# Reflection and Transmission of Plane Waves at Micropolar Piezothermoelastic Solids

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**ABSTRACT**

The present investigation analysis a problem of reflection and transmission at an interface of two micropolar orthotropic piezothermoelastic media. The basic equations and constitutive relations for micropolar orthotropic piezothermoelastic media for G-L theory are derived. The expressions for amplitude ratios corresponding to reflected and transmitted waves are derived analytically. The effect of angle of incidence, frequency, micropolarity, thermopiezoelectric interactions on the reflected and transmitted waves are studied numerically for a specific model. Some special cases of interest one are also deduced.

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**Keywords:** Orthotropic; Micropolar; Piezothermoelastic; Amplitude ratios; Angle of incidence.

## 1 INTRODUCTION

**M**ICROPOLAR elasticity theory which takes into consideration the granular character of the medium, describes deformation by a microrotation and a microdisplacement. Eringen first showed that the classical elasticity theory [4] and the coupled stress theory [1] are two special cases of micropolar elasticity. The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effects by Nowacki [29] and Eringen [2]. A comprehensive review on the micropolar theromoelasticity is given by Eringen [3]. In most of the engineering problems, including the response of soils, geological materials and composites, some significant features of the continuum response may not take into account by the assumptions of isotropic behavior. The formulation and solution of anisotropic problems is far more difficult and cumbersome than their isotropic counterparts. Number of researchers paid attention to the elastodynamic response of an anisotropic continuum in the last few years. In particular, transversely isotropic and orthotropic materials which may not be distinguished from each other in plane strain and plane stress cases, have been more regularly studied. The static problems of plane micropolar strain of a homogeneous and orthotropic elastic solid, torsion problems of homogeneous and orthotropic cylinders in the linear theory of micropolar elasticity and bending of orthotropic micropolar elastic beams by terminals couple were studied by Iesan [11-12-13]. Finite element method for orthotropic micropolar elasticity was developed by Nakamura et al. [28]. Kumar & Choudhary [23-24-25] and [26-27] have studied various problems in orthotropic micropolar continua.

Piezoelectric ceramics and composites find applications in many engineering applications e.g. sensors, actuators, intelligent structures, rocket propelled grenades, ultrasonic imaging, when thermal effects are not considered.

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Piezoelectric ceramics and piezoelectric polymers are pyroelectric media, which are used in small structure and intelligent system. The thermo-piezoelectric material response entails an interaction of three major fields, namely, mechanical, thermal and electric in the macro-physical world. The thermopiezoelectric material has one important application to detect the responses of a structure by measurment of the electric charge, sensing or to reduce excessive responses by applying additional electric forces or thermal forces, actuating. Intelligent structure can be designed by integrating sensing and actuating. The thermopiezoelectric materials are also often used as resonators whose frequencies need to be precisely controlled. It is important to quantify the effect of heat dissipation on the propagation of wave at low and high frequencies, due to the coupling between the thermoelastic and pyroelectric effects. The theory of thermo-piezoelectricity was first developed by Mindlin [22]. The physical laws for the thermo-piezoelectric materials have been explored by Nowacki [30-31-32]. Chandrasekharaiah [14-15] has generalized Mindlin's theory of thermo-piezoelectricity to account for the finite speed of propagation of thermal disturbances. Chen [34] derived the general solution for transversely isotropic piezothermoelastic media. Hou et al.[21] constructed Green's function for a point heat source on the surface of a semi-infinite transversely isotropic pyroelectric media. Abd-Alla et al. [8] investigated reflection and refraction of plane quasilongitudinal waves at an interface of two piezoelectric media under initial stresses. Pang et al. [36] discussed the reflection and refraction of plane waves at the interface between two transversely isotropic piezoelectric and piezomagnetic media. Different researchers have studied the problems of reflection in piezoelectric media notable among them are Sharma et al. [18], Abdalla and Alshaikh [8-9], Kuang and Yuan[37]), Abdalla et al. [10]), Alshaikh [16-17], Abd-Alla, Hamdan, Giorgio and Vescovo [6], Guo and Wei [35], Othman [19], Othman, Atwa, Hasona and Ahmed [20], Abd-Alla, Giorgio, Galantucci, Hamdan and Vescovo [7].

In the present paper, the reflection and transmission phenomenon of plane waves at an interface of two orthotropic micropolar piezothermoelastic media has been discussed. It is found that there exist five plane waves in micropolar orthotropic piezothermoelastic medium namely quasi longitudinal displacement wave, quasi thermal wave, quasi coupled transverse displacement and quasi microrotational waves and one wave mode corresponding to electric potential wave. When a plane quasi wave is incident at an interface, the amplitude ratios of various reflected and transmitted waves are computed numerically and are plotted graphically with angle of incidence.

## 2  BASIC EQUATIONS

Following Green-Lindsay [5], the basic equations of homogeneous orthotropic micropolar piezothermoelastic solid with two relaxation times in the absence of body forces, body couples, electric charge density and heat sources are given by

(a)  Constitutive relations

$$t_{kl} = C_{ijkl}\varepsilon_{kl} + A_{ijkl}w_{kl} - g_{ijk}E_k - \beta_{ij}(T + v\dot{T}),\tag{1}$$

$$m_{ji} = D_{ijkl}w_{kl} + A_{ijkl}\varepsilon_{kl} - e_{ijk}E_k,\tag{2}$$

$$D_i = \in_{ij}E_j + g_{ijk}\varepsilon_{jk} + \lambda_i(T + v\dot{T}),\tag{3}$$

$$q_i = -T_0 b_i \dot{T} + k_{ij}e_j,\tag{4}$$

The deformation and wryness tensor are defined as following:

$$\varepsilon_{ij} = u_{i,j} + \in_{ijk}w_k, \qquad w_{ij} = w_{i,j},\tag{5}$$

(b)  Balance laws

$$t_{kl,k} = \rho\ddot{u}_1,\tag{6}$$

$$m_{kl,k} + \in_{lmn}t_{mn} = \rho J\ddot{w}_1,\tag{7}$$

$$D_{i,i} = 0, \tag{8}$$

$$q_{i,i} = -T_0 \dot{S}, \tag{9}$$

where $S = b_i e_i + \beta_{ij} \varepsilon_{ij} + \lambda_i E_i + \rho c^*(T + v_1 \dot{T})$

where $t_{kl}$, $m_{kl}$ are the stress tensor, couple stress tensor; $D_i$ is the electric displacement vector, $E_i$ is the electric field vector, $q_i$ is the heat flux vector; $S$ is the entropy; $T$ is the thermodynamic temperature; $T_0$ is the absolute temperature; $c^*$ is the specific heat at constant strain; $v$ and $v_1$ are thermal relaxation times; $\rho$ is the bulk mass density; $J$ is the microinertia; $u_l$ and $w_k$ are the components of displacement vector and microrotation vector, respectively; $\in_{ij}$ is dielectric moduli; $\lambda_i$ is pyroelectric moduli; $k_{ij}$ is thermal conductivity tensor; $b_i$ are the coefficients characterizing the lack of a centre of symmetry, $\varepsilon_{ij}$ are the components of micro-strain tensor, $\in_{ijk}$ is the permutation tensor, $\beta_{kl}$ is the thermal elastic coupling tensor; $C_{ijkl}, G_{ijkl}, D_{ijkl}$ are the characteristic constants of material; $g_{ijk}$ is the electro-elastic coupling moduli where $C_{ijkl}, D_{ijkl}, g_{ijk}$ satisfy the symmetric relations

$$C_{ijkl} = C_{klij}, \quad D_{ijkl} = D_{klij}, \quad g_{ijk} = g_{kij}. \tag{10}$$

In a centrosymmetric bodies, all components of $A_{ijkl}$ vanish.

## 3   FORMULATION OF THE PROBLEM

By using the transformations following Slaughter [33] on the set of Eqs. (1) to (9), the equations for micropolar orthotropic piezothermoelastic medium are derived.

We consider an interface of two homogeneous centrosymmetric, orthotropic micropolar piezothermoelastic media initially in an undeformed state and at uniform temperature $T_0$, namely medium $M_1$ and medium $M_2$. We take the origin of coordinate system on the plane interface and $x_3 -$ axis pointing vertically into the medium $M_1$ is taken which is designated as $x_3 \geq 0$. Plane waves are considered such that all the particles on a line parallel to $x_2 -$ axis are equally displaced, so that all the partial derivatives with respect to the variable $x_2$ will be zero. Therefore, we take $\vec{u} = (u_1, 0, u_3), \vec{w} = (0, w_2, 0), \vec{E} = (E_1, 0, E_3), E_i = -\dfrac{\partial \phi}{\partial x_i}, \phi$ is the electric potential and $\dfrac{\partial}{\partial x_2} = 0$, so that the field equations and constitutive relations reduce to the following:

$$C_{11} \frac{\partial^2 u_1}{\partial x_1^2} + C_{73} \frac{\partial^2 u_1}{\partial x_3^2} + (C_{19} + C_{77}) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + (C_{77} - C_{73}) \frac{\partial w_2}{\partial x_3} + g_{13} \frac{\partial^2 \varphi}{\partial x_1 \partial x_3} + g_{71} \frac{\partial^2 \phi}{\partial x_1 \partial x_3} - \beta_1 \frac{\partial}{\partial x_1}(T + v\dot{T}) = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{11}$$

$$C_{37} \frac{\partial^2 u_3}{\partial x_1^2} + C_{99} \frac{\partial^2 u_3}{\partial x_3^2} + (C_{33} + C_{91}) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + (C_{37} - C_{33}) \frac{\partial w_2}{\partial x_1} + g_{31} \frac{\partial^2 \phi}{\partial x_1^2} + g_{93} \frac{\partial^2 \phi}{\partial x_3^2} - \beta_3 \frac{\partial}{\partial x_3}(T + v\dot{T}) = \rho \frac{\partial^2 u_3}{\partial t^2}, \tag{12}$$

$$D_{24} \frac{\partial^2 w_2}{\partial x_1^2} + D_{86} \frac{\partial^2 w_2}{\partial x_3^2} + (C_{73} - C_{33}) \frac{\partial u_1}{\partial x_1} + (C_{77} - C_{37}) \frac{\partial u_3}{\partial x_1} + (C_{73} - C_{33} - 2C_{37}) w_2 + (g_{31} - g_{71}) \frac{\partial \phi}{\partial x_1} = \rho J \frac{\partial^2 w_2}{\partial t^2}, \tag{13}$$

$$- \in_{11} \frac{\partial^2 \phi}{\partial x_1^2} - \in_{33} \frac{\partial^2 \phi}{\partial x_3^2} + (g_{71} + g_{13}) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + g_{31} \frac{\partial^2 u_3}{\partial x_1^2} + g_{93} \frac{\partial^2 u_3}{\partial x_3^2} + \lambda_3 \frac{\partial}{\partial x_3}(T + v\dot{T}) = 0, \tag{14}$$

$$k_1 \frac{\partial^2 T}{\partial x_1^2} + k_3 \frac{\partial^2 T}{\partial x_3^2} - T_0 \frac{\partial}{\partial t}(\beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_1}) - T_0 \frac{\partial}{\partial t}(\lambda_3 \frac{\partial \phi}{\partial x_3}) = \rho c^* \frac{\partial}{\partial t}(T + v_1 \dot{T}), \tag{15}$$

$$t_{33} = C_{91} \frac{\partial u_1}{\partial x_1} + C_{93} \frac{\partial u_3}{\partial x_3} - g_{93} \frac{\partial \phi}{\partial x_3} - \beta_3(T + v\dot{T}), \tag{16}$$

$$t_{31} = C_{73} \frac{\partial u_1}{\partial x_3} + C_{77} \frac{\partial u_3}{\partial x_1} - g_{71} \frac{\partial \phi}{\partial x_1}, \tag{17}$$

$$m_{32} = D_{86} \frac{\partial w_2}{\partial x_3}, \tag{18}$$

where $\beta_1 = C_{11}\alpha_1 + C_{19}\alpha_3$, $\beta_3 = C_{91}\alpha_1 + C_{99}\alpha_3$, $\alpha_1, \alpha_3$ are the coefficients of linear thermal expansion. We have used the notations $11 \to 1, 12 \to 2, 13 \to 3, 21 \to 4, 22 \to 5, 23 \to 6, 31 \to 7, 32 \to 8, 33 \to 9$ for the material constants.

