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Transference of SH-Waves in Fluid Saturated Porous Medium Sandwiched Between Heterogeneous Half-Spaces

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ABSTRACT

A mathematical model is considered to investigate the behavior of horizontally polarized shear waves (SH-waves) in fluid saturated porous medium sandwiched between heterogeneous half-spaces. Heterogeneity in the upper half-space is due to linear variation of elastic parameters, whereas quadratic variation has been considered for lower half-space. The method of separation of variables and Whittaker's function are used to get an analytical solution for the considered problem. Frequency equation of SH waves in considered model has been obtained. Also, frequency equations have been derived for several particular cases. It is observed that the heterogeneity and porosity have significant effect on the phase velocity of SH-waves. In particular, heterogeneity and porosity increases the phase velocity of SH-waves. Obtained result is matched with classical Love wave equation. Graphical representation is done efficiently to explain the findings. Also the surface plot is added to exhibit the velocity profile of SH-waves in different cases.

Keywords: Heterogeneity; SH-waves; Frequency equation; Porosity; Fluid saturated medium; Whittaker function.

1 INTRODUCTION

THE earth consists of layers of different types of material properties. Anisotropy and porosity are basic characteristics of the earth. Several evidences are there in the literature that the study of wave propagation helps to better understand the material properties of layered earth and cause of earthquakes. Lithosphere is the part of earth's interior which includes earth's crust and parts of the upper mantle. Most of the faults and earthquakes occur in the lithosphere because earthquakes result from sudden breaks or shifts in rock. Earth's crust is not homogeneous but has a vertically layered structure. The basic characteristic and Geophysical studies about earth structure motivate to study the propagation of horizontally polarized shear waves (SH waves) in heterogeneous medium.

Geophysical studies have established the fact that among different patterns of heterogeneity beneath the earth, linear variation in elastic parameters appears in great contrast for the crustal part of the earth. Two of the most important hydro-geologic parameters are permeability and porosity. Permeability, the ease of fluid flow through porous medium is a fundamental control on subsurface flow at all depths. Porosity, a measure of the voids spaces in a material controls the quantity of fluid to be stored in a subsurface. Porosity of the medium (soil) plays a key role in global hydro-geological models to represent the subsurface. The existence of ground water reservoirs beneath the land surface is self-evident of the presence of fluid-saturated porous type medium inside the earth. These facts demand for the study of seismic wave behavior in such a zone, where the waves travel through fluid-saturated porous medium bonded by trivial characteristics of the earth medium. A number of attempts have been made to

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explore the understanding of seismic wave behaviour in heterogeneous medium. Detailed information on elastic wave behavior may be found in Ewing et al. [20]. SH wave propagation in laterally heterogeneous medium has been investigated by Roy [13]. Effect of point source and heterogeneity on the propagation of SH-waves has been shown by Chattopadhyay et al. [15]. Kakar and Kakar [17] studied the propagation of Love waves in non-homogeneous elastic media. They observed that the phase velocity depends upon the scaling parameter of non-homogeneous layer and the characteristic velocity approach to the problem that was of waves in homogeneous half space. Propagation of shear waves in anisotropic medium has been investigated by Chattopadhyay et al. [12]. SH-waves in viscoelastic heterogeneous layer over half-space with self-weight has been investigated by Sahu et al. [18]. They found that the heterogeneity, gravity and internal friction have significant effect on the propagation of SH-waves. SH-wave velocity in a fiber-reinforced anisotropic layer overlying a gravitational heterogeneous half-space has been studied by Kakar [19]. Bhattacharya [4] found the exact solution of SH-wave equation for inhomogeneous media. The theory of porous media is important in many problems of engineering including material science, petroleum industry, biomechanics, soil mechanics and other branches. Multiphase flow is an important physical process that underlies many critical activities such as waste disposal, geothermal production, oil and gas production, agriculture, and groundwater management. Geophysical imaging methods are used increasingly to monitor the flow of liquids and gases in the subsurface. Therefore, it is important to have accurate and efficient techniques for modeling wave propagation in heterogeneous and saturated porous medium. There are several ways to approach the coupled modeling of deformation and multiphase fluid flow in a heterogeneous porous medium, each with its own advantages. Whittaker's function is one of the particular cases of hyper geometric function. It has much significance due to its possible applications in different fields. Based on Biot's theory of wave propagation in fluid saturated porous layer some milestone works are there in literature [1, 2, 3]. Chattopadhyay and De [5] investigated the propagation of Love waves in an isotropic fluid saturated porous layer with irregular interface. Chattopadhyay et al. [6] have discussed the propagation of SH-waves in porous layer. Propagation of longitudinal and shear waves in an elastic medium with void pores has been studied by Dey [7]. The propagation behavior of shear waves in an anisotropic fluid saturated porous plate has been discussed by Pradhan et al. [9]. A detailed investigation has been made by Kumar and Hundal [10] to notice the propagation pattern of surface waves in uniform liquid layer overlying a fluid saturated porous half space. The existence and asymptotic behavior of the surface waves at a free interface of a saturated porous medium are investigated in the low frequency range by Edelman [11]. Gaur and Rani [14] studied the surface wave propagation in non-dissipative porous medium. Samal and Chattaraj [16] studied the surface wave propagation in fiber-reinforced anisotropic elastic layer between liquid saturated porous half-spaces. Chattopadhyay et al. [21] have discussed the propagation behavior of shear wave in a fluid saturated porous layer under initial stress. Kundu et al. [22] have studied the Love wave propagation in porous rigid layer lying over an initially stressed half-space. Recently, Love wave dispersion in pre-stressed homogeneous medium over a porous half-space with irregular boundary surfaces has been investigated by Kundu et al. [23]. A number of problems have been solved by taking the structure containing a layer and a half-space of seismic waves but many problems regarding the propagation of seismic waves in layered medium are still unexplored. The aim of the present paper is to characterize the influence of heterogeneity of both half-spaces and the porosity of intermediate layer on the propagation of SH wave in a fluid saturated porous layer sandwiched between two heterogeneous half-spaces. Among the existing literature, the present paper finds its novelty among the existing literature due to its specific geometry/ problem statement and the type of considered surface wave. This problem contributes to enhance the understanding of surface wave behavior in a composite structure with fluid saturated layer bonded between heterogeneous half-spaces.

