

Shape- Dependent Term Investigation of Khan- Liu Yield/ Fracture Criterion as a Function of Plastic Strain for Anisotropic Metals

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ABSTRACT

The current paper primarily aims to suggest a mathematical model for the shape-dependent term of Khan- Liu (KL) Yield/ fracture criterion as a function of Plastic Strain for DP590 steel alloy. The shape-dependent term in the mention criterion can generalize the application of this criterion in order to predict the behavior of other materials. Plane stress case and the first quarter of the stress plane have been specifically studied. Uniaxial stresses in rolling and transverse directions of sheet and also the tensions caused by equal-biaxial tension have been experimentally used. Then, material constants of KL yield/ fracture criterion and Khan- Huang- Liang (KHL) constitutive equation are calculated using genetic algorithm (GA) optimization and the value of the shape-dependent factor in KL criterion is extracted. The same has been repeated for various plastic strains and finally a polynomial mathematical model based on the plastic strain for the KL shape-dependent factor is suggested. Hence, material constants of KL criterion could be calculated using at least tests namely experimental uniaxial stress test, experimental equal-biaxial stress, and one of the optimization models such as GA. Using the given mathematical model based on the plastic strain, correction term can be calculated and the generalized form of KL criterion can be used for various ductile metallic materials.

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Keywords : Yield/ fracture criterion; Constitutive equation; Shape-dependent term; Cruciform specimen; DP590 Steel alloy.

1 INTRODUCTION

AVAILABILITY of appropriate and relatively accurate behavior criteria is one of the main demands of designers and engineers working within the field of warm sheet forming and special ductile metals. This would be a requirement in order to observe the effects of various parameters and their history on both simultaneously and uncoupled, and use them in simulations to reach higher levels of accuracy. In recent years, Khan et al. have presented a constitutive equation and a relatively comprehensive and functional yield/fracture criterion for anisotropic metallic materials by a series of systematic studies [1-3]. The main purpose of the current paper is

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presenting a mathematical model for Khan-Liu (KL) criterion generalizable term by which the accuracy of this criterion for a given material is increased by conducting the minimum number of tests. Developing new metallic alloys, aluminum alloys, magnesium alloys, and superplastic has motivated researchers to develop behavioral criterion of materials [1]. None of the presented behavioral criteria have been able to predict yield and fracture of all materials accurately [4]. This has led to a large number of criteria developed and suggested by researchers having various conditions regarding accuracy and complexity. Selecting an appropriate criterion, therefore, calls for thorough knowledge.

The most important factors to select a behavioral criterion include: accuracy in predicting the loci of yield/fracture, flexibility, generalizability, number of required mechanical parameters, method capabilities, and acceptability in scientific and industrial communities [5]. All desirable criteria are, needless to say, not reachable at the same time. However, a balance in parameters' quantity and quality in one generalizable criterion is achievable. Khan-Huang-Liang (KHL) constitutive equation and the KL yield/fracture criterion are more comprehensive than other criteria mentioned above in this regard [1, 2, 6]. The two constitutive equation and yield/ fracture criterion are the development of other equations like Johnson-Cook constitutive equation and Hill1948's criterion, although these equations consider the effects of strain, strain rate, temperature, and also hydrostatic pressure and tension-compression asymmetry in an uncoupled approach.

In order to explain the nature of yield/ fracture criterion, one might use the concepts of yield surface or fracture surface [7]. The yield surface is a presentation of the yield function [8]. During studying the yield/ fracture criterion; therefore, researchers take advantage of the yield/ fracture surface and loci. Banabic and Barlat et al. have studied the history and the growing trend of yield criteria [5]. They have evaluated the capabilities of each suggested criterion and have finally reasoned to prove the functionality and superiority of Hill1948, Barlat1989, and also BBC2000 reference criteria. Calculating the accuracy index might indicate that BBC2000 have a higher rate of accuracy. However, the Hill1948 has unparalleled convenience compared to other anisotropic yield criteria. Therefore, selecting the anisotropic yield function in the KL criterion is logical in all aspects. Predicting a yield criterion is based on all experimental data of uniaxial and biaxial tension. Advantages and disadvantages of many isotropic, classic anisotropic and advanced anisotropic yield criteria could be studied in the works of Banabic et al. [5, 9, 10]. Generally, an ideal plasticity constitutive model for metals and alloys must be able to describe the material characteristics, including dependence upon the rate of strain, forming temperature, strain history and the strain rate, work hardening behavior, and strain hardening (both isotropic and anisotropic hardening). Never the less, describing and stating all these phenomena in a single constitutional model proves to be an extremely difficult task [1]. Lin and Chen studied the development and evolution of constitutive models [6]. They are basically divided into three categories namely: (1) Phenomenological Models, (2) Physic-Based Models, and (3) Artificial Neural Network (ANN) models.

The KL is basically a phenomenological model, and these models are characterized by the fact that by considering the effect of parameters on the alloys flow behavior. Researchers are able to restate them as functions of strain, strain rate and temperature. A phenomenological model is practically a classic view to state material behavior in which macroscopic mechanical tests match the appropriate mathematical model. The KL model is of the phenomenological type in which a simple equation for stating the strain rate and temperature effects is used. However, the existing limitations in phenomenological models must be considered while development.