For convenient we introduce the following dimensionless quantities

$$x_1' = \frac{\omega^* x_1}{c_1}, \quad x_3' = \frac{\omega^* x_3}{c_1}, \quad u_1' = \frac{\omega^*}{c_1} u_1, \quad u_3' = \frac{\omega^*}{c_1} u_3, \quad w_2' = \frac{C_{11}}{C_{33}} w_2, \quad t_{ij}' = \frac{1}{C_{11}} t_{ij}, \quad m_{ij}' = \frac{c_1}{\omega^* D_{24}} m_{ij},$$

$$T' = \frac{T}{T_0}, \quad \phi' = \frac{\omega^* \in_{11}}{c_1 g_{13}} \phi, \quad D_i' = \frac{1}{g_{13}} D_i, \quad t' = \omega^* t, \quad v' = \omega^* v, \quad v_1' = \omega^* v_1, \quad c_1^2 = \frac{C_{11}}{\rho}, \quad \omega^{*2} = \frac{C_{33}}{\rho J}, \tag{19}$$

where $\omega^*$ is the characteristic frequency of the material and $c_1$ is the longitudinal wave velocity of the medium. By using the dimensionless quantities in Eqs. (11)-(15), we obtain the following equations

$$\frac{\partial^2 u_1}{\partial x_1^2} + a_1 \frac{\partial^2 u_1}{\partial x_3^2} + a_2 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + a_3 \frac{\partial w_2}{\partial x_3} + a_4 \frac{\partial^2 \phi}{\partial x_1 \partial x_3} - a_5 \frac{\partial}{\partial x_1}(T + v\dot{T}) = a_6 \frac{\partial^2 u_1}{\partial t^2}, \tag{20}$$

$$\frac{\partial^2 u_3}{\partial x_1^2} + a_7 \frac{\partial^2 u_3}{\partial x_3^2} + a_8 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + a_9 \frac{\partial w_2}{\partial x_1} + a_{10} \frac{\partial^2 \phi}{\partial x_1^2} + a_{11} \frac{\partial^2 \phi}{\partial x_3^2} - a_{12} \frac{\partial}{\partial x_3}(T + v\dot{T}) = a_{13} \frac{\partial^2 u_3}{\partial t^2}, \tag{21}$$

$$\frac{\partial^2 w_2}{\partial x_1^2} + a_{14} \frac{\partial^2 w_2}{\partial x_3^2} + a_{15} \frac{\partial u_1}{\partial x_1} + a_{16} \frac{\partial u_3}{\partial x_1} + a_{17} w_2 + a_{18} \frac{\partial \phi}{\partial x_1} = a_{19} \frac{\partial^2 w_2}{\partial t^2}, \tag{22}$$

$$\frac{\partial^2 \phi}{\partial x_1^2} + a_{20} \frac{\partial^2 \phi}{\partial x_3^2} - a_{21} \frac{\partial^2 u_1}{\partial x_1 \partial x_3} - a_{22} \frac{\partial^2 u_3}{\partial x_1^2} - a_{23} \frac{\partial^2 u_3}{\partial x_3^2} + a_{23}^* \frac{\partial}{\partial x_3}(T + v\dot{T}) = 0, \tag{23}$$

$$(\frac{\partial^2 T}{\partial x_1^2} + a_{24} \frac{\partial^2 T}{\partial x_3^2}) - \frac{\partial}{\partial t}(a_{25} \frac{\partial u_1}{\partial x_1} + a_{26} \frac{\partial u_3}{\partial x_1}) - \frac{\partial}{\partial t}(a_{27} \frac{\partial \phi}{\partial x_3}) = a_{28} \frac{\partial}{\partial t}(T + v_1 \dot{T}), \tag{24}$$

where

$$a_1 = \frac{C_{73}}{C_{11}}, \quad a_2 = \frac{C_{19}+C_{77}}{C_{11}}, \quad a_3 = \frac{(C_{77}-C_{73})C_{33}}{(C_{11})^2}, \quad a_4 = \frac{(g_{13}+g_{71})g_{13}}{C_{11}\in_{11}}, \quad a_5 = \frac{\beta_1 T_0}{C_{11}}, \quad a_6 = \frac{\rho c_1^2}{C_{11}},$$

$$a_7 = \frac{C_{99}}{C_{37}}, \quad a_8 = \frac{C_{33}+C_{91}}{C_{37}}, \quad a_9 = \frac{(C_{37}-C_{33})C_{33}}{C_{37}C_{11}}, \quad a_{10} = \frac{g_{31}g_{13}}{C_{37}\in_{11}}, \quad a_{11} = \frac{g_{93}g_{13}}{C_{37}\in_{11}}, \quad a_{12} = \frac{\beta_3 T_0}{C_{37}},$$

$$a_{13} = \frac{\rho c_1^2}{C_{37}}, \quad a_{14} = \frac{D_{86}}{D_{24}}, \quad a_{15} = \frac{(C_{73}-C_{33})C_{11}c_1^2}{(C_{33}D_{24})\omega^{*2}}, \quad a_{16} = \frac{(C_{77}-C_{37})C_{11}c_1^2}{(C_{33}D_{24})\omega^{*2}}, \quad a_{17} = \frac{(C_{77}-C_{33}-2C_{37})c_1^2}{D_{24}\omega^{*2}}, \tag{25}$$

$$a_{18} = \frac{(g_{31}-g_{71})C_{11}g_{13}c_1^2}{C_{33}D_{24}\in_{11}\omega^{*2}}, \quad a_{19} = \frac{\rho J c_1^2}{D_{24}}, \quad a_{20} = \frac{\in_{33}}{\in_{11}}, \quad a_{21} = \frac{g_{71}+g_{13}}{g_{13}}, \quad a_{22} = \frac{g_{31}}{g_{13}}, \quad a_{23} = \frac{g_{93}}{g_{13}},$$

$$a_{23}^* = \frac{\lambda_3 T_0}{g_{13}}, \quad a_{24} = \frac{k_3}{k_1}, \quad a_{25} = \frac{c_1^2\beta_1}{\omega^* k_1}, \quad a_{26} = \frac{c_1^2\beta_3}{\omega^* k_1}, \quad a_{27} = \frac{\lambda_3 c_1^2 g_{13}}{\omega^* k_1 \in_{11}}, \quad a_{28} = \frac{\rho c^* c_1^2}{\omega^* k_1}$$

## 4  PLANE WAVE PROPAGATION

We consider harmonic plane wave propagating in the $x_1 x_3 -$ plane at a given frequency $\omega$ as:

$$(u_1, u_3, w_2, \phi, T) = (\overline{u_1}, \overline{u_3}, \overline{w_2}, \overline{\phi}, \overline{T})e^{\iota(\omega t - k x_1)}, \tag{26}$$

where $\overline{u_1}, \overline{u_3}, \overline{w_2}, \overline{\phi}, \overline{T}$ are functions of $x_3$ only, $k$ is the wave number and $c = \frac{v}{\sin\theta_0}$ where $c(=\frac{k}{\omega})$, $v$ is the phase velocity of wave propagating in $x_1 x_3 -$ plane along a direction making an angle $\theta_0$ with $x_3 -$ axis.

Using Eq. (26) in Eqs. (20)-(24), a system of five homogeneous equations is obtained in five unknowns $\overline{u_1}, \overline{u_3}, \overline{w_2}, \overline{\phi}, \overline{T}$, which for non-trivial solution yield

$$\left(A_1 \frac{d^{10}}{dx_3^{10}} + A_2 \frac{d^8}{dx_3^8} + A_3 \frac{d^6}{dx_3^6} + A_4 \frac{d^4}{dx_3^4} + A_5 \frac{d^2}{dx_3^2} + A_6\right) = 0, \tag{27}$$

where

$$A_1 = a_1 a_7 a_{14} a_{20} a_{24},$$

$$A_2 = a_{24}(h_1 a_1 a_7 + h_2 a_{14} a_{20} + h_4 a_{20} - h_6 a_{23}) + h_{19} a_{14} a_{11} + a_1 a_7 a_{14} a_{20} \delta_{17} + h_{18}^* a_{14},$$

$$A_3 = \delta_{17}(h_1 a_1 a_7 + h_2 a_{14} a_{20} + h_4 a_{20} - a_{23} h_6) + a_{24}(-\delta_{10} k^2 a_1 a_7 + \delta_1 \delta_6 a_{14} a_{20} + h_1 h_2 + a_{20} h_3 - h_4 k^2 + h_6 \delta_{13} - a_{23} h_7 + h_9)$$
$$+ a_{20} h_{14} - a_{23} h_{11} + a_{14} h_{17} + \delta_{10}(a_{11} h_{19} + a_7 h_{22}) + a_{14} h_{19} \delta_8 + a_{11} a_{14} h_{20} + a_7 a_{14} h_{23} + \delta_{10} h_{18}^*,$$

$$A_4 = \delta_{17}(-\delta_{10} k^2 a_1 a_7 + \delta_1 \delta_6 a_{14} a_{20} + h_1 h_2 + a_{20} h_3 - k^2 h_4 + \delta_{13} h_6 - a_{23} h_7 + h_9) + a_{24}(h_{10} - a_{23} h_8 - h_3 k^2$$
$$+ h_7 \delta_{13} + a_{20} h_5 - h_2 \delta_{10} k^2 + h_1 \delta_1 \delta_6) + h_{11} \delta_{13} + h_{25} - h_{14} k^2 + a_{20} h_{15} - a_{23} h_{12} + \delta_{10}(a_7 h_{23} + \delta_6 h_{22} + h_{17} +$$
$$a_{11} h_{20} + h_{19} \delta_8) + a_{14}(h_{18} + a_{11} h_{21} + \delta_6 h_{23} + \delta_8 h_{20} + \delta_6 h_{23}),$$