2 FORMULATION OF THE PROBLEM

We have considered an anisotropic fluid saturated porous layer, sandwiched between heterogeneous half-spaces. The heterogeneity has been considered in rigidity and density. Heterogeneity of the upper half-space has been taken as $\mu = \mu_0(1+mz)$ and $\rho = \rho_0(1+mz)$, whereas, the heterogeneity of the lower half-space has been taken as $\mu_3 = \mu_1(1+sz)^2$ and $\rho_3 = \rho_1(1+sz)^2$, where μ and ρ are rigidity and density of the upper half-space whereas μ_3 and ρ_3 represents the rigidity and density of the lower half-space, respectively.

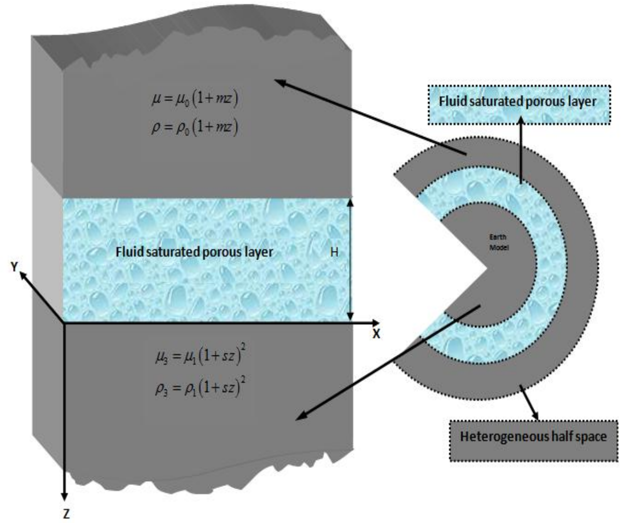


Fig.1
Geometry of the problem.

3 GOVERNING EQUATIONS AND THEIR SOLUTIONS

3.1 Solution for upper half-space

The equation of motion for the isotropic elastic solid medium, without body forces is taken as:

$$\tau_{ij,j} = \rho \frac{\partial^2 v_i}{\partial t^2}, \quad (i,j=1,2,3) \tag{1}$$

where v_i are the components of the displacement and τ_{ij} are the components of the stress tensor and ρ is the density of the medium. (Here coma denotes the differentiation with respect to position).

The constitutive equation for medium M_1 is taken as:

$$\tau_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu \mathrm{e}_{\mathrm{ij}} \tag{2}$$

where λ and μ are Lame's constant, δ_{ij} is the Kronnecker delta, and

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{3}$$

with the help of Eqs. (2) and (3), Eq. (1) becomes

$$\mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) + \frac{\partial \mu}{\partial z} \times \frac{\partial v}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2}. \tag{4}$$

Now for the wave propagating along the *x*-direction, the solution of Eq. (1) may be taken as:

$$v(x,z,t) = V(z) e^{i(kx - \omega t)} \tag{5}$$

We have taken the heterogeneity for upper half-space in the following form

$$\mu = \mu_0 (1 + mz) \tag{6}$$

$$\rho = \rho_0 (1+mz) \tag{7}$$

with the help of Eqs. (5), (6) and (7), Eq. (4) reduces to

$$V''(z)+V'(z)\frac{m}{1+mz}+V(z)\left[\frac{\rho_0}{\mu_0}\omega^2-k^2\right]=0 \tag{8}$$