The more powerful a constitutive model is in simulating the effects of the strain, the strain rate and the temperature, the more complex the equation will be and working with it will, therefore, be more difficult. Consequently, the simple form and ease in use of an equation such as Fields-Backofen model can be a factor contributing to the low level of accuracy [11]. Zhang added a term to Fields-Backofen model for softening to describe softening behavior [12]. Cheng et al. later modified Fields-Backofen model using approaches suggested by Zhang [12, 13]. The resulting equation was more accurate and complex [6]. The same occurred in improving Johnson-Cook constitutive model, so that the improved Johnson-Cook model became mathematically much more complicated and ambiguous while becoming a bit more accurate [6]. The KHL constitutive model is highly similar to the principal Johnson-Cook model [1] and by following the main configuration, the temperature limitation less than the reference temperature is solved. The KHL constitutive model matches the empirical results better than Johnson-Cook's [6, 14]. Khan later applied KHL constitutive equation in the KL yield/ fracture criterion by omitting work hardening effect and considering the effect of hydrostatic pressure. The KL un-generalized yield/ fracture criterion has ten materialistic constants, which by determining them; one can predict the effects of asymmetry in tension- compression, hydrostatic pressure, strain rate and temperature on materials behavior.

Optimization methods have significantly improved to the date. In order to determine coefficients and constants of materials behavioral equations, numerous complex behavioral equations having relatively many materialistic constants can be extracted from optimization methods and one can easily predict the behavior of materials under

various circumstances. In the current paper, therefore, materialistic constants of the KL yield/ fracture criterion in plane stress and RD-TD (rolling and transverse directions) plans for DP590 alloy are determined using the GA optimization which is a branch of evolution algorithms. The empirical results presented by Deng and Kuwabara et al. have been used in order to determine the constants [15]. After determining the constants based on the experimental data of uniaxial and biaxial tension, the shape-dependent term of Yield/ fracture loci is suggested to increase the accuracy of the criterion as a mathematical function in terms of the plastic strain. Therefore, the KL criterion will become independent from all of the material biaxial experimental data for various applications, with the exception of equal-biaxial experimental data by the term.

2 KHAN- LIU (KL) GENERALIZED CRITERION THEORY

The KL generalized criterion mathematical form as following [1]:

$$\left[(2/\sqrt{3})\sin(\theta + \frac{\pi}{3}) \right]^k \times \sqrt{e^{C(\xi+1)} (F\sigma_1^2 + G\sigma_2^2 + H\sigma_3^2 + L\sigma_1\sigma_2 + M\sigma_1\sigma_3 + N\sigma_2\sigma_3)} = e^{C_1 I_1 / \sqrt{3}} \dot{\epsilon}^n T^{*m} \quad (1)$$

where $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses, and shear stresses and constants related to them have been removed. The parameters of θ, K, C , and ξ are Lode angle, the material constant, asymmetry of tension and compression coefficient, and Lode parameter, respectively. The anisotropic parameters are F, G, H, L, M , and N . The hydrostatic pressure coefficient, strain rate sensitivity, and temperature effect are C_1, n , and m respectively. The first stress tensor invariant is I_1 . The strain rate is $\dot{\epsilon}$ and $T^* = (T_m - T) / (T_m - T_r)$ is the dimensionless temperature. The parameters of T_m, T , and T_r are alloy melting temperature, current temperature, and reference temperature at 296 degree of Kelvin, respectively. A schematic of yield/fracture surface of the KL criterion in three-dimensional stress space is plotted in Fig. 1.

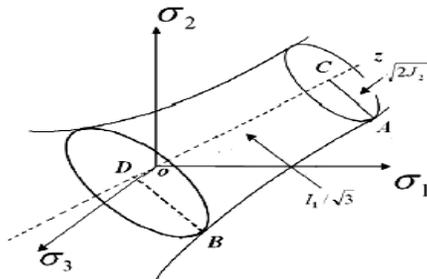


Fig. 1
The schematic of the fracture surface in the principal stress space [1].

The term of $e^{C(\xi+1)}$ indicates the effect of asymmetry in the tension and compression by Lode parameter [1]. Generally, the importance of the Lode parameter for the expression of the proposed yield criteria and evaluation of stress case can be used [16-18]. The term of bracket is the same Hill1948 criterion that can simulate the effects of anisotropy and includes anisotropic coefficients of F, G, H, L, M , and N . The term of $e^{C(\xi+1)}$ can express the effect of hydrostatic pressure for some metals such as Ti-6Al-4V alloys which are sensitive to it [1, 6, 15]. Moreover, a constitutive equation is a requirement to express the effects of the strain rate and temperature. The term of $e^{C_1 I_1 / \sqrt{3}} \dot{\epsilon}^n T^{*m}$ displays the form of constitutive equation in the proposed criterion[1]. The term of $\left[(2/\sqrt{3})\sin(\theta + \pi/3) \right]^k$ describes generalized orientation concept in the KL criterion, where k is a material constant and can be determined by curve fitting the experimental data obtained from the plane strain tests. The appropriate criterion for material will be met by determining the K value [1, 16]. The value of K is $0 \leq K \leq 1$.

Eq. (1) becomes Tresca-type like fracture locus, when K is equal to one, while a quadratic fracture locus can be obtained when k is equal to zero. Other value of K will specify the intermediate state of yield locus. Fig. 2 shows the comparison of proposed fracture surfaces in RD-ND plane with different values of k and under reference strain rate and temperature. ξ denotes to the Lode parameter which is defined as Eq.(2) [1, 2, 17, 18].

$$\xi = \cos 3\theta = \frac{27}{2} \frac{J_3}{(\sqrt{3J_2})^3} \tag{2}$$

where θ is the Lode angle, J_2 and J_3 are the second and third invariants of the deviatoric stress tensor, respectively. It is achieved that under uniaxial tension loading $\xi = +1$ and uniaxial compression loading $\xi = -1$. On the other hand, θ always is in range of $0 \leq \theta \leq \pi/3$ [1]. The corresponding expressions for the anisotropy parameters are presented in Eq. (3) set [1].

