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3D Thermoelastic Interactions in an Anisotropic Lastic Slab Due to Prescribed Surface Temparature

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ABSTRACT

The present paper is devoted to the determination of displacement, stresses and temperature from three dimensional anisotropic half spaces due to presence of heat source. The normal mode analysis technique has been used to the basic equations of motion and generalized heat conduction equation proposed by Green-Naghdi model-II [1]. The resulting equation are written in the form of a vector –matrix differential equation and exact expression for displacement component, stresses, strains and temperature are obtained by using eigen value approach. Finally, temperature, stresses and strain are presented graphically and analyzed.

Keywords: Eigenvalue; Generalized thermoelasticity; Normal mode analysis and vector-matrix; Differential equation.

1 INTRODUCTION

THE drawback of the theory of classical thermoelasticity from the so-called "contradiction of heat conduction equation" i.e, heat equation is mixed parabolic-hyperbolic type, predicting infinite speeds of propagation for heat waves contrary to experimental results. The heat conduction equation of the theories of generalized thermo-elasticity is hyperbolic type and free from this contradiction.The theory of generalized thermo-elasticity was introduced by Lord and Shulman[6] with one relaxation time parameter by modification of Fourier Law. Green and Lindsay [2] modified both the heat conduction equation and equation of motion without violations of Fourier Law by introducing two relaxation time parameters.

Along with these theories, Green and Naghdi [3-4-5] (G-N model) also proposed another generalized theory of thermoelasticity by introducing “thermal displacement gradient” among the independent constitutive variables and named as type I, II, and III. Among these models, type I is same as classical heat equation which is based on Fourier’s law where the theories are linearized. The type II and type III models permit finite speed of wave propagation. The basic difference of type II from type I and type III is that it does not contain dissipation of thermal energy where as type III contains dissipation of energy. The type II and Type III are also known as thermo-elasticity without energy dissipation (TEWOED) and thermo-elasticity with energy dissipation (TEWED). Several investigations relating to TEWOED theory have been studied by RoyChoudhury and Bandyopadhyay [7], RoyChoudhury and Dutta [8], Sharma and Chouhan [12], Chandrasekharaiah and Srinath [1], Sarkar and Lahiri[10-11] .

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The aim of the present research article to study the distribution of stresses, strains and temperature for a anisotropic half space subjected to (i) time dependent heat source and ii) traction free on the boundary of the space. Normal mode analysis technique and eigenvalue approach have been used to solve the problem. Finally numerical results are presented graphically and analyzed.

2 BASIC EQUATIONS

In the absence of body force, the field equations for linear homogeneous anisotropic thermoelastic body in the context of Green-Nagdhi model (G-N model) [3], are as follows:

The equations of motion:

$$\tau_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}$$

Heat- conduction equation of Green- Nagdhi type II:

$$k_{ij}^{*}\nabla^2\theta = \rho c_E \frac{\partial^2\theta}{\partial t^2} + \theta_0\beta_{ij}\frac{\partial}{\partial t^2}(\frac{\partial u_i}{\partial x_j}) + Q$$

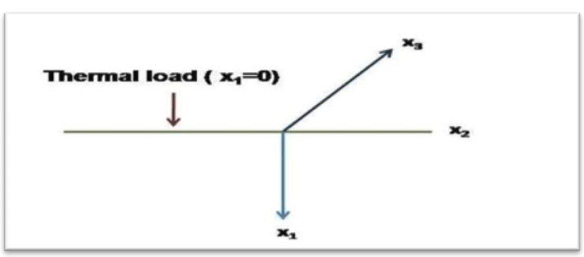
The Duhamel- Neumann constitutive equation are:

$$\tau_{ij} = c_{ijkl}e_{kl} - \beta_{ij}\theta\delta_{ij}$$

Strain – tensor :

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), u_{i,j} = \frac{\partial u_i}{\partial x_j}$$

3 FORMULATION OF THE PROBLEM



Rigidity fixed surface with thermal load.

Consider triclinic thermoelastic half-space occupied in the region Ω defined by $\Omega = \{(x_1, x_2, x_3) : 0 \le x_1 \prec \infty$; $-\infty \le x_2 \prec \infty$; $0 \prec x_3 \prec \infty\}$ subjected to time dependent heat source on the boundary plane to the surface x_1=0. The body is initially at rest and the surface x_1=0 is assumed to be traction free.

For three dimensional plane wave in a homogeneous anisotropic elastic medium, the components of displacement vector are as follows:

$$u_i = u_i(x_1, x_2, x_3, t) \tag{1}$$

$$\begin{aligned}
\tau_{11} &= c_{11}e_{11}+c_{12}e_{22}+c_{13}e_{33}+2(c_{14}e_{23}+c_{15}e_{13}+c_{16}e_{12})-\beta_{11}\theta \\
\tau_{22} &= c_{21}e_{11}+c_{22}e_{22}+c_{23}e_{33}+2(c_{24}e_{23}+c_{25}e_{13}+c_{26}e_{12})-\beta_{22}\theta \\
\tau_{33} &= c_{31}e_{11}+c_{32}e_{22}+c_{33}e_{33}+2(c_{34}e_{23}+c_{35}e_{13}+c_{36}e_{12})-\beta_{33}\theta \\
\tau_{23} &= c_{41}e_{11}+c_{42}e_{22}+c_{43}e_{33}+2(c_{44}e_{23}+c_{45}e_{13}+c_{46}e_{12}) \\
\tau_{13} &= c_{51}e_{11}+c_{52}e_{22}+c_{53}e_{33}+2(c_{54}e_{23}+c_{55}e_{13}+c_{56}e_{12}) \\
\tau_{12} &= c_{61}e_{11}+c_{62}e_{22}+c_{63}e_{33}+2(c_{64}e_{23}+c_{65}e_{13}+c_{66}e_{12})
\end{aligned} \tag{2}$$

where t represents time and $x_i\,(i=1,2,3)$ denotes the respective orthogonal cartesian co-ordinate axes. Using Hooke's law stress-strain-temperature relations in a triclinic medium the equations of motion in absence of body force are given as:

$$\begin{aligned}
\frac{\partial\tau_{11}}{\partial x_1}+\frac{\partial\tau_{12}}{\partial x_2}+\frac{\partial\tau_{13}}{\partial x_3} &= \rho\frac{\partial^2 u_1}{\partial t^2} \\
\frac{\partial\tau_{21}}{\partial x_1}+\frac{\partial\tau_{22}}{\partial x_2}+\frac{\partial\tau_{23}}{\partial x_3} &= \rho\frac{\partial^2 u_2}{\partial t^2} \\
\frac{\partial\tau_{31}}{\partial x_1}+\frac{\partial\tau_{32}}{\partial x_2}+\frac{\partial\tau_{33}}{\partial x_3} &= \rho\frac{\partial^2 u_3}{\partial t^2}
\end{aligned} \tag{3}$$

Using Eqs. (1) and (2) in Eqs. (3) becomes

$$\begin{aligned}
&(c_{11}\frac{\partial^2 u_1}{\partial x_1^2}+c_{66}\frac{\partial^2 u_1}{\partial x_2^2}+c_{55}\frac{\partial^2 u_1}{\partial x_3^2})+2(c_{16}\frac{\partial^2 u_1}{\partial x_1\partial x_2}+c_{15}\frac{\partial^2 u_1}{\partial x_1\partial x_3}+c_{56}\frac{\partial^2 u_1}{\partial x_2\partial x_3})+(c_{16}\frac{\partial^2 u_2}{\partial x_1^2}+c_{26}\frac{\partial^2 u_2}{\partial x_2^2}+c_{45}\frac{\partial^2 u_2}{\partial x_3^2}) \\
&+(c_{12}+c_{66})\frac{\partial^2 u_2}{\partial x_1\partial x_2}+(c_{14}+c_{56})\frac{\partial^2 u_2}{\partial x_1\partial x_3}+(c_{46}+c_{25})\frac{\partial^2 u_2}{\partial x_2\partial x_3}+(c_{15}\frac{\partial^2 u_3}{\partial x_1^2}+c_{46}\frac{\partial^2 u_3}{\partial x_2^2}+c_{35}\frac{\partial^2 u_3}{\partial x_3^2})+ \\
&+(c_{13}+c_{55})\frac{\partial^2 u_3}{\partial x_1\partial x_3}+(c_{36}+c_{45})\frac{\partial^2 u_3}{\partial x_2\partial x_3}\}-\beta_{11}\frac{\partial\theta}{\partial x_1}=\rho\frac{\partial^2 u_1}{\partial t^2}
\end{aligned} \tag{4}$$