$$A_5 = \delta_{17}(-\delta_{10} k^2 h_2 + \delta_1 \delta_6 h_1 + a_{20} h_5 - k^2 h_3 + \delta_{13} h_7 - a_{23} h_8 + h_{10}) + a_{24}(-h_5 k^2 + h_8 \delta_{13} - \delta_1 \delta_6 \delta_{10} k^2) +$$
$$h_{12} \delta_{13} + h_{26} - h_{15} k^2 + a_{20} h_{16} - a_{23} h_{13} + \delta_{10}(a_{11} h_{21} + \delta_8 h_{20} + h_{18}) + a_{14} \delta_8 h_{21} + \delta_6 \delta_{10} h_{23} + h_{24}(\delta_6 a_{14} + \delta_{10} a_7),$$

$$A_6 = \delta_{17}(-h_5 k^2 + h_8 \delta_{13} - \delta_1 \delta_6 \delta_{10} k^2) + h_{13} \delta_{13} - h_{16} k^2 + \delta_{10}(\delta_8 h_{21} + \delta_6 h_{24}),$$

$$h_1 = \delta_{10} a_{20} - k^2 a_{14}, \quad h_2 = a_1 \delta_6 + a_7 \delta_1 - \delta_2 \delta_5, \quad h_3 = (a_1 \delta_7 - a_3 \delta_5)a_{16} \iota k + (\delta_2 \delta_7 - a_3 \delta_6)a_{15}, \quad h_4 = -a_3 a_7 a_{15},$$

$$h_5 = \delta_1 \delta_7 a_{16} \iota k, \quad h_6 = \delta_3 \delta_{15} a_{14} + a_3 a_{11} a_{15} + a_1 a_{11}, \quad h_7 = \delta_7 \delta_{11}(a_1 + \delta_1) + \delta_3(\delta_5 \delta_{10} - \delta_7 a_{15} - \delta_{11} a_3) + a_3 a_{15} \delta_8 + a_{11} \delta_1 + a_1 \delta_8,$$

$$h_8 = \delta_1 \delta_7 \delta_{11} + \delta_1 \delta_8, \quad h_9 = (\delta_{11} a_7 + a_{11} a_{16} \iota k)\delta_{12} a_3, \quad h_{10} = \delta_{11} \delta_{12}(-\delta_2 \delta_7 + a_3 \delta_6) + a_{16} \delta_{12}(a_3 \iota k \delta_8 - \iota \delta_3 \delta_7 k),$$

$h_{11} = -a_3 a_{15} \delta_9 \delta_{16}$, $h_{12} = \delta_9 \delta_{11} \delta_{14} a_3 + \delta_4 \delta_7 \delta_{16} a_{15}$, $h_{13} = -\delta_{11} \delta_{14} \delta_4 \delta_7$, $h_{14} = a_3 a_{15} \delta_9 \delta_{15}$, $h_{15} = a_3 a_{16} \delta_9 \delta_{14} \imath k - \delta_4 \delta_7 \delta_{15} a_{15}$,

$h_{16} = -\delta_4 \delta_7 \delta_{14} a_{16} \imath k$, $h_{17} = -\delta_1 \delta_9 (a_{23} \delta_{16} + \delta_{15} a_{20}) - \delta_2 \delta_9 (\delta_{14} a_{20} - \delta_{12} \delta_{16}) + \delta_3 \delta_9 (\delta_{12} \delta_{15} + \delta_{14} a_{23}) + \delta_4 \delta_5 (a_{20} \delta_{15} + \delta_{16} a_{23}) +$

$\delta_2 \delta_{13}^* (\delta_5 \delta_{16} - \delta_{14} a_{11}) + \delta_3 \delta_{13}^* (\delta_{14} a_7 - \delta_5 \delta_{15})$,

$h_{18} = \delta_9 (\delta_1 \delta_{13} \delta_{16} + \delta_1 \delta_{15} k^2 - \delta_3 \delta_{13} \delta_{14} + \delta_2 \delta_{14} k^2) - \delta_4 \delta_5 (\delta_{15} k^2 + \delta_{13} \delta_{16}) + \delta_{14} \delta_{13}^* (\delta_3 \delta_6 - \delta_2 \delta_8)$,

$h_{19} = \delta_2 \delta_{12} a_{24} + a_3 a_{15} \delta_5 \delta_{13}^*$, $h_{20} = \delta_2 \delta_{12} \delta_{17} - \delta_4 (\delta_{12} \delta_{15} + \delta_{14} a_{23}) - a_{16} \imath k \delta_{14} \delta_{13}^*$,

$h_{21} = \delta_4 \delta_{13} \delta_{14}$, $h_{22} = -\delta_3 \delta_{12} a_{24} + a_3 a_{15} \delta_{16} \delta_{13}^*$, $h_{23} = -\delta_3 \delta_{12} \delta_{17} + \delta_4 (\delta_{12} \delta_{16} - \delta_{14} a_{26}) - a_3 a_{11} \delta_{14} \delta_{13}^*$,

$h_{24} = \delta_4 \delta_{14} k^2$, $h_{25} = -a_3 \delta_9 \delta_{12} (\delta_{11} \delta_{15} + \imath k a_{16} \delta_{16}) + a_{15} \delta_7 \delta_{13}^* (\delta_3 \delta_{15} - \delta_2 \delta_{16}) + a_3 \delta_5 \delta_{13}^* (\delta_{11} \delta_{15} + \imath k a_{16} \delta_{16})$,

$h_{26} = (\delta_4 \delta_7 \delta_{12} - \delta_1 \delta_7 \delta_{13}^*)(\delta_{11} \delta_{15} + \imath k a_{16} \delta_{16}) + \delta_7 \delta_{14} \delta_{13}^* (\delta_{11} \delta_{15} + \imath k a_{16} \delta_{16})$, $h_{18}^* = -\delta_9 a_1 (a_{23} \delta_{16} + a_{20} \delta_{15})$,

$\delta_1 = a_6 \omega^2 - k^2$, $\delta_2 = -a_2 \imath k$, $\delta_3 = -a_4 \imath k$, $\delta_4 = a_5 \imath k (1 + \imath v \omega)$, $\delta_5 = -a_8 \imath k$, $\delta_6 = a_{13} \omega^2 - k^2$,

$\delta_7 = -a_9 \imath k$, $\delta_8 = -a_{10} k^2$, $\delta_9 = -a_{12}(1 + \imath v \omega)$, $\delta_{10} = a_{19} \omega^2 + a_{17} - k^2$, $\delta_{11} = a_{18} \imath k$,

$\delta_{12} = a_{21} \imath k$, $\delta_{13} = a_{22} k^2$, $\delta_{13}^* = a_{23}^*(1 + \imath v \omega)$, $\delta_{14} = -a_{25} k \omega$, $\delta_{15} = -a_{26} \imath \omega$, $\delta_{16} = -a_{27} \imath \omega$,

$\delta_{17} = -a_{28}(\imath \omega - \omega^2 v_1) - k^2$

The roots of the Eq. (27) give the velocities of five plane waves in the decreasing order of the velocities i.e quasi longitudinal displacement wave (quasi LD wave), quasi thermal wave (quasi T wave), quasi CD-I, quasi CD-II wave and electric potential wave (PE wave).

## 5 REFLECTION AND TRANSMISSION

We consider an interface of two homogeneous orthotropic micropolar piezothermoelastic media in contact with each other. A plane wave quasi LD wave or quasi thermal wave is incident making an angle $\theta_0$ with $x_3 -$ axis at an interface. Each incident wave results in five reflected wave modes in medium $M_1$ which is designated as $x_3 > 0$ and five transmitted wave modes in medium $M_2$ which is designated as $x_3 < 0$. In medium $M_1$ and medium $M_2$, reflected and transmitted wave modes are represented by quasi LD wave, quasi thermal wave, quasi CD-I wave, quasi CD-II wave and one other mode corresponding to electric potential wave mode i.e. PE wave mode. All the quantities in medium $M_2$ are denoted by bar.

The values of displacements, microrotation, electric potential and temperature distribution in medium $M_1$ are given by

$$u_1(x_3) = (B_1 e^{-\lambda_4 x_3} + B_2 e^{-\lambda_2 x_3} + B_3 e^{-\lambda_3 x_3} + B_4 e^{-\lambda_4 x_3} + B_5 e^{-\lambda_5 x_3} + B_6 e^{\lambda_4 x_3} + B_7 e^{\lambda_2 x_3} + B_8 e^{\lambda_3 x_3} + B_9 e^{\lambda_4 x_3} + B_{10} e^{\lambda_5 x_3})e^{i(\omega t - k x_1)},$$
(28)

$$u_3(x_3) = (m_1 B_1 e^{-\lambda_4 x_3} + m_2 B_2 e^{-\lambda_2 x_3} + m_3 B_3 e^{-\lambda_3 x_3} + m_4 B_4 e^{-\lambda_4 x_3} + m_5 B_5 e^{-\lambda_5 x_3} + m_1 B_6 e^{\lambda_4 x_3} + m_2 B_7 e^{\lambda_2 x_3} + m_3 B_8 e^{\lambda_3 x_3} + m_4 B_9 e^{\lambda_4 x_3} + m_5 B_{10} e^{\lambda_5 x_3})e^{i(\omega t - k x_1)},$$
(29)

$$w_2(x_3) = (n_1 B_1 e^{-\lambda_4 x_3} + +n_2 B_2 e^{-\lambda_2 x_3} + n_3 B_3 e^{-\lambda_3 x_3} + n_4 B_4 e^{-\lambda_4 x_3} + n_5 B_5 e^{-\lambda_5 x_3} + n_1 B_6 e^{\lambda_4 x_3} + n_2 B_7 e^{\lambda_2 x_3} + n_3 B_8 e^{\lambda_3 x_3} + n_4 B_9 e^{\lambda_4 x_3} + n_5 B_{10} e^{\lambda_5 x_3})e^{i(\omega t - k x_1)},$$
(30)

$$\phi(x_3) = (g_1 B_1 e^{-\lambda_4 x_3} + g_2 B_2 e^{-\lambda_2 x_3} + g_3 B_3 e^{-\lambda_3 x_3} + g_4 B_4 e^{-\lambda_4 x_3} + g_5 B_5 e^{-\lambda_5 x_3} + g_1 B_6 e^{\lambda_4 x_3} + g_2 B_7 e^{\lambda_2 x_3} + g_3 B_8 e^{\lambda_3 x_3} + g_4 B_9 e^{\lambda_4 x_3} + g_5 B_{10} e^{\lambda_5 x_3})e^{i(\omega t - k x_1)},$$
(31)

$$T(x_3) = (l_1 B_{01} e^{-\lambda_4 x_3} + l_2 B_2 e^{-\lambda_2 x_3} + l_3 B_3 e^{-\lambda_3 x_3} + l_4 B_4 e^{-\lambda_4 x_3} + l_5 B_5 e^{-\lambda_5 x_3} + l_1 B_6 e^{\lambda_4 x_3} + l_2 B_7 e^{\lambda_2 x_3} + l_3 B_8 e^{\lambda_3 x_3} + l_4 B_9 e^{\lambda_4 x_3} + l_5 B_{10} e^{\lambda_5 x_3})e^{i(\omega t - k x_1)},$$
(32)