$$F = \sigma_1^{-2} e^{2C_1 I_1 / \sqrt{3}} \tag{3a}$$

$$G = \sigma_2^{-2} e^{2C_1 I_1 / \sqrt{3}} \tag{3b}$$

$$H = \sigma_3^{-2} e^{2C_1 I_1 / \sqrt{3}} \tag{3c}$$

$$L = \sigma_B^{-2} e^{2C_1 I_1 / \sqrt{3}} - F - G \tag{3d}$$

$$M = \sigma_B^{-2} e^{2C_1 I_1 / \sqrt{3}} - F - H \tag{3e}$$

$$N = \sigma_B^{-2} e^{2C_1 I_1 / \sqrt{3}} - H - G \tag{3f}$$

where σ_B is the equal-biaxial stress and it is as following in the case of equal-biaxial tension, and plane stress:

$$\sigma_1 = \sigma_2 = \sigma_B, \sigma_3 = 0 \tag{4}$$

More details can be traced to Khan and colleagues work [1].

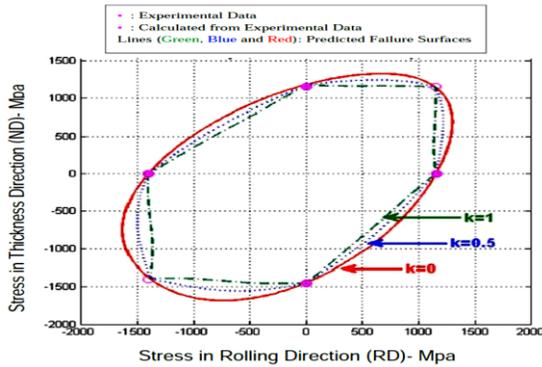


Fig. 2
The comparison between the proposed fracture loci in RD–ND plane by different K values and at 1 sec^{-1} and 296 degree of Kelvin [1].

3 KHAN- HUANG- LIANG (KHL) CONSTITUTIVE EQUATION

The following equation shows the KHL plastic constitutive model which has predicted the isotropic thermo-mechanical behavior of different materials successfully [19-21].

$$\sigma = \left[A + B \left(\frac{\ln \dot{\epsilon}}{\ln D_0^p} \right)^{n_0} \epsilon^{n_1} \right] \left(\frac{\dot{\epsilon}}{\dot{\epsilon}^*} \right)^c \left(\frac{T_m - T}{T_m - T_r} \right)^m \tag{5}$$

where σ, ε , and $\dot{\varepsilon}$ are the flow stress, the plastic strain, and the current plastic strain rate, respectively. T, T_m and T_r are the current, melting, and reference temperatures. $\dot{\varepsilon} = 1 \text{ s}^{-1}$ is the reference strain rate at which the strain rate effect term reduces to unity. $D_0^p = 10^6 \text{ s}^{-1}$ is a constant which is used to non-dimensionalize the strain rate term. In the KHL constitutive model, there are totally six material constants: A, B, n_0, n_1, c , and m .

The six material constants in the KHL constitutive model are determined from the experimental observations by the systematical method proposed by Khan and Liang [22]. Subsequently, these material constants determined from experimental data are imported into a least-squared based optimization program as initial values to obtain the final optimized material constants [23]. It should be noted that only A and B constants; representing yield stress at a strain rate 1 s^{-1} and work hardening coefficient, respectively, are different for different anisotropy directions.

The following equation will be obtained by placing the hydrostatic pressure term, i.e. $e^{C_1 I_1 / \sqrt{3}}$ in Eq. (5):

$$\sigma = e^{C_1 I_1 / \sqrt{3}} \left[A + B \left(\frac{\ln \dot{\varepsilon}}{\ln D_0^p} \right)^{n_0} \varepsilon^{n_1} \right] \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}^*} \right)^c \left(\frac{T_m - T}{T_m - T_r} \right)^m \quad (6)$$

Experimental stress- strain curves are needed to determine the material constants of Eq. (5) and Eq. (6) at different strain rates and temperatures. More details can be traced to Khan and colleagues works and Lin and Chen studies [2, 6].

4 GENETIC ALGORITHM

GA, as one of the most used optimization algorithms, a search algorithm, is around randomly and based on population, which is extracted from genetic science and natural evolution. The GA introduced by John Holland and his students in the early 1970s and their research was published in a book in 1975 [24]. The conventional form of the GA was introduced by Goldberg, one of Holland's students, under the title of a simple GA [25]. In the GA each variable is considered as a gene and each solution is considered to be a chromosome. Algorithm population is composed by a set of chromosomes and evaluated at each iteration and called generation.

After making and evaluating the initial random population, to create the next generation, some of the chromosomes of the current generation as the mating population are selected by the selection mechanism, until they make up their children. At present, the current generation and children populations will be compared in terms of higher fitness and the chromosomes with higher fitness will build the next generation. Each of the mating population chromosomes, which are known as the parent, will generate its children by two mechanisms of crossover and mutation. Selection mechanism determines which chromosomes of the current generation directly or indirectly, be the present in the next generation. Some of the known selection mechanisms are: Roulette wheel selection, tournament selection, stochastic universal sampling, rank-based selection, and Boltzmann selection. One of the major operators is crossover operator to produce new chromosomes by the combination of parent's chromosomes. Single-point, two-point and uniform crossover operators are best known. Since a GA code is implemented in the study, the arithmetic crossover has been used. Mutation, as one of the operators at reproduction using a random change in parental chromosomes, will produce the children chromosomes. Mutation operator decreases convergence to local optimum by increasing search space exploration. One of the most mutation operators is non-uniform mutation operator in continuous-GA [26].