Substituting $V(z)=\varphi(z)(1+mz)^{-\frac{1}{2}}$ in Eq. (8) to eliminate the term $V(z)$, we obtain

$$\varphi''(z)+\varphi(z)\left[\frac{1}{4}\frac{m^2}{(1+mz)^2}+k^2\left(\frac{c^2}{\beta_1^2}-1\right)\right]=0 \tag{9}$$

where $\beta_1=\sqrt{\frac{\mu_0}{\rho_0}}$ is velocity of shear wave in heterogeneous upper half-space. Using $\varphi(z)=\psi(\eta)$ in Eq. (9), we get

$$\frac{d^2\psi}{d\eta^2}+\left[\frac{1}{4\eta^2}-\frac{1}{4}\right]\psi(\eta)=0 \tag{10}$$

where $\eta=\frac{2k}{m}\sqrt{1-\frac{c^2}{\beta_1^2}}(1+mz)$.

Eq. (10) is the Whittaker's equation. The solution of Eq. (10) is given by $\psi(\eta)=D_1W_{0,0}(\eta)+D_2W_{0,0}(-\eta)$, where D_1 and D_2 are arbitrary constants and $W_{0,0}(\eta)$ is the Whittaker's function. Since we have the condition $V(z)\rightarrow 0$ when $z\rightarrow-\infty$ i.e. $\psi(\eta)\rightarrow 0$ when $\eta\rightarrow\infty$. Therefore the solution of Eq. (10), satisfying the above condition may be written as:

$$\psi(\eta)=D_2W_{0,0}(-\eta) \tag{11}$$

Hence, the displacement component in the heterogeneous half-space is given by

$$v(x,z,t)=V(z)e^{i(kx-\omega t)}=\frac{D_2W_{0,0}(-\eta)}{(1+mz)^{\frac{1}{2}}}e^{i(kx-\omega t)} \tag{12}$$

Expanding Whittaker's function up to linear terms, Eq. (12) reduces to

$$v(x,z,t)=D_2\left(\frac{-m_1}{m}\right)^{-\frac{1}{2}}e^{\frac{m_1(1+mz)}{2m}}\left\{1-\frac{m_1(1+mz)}{2m}\right\}. \tag{13}$$

where $m_1=2k\sqrt{1-\frac{c^2}{\beta_1^2}}$

3.2 Solution for porous layer

Equation of motion for fluid saturated porous layer in the absence of body forces are taken as:

$$\sigma_{ij,j} = \frac{\partial^2}{\partial t^2}\left(\rho_{11}u_i + \rho_{12}U_i\right) - b_{ij}\frac{\partial}{\partial t}(U_j - u_j) \tag{14}$$

and

$$\sigma_{ij,j} = \frac{\partial^2}{\partial t^2}\left(\rho_{12}u_i + \rho_{22}U_i\right) + b_{ij}\frac{\partial}{\partial t}(U_j - u_j) \tag{15}$$

σ_{ij} are the components of the stress tensor and u_i are the components of the displacement vector of the solid and U_i are the components of the displacement vector of the fluid, and

$$\sigma = -pf \tag{16}$$

where p is the pressure in the fluid and f is the porosity of the medium. Mass coefficients $\rho_{11}, \rho_{12}, \rho_{22}$ are related to the densities $\rho^{'}, \rho_s, \rho_f$ of the layer, solid and fluid, respectively. We have

$$\rho_{11} + \rho_{12} = (1-f)\rho_s \tag{17}$$

and

$$\rho_{12} + \rho_{22} = f\,\rho_f \tag{18}$$

so the mass density of the aggregate is

$$\rho^{'} = \rho_{11} + 2\times\rho_{12} + \rho_{22} = \rho_s + f\,(\rho_f - \rho_s) \tag{19}$$

The mass coefficients must follow the following inequalities

$$\rho_{11} > 0, \rho_{12} < 0, \rho_{22} > 0, \rho_{11}\rho_{22} - \rho_{22}{}^2 > 0 \tag{20}$$

where ρ_{12} is the coupling parameter. The constitutive equations for anisotropic fluid-saturated porous medium is

$$\sigma_{11} = (A \in +ME) + 2Nu_{1,1} + (F-A)u_{3,3} \tag{21}$$

$$\sigma_{22} = (A \in +ME) + 2Nu_{2,2} + (F-A)u_{3,3} \tag{22}$$

$$\sigma_{33} = (F \in +QE) + (2C-F)u_{3,3} \tag{23}$$

$$\sigma_{23} = G(u_{2,3} + u_{3,2}) \tag{24}$$

$$\sigma_{31} = G(u_{1,3} + u_{3,1}) \tag{25}$$

$$\sigma_{12} = G(u_{2,1} + u_{1,2}) \tag{26}$$

$$\sigma = (M \in +RE) + (Q-M)u_{3,3} \tag{27}$$

where *A, F, C, G, M, Q, N, R* are the material constants and

$$\in_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial u_j}+\frac{\partial u_j}{\partial u_i}\right) \tag{28}$$

