I + \delta W_E = 0 \tag{8}$$

where δ is variational operator, W_I, W_E are internal and external force works, respectively. Using the boundary conditions shown in Fig. 3, stress function is given by:

$$\phi = \frac{\lambda F}{2} x^2 + \frac{F}{2} y^2 \tag{9}$$

In which F is the applied force in x direction and λ is the aspect ratio of forces in two normal directions as shown in Fig. 3.

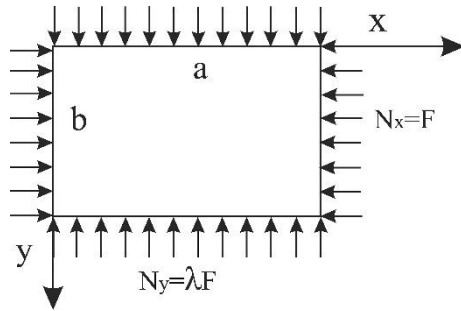


Fig.3
Applied in plane forces on plate edges.

As presented in literature [30], after substituting stress function and moments (in term of displacement) in equilibrium equation the plate displacement field is obtained:

$$-D\nabla^4 w - kw + F \frac{\partial^2 w}{\partial x^2} + \lambda F \frac{\partial^2 w}{\partial y^2} = 0 \tag{10}$$

or,

$$-D \left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) - kw + F \frac{\partial^2 w}{\partial x^2} + \lambda F \frac{\partial^2 w}{\partial y^2} = 0 \tag{11}$$

4 GALERKIN METHOD

There are several numerical solutions for solving the obtained differential equation. In Galerkin method it is assumed that the answer is the sum of a sequence of φ_i functions with homogenous essential and natural boundary conditions in form of:

$$U_N = \sum_{i=1}^N c_i \varphi_i + \varphi_0 \tag{12}$$

where U_N is the approximate solution, c_i are constants connecting functions which must be determined, N the number of functions, φ_i are trial functions and φ_0 is a function which is satisfying the boundary conditions. In this paper, effect of eight different boundary conditions, shown in Fig. 4, is investigated:

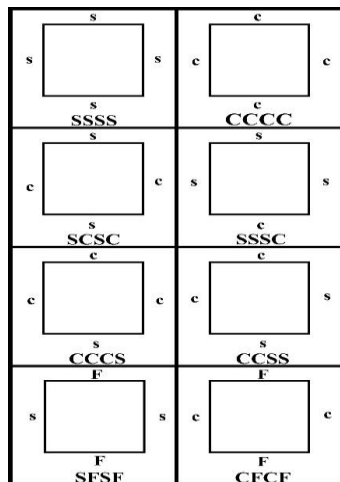


Fig.4
Considered boundary conditions.

In Fig. 4, C , S and F are representing clamped, simply supported and free boundary conditions on entire edge. Approximation functions for each of these boundary conditions are given as:

Four side simply support (ssss):

$$W_{MN} = \sum_{m=1}^M \sum_{n=1}^N w_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (13)$$

Four side clamped support (cccc):

$$W_{MN} = \sum_{m=1}^M \sum_{n=1}^N w_{mn} \left(1 - \cos \frac{2m\pi x}{a}\right) \left(1 - \cos \frac{2n\pi y}{b}\right) \quad (14)$$

Symmetric two side clamped and two side simply supported (scsc):

$$W_{MN} = \sum_{m=1}^M \sum_{n=1}^N w_{mn} \sin \frac{m\pi x}{a} \left(1 - \cos \frac{2n\pi y}{b}\right) \quad (15)$$

One side clamped and three side simply supported (sssc):

$$W_{MN} = \sum_{m=1}^M \sum_{n=1}^N w_{mn} \sin \frac{m\pi x}{a} \cos \frac{\pi y (2n-1)}{2b} \left(\sin \frac{\pi y (2n-1)}{2b}\right)^2 \quad (16)$$

Three side clamped and one side simply supported (cccs):

$$W_{MN} = \sum_{m=1}^M \sum_{n=1}^N w_{mn} \left(1 - \cos \frac{2m\pi x}{a}\right) \cos \frac{\pi y (2n-1)}{2b} \left(\sin \frac{\pi y (2n-1)}{2b}\right)^2 \quad (17)$$