5 EXPERIMENTAL TESTS RESULTS

The points on the loci of yield/ fracture in various conditions are necessary to estimate the KL criterion coefficients. Fig. 3 shows the required tests to determine the loci of yield/ fracture of a ductile alloys as a sheet. These points could be determined by uniaxial tension and compression tests, biaxial tension and compression tests, and torsion tests. At first, the six points are needed to draw yield/ fracture loci. The four points are related to uniaxial experimental tests and the other two points are related to equal biaxial tests. The uniaxial points are usually obtained easily. However, the equal biaxial points can be obtained by calculation of experimental data [1]. In order to determine the points

related to biaxial tensions in the first quarter of σ_1 - σ_2 plane, cruciform specimen tests are used as shown in Fig. 4 [27, 28]. Fig. 5 shows the loading ratio on the cruciform specimen. Fig. 6 shows the biaxial tension tests results and yield geometrical loci of DP590 alloy in the first quarter of tension plane and cross-rolling plane in room temperature and the average strain rate of 6×10^{-4} per second which was extracted by Deng et al. using cruciform specimen [15]. The extracted data from Fig. 6 are used for determining the KL criterion coefficients in order to have an acceptable strategy in this regard.

The experimental stress- strain curve is need for estimating KHL constitutive equation material constants at a specific strain rate and temperature. Deng et.al. presented experimental stress- strain curves for DP590 steel alloy [15]. The results are shown in Fig. 7. Therefore, the curve of Fig. 7 will be used in the research.

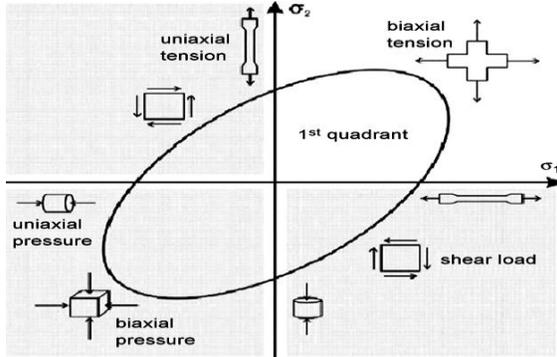


Fig. 3 Connection of yield loci and corresponding experiments [27, 28].

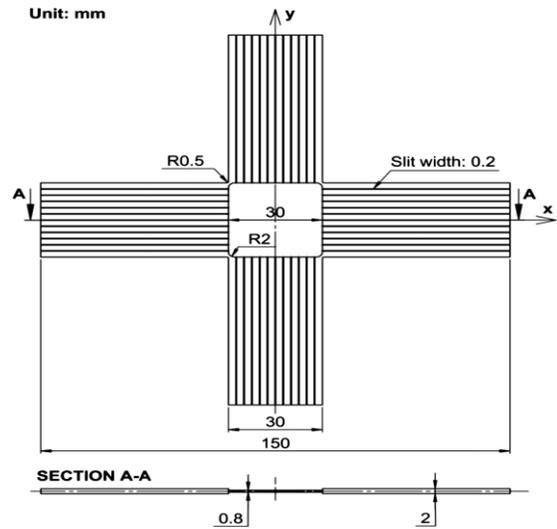


Fig. 4 Proposed cruciform specimen by Deng et al. for obtaining tension intermediate points if biaxial for DP590 alloy in first quarter of stress plane [15].

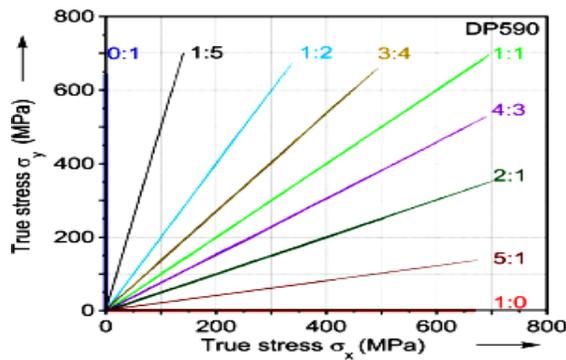


Fig. 5 Prescribed radial paths or loading ratios in the true stress space for biaxial tests [15].

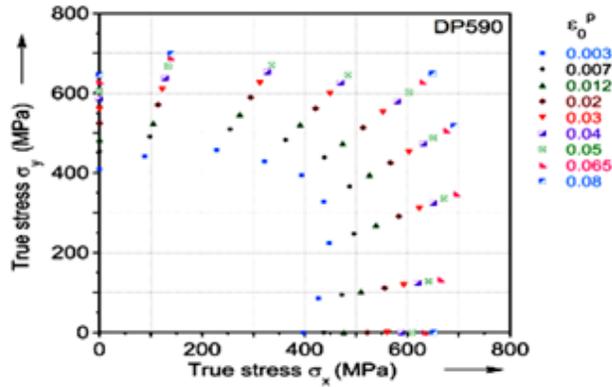


Fig. 6
Experimental results of biaxial tension by cruciform specimen in RD-TD plane, at 296 degree of Kelvin, average strain rate 6×10^{-4} per second for DP590 alloy [15].

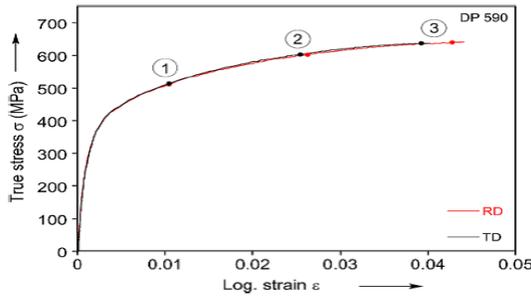


Fig. 7
The true stress- true strain curve of DP590 steel alloy [15].