Eq. (14) and Eq. (15) with the help of Eq. (21) and Eq. (28) reduces to

$$N\frac{\partial^2 u}{\partial x^2}+G\frac{\partial^2 u}{\partial z^2}=\frac{\partial^2}{\partial t^2}\left(\rho_{11}u+\rho_{12}U\right)-b_{11}\frac{\partial}{\partial t}(U-u) \tag{29}$$

and

$$\frac{\partial^2}{\partial t^2}\left(\rho_{12}u+\rho_{22}U\right)+b_{11}\frac{\partial}{\partial t}(U-u)=0 \tag{30}$$

The solution of Eq. (29) and Eq. (30) may be taken as:

$$u(x,z,t)=u(z)e^{i(kx-\omega t)} \tag{31}$$

and

$$U(x,z,t)=U(z)e^{i(kx-\omega t)} \tag{32}$$

with the help of Eqs. (31) and (32), the Eq. (29) and Eq. (30) becomes

$$\left(\frac{\partial^2}{\partial z^2}+L_2^{\ 2}\right)\binom{u}{U}=0 \tag{33}$$

where $L_2^{\ 2}=\xi^2-\frac{N}{G}k^2,\xi^2=\left(F+iR\right)\frac{\omega^2}{\beta_a^{\ 2}},F=\left(\frac{b_{11}^{\ 2}+\gamma_{22}d\rho'^2\omega^2}{b_{11}^{\ 2}+(\rho'\gamma_{22}\omega)^2}\right)\frac{\gamma_{22}}{d},\beta=\sqrt{\frac{G}{d'}},\ d'=\rho_{11}-\frac{\rho_{12}^{\ 2}}{\rho_{22}},d=\gamma_{11}-\frac{\gamma_{12}^{\ 2}}{\gamma_{22}},$
$\gamma=\frac{N}{G},\beta_a=\sqrt{\frac{N}{\rho'}},\rho'=\rho_{11}+2\rho_{12}+\rho_{22}$

The solution of Eq. (33) may be taken as:

$$u(x,z,t)=(A_1\cos L_2 z+A_2\sin L_2 z)e^{i(kx-\omega t)} \tag{34}$$

and

$$U(x,z,t)=\left(\overline{A_1}\cos L_2 z+\overline{A_2}\sin L_2 z\right)e^{i(kx-\omega t)}. \tag{35}$$

3.3 Solution for lower half-space

The equation of motion for the isotropic elastic half space, without body forces is taken as:

$$t_{ij,j}=\rho_3\frac{\partial^2 w_i}{\partial t^2},\quad \text{(i,j=1,2,3)} \tag{36}$$

where w_i are the components of the displacement and t_{ij} are the components of the stress tensor in medium M_3 and ρ_3 is the density of the medium in M_3.

The constitutive equations for medium M_3 are taken as:

$$t_{ij} = \lambda_3 \delta_{ij} \overline{e_{kk}} + 2\mu_3 \overline{\mathrm{e_{ij}}} \tag{37}$$

where λ_3 and μ_3 are Lame's constant, δ_{ij} is the Kronnecker delta and

$$\overline{e_{ij}} = \frac{1}{2}\left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i}\right) \tag{38}$$

with the help of Eqs. (37) and (38), Eq. (36) reduces to

$$\mu_3\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{\partial \mu_3}{\partial z} \times \frac{\partial w}{\partial z} = \rho_3 \frac{\partial^2 w}{\partial t^2} \tag{39}$$

The solution of Eq. (39) may be taken as:

$$w(x,z,t) = W(z) e^{i(kx-\omega t)} \tag{40}$$

and we have taken the heterogeneity for lower half-space in the following form

$$\mu_3 = \mu_1 (1+sz)^2 \tag{41}$$

$$\rho_3 = \rho_1 (1+sz)^2 \tag{42}$$

Introducing Eqs. (40), (41) and (42) in Eq. (39), we have

$$W''(z) + W'(z)\frac{2s}{1+sz} + W(z)\left[\frac{\rho_1}{\mu_1}\omega^2 - k^2\right] = 0 \tag{43}$$