Asymmetric two side clamped and two side simply supported (ccss):

$$W_{MN} = \sum_{m=1}^M \sum_{n=1}^N w_{mn} \cos \frac{\pi x (2m-1)}{2a} \left(\sin \frac{\pi x (2m-1)}{2a}\right)^2 \cos \frac{\pi y (2n-1)}{2b} \left(\sin \frac{\pi y (2n-1)}{2b}\right)^2 \quad (18)$$

Symmetric two sides simply supported and two sides free (sfsf):

$$W_{MN} = \sum_{m=1}^M \sum_{n=1}^N w_{mn} \sin \frac{m\pi x}{a} \left(\cos \frac{2n\pi y}{b} + 2n^2\pi^2 \left(\left(\frac{y}{b}\right)^2 - \frac{y}{b} + 1\right)\right) \quad (19)$$

Symmetric two sides clamped and two sides free (cfcf):

$$W_{MN} = \sum_{m=1}^M \sum_{n=1}^N w_{mn} \left(1 - \cos \frac{2m\pi x}{a}\right) \left(\cos \frac{2n\pi y}{b} + 2n^2\pi^2 \left(\left(\frac{y}{b}\right)^2 - \frac{y}{b} + 1\right)\right) \quad (20)$$

In which w_{mn} are plate displacement constants, m and n are number of half waves in x and y directions and a and b are length of plate in these directions. Based on Galerkin method, after rearrangement of displacement field (Eq. (11)) and substitution of approximation functions (w_{MN}), constants and other unknowns are calculated using the following procedure:

$$A(w) = f \quad (21)$$

$$R_{MN} = A(W_{MN}) - f \neq 0 \quad (22)$$

$$\int_0^b \int_0^a \varphi_{ij} R_{MN} dx dy = 0 \begin{cases} i = 1, 2, 3, \dots, M \\ j = 1, 2, 3, \dots, N \end{cases} \quad (23)$$

In which φ_{ij} are the approximation functions or trial functions (in two dimensional form), R_{MN} are error functions (residuals). Eq. (23) is in fact representing $M \times N$ equation and have the same number of unknown which rearranging them, having Eq. (11) in mind, gives out the matrix equation as below:

$$[B]_{(M \times N) \times (M \times N)} [w_{mn}]_{(M \times N) \times 1} = [0]_{(M \times N) \times 1} \quad (24)$$

Nontrivial solution exists only if determinant of coefficient matrix (B) is zero. Critical load, F in Eq. (11), is reached by taking determinant of B equal to zero. For each boundary conditions, this equation is solved and its results are presented in the next section.

5 NUMERICAL EXAMPLE

Effect of power law index, plate aspect ratio, elastic foundation stiffness and metal core thickness on critical buckling load is examined through series of numerical examples and their results are shown in following graphs. These calculations are done for a 5 mm thick plate made of aluminum and alumina ceramic.

5.1 Effect of FGM power law index

Using previously obtained equations, effect of FGM power law index on critical buckling load in case of quadratic plate, symmetric biaxial in-plane pressure, non-elastic condition, 1 mm thick metal core and $a/h = 20$ for four types of boundary conditions is investigated (Fig. 5). Results show that any increase in power law index will reduce buckling load, because in higher power law indexes the amount of ceramic will reduce and the plate will be softer mechanically. However this decreasing effect has more influence on solid supports and it also wears off in higher power law indexes for all boundary conditions. Fig. 5 also shows that SSSS and CFCF graphs are fairly close and so it can give the impression that rigidity of these two conditions is similar.

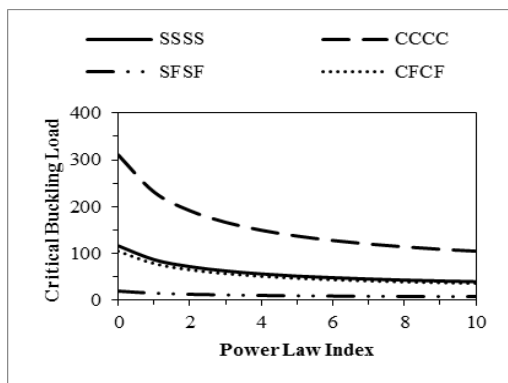


Fig.5
Effect of FGM power law index on critical buckling load.

In case of SFSF (Symmetric two sides simply supported and two sides free) critical buckling load is very low because there is no boundary stiffness resisting loads and keeping the plate at its original state. It should be stated that the critical buckling load is not zero but rather very low.