6 DETERMINATION OF KHAN- LIU CRITERION MATERIAL CONSTANTS

The stress- strain curve of DP590 shows that the material considerably depends on work hardening at ambient temperature. Therefore, KHL constitutive equation, Eq. (6), is placed in the KL criterion for considering work hardening, strain rate, temperature and hydrostatic pressure effects. Hence, the KL criterion will be as follows:

$$\left[\left(\frac{2}{\sqrt{3}} \right) \sin(\theta + \frac{\pi}{3}) \right]^K \times \sqrt{e^{C(\xi+1)} (F\sigma_1^2 + G\sigma_2^2 + H\sigma_3^2 + L\sigma_1\sigma_2 + M\sigma_1\sigma_3 + N\sigma_2\sigma_3)}$$

$$= e^{C_1 t_1 / \sqrt{3}} \left[A + B \left(\frac{\ln \dot{\epsilon}}{\ln D_0^p} \right)^{n_0} \epsilon^{n_1} \right] \left(\frac{\dot{\epsilon}}{\dot{\epsilon}^*} \right)^c \left(\frac{T_m - T}{T_m - T_r} \right)^m \tag{7}$$

In Eq. (1), the right side is the KHL material constitutive equation and in the KL equation the effects of work hardening are ignored. However, the effects of hydrostatic pressure, strain rate, and temperature are considered. KHL equation introduces the size of yield/ fracture geometrical loci and the mentioned factors affect the size of yield/ fracture loci or surface. The predicted curve obtained by the constitutive equation must be correlated with true stress-true strain curve in a certain strain rate and temperature. However, since the intended criterion is stated with yield/ fracture loci in plane stress case, this match is ignored and all coefficients based on the six points on the experimental yield/ fracture loci are extracted at the same time.

The left side of the equation introduces the yield/ fracture geometrical loci, consists of two main parts. The first term, $\left[(2/\sqrt{3})\sin(\theta + \pi/3) \right]^K$, is the correction coefficient which modifies the shape of yield/ fracture loci due to possible difficulties in conducting numerous tests and extracting the accurate yield/ fracture loci. Therefore, the first term cannot enter the calculation when determining the constants and coefficients of the KL criteria. The second term consists of asymmetry in tension and compression, $e^{C(\xi+1)}$, and Hill1948 anisotropic yield function.

Given the plane stress case, i.e. $\sigma_3 = 0$, the cost function for curve fitting problem is considered to be Eq. (8).

$$Cost\ Function = RMSE = \sqrt{\frac{1}{Q} \sum_{k=1}^Q e^2(k)} \tag{8}$$

Q is the number of points by which curve fitting is done and e is the difference between the fitted function and the actual amount which is defined as Eq. (9).

$$e = \sqrt{e^{C(\xi+1)} (F\sigma_1^2 + G\sigma_2^2 + L\sigma_1\sigma_2)} - e^{\frac{C_1 I_1}{\sqrt{\beta}}} \left[A + B \left(\frac{\ln \dot{\epsilon}}{\ln D_0^p} \right)^{n_0} \epsilon^{n_1} \right] \left(\frac{\dot{\epsilon}}{\dot{\epsilon}^*} \right)^c \left(\frac{T_m - T}{T_m - T_r} \right)^m \tag{9}$$

6.1 Determining material constants of Khan-Huang- Liang constitutive equation

The cost function has 11 variables that are presented in Table 1., and determined by GA codes in “MATLAB” software. The material constants of C₁, A, B, n₀, n₁, c and m are determined by experimental stress- strain curve of DP590 steel alloy, Fig. 7, and by the GA optimization method. So that, there is no hydrostatic pressure in the present tests, its term is ignored i.e. C₁ = 0. Data of stress- strain curve DP590 is extracted from Fig. 7 by “xyExtract” software. The data are intercalated in Table 2. indicates the stress in terms of the material plastic strain.

Table1

Problem variables (x_i are variables which are labeled in the developed codes for coefficients).

SYMBOL OF VARIABLES IN GA	x ₁	x ₂	x ₃	x ₄	x ₅	x ₆	x ₇	x ₈	x ₉	x ₁₀	x ₁₁
PARAMETERS	C	F	G	L	c ₁	A	B	n ₀	n ₁	c	m

Table2

Extracted data from stress-strain curve of dp590 (Fig. 7). At ambient temperature and strain rate (4-8)*e-4.

Segments	Strain	Stress
1	0.00E+00	4.00E+02
2	8.81E-04	4.29E+02
3	2.64E-03	4.60E+02
4	4.74E-03	4.85E+02
5	7.60E-03	5.13E+02
6	9.47E-03	5.29E+02
7	1.28E-02	5.53E+02
8	1.56E-02	5.71E+02
9	1.87E-02	5.84E+02
10	2.25E-02	6.04E+02
11	2.65E-02	6.15E+02
12	3.00E-02	6.25E+02
13	3.31E-02	6.32E+02
14	3.63E-02	6.36E+02
15	4.00E-02	6.36E+02
16	4.10E-02	6.36E+02

The GA structure input data is created to optimize the cost function according to Table 3. After implementation of GA codes in “MATLAB” software, obtained optimization constants are intercalated in Table 4. The GA analysis convergence cases, i.e. log mean and mean cost curves, are based on Fig. 8. Also, Fig. 9 shows the aim curve for KHL optimized constants compared to experimental data. It is clear from Fig. 9 that there is an excellent correlation between KHL prediction and experimental results for work hardening case.