Substituting $W(z) = \psi_2(z)(1+sz)^{-1}$ in Eq. (43) to eliminate the term $W(z)$, we obtain

$$\psi_2''(z) + \psi(z)\left[k^2\left(\frac{c^2}{\beta_2^2} - 1\right)\right] = 0 \tag{44}$$

where $\beta_2 = \sqrt{\frac{\mu_1}{\rho_1}}$ is velocity of shear wave in heterogeneous lower half-space. The solution of Eq. (44) may be taken as:

$$\psi(z) = A e^{L_1 z} + B e^{-L_1 z} \tag{45}$$

where $L_1 = k\sqrt{1-\frac{c^2}{\beta_2^2}}$, $\psi_2(\eta) = D_1 W_{0,0}(\eta) + D_2 W_{0,0}(-\eta)$, where D_1 and D_2 are arbitrary constants and $W_{0,0}(\eta)$ is the Whittaker's function. Since we have the condition $V(z) \to 0$ when $\lim z \to \infty$ i.e. $\psi_2(\eta) \to 0$ when $\lim \eta \to \infty$.

$$\psi_2(z) = B e^{-L_1 z} \tag{46}$$

Hence, the displacement for the lower half-space is

$$w(x,z,t) = Be^{-L_1 z}(1+sz)^{-1}e^{i(kx-\omega t)}. \tag{47}$$

4 BOUNDARY CONDITIONS

For the propagation of SH-wave in fluid saturated porous medium sandwiched between two half-spaces, the following boundary conditions to be satisfied

At the interface $z=-H$, the continuity of shear displacement and stress component,

$$v(x,z,t) = u(x,z,t), \tag{48}$$

$$\tau_{32}(x,z,t) = \sigma_{32}(x,z,t). \tag{49}$$

At the interface $z=0$, the continuity of shear displacement and stress component,

$$u(x,z,t) = w(x,z,t), \tag{50}$$

$$\sigma_{32}(x,z,t) = t_{32}(x,z,t). \tag{51}$$

Using all boundary conditions, we have the following equations

$$-A_1\cos L_2H + A_2\sin L_2H + D_2\left(\frac{-m_1}{m}\right)^{\frac{1}{2}} e^{\frac{m_1(1-mH)}{2m}}\left\{1-\frac{m_1(1-mH)}{2m}\right\} = 0 \tag{52}$$

$$-A_1G\sin L_2H - A_2L_2G\cos L_2H + D_2\mu m_1\left(\frac{-m_1}{m}\right)^{\frac{1}{2}} e^{\frac{m_1(1-mH)}{2m}}\left(1-\frac{m_1(1-mH)}{2m}\right)\left[\frac{1}{2}-\frac{1}{2-\frac{m_1(1-mH)}{m}}\right] = 0 \tag{53}$$

$$A_1 = B \tag{54}$$

$$B\mu(L_1+s) + A_2GL_2 = 0 \tag{55}$$

$$A_1\mu(L_1+s) + A_2GL_2 = 0 \tag{56}$$

Now eliminate the constants from Eqs. (52), (53) and (56), we obtain

$$\tan L_2H = \frac{G}{\mu}\left[\frac{L_2\left(k\sqrt{1-\frac{c^2}{\beta_2^2}}+s\right)+2L_2k\sqrt{1-\frac{c^2}{\beta_1^2}}\left(\frac{1}{2}-\frac{1}{2-\frac{m_1(1-mH)}{m}}\right)}{\frac{G^2}{\mu^2}L_2^2-2\left(k\sqrt{1-\frac{c^2}{\beta_2^2}}+s\right)k\sqrt{1-\frac{c^2}{\beta_1^2}}\left(\frac{1}{2}-\frac{1}{2-\frac{m_1(1-mH)}{m}}\right)}\right] \tag{57}$$

Eq. (57) is the required dispersion relation for shear wave propagation in anisotropic fluid saturated porous medium sandwiched between two heterogeneous half-spaces.

5 PARTICULAR CASES

Case1

when upper half-space is homogeneous i.e. $m = 0$, then Eq. (57) reduces to

$$tanL_2 H = \frac{G}{\mu}\left[\frac{L_2\left(k\sqrt{1-\frac{c^2}{\beta_2{}^2}}+s\right)+L_2 k\sqrt{1-\frac{c^2}{\beta_1^2}}}{\frac{G^2}{\mu^2}L_2{}^2-\left(k\sqrt{1-\frac{c^2}{\beta_2{}^2}}+s\right)k\sqrt{1-\frac{c^2}{\beta_1^2}}}\right] \tag{58}$$

Eq. (58) is the frequency equation for SH-wave propagation in fluid saturated layer sandwich between a homogeneous upper half-space and heterogeneous lower half-space.