$$\begin{aligned}
&(c_{16}\frac{\partial^2 u_1}{\partial x_1^2}+c_{26}\frac{\partial^2 u_1}{\partial x_2^2}+c_{45}\frac{\partial^2 u_1}{\partial x_3^2})+(c_{12}+c_{66})\frac{\partial^2 u_1}{\partial x_1\partial x_2}+(c_{14}+c_{56})\frac{\partial^2 u_1}{\partial x_1\partial x_3}+(c_{46}+c_{25})\frac{\partial^2 u_1}{\partial x_2\partial x_3} \\
&+(c_{66}\frac{\partial^2 u_2}{\partial x_1^2}+c_{22}\frac{\partial^2 u_2}{\partial x_2^2}+c_{44}\frac{\partial^2 u_2}{\partial x_3^2})+2(c_{26}\frac{\partial^2 u_2}{\partial x_1\partial x_2}+c_{46}\frac{\partial^2 u_2}{\partial x_1\partial x_3}+c_{24}\frac{\partial^2 u_2}{\partial x_2\partial x_3})+c_{56}\frac{\partial^2 u_3}{\partial x_1^2}+c_{24}\frac{\partial^2 u_3}{\partial x_2^2}+c_{34}\frac{\partial^2 u_3}{\partial x_3^2} \\
&+(c_{46}+c_{25})\frac{\partial^2 u_3}{\partial x_1\partial x_2}+(c_{36}+c_{45})\frac{\partial^2 u_3}{\partial x_1\partial x_3}+(c_{23}+c_{44})\frac{\partial^2 u_3}{\partial x_2\partial x_3}-\beta_{22}\frac{\partial\theta}{\partial x_2}=\rho\frac{\partial^2 u_2}{\partial t^2}
\end{aligned} \tag{5}$$

$$\begin{aligned}
&(c_{15}\frac{\partial^2 u_1}{\partial x_1^2}+c_{46}\frac{\partial^2 u_1}{\partial x_2^2}+c_{35}\frac{\partial^2 u_1}{\partial x_3^2})+(c_{56}+c_{14})\frac{\partial^2 u_1}{\partial x_1\partial x_2}+(c_{55}+c_{13})\frac{\partial^2 u_1}{\partial x_1\partial x_3}+(c_{45}+c_{36})\frac{\partial^2 u_1}{\partial x_2\partial x_3} \\
&+(c_{56}\frac{\partial^2 u_2}{\partial x_1^2}+c_{24}\frac{\partial^2 u_2}{\partial x_2^2}+c_{34}\frac{\partial^2 u_2}{\partial x_3^2})+(c_{25}+c_{46})\frac{\partial^2 u_2}{\partial x_1\partial x_2}+(c_{45}+c_{36})\frac{\partial^2 u_2}{\partial x_1\partial x_3}+(c_{44}+c_{23})\frac{\partial^2 u_2}{\partial x_2\partial x_3}+ \\
&(c_{55}\frac{\partial^2 u_3}{\partial x_1^2}+c_{44}\frac{\partial^2 u_3}{\partial x_2^2}+c_{33}\frac{\partial^2 u_3}{\partial x_3^2}+2(c_{45}\frac{\partial^2 u_3}{\partial x_1\partial x_2}+c_{35}\frac{\partial^2 u_3}{\partial x_1\partial x_3}+c_{34}\frac{\partial^2 u_3}{\partial x_2\partial x_3})-\beta_{33}\frac{\partial\theta}{\partial x_3}=\rho\frac{\partial^2 u_3}{\partial t^2}
\end{aligned} \tag{6}$$

The generalized heat conduction equation in G-N –II model is given by

$$k_{11}^{*}\frac{\partial^2\theta}{\partial x_1^2}+k_{22}^{*}\frac{\partial^2\theta}{\partial x_2^2}+k_{33}^{*}\frac{\partial^2\theta}{\partial x_3^2}=\rho c_E\frac{\partial^2\theta}{\partial t^2}+\theta_0\frac{\partial^2}{\partial t^2}(\beta_{11}\frac{\partial u_1}{\partial x_1}+\beta_{22}\frac{\partial u_2}{\partial x_2}+\beta_{33}\frac{\partial u_3}{\partial x_3})+Q \tag{7}$$

To transform the above equations in non-dimensional form,we introduce the following non-dimensional variables:

$$\begin{aligned}&(x_1^{'},x_2^{'},x_3^{'})=\frac{1}{l}(x_1,x_2,x_3),t^{'}=\frac{c_1 t}{l},(u_1^{'},u_2^{'},u_3^{'})=\frac{c_{11}}{l\beta_{11}\theta_0}(u_1,u_2,u_3),\\&c_1^2=\frac{c_{11}}{\rho},\theta^{'}=\frac{\theta}{\theta_0},\tau_{ij}^{'}=\frac{\tau_{ij}}{\beta_{11}\theta_0},Q^{'}=\frac{l^2}{\theta_0 c_{11}C_E}Q\end{aligned} \tag{8}$$

where l=some standard length and $c_1^2=\frac{c_{11}}{\rho}$, c_1 represent the dilation wave velocity. Eliminating primes we obtain the non-dimensional equations of motion and heat conduction equation as follows:

$$\begin{aligned}&(\frac{\partial^2 u_1}{\partial x_1^2}+\frac{c_{66}}{c_{11}}\frac{\partial^2 u_1}{\partial x_2^2}+\frac{c_{55}}{c_{11}}\frac{\partial^2 u_1}{\partial x_3^2})+2(\frac{c_{16}}{c_{11}}\frac{\partial^2 u_1}{\partial x_1\partial x_2}+\frac{c_{15}}{c_{11}}\frac{\partial^2 u_1}{\partial x_1\partial x_3}+\frac{c_{56}}{c_{11}}\frac{\partial^2 u_1}{\partial x_2\partial x_3})+(\frac{c_{16}}{c_{11}}\frac{\partial^2 u_2}{\partial x_1^2}+\frac{c_{26}}{c_{11}}\frac{\partial^2 u_2}{\partial x_2^2}\\&+\frac{c_{45}}{c_{11}}\frac{\partial^2 u_2}{\partial x_3^2})+(\frac{(c_{12}+c_{66})}{c_{11}}\frac{\partial^2 u_2}{\partial x_1\partial x_2}+\frac{(c_{14}+c_{56})}{c_{11}}\frac{\partial^2 u_2}{\partial x_1\partial x_3}+\frac{(c_{46}+c_{25})}{c_{11}}\frac{\partial^2 u_2}{\partial x_2\partial x_3})+(\frac{c_{15}}{c_{11}}\frac{\partial^2 u_3}{\partial x_1^2}\\&+\frac{c_{46}}{c_{11}}\frac{\partial^2 u_3}{\partial x_2^2}+\frac{c_{35}}{c_{11}}\frac{\partial^2 u_3}{\partial x_3^2}+\frac{(c_{14}+c_{56})}{c_{11}}\frac{\partial^2 u_3}{\partial x_1\partial x_2}+\frac{(c_{13}+c_{55})}{c_{11}}\frac{\partial^2 u_3}{\partial x_1\partial x_3}+\frac{(c_{36}+c_{45})}{c_{11}}\frac{\partial^2 u_3}{\partial x_2\partial x_3})-\frac{\partial\theta}{\partial x_1}=\frac{\partial^2 u_1}{\partial t^2}\end{aligned} \tag{9}$$

$$\begin{aligned}&(\frac{c_{16}}{c_{11}}\frac{\partial^2 u_1}{\partial x_1^2}+\frac{c_{26}}{c_{11}}\frac{\partial^2 u_1}{\partial x_2^2}+\frac{c_{45}}{c_{11}}\frac{\partial^2 u_1}{\partial x_3^2}+\frac{(c_{12}+c_{66})}{c_{11}}\frac{\partial^2 u_1}{\partial x_1\partial x_2}+\frac{(c_{14}+c_{56})}{c_{11}}\frac{\partial^2 u_1}{\partial x_1\partial x_3}+\frac{(c_{46}+c_{25})}{c_{11}}\frac{\partial^2 u_1}{\partial x_2\partial x_3})\\&+(\frac{c_{66}}{c_{11}}\frac{\partial^2 u_2}{\partial x_1^2}+\frac{c_{22}}{c_{11}}\frac{\partial^2 u_2}{\partial x_2^2}+\frac{c_{44}}{c_{11}}\frac{\partial^2 u_2}{\partial x_3^2})+2(\frac{c_{26}}{c_{11}}\frac{\partial^2 u_2}{\partial x_1\partial x_2}+\frac{c_{46}}{c_{11}}\frac{\partial^2 u_2}{\partial x_1\partial x_3}+\frac{c_{24}}{c_{11}}\frac{\partial^2 u_2}{\partial x_2\partial x_3})\\&+(\frac{c_{56}}{c_{11}}\frac{\partial^2 u_3}{\partial x_1^2}+\frac{c_{24}}{c_{11}}\frac{\partial^2 u_3}{\partial x_2^2}+\frac{c_{34}}{c_{11}}\frac{\partial^2 u_3}{\partial x_3^2}+\frac{(c_{46}+c_{25})}{c_{11}}\frac{\partial^2 u_3}{\partial x_1\partial x_2}+\frac{(c_{36}+c_{45})}{c_{11}}\frac{\partial^2 u_3}{\partial x_1\partial x_3}+\frac{(c_{23}+c_{44})}{c_{11}}\frac{\partial^2 u_3}{\partial x_2\partial x_3})\\&-\beta_2\frac{\partial\theta}{\partial x_1}=\frac{\partial^2 u_2}{\partial t^2}\end{aligned} \tag{10}$$