and for medium $M_2$:

$$\overline{u}_1(x_3) = (B_{11}e^{-\overline{\lambda}_1 x_3} + B_{12}e^{-\overline{\lambda}_2 x_3} + B_{13}e^{-\overline{\lambda}_3 x_3} + B_{14}e^{-\overline{\lambda}_4 x_3} + B_{15}e^{-\overline{\lambda}_5 x_3})e^{i(\omega t - kx_1)}, \tag{33}$$

$$\overline{u}_3(x_3) = (\overline{m_1}B_{11}e^{-\overline{\lambda}_1 x_3} + \overline{m_2}B_{12}e^{-\overline{\lambda}_2 x_3} + \overline{m_3}B_{13}e^{-\overline{\lambda}_3 x_3} + \overline{m_4}B_{14}e^{-\overline{\lambda}_4 x_3} + \overline{m_5}B_{15}e^{-\overline{\lambda}_5 x_3})e^{i(\omega t - kx_1)}, \tag{34}$$

$$\overline{w}_2(x_3) = (\overline{n_1}B_{11}e^{-\overline{\lambda}_1 x_3} + \overline{n_2}B_{12}e^{-\overline{\lambda}_2 x_3} + \overline{n_3}B_{13}e^{-\overline{\lambda}_3 x_3} + \overline{n_4}B_{14}e^{-\overline{\lambda}_4 x_3} + \overline{n_5}B_{15}e^{-\overline{\lambda}_5 x_3})e^{i(\omega t - kx_1)}, \tag{35}$$

$$\overline{\phi}(x_3) = (\overline{g_1}B_{11}e^{-\overline{\lambda}_1 x_3} + \overline{g_2}B_{12}e^{-\overline{\lambda}_2 x_3} + \overline{g_3}B_{13}e^{-\overline{\lambda}_3 x_3} + \overline{g_4}B_{14}e^{-\overline{\lambda}_4 x_3} + \overline{g_5}B_{15}e^{-\overline{\lambda}_5 x_3})e^{i(\omega t - kx_1)}, \tag{36}$$

$$\overline{T}(x_3) = (\overline{l_1}B_{11}e^{-\overline{\lambda}_1 x_3} + \overline{l_2}B_{12}e^{-\overline{\lambda}_2 x_3} + \overline{l_3}B_{13}e^{-\overline{\lambda}_3 x_3} + \overline{l_4}B_{14}e^{-\overline{\lambda}_4 x_3} + \overline{l_5}B_{15}e^{-\overline{\lambda}_5 x_3})e^{i(\omega t - kx_1)}, \tag{37}$$

where $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_5$ are the velocities of reflected quasi LD wave, quasi T wave, quasi CD-I wave quasi CD-II wave and PE wave mode respectively in medium $M_1$ and $\overline{\lambda}_1$, $\overline{\lambda}_2$, $\overline{\lambda}_3$, $\overline{\lambda}_4$, $\overline{\lambda}_5$ are the velocities of transmitted quasi LD wave, quasi T wave, quasi CD-I, quasi CD-II wave and PE wave mode respectively in medium $M_2$.
and

$$m_i = \frac{\Delta_1}{\Delta}, \quad n_i = \frac{\Delta_2}{\Delta}, \quad g_i = \frac{\Delta_3}{\Delta}, \quad l_i = \frac{\Delta_4}{\Delta}, \quad i = 1,2,3,4,5$$

$$\Delta = \begin{vmatrix} a_7\lambda_i^2 + \delta_6 & \delta_7 & a_{11}\lambda_i^2 + \delta_8 & \delta_9\lambda_i \\ -a_{16}\iota k & a_{14}\lambda_i^2 + \delta_{10} & \delta_{11} & 0 \\ -a_{23}\lambda_i^2 + \delta_{13} & 0 & a_{20}\lambda_i^2 - k^2 & a_{23}^*(1+\iota v\omega)\lambda_i \\ \delta_{15}\lambda_i & 0 & \delta_{16}\lambda_i & a_{24}\lambda_i^2 + \delta_{17} \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} \delta_5\lambda_i & \delta_7 & a_{11}\lambda_i^2 + \delta_8 & \delta_9\lambda_i \\ a_{15}\lambda_i & a_{14}\lambda_i^2 + \delta_{10} & \delta_{11} & 0 \\ \delta_{12}\lambda_i & 0 & a_{20}\lambda_i^2 - k^2 & a_{23}^*(1+\iota v\omega)\lambda_i \\ \delta_{14} & 0 & \delta_{16}\lambda_i & a_{24}\lambda_i^2 + \delta_{17} \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} \delta_5\lambda_i & a_7\lambda_i^2 + \delta_6 & a_{11}\lambda_i^2 + \delta_8 & \delta_9\lambda_i \\ a_{15}\lambda_i & -a_{16}\iota k & \delta_{11} & 0 \\ \delta_{12}\lambda_i & -a_{23}\lambda_i^2 + \delta_{13} & a_{20}\lambda_i^2 - k^2 & a_{23}^*(1+\iota v\omega)\lambda_i \\ \delta_{14} & \delta_{15}\lambda_i & \delta_{16}\lambda_i & a_{24}\lambda_i^2 + \delta_{17} \end{vmatrix} \tag{38}$$

$$\Delta_3 = \begin{vmatrix} \delta_5\lambda_i & a_7\lambda_i^2 + \delta_6 & \delta_7 & \delta_9\lambda_i \\ a_{15}\lambda_i & -a_{16}\iota k & a_{14}\lambda_i^2 + \delta_{10} & 0 \\ \delta_{12}\lambda_i & -a_{23}\lambda_i^2 + \delta_{13} & 0 & a_{23}^*(1+\iota v\omega)\lambda_i \\ \delta_{14} & \delta_{15}\lambda_i & 0 & a_{24}\lambda_i^2 + \delta_{17} \end{vmatrix}$$

$$\Delta_4 = \begin{vmatrix} \delta_5\lambda_i & a_7\lambda_i^2 + \delta_6 & \delta_7 & a_{11}\lambda_i^2 + \delta_8 \\ a_{15}\lambda_i & -a_{16}\iota k & a_{14}\lambda_i^2 + \delta_{10} & \delta_{11} \\ \delta_{12}\lambda_i & -a_{23}\lambda_i^2 + \delta_{13} & 0 & a_{20}\lambda_i^2 - k^2 \\ \delta_{14} & \delta_{15}\lambda_i & 0 & \delta_{16}\lambda_i \end{vmatrix}$$

## 6   BOUNDARY CONDITIONS

The appropriate boundary conditions at an interface $x_3 = 0$ are given by

$$t_{33} = \overline{t_{33}}, \quad t_{31} = \overline{t_{31}}, \quad m_{32} = \overline{m_{32}}, \quad k_3\frac{\partial T}{\partial x_3} = \overline{k_3}\frac{\partial \overline{T}}{\partial x_3}, \quad u_1 = \overline{u_1}, \quad u_3 = \overline{u_3}, \quad w_2 = \overline{w_2}, \quad T = \overline{T}, \quad E_1 = \overline{E_1}, \quad D_3 = \overline{D_3} \qquad (39)$$

Making use of Eqs. (28) to (37) in boundary conditions given by Eq. (39), we obtain a system of ten homogeneous equations as:

$$\sum_{j=1}^{15} a_{ij} B_j = 0 \, ; \, \left( i = 1, 2, \ldots\ldots, 10 \right) \tag{40}$$

where

$$
\begin{aligned}
&a_{1i} = -d_1 \iota k - d_2\lambda_i m_i + d_3\lambda_i g_i - d_4(1+\iota\omega v)l_i, \quad a_{1j} = -d_1\iota k + d_2\lambda_i m_i - d_3\lambda_i g_i - d_4(1+\iota\omega v)l_i, \\
&a_{1k} = \overline{d_1}\iota k + (\overline{d_2\, m_i} - \overline{d_3\, g_i})\overline{\lambda_i} + \overline{d_4}(1+\iota\omega v)\overline{l_i}, \\
&a_{2i} = -a_1\lambda_i + (d_6 g_i - d_5 m_i)\iota k, \quad a_{2j} = a_1\lambda_i + (d_6 g_i - d_5 m_i)\iota k, \quad a_{2k} = \overline{a_1\lambda_i} + (\overline{d_5 m_i} - \overline{d_6 g_i})\iota k, \\
&a_{3i} = -d_7\lambda_i n_i, \quad a_{3j} = d_7\lambda_i n_i, \quad a_{3k} = \overline{d_7\lambda_i n_i}, \quad a_{4i} = -\lambda_i l_i, \quad a_{4j} = \lambda_i l_i, \quad a_{4k} = d_8\overline{\lambda_i l_i}, \\
&a_{5i} = 1, \quad a_{5j} = 1, \quad a_{5k} = -1, \quad a_{6i} = m_i, \quad a_{6j} = 0, \quad a_{6k} = -\overline{m_i}, \\
&a_{7i} = n_i, \quad a_{7j} = 0, \quad a_{7k} = -\overline{n_i}, \quad a_{8i} = l_i, \quad a_{8j} = 0, \quad a_{8k} = -\overline{l_i}, \\
&a_{9i} = -\iota k g_i, \quad a_{9j} = 0, \quad a_{9k} = \iota k\,\overline{g_i}, \quad a_{10i} = -d_9\lambda_i g_i - \iota k d_{10} - d_{11}\lambda_i m_i + a_{23}^*(1+\iota v\omega), \\
&a_{10j} = d_9\lambda_i g_i - \iota k d_{10} + d_{11}\lambda_i m_1 + a_{23}^*(1+\iota v\omega), \quad a_{10k} = \overline{d_9\,\overline{\lambda_i}\, g_i} - \iota k\,\overline{d_{10}} + \overline{d_{11}\lambda_i\, m_i} + \overline{a_{23}^*}(1+\iota v\omega), \\
&i = 1,2,3,4,5, \quad j = 6,7,8,9,10, \quad k = 11,12,13,14,15
\end{aligned}
\tag{41}
$$

where

$$
\begin{aligned}
&d_1 = \frac{C_{91}}{C_{11}}, \quad d_2 = \frac{C_{93}}{C_{11}}, \quad d_3 = \frac{g_{93}g_{13}}{\epsilon_{11}C_{11}}, \quad d_4 = \frac{\beta_3 T_0}{C_{11}}, \quad d_5 = \frac{C_{77}}{C_{11}}, \quad d_6 = \frac{g_{71}g_{13}}{\epsilon_{11}C_{11}}, \\
&d_7 = \frac{D_{86}C_{33}}{D_{24}C_{11}}, \quad d_8 = \frac{\overline{k_3}}{k_3}, \quad d_9 = \frac{\epsilon_{33}}{\epsilon_{11}}, \quad d_{10} = \frac{g_{71}}{g_{13}}, \quad d_{11} = \frac{g_{93}}{g_{13}} \\
&\overline{d_1} = \frac{\overline{C_{91}}}{C_{11}}, \quad \overline{d_2} = \frac{\overline{C_{93}}}{C_{11}}, \quad \overline{d_3} = \frac{\overline{g_{93}g_{13}}}{\epsilon_{11}C_{11}}, \quad \overline{d_4} = \frac{\overline{\beta_3 T_0}}{C_{11}}, \quad \overline{d_5} = \frac{\overline{C_{77}}}{C_{11}}, \quad \overline{d_6} = \frac{\overline{g_{71}g_{13}}}{\epsilon_{11}C_{11}}, \\
&\overline{d_7} = \frac{\overline{D_{86}C_{33}}}{D_{24}C_{11}}, \quad \overline{d_9} = \frac{\overline{\epsilon_{33}}}{\epsilon_{11}}, \quad \overline{d_{10}} = \frac{\overline{g_{71}}}{g_{13}}, \quad \overline{d_{11}} = \frac{\overline{g_{93}}}{g_{13}}
\end{aligned}
$$