Case2

when lower half-space is homogeneous, i.e. $s = 0$, then Eq. (57) becomes

$$tanL_2 H = \frac{G}{\mu}\left[\frac{L_2\left(k\sqrt{1-\frac{c^2}{\beta_2^2}}\right)+2L_2 k\sqrt{1-\frac{c^2}{\beta_1^2}}\left(\frac{1}{2}-\frac{1}{2-\frac{m_1(1-mH)}{m}}\right)}{\frac{G^2}{\mu^2}L_2{}^2-2\left(k\sqrt{1-\frac{c^2}{\beta_2^2}}\right)k\sqrt{1-\frac{c^2}{\beta_1^2}}\left(\frac{1}{2}-\frac{1}{2-\frac{m_1(1-mH)}{m}}\right)}\right] \tag{59}$$

Eq. (59) is the frequency equation for SH-wave propagation in fluid saturated layer sandwich between a heterogeneous upper half-space and homogeneous lower half-space.

Case3

when both of the half-spaces are homogeneous, i.e. $s = 0, m = 0$, then Eq. (57) becomes,

$$tanL_2 H = \frac{GL_2}{\mu}\left[\frac{\left(k\sqrt{1-\frac{c^2}{\beta_2^2}}\right)+k\sqrt{1-\frac{c^2}{\beta_1^2}}}{\frac{G^2}{\mu^2}L_2{}^2-\left(k\sqrt{1-\frac{c^2}{\beta_2^2}}\right)k\sqrt{1-\frac{c^2}{\beta_1^2}}}\right] \tag{60}$$

Eq. (60) is the frequency equation for SH wave propagation in fluid saturated layer sandwich between two homogeneous half-spaces.

Case4

when the upper half-space is absent, lower half-spaces is homogeneous and intermediate layer is free of porosity i.e. $\gamma \to 1, d \to 1, N \to \mu_1, G \to \mu_1$ then Eq. (57) reduces to Ewing et al. [21]

$$\tan\left[kH\sqrt{\frac{c^2}{\beta_a^2}-1}\right]=\frac{\mu}{\mu_1}\frac{\sqrt{1-\frac{c^2}{\beta_2^2}}}{\sqrt{\frac{c^2}{\beta_a^2}-1}}. \tag{61}$$

where $\beta_a=\sqrt{\frac{N}{\rho'}}$. Eq. (61) is the velocity equation of Love waves in a homogeneous isotropic layer lying over a homogeneous isotropic half-space.

6 NUMERICAL EXAMPLES AND DISCUSSION

In order to show the effect of heterogeneity on SH-wave propagation for considered model, we use the following data

For isotropic heterogeneous upper half-space (Gubbins [8])

$$\mu_1=7.1\times10^{10}N/m^2,\ \ \rho_1=3321\ \text{kg/m}^3$$

For isotropic heterogeneous half-space (Gubbins [8])

$$\mu_2=6.77\times10^{10}N/m^2,\ \ \rho_1=3323\ \text{kg/m}^3$$

For anisotropic fluid saturated porous medium we are using following data (Samal and Chattaraj [16])

$$b_{11}=0.2,\ N=0.2774\times10^{10}N/m^2,\ G=0.1387\times10^{10}N/m^2$$

Numerical results are obtained by using Eq. (57) to represent the effect of heterogeneity on propagation of shear waves. In all the figures, curves are plotted to exhibit the variation of dimensionless phase velocity (c/β) against dimensionless wave number (kh).

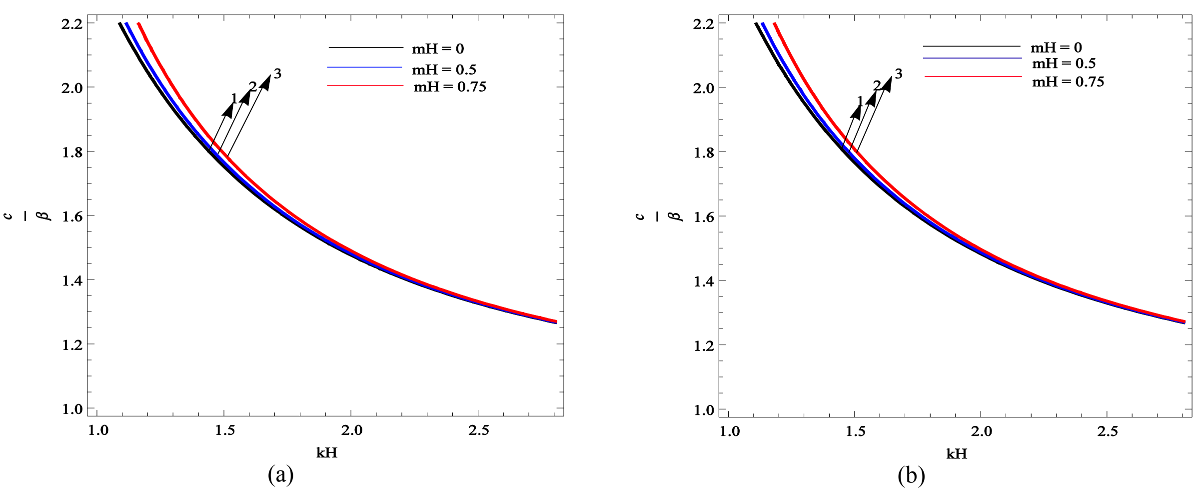


Fig.2
Variation in dimensionless phase velocity (c/β) against dimensionless wave number (kh) for different values of heterogeneity parameter (mH) of half-space (a) when $sH=0.0$ (b) when $sH\neq0$.