$$\begin{aligned}&(\frac{c_{15}}{c_{11}}\frac{\partial^2 u_1}{\partial x_1^2}+\frac{c_{46}}{c_{11}}\frac{\partial^2 u_1}{\partial x_2^2}+\frac{c_{35}}{c_{11}}\frac{\partial^2 u_1}{\partial x_3^2}+\frac{(c_{56}+c_{14})}{c_{11}}\frac{\partial^2 u_1}{\partial x_1\partial x_2}+\frac{(c_{55}+c_{13})}{c_{11}}\frac{\partial^2 u_1}{\partial x_1\partial x_3}+\frac{(c_{45}+c_{36})}{c_{11}}\frac{\partial^2 u_1}{\partial x_2\partial x_3})\\&+(\frac{c_{56}}{c_{11}}\frac{\partial^2 u_2}{\partial x_1^2}+\frac{c_{24}}{c_{11}}\frac{\partial^2 u_2}{\partial x_2^2}+\frac{c_{34}}{c_{11}}\frac{\partial^2 u_2}{\partial x_3^2}+\frac{(c_{25}+c_{46})}{c_{11}}\frac{\partial^2 u_2}{\partial x_1\partial x_2}+\frac{(c_{45}+c_{36})}{c_{11}}\frac{\partial^2 u_2}{\partial x_1\partial x_3}+\frac{(c_{44}+c_{23})}{c_{11}}\frac{\partial^2 u_2}{\partial x_2\partial x_3})\\&+(\frac{c_{55}}{c_{11}}\frac{\partial^2 u_3}{\partial x_1^2}+\frac{c_{44}}{c_{11}}\frac{\partial^2 u_3}{\partial x_2^2}+\frac{c_{33}}{c_{11}}\frac{\partial^2 u_3}{\partial x_3^2})+2(\frac{c_{45}}{c_{11}}\frac{\partial^2 u_3}{\partial x_1\partial x_2}+\frac{c_{35}}{c_{11}}\frac{\partial^2 u_3}{\partial x_1\partial x_3}+\frac{c_{34}}{c_{11}}\frac{\partial^2 u_3}{\partial x_2\partial x_3})-\beta_3\frac{\partial\theta}{\partial x_3}=\rho\frac{\partial^2 u_3}{\partial t^2}\end{aligned} \tag{11}$$

$$c_T^2(\frac{\partial^2\theta}{\partial x_1^2}+k_2\frac{\partial^2\theta}{\partial x_2^2}+k_3\frac{\partial^2\theta}{\partial x_3^2})=\frac{\partial^2\theta}{\partial t^2}+\varepsilon_1\frac{\partial^2}{\partial t^2}(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3})-Q \tag{12}$$

where, $\varepsilon_1=\frac{\beta_{11}^2\theta_0}{\rho C_{11}C_E}$, $c_T^2=\frac{k_{11}^{*}}{c_E c_{11}}=\frac{k^{*}_{11}}{\rho c_E c_1^2}$, $k_2=\frac{k_{22}^{*}}{k_{11}^{*}}$, $k_3=\frac{k_{33}^{*}}{k_{11}^{*}}$, $\beta_2=\frac{\beta_{22}}{\beta_{11}}$ *and* $\beta_3=\frac{\beta_{33}}{\beta_{11}}$

Non-dimensional stress components are

$$\tau_{11}=\frac{1}{c_{11}}[c_{11}e_{11}+c_{12}e_{22}+c_{13}e_{33}+2(c_{14}e_{23}+c_{15}e_{13}+c_{16}e_{12})]-\theta$$
$$\tau_{22}=\frac{1}{c_{11}}[c_{21}e_{11}+c_{22}e_{22}+c_{23}e_{33}+2(c_{24}e_{23}+c_{25}e_{13}+c_{26}e_{12})]-\beta_2\theta$$
$$\tau_{33}=\frac{1}{c_{11}}[c_{31}e_{11}+c_{32}e_{22}+c_{33}e_{33}+2(c_{34}e_{23}+c_{35}e_{13}+c_{36}e_{12})]-\beta_3\theta \quad (13)$$
$$\tau_{23}=\frac{1}{c_{11}}[c_{41}e_{11}+c_{42}e_{22}+c_{43}e_{33}+2(c_{44}e_{23}+c_{45}e_{13}+c_{46}e_{12})]$$
$$\tau_{13}=\frac{1}{c_{11}}[c_{51}e_{11}+c_{52}e_{22}+c_{53}e_{33}+2(c_{54}e_{23}+c_{55}e_{13}+c_{56}e_{12})]$$
$$\tau_{12}=\frac{1}{c_{11}}[c_{61}e_{11}+c_{62}e_{22}+c_{63}e_{33}+2(c_{64}e_{23}+c_{65}e_{13}+c_{66}e_{12})]$$

where, $\beta_i=\frac{\beta_{ii}}{\beta_{11}},\ (i=2,3)$

4 SOLUTION PROCEDURE

4.1 Formulation of vector-matrix differental equation

For the solution of the Eqs (9)-(13), the physical variables can be decomposed in terms of normal modes in the following form :

$$[u_1,u_2,u_3,e_{ij},\theta,\tau_{ij},Q](x_1,x_2,x_3,t)=[u_1^*,u_2^*,u_3^*,e_{ij}^*,\theta^*,\tau_{ij}^*,Q^*](x_1)e^{\omega t+i(ax_2+bx_3)} \quad (14)$$

where $i=\sqrt{-1},\omega$ is the angular frequency and *a, b* are the wave numbers along x_2 and x_3 direction respectively. Using Eq. (14), Eqs. (9)-(13) can be as (omitting '*' for convenience)

$$\frac{d^2u_1}{dx_1^2}+a_{11}\frac{du_1}{dx_1}+a_{12}u_1+a_{21}\frac{d^2u_2}{dx_1^2}+a_{22}\frac{du_2}{dx_1}+a_{23}u_2+a_{31}\frac{d^2u_3}{dx_1^2}+a_{32}\frac{du_3}{dx_1}+a_{33}u_3-\frac{d\theta}{dx_1}=0 \quad (15)$$

$$b_{11}\frac{d^2u_1}{dx_1^2}+b_{12}\frac{du_1}{dx_1}+b_{13}u_1+\frac{d^2u_2}{dx_1^2}+b_{21}\frac{du_2}{dx_1}+b_{22}u_2+b_{31}\frac{d^2u_3}{dx_1^2}+b_{32}\frac{du_3}{dx_1}+b_{33}u_3-b_{34}\theta=0 \quad (16)$$

$$m_{11}\frac{d^2u_1}{dx_1^2}+m_{12}\frac{du_1}{dx_1}+m_{13}u_1+m_{21}\frac{d^2u_2}{dx_1^2}+m_{22}\frac{du_2}{dx_1}+m_{23}u_2+\frac{d^2u_3}{dx_1^2}+m_{31}\frac{du_3}{dx_1}+m_{32}u_3-m_{33}\theta=0 \quad (17)$$

$$c_T^2\frac{d^2\theta}{dx_1^2}-\varepsilon_1\omega^2\frac{du_1}{dx_1}-ia\omega^2\varepsilon_2u_2-ib\omega^2\varepsilon_3u_3-[c_T^2(a^2+b^2)-\omega^2]\theta-Q=0 \quad (18)$$

where, $\varepsilon_1=\frac{\beta_{11}^2\theta_0}{C_{11}C_E\rho},\ \varepsilon_2=\frac{\beta_{11}\beta_{22}\theta_0}{C_{11}C_E\rho},\ \varepsilon_3=\frac{\beta_{11}\beta_{33}\theta_0}{C_{11}C_E\rho}$

Stress components are:

$$\tau_{11}=\frac{du_1}{dx_1}+h_{12}\frac{du_2}{dx_1}+h_{13}\frac{du_3}{dx_1}+h_{14}u_1+h_{15}u_2+h_{16}u_3-\beta_1\theta \tag{19}$$

$$\tau_{22}=h_{21}\frac{du_1}{dx_1}+h_{22}\frac{du_2}{dx_2}+h_{23}\frac{du_3}{dx_3}+h_{24}u_1+h_{25}u_2+h_{26}u_3-\beta_2\theta \tag{20}$$

$$\tau_{33}=h_{31}\frac{du_1}{dx_1}+h_{32}\frac{du_2}{dx_2}+h_{33}\frac{du_3}{dx_3}+h_{34}u_1+h_{35}u_2+h_{36}u_3-\beta_3\theta \tag{21}$$

$$\tau_{23}=h_{41}\frac{du_1}{dx_1}+h_{42}\frac{du_2}{dx_2}+h_{43}\frac{du_3}{dx_3}+h_{44}u_1+h_{45}u_2+h_{46}u_3 \tag{22}$$

$$\tau_{13}=h_{51}\frac{du_1}{dx_1}+h_{52}\frac{du_2}{dx_2}+h_{53}\frac{du_3}{dx_3}+h_{54}u_1+h_{55}u_2+h_{56}u_3 \tag{23}$$

$$\tau_{12}=h_{61}\frac{du_1}{dx_1}+h_{62}\frac{du_2}{dx_2}+h_{63}\frac{du_3}{dx_3}+h_{64}u_1+h_{65}u_2+h_{66}u_3 \tag{24}$$

where a_{ij} and b_{ij} , m_{ij} and h_{ij} $(i, j = 1,2,3)$ are given in the Appendix A.