5.2 Effect of plate aspect ratio

Effect of plate aspect ratio on critical buckling load in non-elastic condition, uniaxial pressure, power law index of unit and without metal core have been derived (Fig. 6). In case of SFSF, the critical buckling load is very low with the same reasoning as in section 5.1.

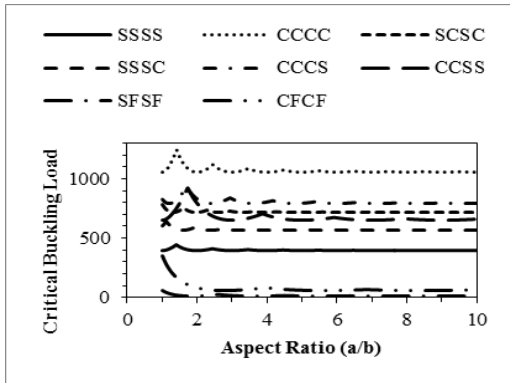


Fig.6
Effect of aspect ratio on plate critical buckling load.

Results indicate that increasing the aspect ratio has a paternal effect on critical buckling load. The reason for this is when plate aspect ratio (a/b) increases, buckling mode jumps to the next shape (with more half waves) (Fig. 7).

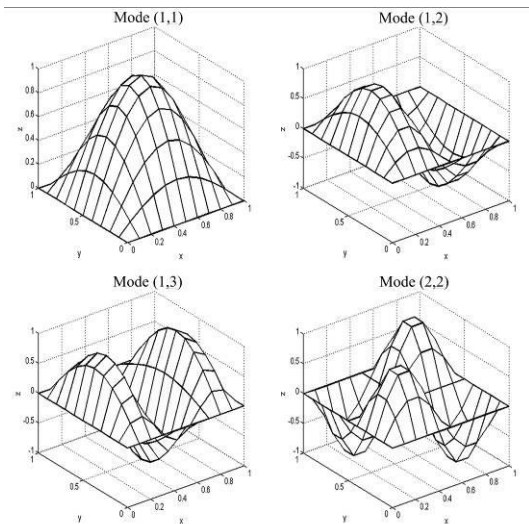


Fig.7
Different mode shape caused by changing plate aspect ratio.

In present method of calculating plate critical load, it is calculated for different modes and then their minimum is chosen as the critical buckling load. Each curve is obtained by putting together all the minimum forces in each domain. In each specific boundary condition, minimum buckling load for all modes is constant. Also, as expected, this graphs show that more boundary condition rigidity and symmetricity will result in higher critical buckling load.

5.3 Effect of elastic foundation stiffness

Effect of elastic foundation stiffness on critical buckling load in quadratic plate with symmetric biaxial loading, 1 mm thick metal core and unit power law index is derived (Fig. 8). Obtained graphs show that for all of the boundary conditions any increase in elastic foundation stiffness will result in higher critical buckling load. However, the influence of elastic foundation on plates with two free sides is more than other cases and it simply can be attributed to the difference in rigidity of boundary conditions for different cases.

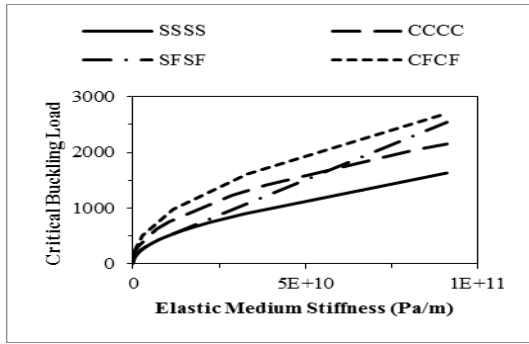


Fig.8
Effect of elastic foundation stiffness on critical buckling load.

5.4 Effect of metal core thickness

Effect of metal core thickness on critical buckling load in a quadratic plate made of aluminum as core and ceramic as the outer layers have been investigated (Fig. 9). Results show that any increase in thickness of metal core will result in lower buckling load.

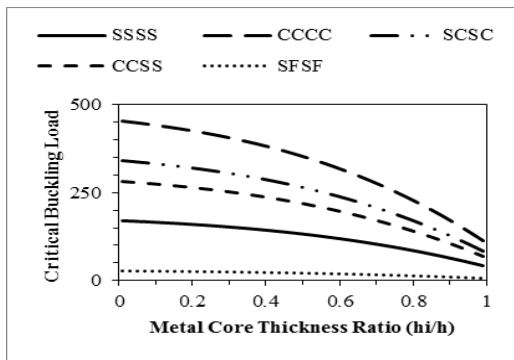


Fig.9
Effect of metal core thickness on critical buckling load.