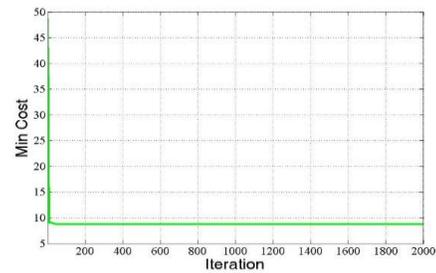
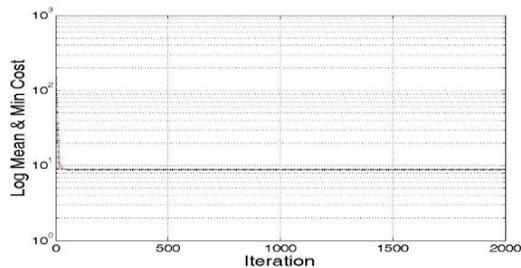
Table3

Data used in GA for KHL constitutive equation.

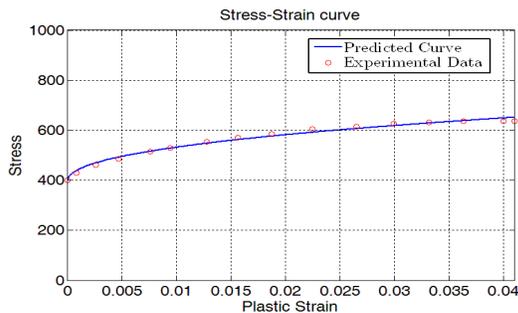
Parameters	Quantities
Number of variables	7
Maximum repetition	2000
Population size	80
Crossover percent	80%
Mutation percent	30%
Mutation rate	125%
Selection mechanism of parents	Roulette wheel

Table4Obtained optimization constants by GA method for KHL constitutive equation for DP590 steel alloy at ambient temperature and the strain rate of $(4-6) \times 10^{-4}$.

c_1	A	B	n_1	n_0	c	m	Best Cost (RMSE)
0	1286.6	563.6939	0.4363	4.7883	0.1608	1.2994	8.7792

**Fig. 8**

The GA convergence curves corresponding to Table 4.

**Fig. 9**Predicted diagram of stress- strain of DP590 steel alloy by KHL constitutive equation and by GA method, at ambient temperature and the strain rate of $(4-8) \times 10^{-4}$.

The cost function amount can be more decreased by balancing GA structure parameters and changing the range of problem variables. The smaller the cost function, the more preferable the accuracy of optimization. The authors considered that KHL constitutive equation has a desirable correlation with experimental results when the material has work hardening. However, the constitutive equation is inefficient for softening and perfect plastic behaviors of materials [14]. Therefore, viscoplastic constitutive equations must be used for such behaviors.

6.2 Determining correction coefficient of khan-liu criterion

Now, other material constants of KL, i.e. C , F , G , and L , are determine by the GA optimization method. For this purpose, the data used in GA codes are presented in Table 5., corresponding to Fig. 6. The GA structure for the material constants is same Table 3 contents. Of course, the variables number is four in here. After running the algorithm codes in “MATLAB” software, optimized solutions for the KL criterion constants in each plastic strain,

the average strain rate of 6×10^{-4} and with loading ratios of 0:1, 1:1 and 1:0 are obtained as shown in Table 6. The convergence curves were almost the same and had similar trends for all conditions. It is worth mentioning that zero amount of RMSE indicates the high level of accuracy for material constants.

Finally, K amount will be calculated in terms of any plastic strain by determining the KL material constants. The results of K are presented in Table 7.

Table 5

Experiment points for curve fitting, from Fig. 6 related to yield loci, at various plastic strains, at room temperature and average strain rate of 6×10^{-4} per second in *Mpa*.

Loading Paths $\sigma_1 : \sigma_2$	ϵ_0^p									
	0.003		0.007		0.012		0.02		0.03	
	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
0:1	0.00	410.51	0.00	454.14	0.00	480.00	0.00	526.87	0.00	562.42
1:5	90.32	441.21	99.01	491.31	106.11	521.81	114.75	573.51	122.83	610.66
1:2	230.65	458.99	255.42	510.71	275.84	542.47	295.76	590.92	313.54	626.44
3:4	322.58	429.90	363.54	483.23	392.16	518.00	421.82	563.19	450.91	598.69
1:1	395.16	395.16	437.82	437.82	476.10	476.10	513.94	513.94	552.73	552.73
4:3	438.71	328.08	487.97	366.87	527.66	391.66	567.27	427.14	604.45	454.54
2:1	448.39	224.65	496.27	248.89	540.34	265.58	583.43	292.97	623.84	312.28
5:1	427.42	85.66	472.39	95.35	510.91	99.17	555.96	113.62	594.75	120.01
1:0	398.39	0.00	438.71	0.00	476.77	0.00	522.02	0.00	562.42	0.00

Continuation of Table 5.

Loading Paths $\sigma_1 : \sigma_2$	ϵ_0^p							
	0.04		0.05		0.065		0.08	
	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
0:1	0.00	586.67	0.00	606.06	0.00	625.46	0.00	646.77
1:5	127.68	638.13	134.14	668.82	138.99	686.59	140.61	698.10
1:2	328.08	653.88	335.75	668.41	----	----	----	----
3:4	470.30	626.12	484.18	645.49	----	----	----	----
1:1	580.20	580.20	602.01	602.01	630.30	630.30	649.70	649.70
4:3	630.30	472.26	650.63	486.77	677.17	502.87	691.72	517.96
2:1	651.31	325.15	671.90	336.42	696.57	344.45	----	----
5:1	620.61	123.19	641.68	128.00	664.24	131.18	----	----
1:0	589.90	0.00	609.68	0.00	633.54	0.00	651.31	0.00

Table 6

Obtained optimization material constants by GA method for the KL criterion for DP590 steel alloy at ambient temperature and strain rate of $(4-6) \times 10^{-4}$.