when quasi LD wave is incident: $B_2 = B_3 = B_4 = B_5 = 0$. Dividing the set of equations throughout by $B_1$, we obtain a system of ten non-homogeneous equations in ten unknowns which can be solved by Crammer's rule and we have

$$Z_i = \frac{B_{i+5}}{B_1} = \frac{\Gamma_i^1}{\Gamma} \, ; \, i = 1,2,3,4,5,6,7,8,9,10$$

when quasi T wave is incident: $B_1 = B_3 = B_4 = B_5 = 0$. Dividing the set of equations throughout by $B_2$, we obtain a

system of ten non-homogeneous equations in ten unknowns which can be solved by Crammer's rule and we have

$$Z_i = \frac{B_{i+5}}{B_2} = \frac{\Gamma_i^2}{\Gamma}, \quad ; i = 1,2,3,4,5,6,7,8,9,10$$

where $\Gamma = \left| a_{ii+5} \right|_{10 \times 10}$, and $\Gamma_i^p \, (i = 1,2,3,........,10) \, (p = 1,2,3,........,10)$ can be obtained by replacing, respectively the 1$^{st}$, 2$^{nd}$,............, 10$^{th}$ columns of $\Gamma$ by $\left[ -a_{1p}, -a_{2p}, a_{3p}, a_{4p}, .........., a_{10p} \right]^T$.

## 7  PARTICULAR CASES

If we neglect piezoelectric effect in medium $M_2$, we obtain amplitude ratios at an interface of orthotropic micropolar piezothermoelastic solid and orthotropic micropolar thermoelastic solid with the values of $a_{ij}$ as:

$$a_{1i} = -d_1 \iota k - d_2 \lambda_i m_i + d_3 \lambda_i g_i - d_4(1 + \iota \omega v)l_i, \quad a_{1j} = -d_1 \iota k + d_2 \lambda_i m_i - d_3 \lambda_i g_i - d_4(1 + \iota \omega v)l_i,$$

$$a_{1k} = \overline{d_1} \iota k + (\overline{d_2 m_i} - \overline{d_3 g_i})\overline{\lambda_i} + \overline{d_4}(1 + \iota \omega v)\overline{l_i},$$

$$a_{2i} = -a_1 \lambda_i + (d_6 g_i - d_5 m_i)\iota k, \quad a_{2j} = a_1 \lambda_i + (d_6 g_i - d_5 m_i)\iota k, \quad a_{2k} = \overline{a_1 \lambda_i} + (\overline{d_5 m_i} - \overline{d_6 g_i})\iota k,$$

$$a_{3i} = -d_7 \lambda_i n_i, \quad a_{3j} = d_7 \lambda_i n_i, \quad a_{3k} = \overline{d_7 \lambda_i n_i}, \quad a_{4i} = -\lambda_i l_i, \quad a_{4j} = \lambda_i l_i, \quad a_{4k} = d_8 \overline{\lambda_i l_i},$$

$$a_{5i} = 1, \quad a_{5j} = 1, \quad a_{5k} = -1, \quad a_{6i} = m_i, \quad a_{6j} = 0, \quad a_{6k} = -\overline{m_i},$$

$$a_{7i} = n_i, \quad a_{7j} = 0, \quad a_{7k} = -\overline{n_i}, \quad a_{8i} = l_i, \quad a_{8j} = 0, \quad a_{8k} = -\overline{l_i},$$

$$a_{9i} = -d_9 \lambda_i g_i - \iota k d_{10} - d_{11} \lambda_i m_i + a_{23}^*(1 + \iota v \omega), \quad a_{9j} = d_9 \lambda_i g_i - \iota k d_{10} + d_{11} \lambda_i m_1 + a_{23}^*(1 + \iota v \omega), \quad a_{9k} = 0,$$

$$i = 1,2,3,4,5, \quad j = 6,7,8,9,10, \quad k = 11,12,13,14$$

By neglecting the micropolarity effect in medium $M_2$, we obtain amplitude ratios at an interface of orthotropic micropolar piezothermoelastic solid and orthotropic piezothermoelastic solid with the values of $a_{ij}$ as:

$$a_{1i} = -d_1 \iota k - d_2 \lambda_i m_i + d_3 \lambda_i g_i - d_4(1 + \iota \omega v)l_i, \quad a_{1j} = -d_1 \iota k + d_2 \lambda_i m_i - d_3 \lambda_i g_i - d_4(1 + \iota \omega v)l_i,$$

$$a_{1k} = \overline{d_1} \iota k + (\overline{d_2 m_i} - \overline{d_3 g_i})\overline{\lambda_i} + \overline{d_4}(1 + \iota \omega v)\overline{l_i},$$

$$a_{2i} = -a_1 \lambda_i + (d_6 g_i - d_5 m_i)\iota k, \quad a_{2j} = a_1 \lambda_i + (d_6 g_i - d_5 m_i)\iota k, \quad a_{2k} = \overline{a_1 \lambda_i} + (\overline{d_5 m_i} - \overline{d_6 g_i})\iota k,$$

$$a_{3i} = -d_7 \lambda_i n_i, \quad a_{3j} = d_7 \lambda_i n_i, \quad a_{3k} = 0, \quad a_{4i} = -\lambda_i l_i, \quad a_{4j} = \lambda_i l_i, \quad a_{4k} = d_8 \overline{\lambda_i l_i},$$

$$a_{5i} = 1, \quad a_{5j} = 1, \quad a_{5k} = -1, \quad a_{6i} = m_i, \quad a_{6j} = 0, \quad a_{6k} = -\overline{m_i},$$

$$a_{7i} = l_i, \quad a_{7j} = 0, \quad a_{7k} = -\overline{l_i}, \quad a_{8i} = -\iota k g_i, \quad a_{8j} = 0, \quad a_{8k} = \iota k \overline{g_i},$$

$$a_{9i} = -d_9 \lambda_i g_i - \iota k d_{10} - d_{11} \lambda_i m_i + a_{23}^*(1 + \iota v \omega), \quad a_{9j} = d_9 \lambda_i g_i - \iota k d_{10} + d_{11} \lambda_i m_1 + a_{23}^*(1 + \iota v \omega),$$

$$a_{9k} = \overline{d_9 \lambda_i g_i} - \iota k \overline{d_{10}} + \overline{d_{11} \lambda_i m_i} + \overline{a_{23}^*}(1 + \iota v \omega),$$

$$i = 1,2,3,4,5, \quad j = 6,7,8,9,10, \quad k = 11,12,13,14$$

If we neglect piezoelectric and micropolarity effects in medium $M_1$ and $M_2$, our results tally with those obtained for perfect bonding case.

## 8  NUMERICAL RESULTS AND DISCUSSION

In order to determine the amplitude ratios, the method of crammer's rule has been used and computer program in Matlab 7.8 has been developed.

The physical data for medium $M_1$ is given by

$$C_{11} = 7.46 \times 10^{10} Nm^{-2}, \quad C_{19} = 3.9 \times 10^{10} Nm^{-2}, \quad C_{33} = 1.37 \times 10^9 Nm^{-2}, \quad C_{99} = 8.39 \times 10^9 Nm^{-2},$$

$$C_{91} = 0.399 \times 10^9 Nm^{-2}, \quad C_{77} = 0.0138 \times 10^9 Nm^{-2}, \quad C_{37} = 0.134 \times 10^9 Nm^{-2}, \quad C_{73} = 1.32 \times 10^9 Nm^{-2},$$

$$g_{13} = -0.142 \times 10^{-3} cm^{-2}, \quad g_{71} = -0.165 \times 10^{-3} cm^{-2}, \quad g_{93} = 0.351 \times 10^{-3} cm^{-2}, \quad g_{31} = -0.139 \times 10^{-3} cm^{-2},$$

$$\varepsilon_{11} = 8.29 \times 10^{-11} Nm^{-2}/K, \quad \varepsilon_{33} = 9.07 \times 10^{-11} Nm^{-2}/K, \quad \lambda_3 = 7.6 \times 10^{-6} cm^{-2}/K, \quad \nu_1 = 0.8s,$$

$$k_1 = 9.5 Wm^{-1}K^{-1}, \quad k_3 = 9.7 Wm^{-1}K^{-1}, \quad \beta_1 = 0.670 \times 10^5 C^2 N^{-1}m^{-2}, \quad \beta_3 = 0.581 \times 10^5 C^2 N^{-1}m^{-2},$$

$$\nu = 0.26s, \quad T_0 = 298K, \quad \rho = 5504 Kgm^{-3}, \quad c^* = 2.64 \times 10^2 NmKg^{-1}K^{-1}, \quad J = 0.02 \times 10^{-11} m^{-2},$$

$$D_{24} = 0.134N, \quad D_{86} = 0.243N,$$

and for medium $M_2$ is given by

$$\overline{C_{11}} = 0.141 \times 10^9 Nm^{-2}, \quad \overline{C_{19}} = 0.786 \times 10^9 Nm^{-2}, \quad \overline{C_{33}} = 26.1 \times 10^9 Nm^{-2}, \quad \overline{C_{99}} = 1.81 \times 10^9 Nm^{-2},$$

$$\overline{C_{91}} = 75.8 \times 10^9 Nm^{-2}, \quad \overline{C_{77}} = 26.9 \times 10^9 Nm^{-2}, \quad \overline{C_{37}} = 26.6 \times 10^9 Nm^{-2}, \quad \overline{C_{73}} = 26.3 \times 10^9 Nm^{-2},$$

$$\overline{g_{13}} = 0.013 \times 10^{-3} cm^{-2}, \quad \overline{g_{71}} = -0.061 \times 10^{-3} cm^{-2}, \quad \overline{g_{93}} = 0.00157 \times 10^{-3} cm^{-2}, \quad \overline{g_{31}} = 1.28 \times 10^{-3} cm^{-2},$$

$$\overline{\varepsilon_{11}} = 0.530 \times 10^{-11} Nm^{-2}/K, \quad \overline{\varepsilon_{33}} = 0.708 \times 10^{-11} Nm^{-2}/K, \quad \overline{\lambda_3} = 8.2 \times 10^{-6} cm^{-2}/K, \quad \overline{\nu_1} = 0.6s,$$

$$\overline{k_1} = 1.1 Wm^{-1}K^{-1}, \quad \overline{k_3} = 1.1 Wm^{-1}K^{-1}, \quad \overline{\beta_1} = 0.526 \times 10^5 C^2 N^{-1}m^{-2}, \quad \overline{\beta_3} = 0.355 \times 10^5 C^2 N^{-1}m^{-2},$$

$$\overline{\nu} = 0.25s, \quad \overline{T_0} = 330K, \quad \overline{\rho} = 7500 Kgm^{-3}, \quad \overline{c^*} = 350 NmKg^{-1}K^{-1}, \quad \overline{J} = 2.0 \times 10^{-11} m^{-2},$$

$$\overline{D_{24}} = 0.119N, \quad \overline{D_{86}} = 0.127N,$$

Figs. 1-20 show the variations of amplitude ratios with angle of incidence for incidence of plane waves at an interface for GL-theory. In Figs. 1-20 MPT corresponds to amplitude ratios in orthotropic micropolar piezothermoelastic solid, WPE corresponds to amplitude ratios in orthotropic micropolar thermoelastic solid, WMP corresponds to amplitude ratios in orthotropic piezothermoelastic solid.