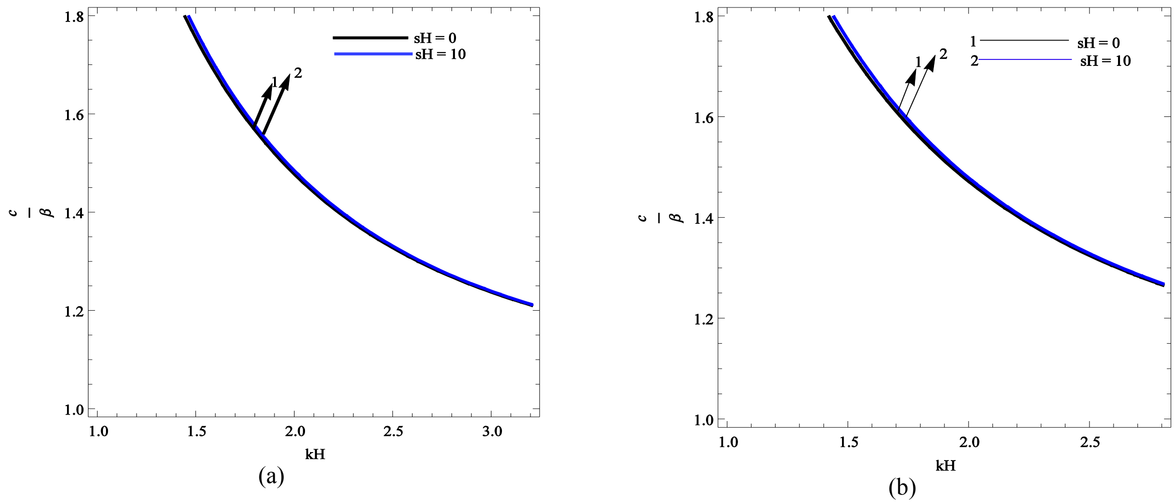


Fig.3
Variation in dimensionless phase velocity (c/β) against dimensionless wave number (kh) for different values of heterogeneity parameter sH of upper layer (a) when $mH = 0.0$ (b) when $mH \neq 0$.

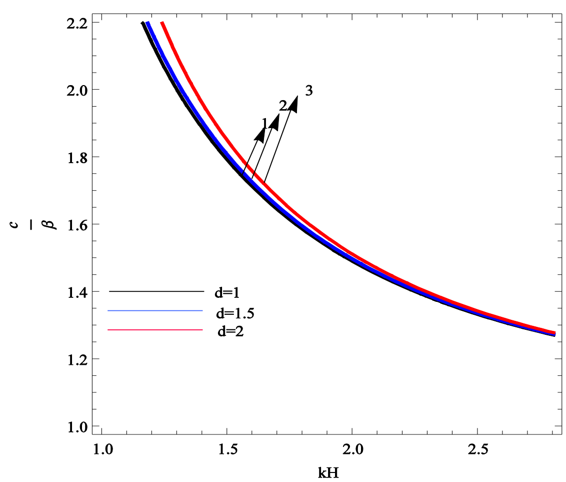


Fig.4
Variation in dimensionless phase velocity (c/β) against dimensionless wave number (kh) for different values of porosity *(d)*.