Eqs. (15)-(18) can be written in the vector-matrix differential equation as [Sarkar and Lahiri, [11]],

$$\frac{d\underline{v}}{dx_1}=A\underline{v}+\underline{f} \tag{25}$$

where, $\underline{v}(x_1)=\begin{bmatrix}u_1 & u_2 & u_3 & \theta & u_1' & u_2' & u_3' & \theta'\end{bmatrix}^T$, $A=\begin{bmatrix}L_{11} & L_{12}\\ L_{21} & L_{22}\end{bmatrix}$, $\underline{f}=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q\end{bmatrix}^T$

where L_{11} and L_{12} are null matrix and identity matrix of order 4x4 respectively and L_{21}, L_{22} are given in the Appendix A.

4.2 Solution of the vector-matrix differential equation

For the solution of the vector-matrix differential Eq. (25), we apply the method of eigenvalue approach as in [1].The characteristic equation of matrix A is given by

$$|A-\lambda I|=0 \tag{26}$$

The roots of the characteristic Eq. (26) are $\lambda=\lambda_i\,(i=1(1)8)$. There exists four waves corresponding to descending order of their velocities namely a quasi P-wave(qP_1) and two quasi transverse (qS_1,qS_2) and a quasi-thermal wave(qP_2), which are obtained from the corresponding eigen values. The expression of phase velocity, attenuation coefficient, specific loss and penetration depth of these type of waves are given in Appendix C.

The eigenvector X corresponding to the eigenvalue λ can be calculated as:

$$X=\begin{bmatrix}\delta_1 & \delta_2 & \delta_3 & \delta_4 & \lambda\delta_1 & \lambda\delta_2 & \lambda\delta_3 & \lambda\delta_4\end{bmatrix}^T \tag{27}$$

where,

$$\delta_1 = (f_{24}f_{13} - f_{14}f_{23})(f_{22}f_{33} - f_{32}f_{23}) - (f_{34}f_{23} - f_{24}f_{33})(f_{12}f_{23} - f_{22}f_{13}),$$
$$\delta_2 = (f_{34}f_{23} - f_{24}f_{33})(f_{11}f_{23} - f_{21}f_{13}) - (f_{24}f_{13} - f_{14}f_{23})(f_{21}f_{33} - f_{31}f_{23}),$$
$$\delta_3 = (f_{12}f_{21} - f_{11}f_{22})(f_{21}f_{34} - f_{31}f_{24}) - (f_{22}f_{31} - f_{21}f_{32})(f_{11}f_{24} - f_{14}f_{21}),$$
$$\delta_4 = (f_{11}f_{23} - f_{21}f_{13})(f_{22}f_{33} - f_{32}f_{23}) - (f_{12}f_{23} - f_{22}f_{13})(f_{21}f_{33} - f_{31}f_{23}).$$

where $f_{ij}\ (i, j = 1,2,3)$ are given in the Appendix A.

From Eq. (27), we can calculate the eigenvalue $X_i\ (i = 1(1)8)$ corresponding to the eigenvalue $\lambda = \lambda_i\ (i = 1(1)8)$. For our further reference, we use the following notations

$$X_i = \begin{cases} [X]_{\lambda=\lambda_{\left(\frac{i+1}{2}\right)}} & for \quad i = 1(2)7 \\ [X]_{\lambda=-\lambda_{\left(\frac{i}{2}\right)}} & for \quad i = 2(2)8 \end{cases} \tag{28}$$

The general solution of Eq. (25) can be written as (Appendix D):

$$v(x_1) = \sum_{i=1}^{8} X_i \left(A_i e^{\lambda_i x_1} + e^{\lambda_i x_1} \int Q_i e^{-\lambda_i x_1} dx_1\right) \tag{29}$$

where A_i are arbitrary constants. Using Eq. (29) the displacement components are obtained as follows:

$$u_j = \sum_{i=1}^{8} x_{ji} \left(A_i e^{\lambda_i x_1} - \frac{Q_i}{\lambda_i}\right), \quad \text{for} \quad j = 1, 2, 3 \tag{30}$$

and stress components and temperature are as follows :

$$\begin{aligned} \tau_{11} = &\sum_{i=1}^{8}(h_{14} + \lambda_i) A_i x_{1i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{15} + h_{12}\lambda_i) A_i x_{2i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{16} + h_{13}\lambda_i) A_i x_{3i} e^{\lambda_i x_1} \\ &- \sum_{i=1}^{8} x_{4i}\left(A_i e^{\lambda_i x_1} - \frac{Q_i}{\lambda_i}\right) + \sum_{i=1}^{8} h_{14} x_{1i} \frac{Q_i}{\lambda_i} - \sum_{i=1}^{8} h_{15} x_{2i} \frac{Q_i}{\lambda_i} - \sum_{i=1}^{8} h_{16} x_{3i} \frac{Q_i}{\lambda_i} \end{aligned} \tag{31}$$

$$\begin{aligned} \tau_{22} = &\sum_{i=1}^{8}(h_{21}\lambda_i + h_{24}) A_i x_{1i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{22}\lambda_i + h_{25}) A_i x_{2i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{23}\lambda_i + h_{26}) A_i x_{3i} e^{\lambda_i x_1} \\ &- \beta_2 \sum_{i=1}^{8} x_{4i}\left(A_i e^{\lambda_i x_1} - \frac{Q_i}{\lambda_i}\right) - h_{24}\sum_{i=1}^{8} x_{1i} \frac{Q_i}{\lambda_i} - h_{25}\sum_{i=1}^{8} x_{2i} \frac{Q_i}{\lambda_i} - h_{26}\sum_{i=1}^{8} x_{3i} \frac{Q_i}{\lambda_i} \end{aligned} \tag{32}$$

$$\begin{aligned} \tau_{33} = &\sum_{i=1}^{8}(h_{34} + \lambda_i h_{31}) A_i x_{1i} e^{\lambda_i x} + \sum_{i=1}^{8}(h_{35} + \lambda_i h_{32}) A_i x_{2i} e^{\lambda_i x} + \sum_{i=1}^{8}(h_{36} + h_{33}\lambda_i) A_i x_{3i} e^{\lambda_i x} \\ &- \beta_3 \sum_{i=1}^{8} x_{4i} A_i e^{\lambda_i x_1} + h_{34}\sum_{i=1}^{8} x_{1i} \frac{Q_i}{\lambda_i} - h_{35}\sum_{i=1}^{8} x_{2i} \frac{Q_i}{\lambda_i} - h_{36}\sum_{i=1}^{8} x_{3i} \frac{Q_i}{\lambda_i} + \beta_3 \sum_{i=1}^{8} x_{4i} \frac{Q_i}{\lambda_i} \end{aligned} \tag{33}$$

$$\tau_{23} = \sum_{i=1}^{8}(h_{44} + \lambda_i h_{41})A_i x_{1i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{45} + \lambda_i h_{42})A_i x_{2i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{46} + \lambda_i h_{43})A_i x_{3i} e^{\lambda_i x_1} \quad (34)$$

$$\tau_{13} = \sum_{i=1}^{8}(h_{54} + \lambda_i h_{51})A_i x_{1i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{55} + \lambda_i h_{52})A_i x_{2i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{56} + \lambda_i h_{53})A_i x_{3i} e^{\lambda_i x_1} \quad (35)$$

$$\tau_{12} = \sum_{i=1}^{8}(h_{64} + \lambda_i h_{61})A_i x_{1i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{65} + \lambda_i h_{62})A_i x_{2i} e^{\lambda_i x_1} + \sum_{i=1}^{8}(h_{66} + \lambda_i h_{63})A_i x_{3i} e^{\lambda_i x_1} \quad (36)$$

$$\theta = \sum_{i=1}^{8} A_i x_{4i} e^{\lambda_i x_1} - \sum_{i=1}^{8} \frac{Q_i}{\lambda_i} \quad (37)$$

The simplified form of Eqs. (31)-(36) can be written as:

$$\tau_{11} = \sum_{i=1}^{8} A_i R_{1i}(x_1) - \sum_{i=1}^{8} Q_i N_{1i}, \tau_{22} = \sum_{i=1}^{8} A_i R_{2i}(x_1) - \sum_{i=1}^{8} Q_i N_{2i}, \tau_{33} = \sum_{i=1}^{8} A_i R_{3i}(x_1) - \sum_{i=1}^{8} Q_i N_{3i},$$
$$\tau_{23} = \sum_{i=1}^{8} A_i R_{4i}(x_1), \tau_{13} = \sum_{i=1}^{8} A_i R_{5i}(x_1), \tau_{12} = \sum_{i=1}^{8} A_i R_{6i}(x_1) \quad (38)$$

where $R_{ij} = R_{ij}(x)(i = 1(1)6, j = 1(1)4), N_{ji}, j = 1,2,3$ and $i = 1,2,...,8$ are given in the Appendix B and Q_i are the ith component of f for $\lambda = \lambda_i$.