*8.1 Quasi LD wave incidence*

Figs. 1-10 represent the variations of amplitude ratios $|Z_1|; 1 \le i \le 10$ with angle of incidence $\theta_0$ for incidence of quasi LD wave.

Fig. 1 shows that the values of amplitude ratio $|Z_1|$ for MPT, WMP and WPE increase from normal incidence to grazing incidence. It is noticed that the values of amplitude ratio for WMP are less than the values for WPE. It shows that the micropolarity effect increases the magnitude of amplitude ratio. The values of amplitude ratio $|Z_1|$ for WMP are magnified by multiplying by $10^2$.

It is noticed clearly from Fig. 2 that the values of amplitude ratio $|Z_2|$ for WPE start with minimum value at normal incidence and then increase gradually to attain maximum value at the grazing incidence, while the values for WMP decrease with increase in angle of incidence. The values of amplitude ratio for orthotropic piezothermoelastic solid are greater than the values for orthotropic micropolar thermoelastic solid that reveals the micropolarity effect.

From Fig. 3 it is clearly revealed that the values of amplitude ratio $|Z_3|$ for MPT increase in the whole range. It is observed that the values of amplitude ratio for orthotropic micropolar piezothermoelastic solid are greater than the values for orthotropic piezothermoelastic solid and orthotropic micropolar thermoelastic solid.

Fig. 4 depicts the variation of amplitude ratio $|Z_4|$ with angle of incidence. The values of amplitude ratio for MPT and WPE increase, while WMP oscillate with increase in angle of incidence. The values of amplitude ratio for WPE are greater than the values of amplitude ratio for WMP.

Fig. 5 shows that the values of amplitude ratio $|Z_5|$ for MPT, WPE and WMP attain minimum value at normal incidence and then the values increase sharply as $\theta_0$ increases. The values for MPT are greater than the values for WMP and WPE in the whole range.

Fig. 6 shows that the values of amplitude ratio $|Z_6|$ for WPE increase with angle of incidence, while the values for WMP decrease. In this case, the removal of micropolarity effect increases the magnitude of amplitude ratio. Fig. 7 depicts that the values of amplitude ratio $|Z_7|$ for MPT and WMP increase as angle of incidence increases, while the values for WPE decreases.

It is noticed from Fig. 8 that the values of amplitude ratio $|Z_8|$ for MPT, WMP and WPE increase with increase in angle of incidence. The values for WMP are greater than the values for MPT in the whole range.

Fig. 9 reveals that the values of amplitude ratio $|Z_9|$ for WPE attain maximum value at normal incidence and then decrease in the further range and are greater than the values for MPT in the whole range. It is depicted from Fig. 10 that the values of amplitude ratio $|Z_{10}|$ for MPT increase in the whole range. The values for MPT are less than the values for WMP in the whole range.
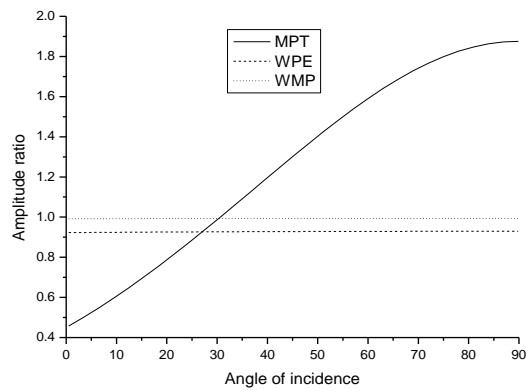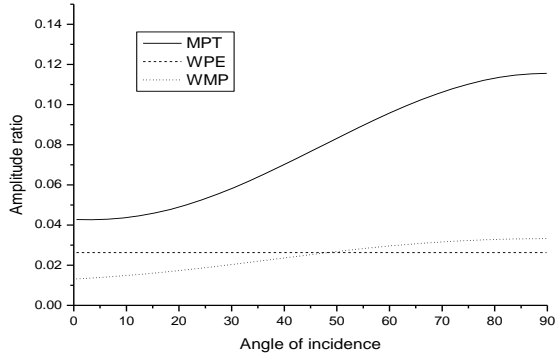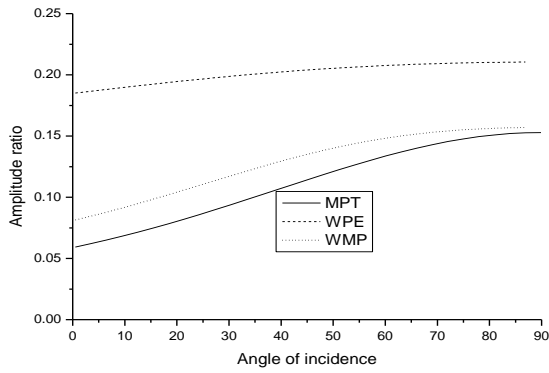


**Fig.1**
Variation of amplitude ratio $|Z_1|$ with angle of incidence.
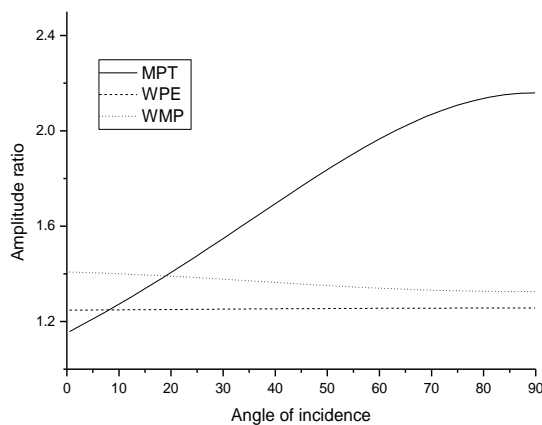


**Fig.2**
Variation of amplitude ratio $|Z_2|$ with angle of incidence.
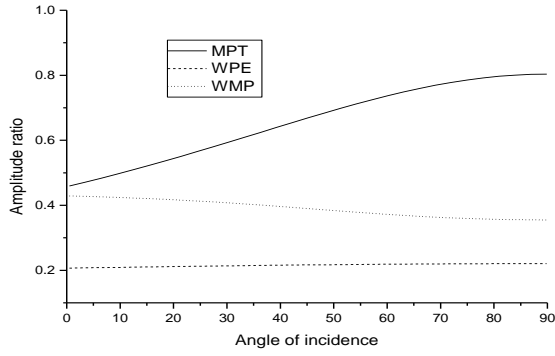
**Fig.3**
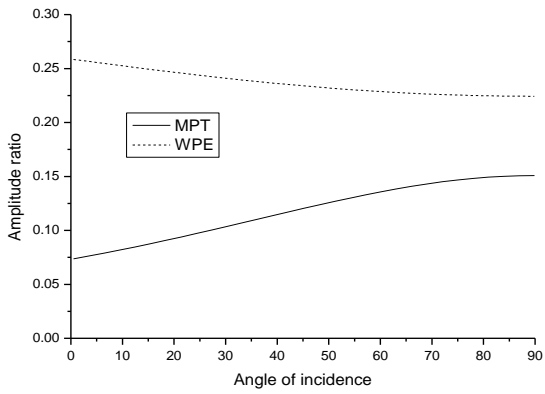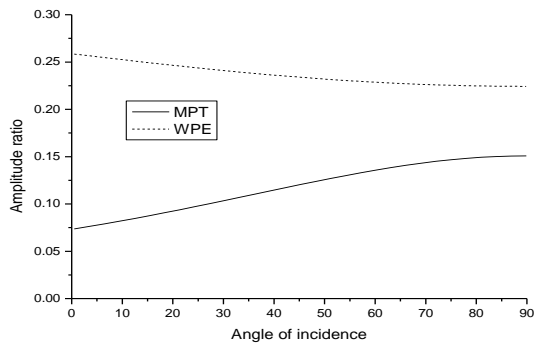Variation of amplitude ratio $|Z_3|$ with angle of incidence.
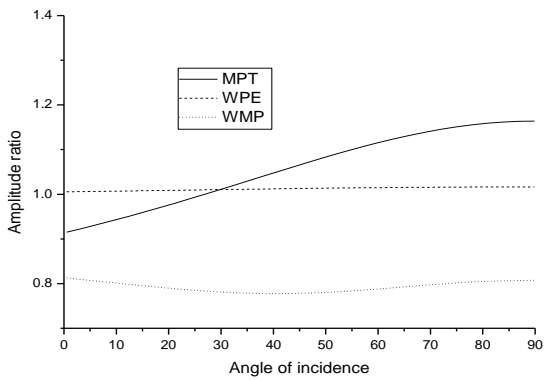


**Fig.4**
Variation of amplitude ratio $|Z_4|$ with angle of incidence.
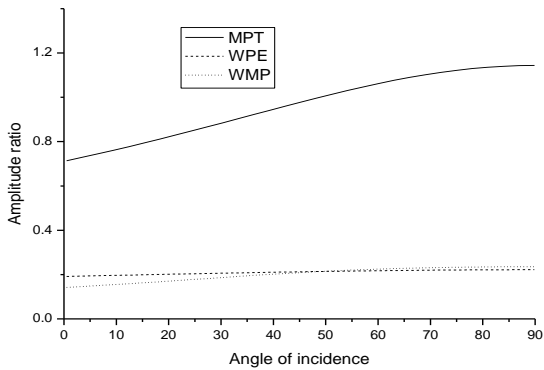


**Fig.5**
Variation of amplitude ratio $|Z_5|$ with angle of incidence.
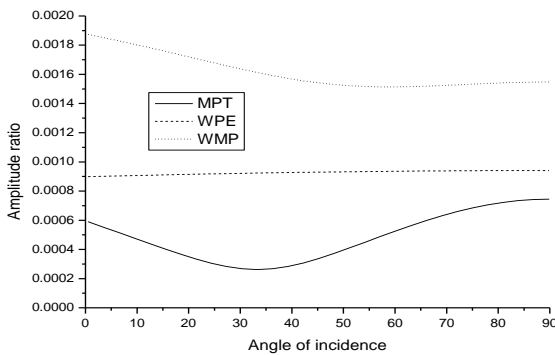


**Fig.6**
Variation of amplitude ratio $|Z_6|$ with angle of incidence.

**Fig.7**
Variation of amplitude ratio $\left|Z_7\right|$ with angle of incidence.



**Fig.8**
Variation of amplitude ratio $\left|Z_8\right|$ with angle of incidence.



**Fig.9**
Variation of amplitude ratio $\left|Z_9\right|$ with angle of incidence.



**Fig.10**
Variation of amplitude ratio $\left|Z_{10}\right|$ with angle of incidence.

*8.2 Quasi T wave incidence*

Figs. 11-20 represent the variations of amplitude ratios $|Z_1|; 1 \leq i \leq 10$ with angle of incidence $\theta_0$ for incidence of quasi T wave.