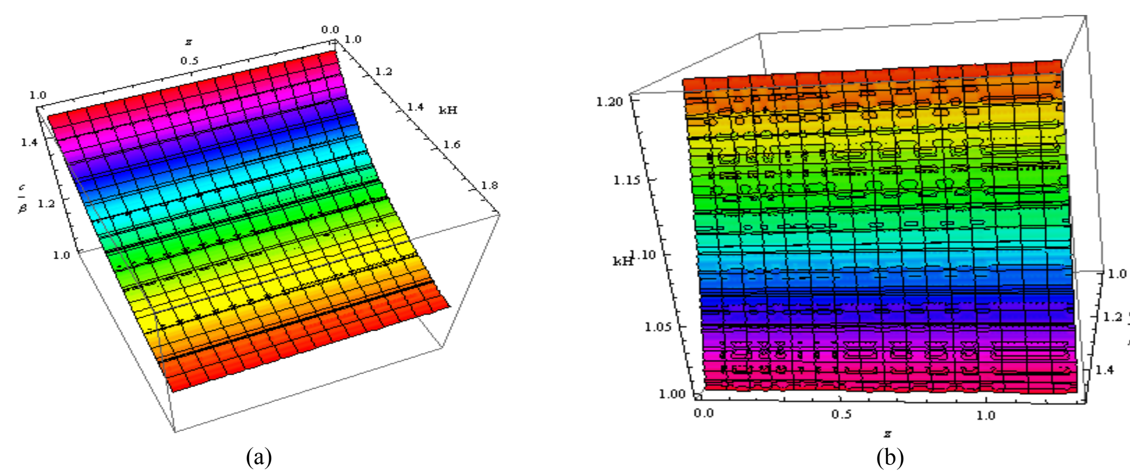
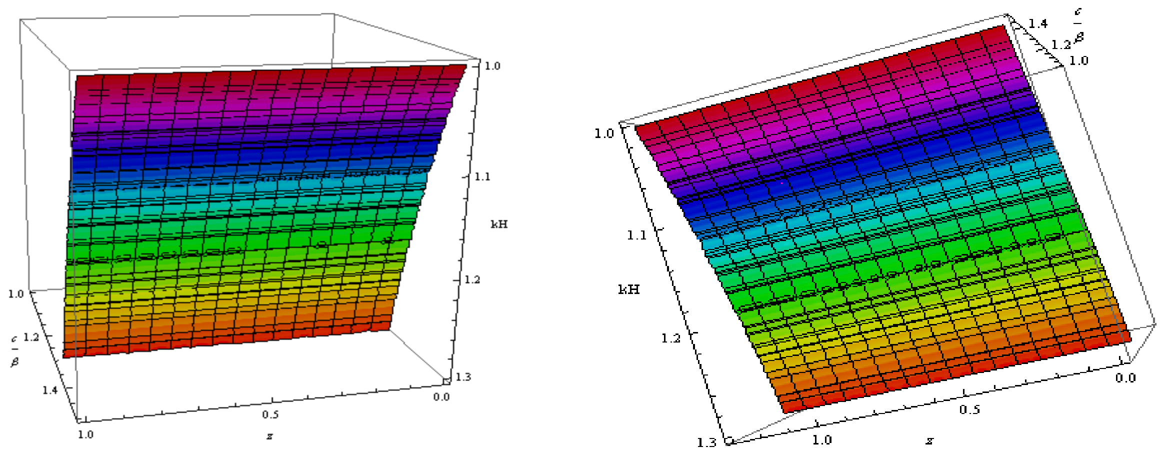


Fig.5
Surface plot for variation of dimensionless phase velocity (c/β) with respect to dimensionless wave number (kH) heterogeneity parameter sH (taken as z), (a) when $mH = 0.0$ (b) when $sH = 0.0$ respectively.



(a) (b)

Fig.6
Surface plot for variation of dimensionless phase velocity (c/β) with respect to dimensionless wave number (kH) and heterogeneity parameter sH (taken as z) (a) when $mH \neq 0$ (b) when $sH \neq 0$.

In Figs. 2-3 the variation of phase velocity of SH-waves with respect to wave number is shown. It may be observed through figures that the phase velocity of SH- wave decreases with increment in wave number. Figs. 2-3 represent the effect of heterogeneity of half-spaces on phase velocity of SH waves. Fig. 4 represents the effect of porosity on phase velocity of SH waves. Through Figs. 2-4 it may be observed that the heterogeneity (mH and sH) of the half-spaces and porosity of intermediate layer increases the phase velocity of SH waves. Figs. 5 - 6 represent the surface plot to exhibit the variation of phase velocity with respect to wave number and heterogeneity parameters (mH and sH).

7 CONCLUSIONS

Propagation of SH-waves in fluid saturated porous medium sandwiched between two heterogeneous half-spaces has been studied. Frequency equation has been obtained in closed form. Frequency equation is found to be in well agreement with the classical Love wave equation when upper half-space, porosity of the layer and heterogeneity of lower half-space are neglected. It is observed that the heterogeneity has significant effect on the propagation of SH-waves. Findings have been shown by the means of graphs. This study may have possible applications in the field of seismology and geophysics. The major outcomes of the study may be listed as:

- Dispersion relation for SH-waves in fluid saturated porous medium sandwiched between two heterogeneous half-spaces is obtained.
- Effect of heterogeneity and wave number on phase velocity of SH-waves has been shown by surface plot.
- The phase velocity of SH-waves increases with increasing value of heterogeneity parameters.
- The phase velocity of SH-waves increases with increment in porosity of the intermediate fluid saturated porous layer.
- Propagation of SH-waves in fluid saturated porous medium compact between two isotropic homogeneous half-spaces is studied as a special case of the present study.
- Other particular cases have been discussed.

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