5 BOUNDARY CONDITIONS

Due to the regularity condition of the solution at infinity,there are four terms containing exponentials of growing in nature in the space variables x_1 has been discarded and the remaining arbitrary constants $A_i, (i = 1,2,...,4)$, are to be determined from the following boundary conditions.

5.1 Mechanical boundary condition

The boundary of the half-space $x_1 = 0$ has no traction everywhere i.e.,

$$\tau_{11}(0,x_2,x_3,t) = \tau_{22}(0,x_2,x_3,t) = \tau_{33}(0,x_2,x_3,t) = 0 \quad (39)$$

5.2 The thermal boundary condition

$$\theta(0,x_2,x_3,t) = r(x_2,x_3,t)$$

with the help of Eq. (14), Eqs. (38) we get,

$$\tau_{11}(0,x_2,x_3,t) = \tau_{22}(0,x_2,x_3,t) = \tau_{33}(0,x_2,x_3,t) = 0 \quad (40)$$

and

$$\theta(0,x_2,x_3,t) = r^* \quad (41)$$

Using the boundary conditions (40) and (41) in Eqs. (31) to (33) and (37), we get following simultaneous equations

$$\sum_{i=1}^{8} A_i R_{1i}(0)=0; \sum_{i=1}^{8} A_i R_{2i}(0)=0; \sum_{i=1}^{8} A_i R_{3i}(0)=0; \sum_{i=1}^{8} A_i R_{7i}(0)=r^* \tag{42}$$

Solving the above four Eqs. (42),we get arbitrary constants $A_i\ (i=1,2,3,4)$ are as follows:

$A_i=\dfrac{D_i}{\Delta}; i=1(1)4$ where D_i and Δ are given in Appendix B.

6 NUMERICAL SOLUTION

With a view of illustrating the problem, we now consider a numerical example for which computational results are presented. To study the effect of wave propagation, we use the following physical parameters in SI units given in Chattopadhyay and Rogerson [9].

$$c_{11}=16.248GPa\ ; c_{22}=11.88GPa\ ; c_{33}=12.216GPa\ ; c_{12}=1.48GPa\ ; c_{13}=2.4GPa\ ;$$
$$c_{14}=-1.152GPa\ ; c_{15}=0\ ; c_{16}=-0.561GPa\ ; c_{23}=1.032GPa\ ; c_{24}=0.912GPa\ ;$$
$$c_{25}=1.608GPa\ ; c_{26}=1.248GPa\ ; c_{34}=-0.672GPa\ ; c_{35}=0.216GPa\ ;$$
$$c_{36}=-0.216GPa\ ; c_{44}=5.64GPa\ ; c_{45}=2.16GPa\ ; c_{46}=0; c_{55}=5.88GPa\ ;$$
$$c_{66}=6.91GPa\ ; c_{56}=0\ ; \rho=2.4\ ; \varepsilon_1=0.0221\ ; \varepsilon_2=0.0143\ ; \varepsilon_3=0.0174. r^*=20$$

7 CONCLUSIONS

In order to study the characteristic of stresses, strains and temperature, we have drawn several graphs for different values of the space variable (x_1), time *t*, and heat source Q . we conclude the following observations:

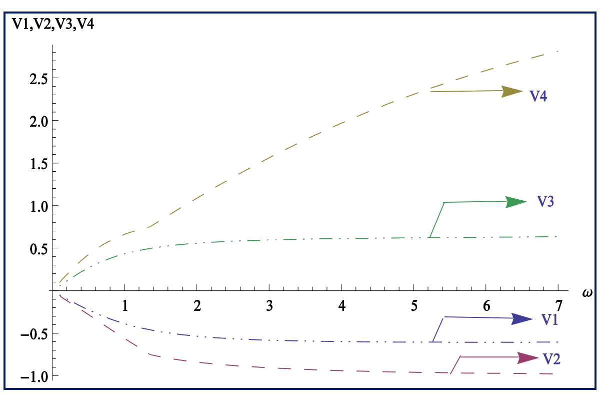


Fig.1
Distribution of V1, V2, V3, V4 for different values of ω .

In Fig. 1 we have shown the variation of different phase velocity (V1, V2, V3, and V4) with frequency (ω).

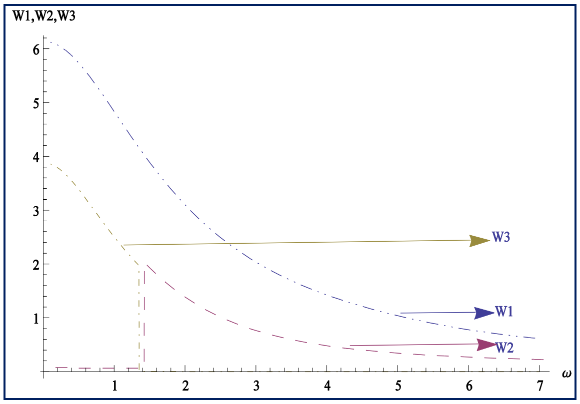


Fig.2
Distribution of W1, W2, W3 for different values of ω .

In Fig. 2 we have shown the variation of different specific loss (W1, W2 and W3) with frequency (ω). W1 steadily decreases as ω increases. There is no specific loss for W2 when $0 \le \omega \le 1.3$ and W_2 steadily decreases for $1 1.3 \le \omega \le 7$. W3 steadily dreases and there is sudden loss at $\omega = 1.2$ .There is point of discotiniuty at $\omega = 1.2$ for W2 and W2 but opposite in nature.

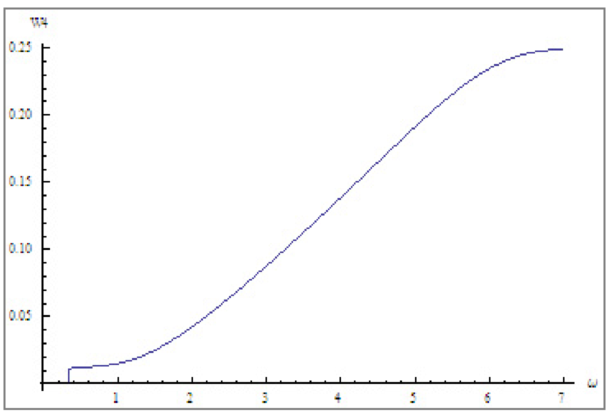


Fig.3
Distribution of W4 for different values of ω .

In Fig. 3 we have shown the variation of specific loss (W4) with frequency (ω), where value of specific loss gradually increases as the value of frequency increases.

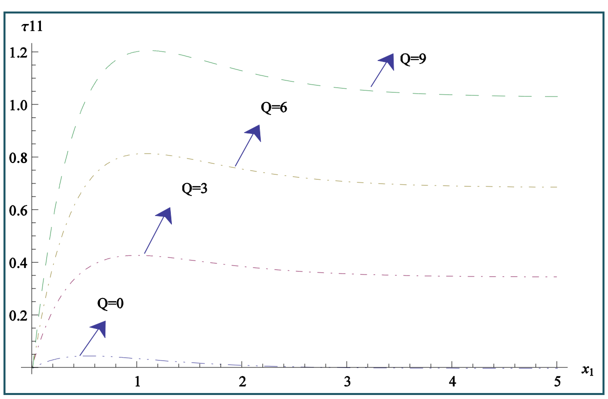


Fig.4
Distribution of τ_{11} versus x_1 for different values of Q.

In Fig.4 we have shown the variation of stress component (τ_{11}) with space variable x_1 for the different values of heat source. For different values of Q , τ_{11} initially increases then becomes steady as x_1 increases. Numerical value of τ_{11} increases as heat source Q increases.

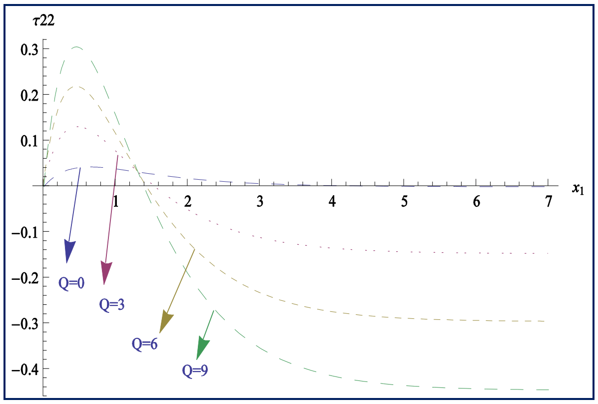


Fig.5
Distribution of τ_{22} versus x_1 for different values of Q.