Fig. 11 shows that the values of amplitude ratio $|Z_1|$ for WPE start with maximum value at normal incidence and then the values decrease to attain minimum value at the grazing incidence. It is seen that the values for MPT increase with angle of incidence.

Fig. 12 depicts the variation of amplitude ratio $|Z_2|$ with angle of incidence. The values of amplitude ratio for MPT decrease with angle of incidence to attain minimum value at the grazing incidence. In this case, the removal of piezoelectric effect and the removal of micropolarity effect increases the magnitude of amplitude ratio.

It is noticed from Fig. 13 that the values of amplitude ratio $|Z_3|$ for WPE get increased, while the values for MPT get decreased with angle of incidence. It is seen that absence of micropolarity effect raises the magnitude of amplitude. Fig. 14 shows that the values of amplitude ratio $|Z_4|$ for WPE remain less than the values for MPT and WMP in the whole range.

Fig. 15 depicts that the values of amplitude ratio $|Z_5|$ for MPT decrease in the whole range, except the initial range where the values of amplitude ratio get increased. The values of amplitude ratios for MPT are greater than the values for WMP in the whole range.

Fig. 16 shows that the values of amplitude ratio $|Z_6|$ for MPT, WPE and WMP are very small in magnitude and oscillate from normal incidence to grazing incidence. The absence of micropolarity effect increases the magnitude of amplitude ratio in this case. It is seen from Fig. 17 that the values of amplitude ratio $|Z_7|$ for WPE get increased with increase in $\theta_0$. The values for MPT increase in the whole range, except the initial range, where the values decrease.

It is depicted from Fig. 18 that the values of amplitude ratio $|Z_8|$ for MPT start with minimum value at the normal incidence and then increase to attain maximum value near the grazing incidence. The values for WPE decrease in the whole range. The values of amplitude ratio in the absence of piezoelectric effect are smaller than the values in the presence of piezoelectric effect.

Fig. 19 shows that the values of amplitude ratio $|Z_9|$ for MPT and WPE attain maximum value at the normal incidence and then decrease to attain minimum value at grazing incidence. Fig. 20 depicts that the values of amplitude ratio $|Z_{10}|$ for MPT increase sharply in the initial range and then decrease as long as $\theta_0$ increases. In this case, the micropolarity effect increases the magnitude of amplitude ratio in the whole range.
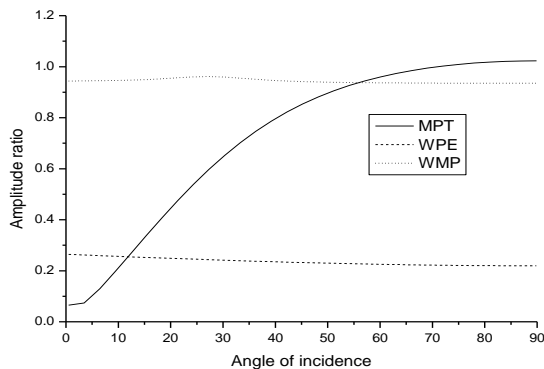


**Fig.11**
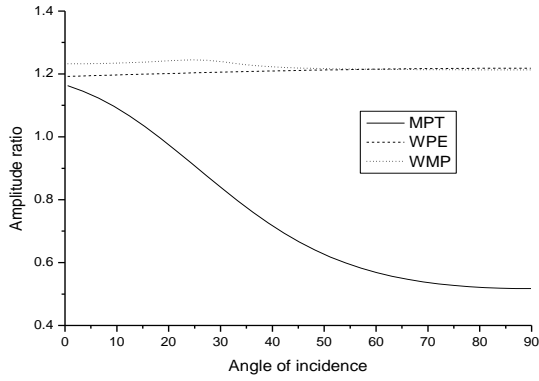Variation of amplitude ratio $|Z_1|$ with angle of incidence.

**Fig.12**
Variation of amplitude ratio $|Z_2|$ with angle of incidence.
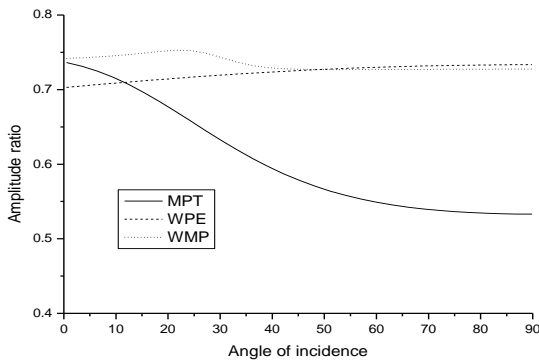


**Fig.13**
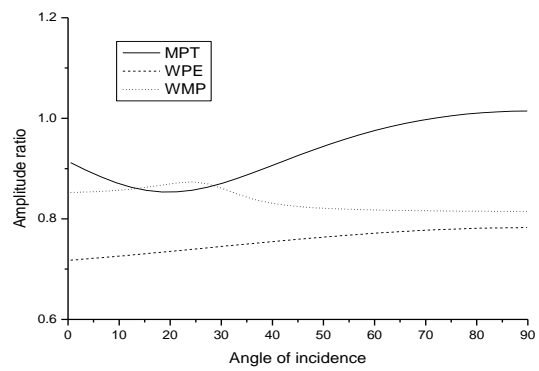Variation of amplitude ratio $|Z_3|$ with angle of incidence.



**Fig.14**
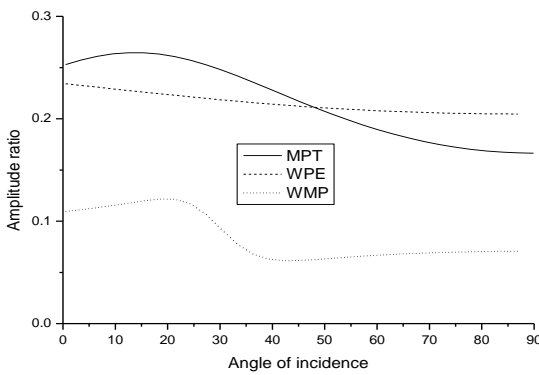Variation of amplitude ratio $|Z_4|$ with angle of incidence.



**Fig.15**
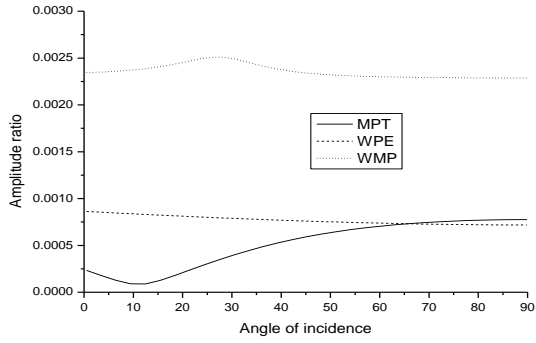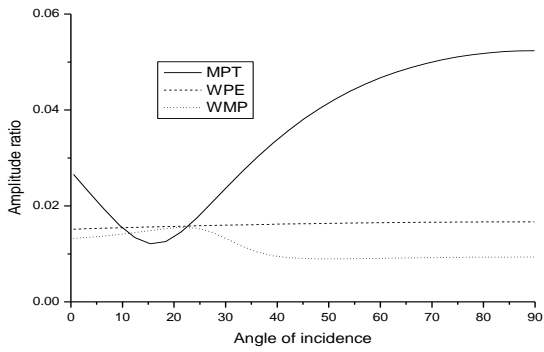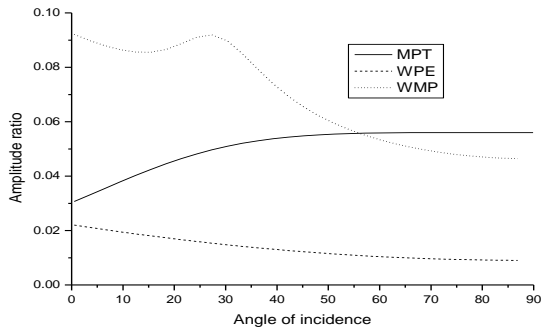Variation of amplitude ratio $|Z_5|$ with angle of incidence.
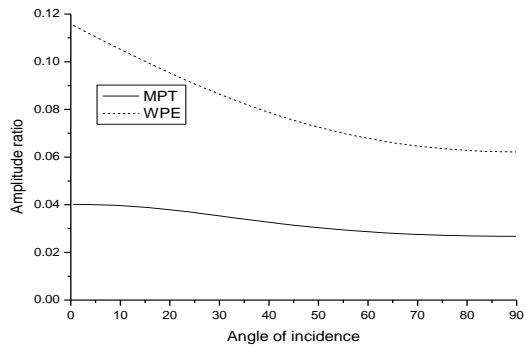
**Fig.16**
Variation of amplitude ratio $\left|Z_6\right|$ with angle of incidence.



**Fig.17**
Variation of amplitude ratio $\left|Z_7\right|$ with angle of incidence.



**Fig.18**
Variation of amplitude ratio $\left|Z_8\right|$ with angle of incidence.



**Fig.19**
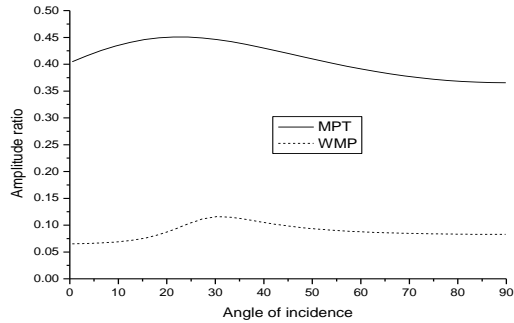Variation of amplitude ratio $\left|Z_9\right|$ with angle of incidence.

**Fig.20**
Variation of amplitude ratio $|Z_{10}|$ with angle of incidence.

Figs. 21-24 show the variations of wavefronts of displacement, microrotation and temperature with respect to $x_1$-axis for time instants $t = 1$ and $t = 5$ secs in orthotropic micropolar piezothermoelastic half-space.

Figs. 21 and 23 depict that the amplitude of horizontal displacement $u_1$ and microrotation $w_2$ decreases with increase in the value of $x_1$. The values for $t = 5$ are greater than the values for $t = 1$, which shows that as the time increases the amplitude increases.