Plastic Strain	C	F	G	L	Best Cost(RMSE)
0.003	-0.0845	1.1362.6	1.0701	-1.2311	0
0.007	-0.0119	0.8072	0.7533	-0.7691	0
0.012	-0.0536	0.7396	0.7297	-0.8030	0
0.02	-0.0228	0.5766	0.5661	-0.5743	0
0.03	-0.0095	0.4802	0.4802	-0.4726	0
0.04	0.0370	0.3952	0.3995	-0.3548	0
0.05	-0.0266	0.4175	0.4225	-0.4340	0
0.065	0.0127	0.3543	0.3635	-0.3202	0
0.08	-0.0619	0.3859	0.3913	-0.4346	0

Table7

The amounts of K obtained from each analysis for various plastic strains by GA.

ε^p	0.003	0.007	0.012	0.02	0.03	0.04	0.05	0.065	0.08
K	0.1915	0.0706	0.1615	0.1356	0.1919	0.1716	0.2778	0.0772	0.3542

By curve fitting through GA and using experimental data of Fig. 6 or Table 5., the curves related to each K are drawn in Fig. 10. The figure shows that for each K value in terms of ε_0^p , the yield loci is closer to experimental points compared to $K=1$ or $K=0$. Therefore, the criterion accuracy increases by using the shape-dependent term. Table 7. data could be fitted using “MATLAB” software as following and a mathematical relation can be extracted for K in terms of plastic strain for instance smoothing spline type or each other mathematical function. The curve related to Eq. (10) is shown in Fig. 11. Fig. 12 also show material constants of KL criterion in terms of plastic strain. A mathematical relationship can be obtained between every material constants and plastic strain by a simple curve fitting one dimension.

Smoothing spline:

$f(x) =$ piecewise polynomial computed from p

Smoothing parameter: $p = 0.99999994$

Goodness of fit:

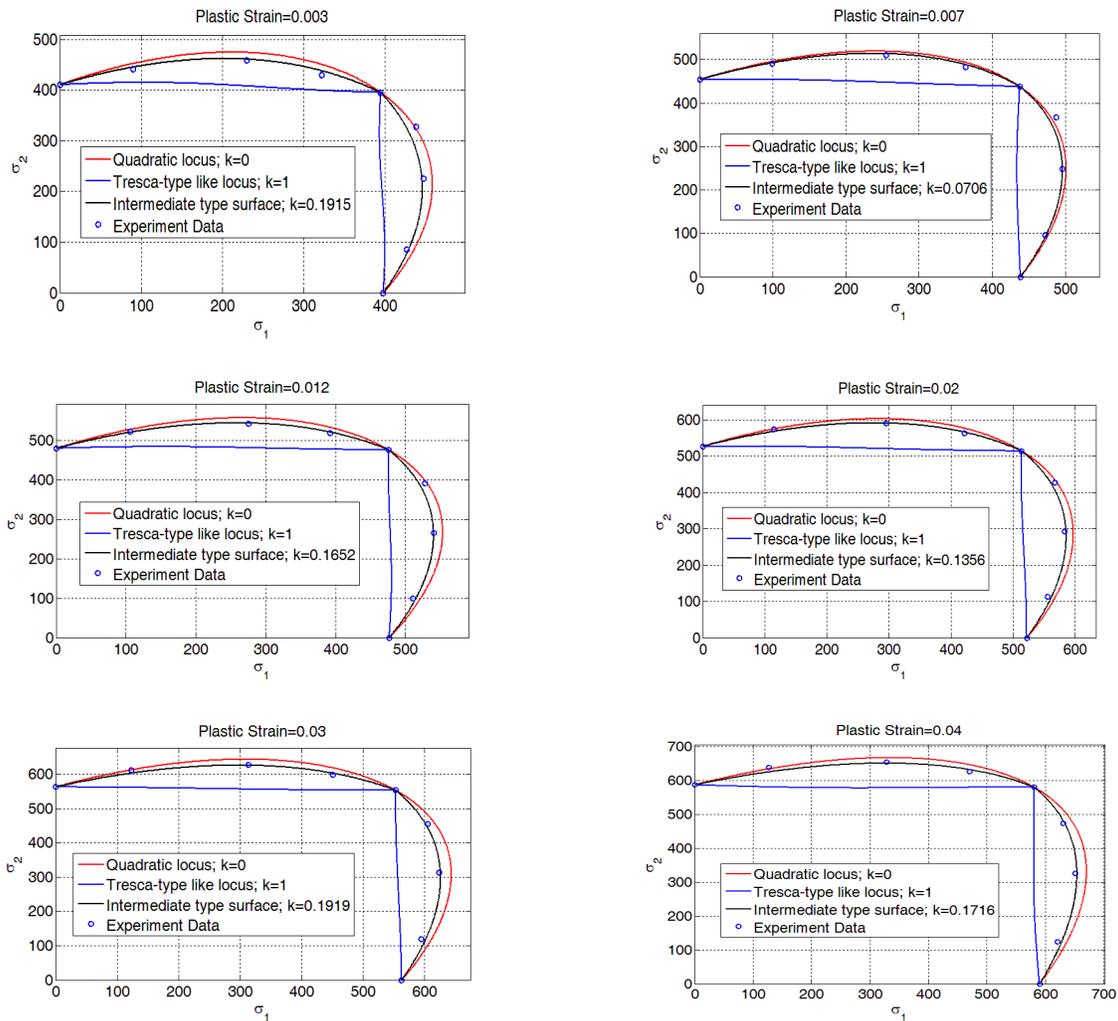
; SSE: 0.01011

; R-square: 0.8447

; Adjusted R-square: 0.5409

; RMSE: 0.06111

(10)