In Fig. 5 τ_{22} is extensive for $0 \le x_1 \le 1.4$. The numerical values of τ_{22} increases as Q increases. All the values of τ_{22} remain unchanged after $x_1 = 5$.

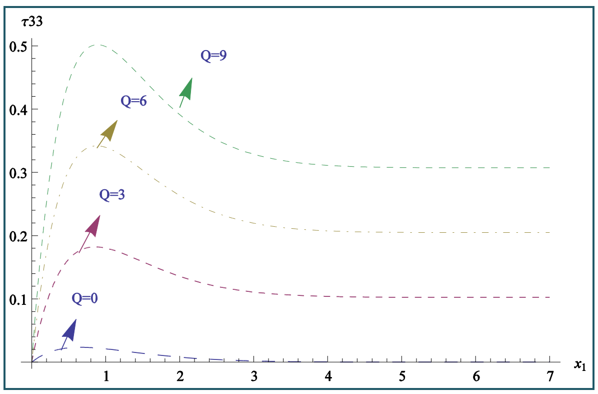


Fig.6
Distribution of τ_{33} versus x_1 for different values of Q.

In Fig. 6 we have shown the variation of stress component (τ_{33}) with space variable x_1 for the different values of heat source. For different values of Q, τ_{33} initially increase, then decreases and then remain unchanged as x_1 increases.

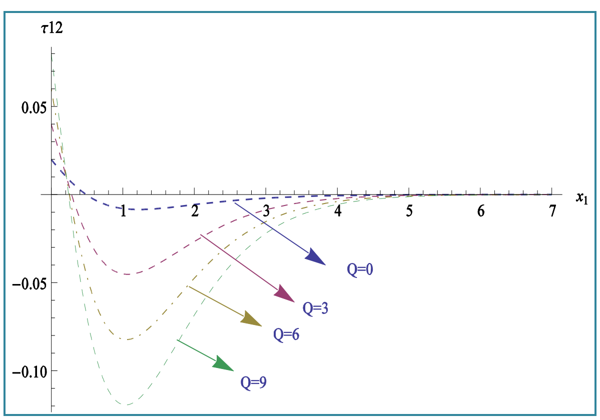


Fig.7
Distribution of τ_{12} versus x_1 for different values of Q.

Fig.7 represents variationf for different values of x_1. τ_{12} is extensive for $0 \le x_1 \le 0.2$, then compressive, finally it becomes zero as space variable x_1 increases. Numerical values of τ_{12} tend to zero at $x_1 = 5.6$ for all values of Q.

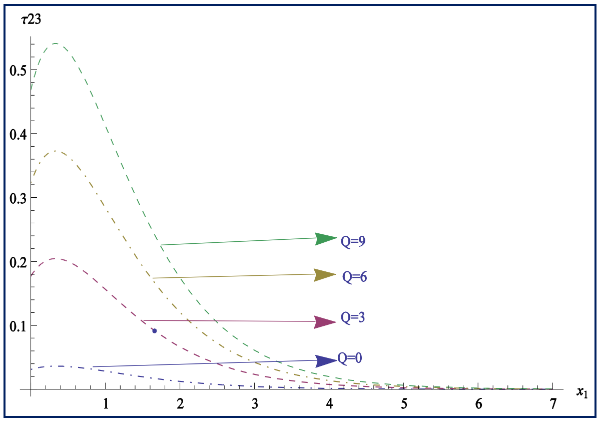


Fig.8
Distribution of τ_{23} versus x_1 for different values of Q.

In Fig. 8 we have shown the variation of stress component (τ_{23}) with space variable x_1 for the different values of heat source. For different values of Q, τ_{23} gradually decrease as x_1 increases and finally becomes zero. Effect of rotation is prominant for $0 \le x_1 \le 3$.

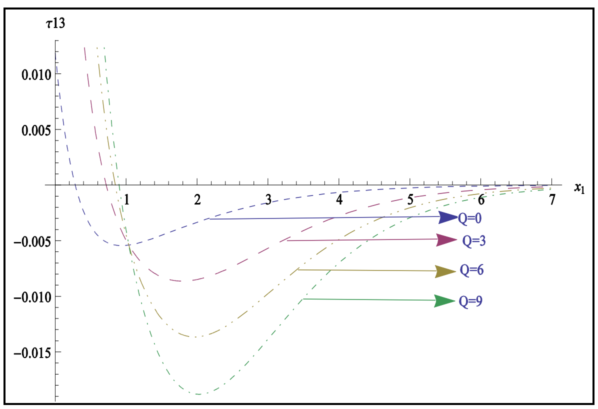


Fig.9
Distribution of τ_{13} versus x_1 for different values of Q.

In Fig. 9, the numerical value of τ_{13} is very very small than the other sherring stresses. Near the heated region, it is positive, then becomes negative and finally tends to zero as x_1 increases.

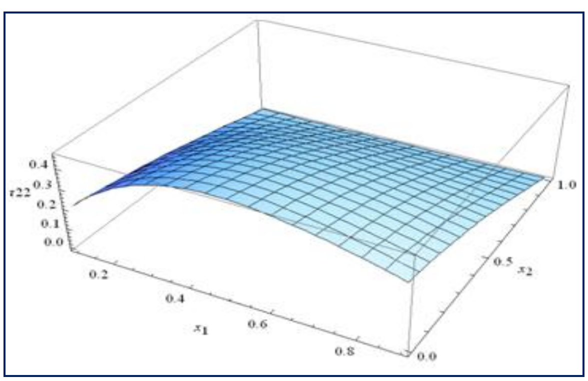


Fig.10
Distribution of τ_{22} versus x_1 and x_2 for fixed time and Q.

Fig. 10 represents the the variation of stress component τ_{22} for different values of x_1 and x_2 when $Q = 4$. For fixed x_1, τ_{22} is maximun when $x_1 = 0.3$.

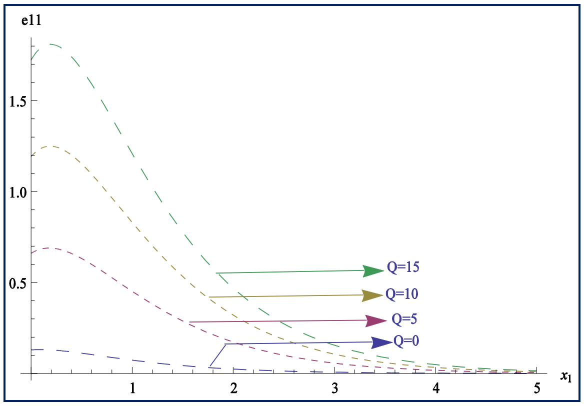


Fig.11
Distribution of strain component e_{11} verses x_1 for different values of Q.

Absolute value of the strain component e_{11} gradually decreases as x_1 increases and tend to zero as x_1 increases. For fixed x_1, e_{11} increases as Q increases.

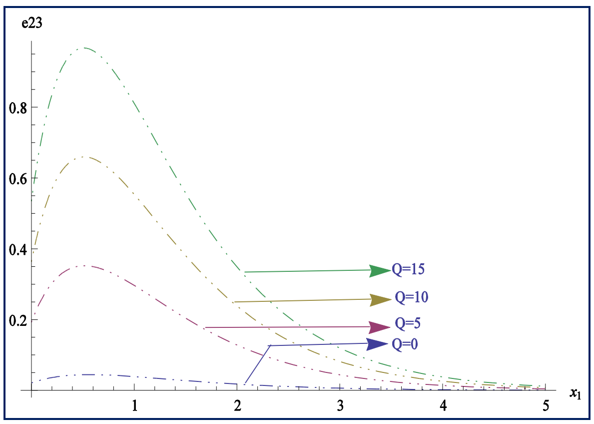


Fig.12
Distribution of strain component e_{23} verses x_1 for different values of Q.

In Fig. 12 we have shown the variation of strain component with space variable x_1 for the different values of heat source. Absolute values of e_{23} gradually increases as increase of heat source. e_{23} is maximum near the heated region . For all values Q , e_{23} finally tends to zero as x_1 increases.

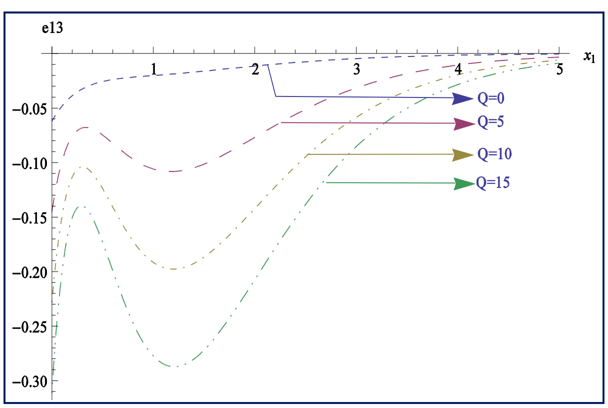


Fig.13
Distribution of strain component e_{13} verses x_1 for different values of Q.