It should be noted that in other cases, except when plate aspect ratio is changing, first buckling mode (which is of value here) contains only one half wave in two directions (Fig. 10).

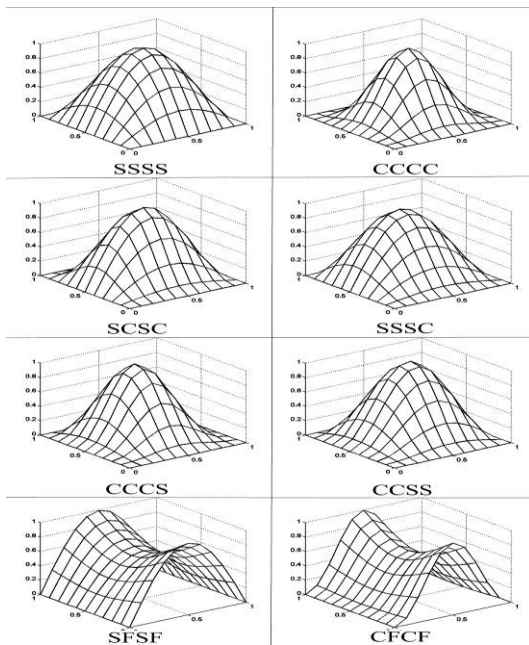


Fig.10
First mode shape of plate with different boundary conditions.

6 VALIDATION

For validating the obtained results and estimating the approximation error, a comparison between the present research results and results of an investigation performed by Latifi et al. [31], which had used Fourier series expansion, is presented. As shown in Table 1. The results of present approach are fairly close to the previous data. Table 1. Also shows that the difference between two researches is under 1 percent in most cases and so recent approach is reliable.

Table 1
Comparison of buckling force of FGM plate.

| p | b/a | Present | Latifi et.al[31] | Error % |
|-----|-------|----------|------------------|----------|
| 0 | 1 | 6.868929 | 6.811 | 0.850519 |
| | 1.5 | 8.061451 | 7.981 | 1.008034 |
| | 2 | 10.7327 | 10.63 | 0.966146 |
| | 2.5 | 14.44192 | 14.311 | 0.914841 |
| | 3 | 19.08036 | 18.921 | 0.842228 |
| 1 | 1 | 3.423766 | 3.392 | 0.936491 |
| | 1.5 | 4.01817 | 3.97 | 1.213339 |
| | 2 | 5.349634 | 5.312 | 0.708472 |
| | 2.5 | 7.198468 | 7.132 | 0.931962 |
| | 3 | 9.51046 | 9.431 | 0.842545 |
| 2 | 1 | 2.671631 | 2.65 | 0.816258 |
| | 1.5 | 3.135456 | 3.112 | 0.753716 |
| | 2 | 4.174423 | 4.13 | 1.075622 |
| | 2.5 | 5.617104 | 5.564 | 0.954419 |
| | 3 | 7.421197 | 7.352 | 0.941197 |
| 5 | 1 | 2.259323 | 2.241 | 0.817626 |
| | 1.5 | 2.651567 | 2.621 | 1.166218 |
| | 2 | 3.530192 | 3.512 | 0.518001 |
| | 2.5 | 4.750227 | 4.723 | 0.576468 |
| | 3 | 6.275897 | 6.223 | 0.850027 |

7 CONCLUSIONS

Governing equations of symmetric FGM cored plate have been derived using classic plate theory (CPT). Using Galerkin method these equations were solved for eight different boundary conditions and critical buckling load were calculated. Numerical examples were presented to investigate the effect of different parameters on critical buckling load. Based on graphs obtained from these numerical examples, results of this research can be pointed out as follow:

- Increasing power law index in ceramic-metal-ceramic FGMs will decrease critical buckling load,
- Although increasing plate aspect ratio will change buckling mode, minimum buckling load for all modes is constant,
- Increasing elastic foundation stiffness will increase critical buckling load,
- Lower boundary condition rigidity will result in lower critical buckling load,
- In absence of elastic foundation, in boundary conditions with lower rigidity, critical buckling load is lower, and
- Increasing thickness of metal core will decrease critical buckling load.

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