Figs. 22 and 24 show the variation of vertical displacement $u_3$ and temperature $T$ with respect to $x_1$-axis. It is observed that the values of vertical displacement $u_3$ and temperature $T$ first decrease and then increase.
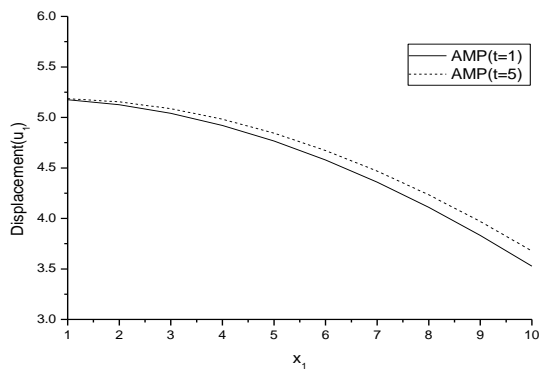


**Fig.21**
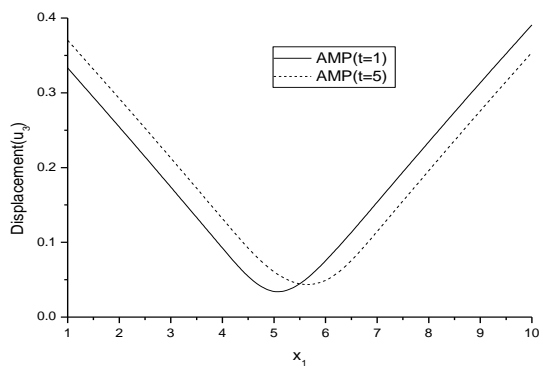Variation of horizontal displacement $u_1$ with $x_1$-axis for incidence of quasi LD wave.



**Fig.22**
Variation of vertical displacement $u_3$ with $x_1$-axis for incidence of quasi LD wave.
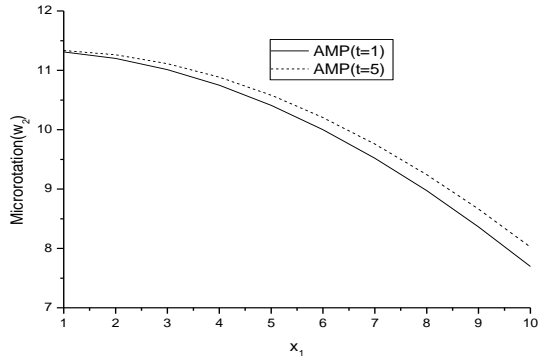
**Fig.23**
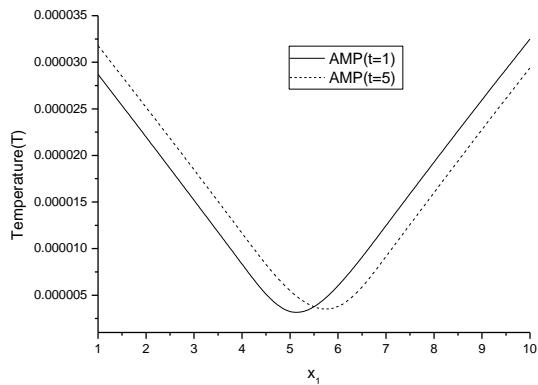Variation of microrotation $w_2$ with $x_1$-axis for incidence of quasi LD wave.



**Fig.24**
Variation of temperature $T$ with $x_1$-axis for incidence of quasi LD wave.

## 9    CONCLUSIONS

The reflection and transmission coefficients of various plane quasi waves on incidence of quasi LD wave and quasi T wave at an interface of two orthotropic micropolar piezothermoelastic media are obtained in the present paper. It is noticed that reflection and transmission coefficients are influenced by piezoelectric and micropolarity effect. It is seen that when quasi LD wave is incident, the values of amplitude ratios of reflected quasi T wave in the absence of micropolarity effect are greater that reveals the effect of micropolarity. When quasi T wave is incident, the piezoelectric effect increases the magnitude of amplitude ratio of reflected quasi CD-II wave and transmitted quasi CD-II wave modes. The problem investigated in this paper has wide applications in signal processing and wireless communication in addition to improvement of SAW wave devices and defence equipment.

## REFERENCES

[1]     Eringen A.C.,1996, Linear theory of micropolar elasticity, *Journal of Mathematics and Mechanics* **15**: 909-923.
[2]     Eringen A.C., 1970, *Foundations of Micropolar Thermoelasticity*, Course Held at the Department for Mechanics of Deformable Bodies, Springer.
[3]     Eringen A.C., 1992, *Microcontinuum Field Theory I, Foundations and Solids*, Springer, New York.
[4]     Eringen A.C., Suhubi E.S., 1964, Non-linear theory of aimple micro-elastic solids, *International Journal of Engineering Science* **2**:189-203.
[5]     *Green A.E.,* Lindsay K.A., 1972, Thermoelasticity, *Journal of Elasticity* **2**: 1-7.
[6]     Abd-Alla A.N., Hamdan A.M., Giorgio I., Del Vescovo D., 2014, The mathematical model of reflection and refraction of longitudinal waves in thermo-piezoelectric materials, *Archive of Applied Mechanics* **84**(9): 1229-1248.
[7]     Abd-Alla A.N., Giorgio I., Galantucci L., Hamdan A.M., Del Vescovo D., 2016, Wave reflection at a free interface in an anisotropic pyroelectric medium with nonclassical thermoelasticity, *Continuum Mechanics and Thermodynamics* **28**(1-2): 67-84.

[8]     Abd-Alla A.N., Alshaikh F.A., 2009, Reflection and refraction of plane quasi-longitudinal waves at an interface of two piezoelectric media under initial stresses, *Archive of Applied Mechanics* **79**(9): 843-857.

[9]     Abd-Alla A.N., Alshaikh F.A., 2009, The effect of the initial stresses on the reflection and transmission of plane quasi-vertical transverse waves in piezoelectric materials, *Proceedings of World Academy of Science, Engineering and Technology* **38**: 660-668.

[10]    Abd-Alla A.N., Alshaikh F.A., Al-Hossain A.Y., 2012, The reflection phenomena of quasi-vertical transverse waves in piezoelectric medium under initial stresses, *Meccanica* **47**(3): 731-744.

[11]    Iesan D., 1973, The plane micropolar strain of orthotropic elastic solids, *Archiwum Mechaniki Stosowanej* **25**: 547-561.

[12]    Iesan D., 1974, Torsion of anisotropic micropolar elastic cylinders, *Zeitschrift für Angewandte Mathematik und Mechanik* **54**: 773-779.

[13]    Iesan D., 1974, Bending of orthotropic micropolar elastic beams by terminal couples, *Analele Stiintifice Ale Universitatii Iasi* **20**: 411-418.

[14]    Chandrasekharaiah D.S., 1984, A temperature-rate-dependent theory of thermopiezoelectricity, *Journal of  Thermal Stresses* **7**: 293-306.

[15]    Chandrasekharaiah D.S.,1988, A generalized linear thermoelasticity theory for piezoelectric media, *Acta Mechanica* **71**: 39-49.

[16]    Alshaikh F.A., 2012, The mathematical modelling for studying the influence of the initial stresses and relaxation times on reflection and refraction waves in piezothermoelastic half-space, *Applied Mathematics* **3**(8): 819-832.

[17]    Alshaikh F.A., 2012, Reflection of quasi vertical transverse waves in the thermo-piezoelectric material under initial stress (Green- Lindsay Model), *International Journal of Pure and Applied Sciences and Technology* **13**: 27-39.

[18]    Sharma J.N., Walia V., Gupta S.K., 2008, Reflection of piezo-thermoelastic waves from the charge and stress free boundary of a transversely isotropic half space, *International Journal of Engineering Science* **46**(2):131-146.

[19]    Othman M.I.A., 2015,The effect of rotation on piezothermoelastic medium using different theories, *Structural Engineering and Mechanics* **56**(4): 649-665.

[20]    Othman M.I.A., Atwa S.Y., Hasona W.M., Ahmed E.A.A., 2015, Propagation of plane waves in generalized piezo-thermoelastic medium: Comparison of different theories, *International Journal of Innovative Research in Science, Engineering and Technology* **4**(4): 2292-2300.

[21]    Hou P.F., Luo W., Leung Y.T., 2008, A point heat source on the surface of a semi-infinite transverse isotropic piezothermoelastic material, *SME Journal of  Applied Mechanics* **75**:1-8.

[22]    Mindlin R.D., 1961, *On the Equations of Motion of Piezoelectric Crystals, Problems of Continuum Mechanics*, SIAM, Philadelphia.

[23]    Kumar R., Choudhary S., 2002, Mechanical sources in orthotropic micropolar continua, *Proceedings of the Indian Academy of Sciences (Earth and Planetary Sciences)* **111**: 133-141.

[24]    Kumar R., Choudhary S., 2002, Influence of Green's function for orthotropic micropolar continua, *Archive of Mechanics* **54**: 185-198.

[25]    Kumar R., Choudhary S., 2002, Dynamical behavior of orthotropic micropolar elastic medium, *Journal of Vibration and Control* **8**: 1053-1069.

[26]    Kumar R., Choudhary S., 2003, Response of orthotropic microploar elastic medium due to various sources, *Meccanica* **38**: 349-368.

[27]    Kumar R., Choudhary S., 2004, Response of orthotropic micropolar elastic medium due to time harmonic sources*, Sadhana* **29**: 83-92.

[28]    Nakamura S., Benedict R., Lakes R., 1984, Finite element method for orthotropic micropolar elasticity, International, *Journal of Engineering Science* **22**: 319-330.

[29]    Nowacki W., 1966, Couple stress in the theory of thermoelasticity, *Irreversible Aspects of Continuum Mechanics and Transfer of Physical Characteristics in Moving Fluids,* Springer,Verlag.

[30]    Nowacki W.,1978, Some general theorems of thermo-piezoelectricity, *Journal of Thermal Stresses* **1**: 171-182.

[31]    Nowacki W., 1979, *Foundations of Linear Piezoelectricity, Electromagnetic Interactions in Elastic Solids*, Springer, Wein.

[32]    Nowacki W., 1983, *Mathematical Models of Phenomenological Piezo-Electricity*, New Problems in Mechanics of Continua, University of Waterloo Press, Waterloo, Ontario.

[33]    Slaughter W.S., 2002, *The Linearized Theory of Elasticity*,  Birkhauser, Basel.

[34]    Chen W.Q., 2000, On the general solution for piezothermoelastic for transverse isotropy with application, *ASME, Journal of Applied Mechanics* **67**: 705-711.

[35]    Guo X., Wei P., 2014, Effects of initial stress on the reflection and transmission waves at the interface between two piezoelectric half spaces, *International Journal of Solids and Structures* **51**(21): 3735-3751.

[36]    Pang Y., Wang Y. S., Liu J.X., Fang D. N., 2008, Reflection and refraction of plane waves at the interface between piezoelectric and piezomagnetic media, *International Journal of Engineering Science* **46**: 1098-1110.

[37]    Kuang Z.B., Yuan X.G.,2011, Reflection and transmission of waves in pyroelectric and piezoelectric materials , *Journal of Sound and Vibration* **330**(6):1111-1120.