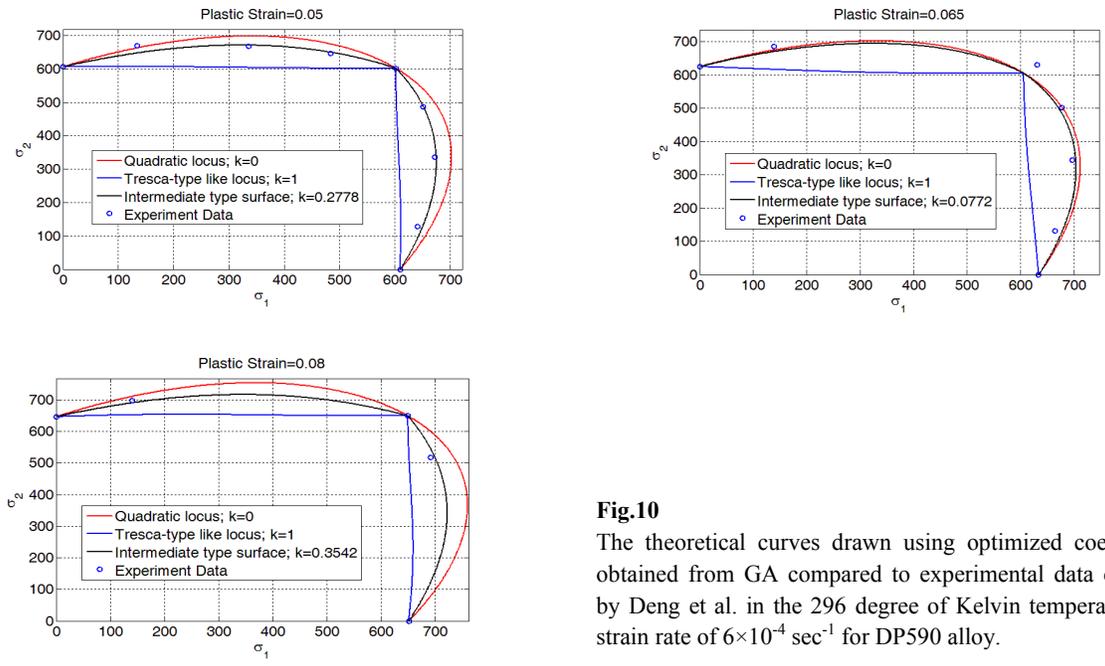


Fig.10
The theoretical curves drawn using optimized coefficients obtained from GA compared to experimental data obtained by Deng et al. in the 296 degree of Kelvin temperature and strain rate of $6 \times 10^{-4} \text{ sec}^{-1}$ for DP590 alloy.

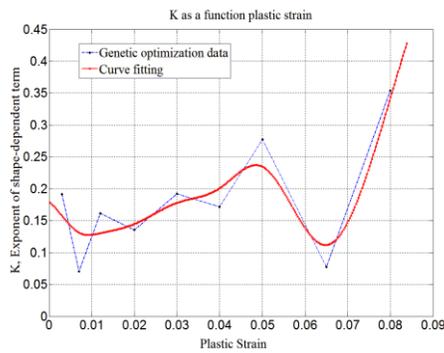


Fig.11
Curve fitting of K vs. plastic strain.

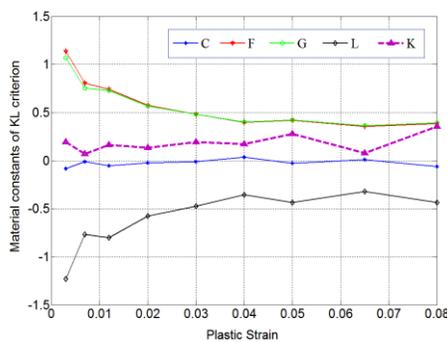


Fig.12
The material constants of KL criterion in terms of plastic strain.

7 CONCLUSIONS

The economy of scale by conducting the minimum number of experimental tests in order to determine the behavior of the material is of a paramount significance for designers. The Khan-Liu (KL) yield/ fracture criterion, which is the development of Hill1948 and Johnson-Cook criteria, is able to describe the behavior of a wide range of materials appropriately. Therefore, a correction coefficient is used to generalize this criterion which can be determined through conducting strain plane tests using cruciform specimen. Currently, the mentioned correction coefficient is

determined based on the plastic strain and for the average strain rate of 6×10^{-4} at room temperature for DP590 steel alloy and through a genetic algorithm (GA). The KL criterion accuracy will be increased by the correction coefficient and data of three experimental tests of (1) uniaxial tension in rolling direction, (2) uniaxial tension in the transverse direction and (3) equal biaxial tension. In the present work, the correction coefficient is determined in terms of plastic strain. The coefficient can be also determined in two variables terms of strain rate and temperature for warm sheet forming of ductile materials. DP590 steel alloy significantly has work hardening. Therefore, Khan-Huang- Liang (KHL) constitutive equation is used in the KL yield/ fracture criterion to consider the work hardening effect in addition to strain rate and temperature effects.

The following efforts can be performed as future works:

- 1- Determination of KHL material constants in terms of strain rate and temperature
- 2- Consideration of simultaneous effects of grain size and hydrostatic pressure in KHL constitutive equation
- 3- Determination of KHL accuracy by accuracy indexes for different strain rates and temperature ranges
- 4- Making ready subroutines for KHL and the KL in “ABAQUS” software
- 5- Improvement of the shape- dependent term of the KL criterion by plane strain tests at different strain rates and temperatures and for some material in terms of plastic strains

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