In Fig. 13, we have shown the variation of strain component (e_{13}) with space variable x_1 for the different values of heat source (Q). For different values of Q, e_{13} initially increase, then decrease and then tend to zero as x_1 increases.

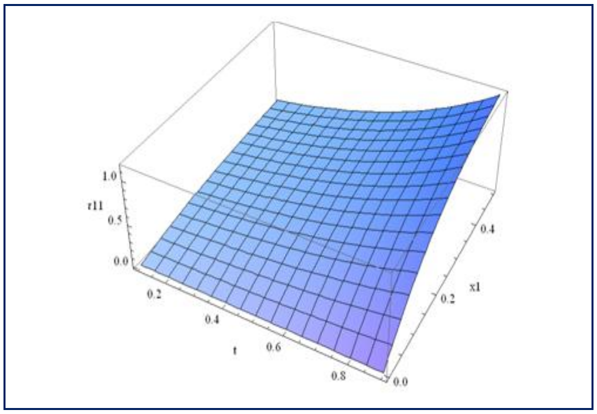


Fig.14
Distribution of stress component τ_{11} verses x_1 and *t*.

Fig. 14 shows the variation of stress component, τ_{11} for different values of space variable (x_1) and time (*t*). Value of τ_{11} gradually increases at $t = 1$ as x_1 increases.

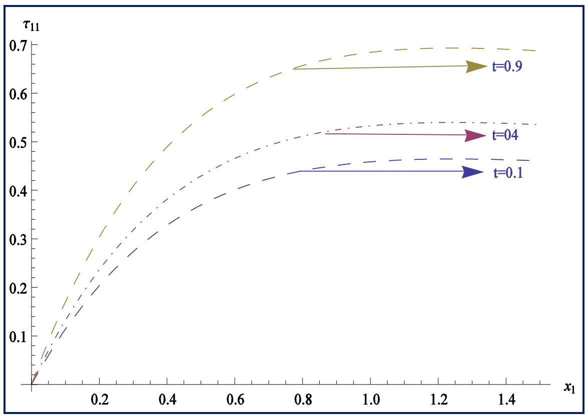


Fig.15
Distribution of τ_{11} versus x_1 for different values of *t*.

In Fig. 15 we have shown the variation of stress component (τ_{11}) with space variable x_1 for the different values of time(*t*). For different values of *Q*, τ_{11} initially increase and then remain unchanged as x_1 increases.

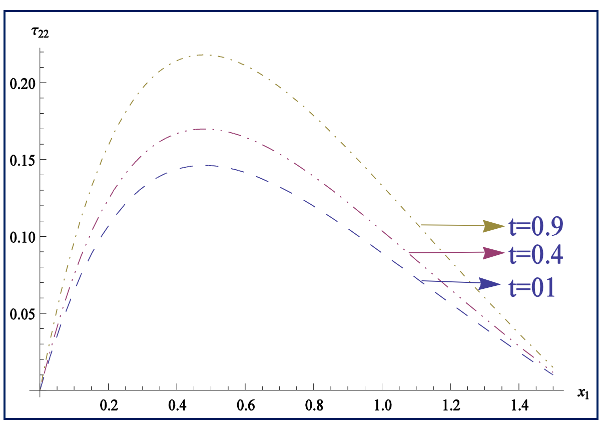


Fig.16
Distribution of τ_{22} versus x_1 for different values of *t*.

Here, we have shown the variation of stress component (τ_{22}) with space variable x_1 for the different values of time(*t*). For different values of *Q*, τ_{22} initially increases and attain heighest value for $0.4 \le x_1 \le 0.6$ and then tend to zero as x_1 increases.

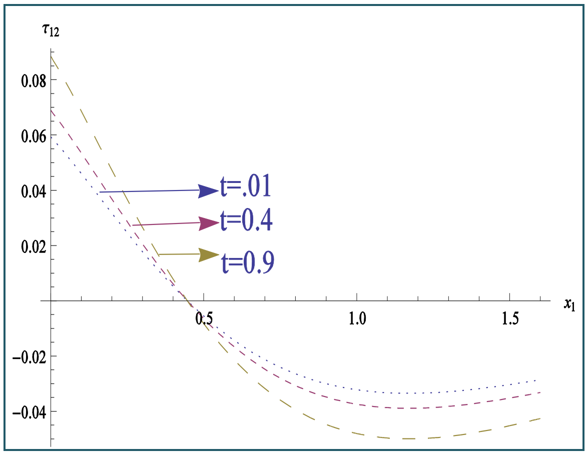


Fig.17
Distribution of τ_{12} versus x_1 for different values of *t*.

Fig. 17 represents variation of strees component τ_{12} verses x_1 for fixed heat source, $Q=4$ for different time. Absolute value of τ_{12} increases as time increases for fixed x_1. It is positive in the region $0 \le x_1 \le 0.45$ and negative in the region $0.45 \le x_1 \le 1.5$.

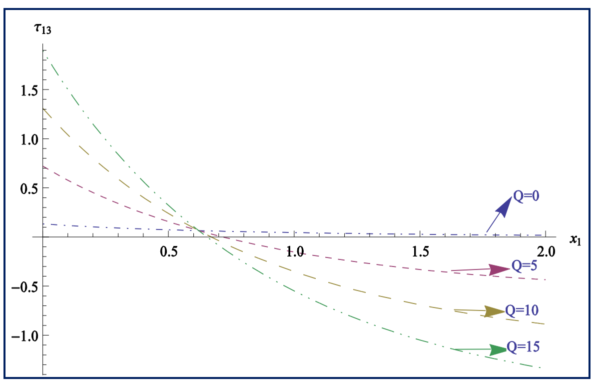


Fig.18
Distribution of τ_{13} versus x_1 for different values of heat source Q.

Absolute value of τ_{13} increases as Q increases .For all Q , τ_{13} is extensive when $0 \le x_1 \le 0.7$ and become compressive within the region $0.7 \le x_1 \le 2$.

APPENDIX A

$$a_{11}=2ia\frac{c_{16}}{c_{11}}+2ib\frac{c_{15}}{c_{11}},a_{12}=-a^2\frac{c_{66}}{c_{11}}-b^2\frac{c_{55}}{c_{11}}-2ab\frac{c_{55}}{c_{11}}-\omega^2,a_{21}=\frac{c_{16}}{c_{11}},a_{22}=ia\frac{c_{12}+c_{66}}{c_{11}}+ib\frac{c_{14}+c_{56}}{c_{11}},$$

$$a_{23}=-a^2\frac{c_{26}}{c_{11}}-b^2\frac{c_{45}}{c_{11}}-ab\frac{c_{46}+c_{25}}{c_{11}},a_{31}=\frac{c_{15}}{c_{11}},a_{32}=ia\frac{c_{14}+c_{56}}{c_{11}}+ib\frac{c_{13}+c_{55}}{c_{11}},a_{34}=-(1+T_1\omega)\frac{d\theta}{dx_1},$$

$$a_{33}=-a^2\frac{c_{46}}{c_{11}}-b^2\frac{c_{35}}{c_{11}}-ab\frac{c_{36}+c_{45}}{c_{11}},b_{11}=\frac{c_{16}}{c_{66}},b_{12}=ia\frac{c_{12}+c_{66}}{c_{66}}+ib\frac{c_{14}+c_{56}}{c_{66}},$$

$$b_{13}=-a^2\frac{c_{26}}{c_{66}}-b^2\frac{c_{45}}{c_{66}}-ab\frac{c_{46}+c_{25}}{c_{66}},b_{21}=2ia\frac{c_{26}}{c_{66}}+2ib\frac{c_{46}}{c_{66}},b_{22}=-a^2\frac{c_{22}}{c_{66}}-b^2\frac{c_{44}}{c_{66}}-2ab\frac{c_{24}}{c_{66}}-\frac{c_{11}}{c_{66}}\omega^2;$$

$$b_{31}=\frac{c_{56}}{c_{66}},b_{32}=ia\frac{c_{46}+c_{25}}{c_{66}}+ib\frac{c_{36}+c_{45}}{c_{66}},b_{33}=-a^2\frac{c_{24}}{c_{66}}-b^2\frac{c_{34}}{c_{66}}-ab\frac{c_{23}+c_{44}}{c_{66}},b_{34}=\frac{ia\beta_2 c_{11}(1+T_1\omega)}{c_{66}},$$

$$m_{11}=\frac{c_{15}}{c_{55}},m_{12}=ia\frac{c_{56}+c_{14}}{c_{55}}+ib\frac{c_{55}+c_{13}}{c_{55}},m_{13}=-a^2\frac{c_{46}}{c_{55}}-b^2\frac{c_{35}}{c_{55}}-ab\frac{c_{45}+c_{36}}{c_{55}},m_{21}=\frac{c_{56}}{c_{55}},$$

$$m_{22}=ia\frac{c_{25}+c_{46}}{c_{55}}+ib\frac{c_{45}+c_{36}}{c_{55}},m_{23}=-a^2\frac{c_{24}}{c_{55}}-b^2\frac{c_{34}}{c_{55}}-ab\frac{c_{44}+c_{23}}{c_{55}},m_{31}=2ia\frac{c_{45}}{c_{55}}+2ib\frac{c_{35}}{c_{55}},$$

$$m_{32}=-a^2\frac{c_{44}}{c_{55}}-b^2\frac{c_{33}}{c_{55}}-2ab\frac{c_{34}}{c_{55}}-\frac{c_{11}}{c_{55}}\omega^2,m_{33}=\frac{ib\beta_3c_{11}}{c_{55}},$$

$$h_{12}=\frac{c_{16}}{c_{11}},h_{13}=\frac{c_{15}}{c_{11}},h_{14}=\frac{i(bc_{15}+ac_{16})}{c_{11}},h_{15}=\frac{i(bc_{14}+ac_{12})}{c_{11}},h_{16}=\frac{i(bc_{13}+ac_{14})}{c_{11}},h_{21}=\frac{c_{21}}{c_{11}},$$

$$h_{22}=\frac{c_{26}}{c_{11}},h_{23}=\frac{c_{25}}{c_{11}},h_{24}=\frac{i(bc_{25}+ac_{26})}{c_{11}},h_{25}=\frac{i(bc_{24}+ac_{22})}{c_{11}},h_{26}=\frac{i(bc_{23}+ac_{24})}{c_{11}},h_{31}=\frac{c_{31}}{c_{11}},$$

$$h_{32}=\frac{c_{36}}{c_{11}},h_{33}=\frac{c_{35}}{c_{11}},h_{34}=\frac{i(bc_{35}+ac_{26})}{c_{11}},h_{35}=\frac{i(bc_{34}+ac_{32})}{c_{11}},h_{36}=\frac{i(bc_{33}+ac_{34})}{c_{11}},h_{41}=\frac{c_{41}}{c_{11}},$$

$$h_{42}=\frac{c_{46}}{c_{11}},h_{43}=\frac{c_{45}}{c_{11}},h_{44}=\frac{i(bc_{45}+ac_{46})}{c_{11}},h_{45}=\frac{i(bc_{44}+ac_{42})}{c_{11}},h_{46}=\frac{i(bc_{43}+ac_{44})}{c_{11}},h_{51}=\frac{c_{51}}{c_{11}},$$

$$h_{52}=\frac{c_{56}}{c_{11}},h_{53}=\frac{c_{55}}{c_{11}},h_{54}=\frac{i(bc_{55}+ac_{56})}{c_{11}},h_{55}=\frac{i(bc_{54}+ac_{52})}{c_{11}},h_{56}=\frac{i(bc_{53}+ac_{54})}{c_{11}},h_{61}=\frac{c_{61}}{c_{11}},$$

$$h_{62}=\frac{c_{66}}{c_{11}},h_{63}=\frac{c_{65}}{c_{11}},h_{64}=\frac{i(bc_{65}+ac_{66})}{c_{11}},h_{65}=\frac{i(bc_{64}+ac_{62})}{c_{11}},h_{66}=\frac{i(bc_{63}+ac_{64})}{c_{11}},$$

$$d_{11}=1+\frac{a_{21}(b_{31}m_{11}-b_{11})}{1-b_{31}m_{21}}+\frac{a_{31}(m_{21}b_{11}-m_{11})}{1-b_{31}m_{21}},d_{12}=a_{11}+\frac{a_{21}(b_{31}m_{12}-b_{12})}{1-b_{31}m_{21}}+\frac{a_{31}(m_{21}b_{12}-m_{12})}{1-b_{31}m_{21}},$$

$$d_{13}=a_{12}+\frac{a_{21}(b_{31}m_{13}-b_{13})}{1-b_{31}m_{21}}+\frac{a_{31}(m_{21}b_{13}-m_{13})}{1-b_{31}m_{21}},d_{14}=a_{22}+\frac{a_{21}(b_{31}m_{22}-b_{21})}{1-b_{31}m_{21}}+\frac{a_{31}(m_{21}b_{21}-m_{22})}{1-b_{31}m_{21}},$$

$$d_{15}=a_{23}+\frac{a_{21}(b_{31}m_{23}-b_{22})}{1-b_{31}m_{21}}+\frac{a_{31}(m_{21}b_{22}-m_{23})}{1-b_{31}m_{21}},d_{16}=a_{32}+\frac{a_{21}(b_{31}m_{31}-b_{32})}{1-b_{31}m_{21}}+\frac{a_{31}(m_{21}b_{32}-m_{31})}{1-b_{31}m_{21}},$$

$$d_{17}=a_{33}+\frac{a_{21}(b_{31}m_{32}-b_{33})}{1-b_{31}m_{21}}+\frac{a_{31}(m_{21}b_{33}-m_{31})}{1-b_{31}m_{21}},d_{18}=-\frac{a_{21}(b_{31}m_{33}+b_{34})}{1-b_{31}m_{21}}+\frac{a_{31}(m_{33}-m_{21}b_{34})}{1-b_{31}m_{21}},$$

$$d_{21}=(-b_{12}-\frac{b_{31}(m_{21}b_{12}-m_{12})}{1-b_{31}m_{21}})-\frac{d_{12}}{d_{11}}(-b_{11}-\frac{b_{31}(m_{21}b_{11}-m_{11})}{1-b_{31}m_{21}}),$$

$$d_{22}=(-b_{13}-\frac{b_{31}(m_{21}b_{13}-m_{13})}{1-b_{31}m_{21}})-\frac{d_{13}}{d_{11}}(-b_{11}-\frac{b_{31}(m_{21}b_{11}-m_{11})}{1-b_{31}m_{21}}),$$

$$d_{23}=(-b_{21}-\frac{b_{31}(m_{21}b_{21}-m_{22})}{1-b_{31}m_{21}})-\frac{d_{14}}{d_{11}}(-b_{11}-\frac{b_{31}(m_{21}b_{11}-m_{11})}{1-b_{31}m_{21}}),$$

$$d_{24}=(-b_{22}-\frac{b_{31}(m_{21}b_{22}-m_{23})}{1-b_{31}m_{21}})-\frac{d_{15}}{d_{11}}(-b_{11}-\frac{b_{31}(m_{21}b_{11}-m_{11})}{1-b_{31}m_{21}}),$$

$$d_{25}=(-b_{32}-\frac{b_{31}(m_{21}b_{32}-m_{31})}{1-b_{31}m_{21}})-\frac{d_{16}}{d_{11}}(-b_{11}-\frac{b_{31}(m_{21}b_{11}-m_{11})}{1-b_{31}m_{21}}),$$

$$d_{26}=(-b_{33}-\frac{b_{31}(m_{21}b_{33}-m_{32})}{1-b_{31}m_{21}})-\frac{d_{17}}{d_{11}}(-b_{11}-\frac{b_{31}(m_{21}b_{11}-m_{11})}{1-b_{31}m_{21}}),$$

$$d_{27}=\frac{1}{d_{11}}(-b_{11}-\frac{b_{31}(m_{21}b_{11}-m_{11})}{1-b_{31}m_{21}}),d_{31}=\frac{m_{11}d_{12}}{d_{11}}-m_{12}-m_{21}d_{21},d_{32}=\frac{m_{11}d_{13}}{d_{11}}-m_{13}-m_{21}d_{22},$$

$$d_{33}=\frac{m_{11}d_{14}}{d_{11}}-m_{22}-m_{21}d_{23},d_{34}=\frac{m_{11}d_{15}}{d_{11}}-m_{23}-m_{21}d_{24},d_{35}=\frac{m_{11}d_{16}}{d_{11}}-m_{31}-m_{21}d_{25},$$

$$d_{36}=\frac{m_{11}d_{17}}{d_{11}}-m_{32}-m_{21}d_{26},d_{37}=-(\frac{m_{11}\beta_1}{d_{11}}+m_{21}d_{27}),d_{38}=\frac{m_{11}d_{18}}{d_{11}}+m_{33}-m_{21}d_{28},$$

$$d_{41}=\frac{\varepsilon_1\omega^2}{C_T^2},d_{42}=0,d_{43}=0,d_{44}=\frac{ia\varepsilon_2}{C_T^2},d_{45}=0,d_{46}=\frac{ib\varepsilon\beta_3}{C_T^2},d_{47}=0d_{48}=\frac{C_T^2(a^2+b^2)+\omega^2}{C_T^2},$$

$$g_{51}=-\frac{d_{13}}{d_{11}},g_{52}=-\frac{d_{15}}{d_{11}},g_{53}=-\frac{d_{17}}{d_{11}},g_{54}=-\frac{d_{18}}{d_{11}},g_{55}=-\frac{d_{12}}{d_{11}},g_{56}=-\frac{d_{14}}{d_{11}},g_{57}=-\frac{d_{16}}{d_{11}},g_{58}=\frac{1}{d_{11}},$$

$$g_{61}=d_{22},g_{62}=d_{24},g_{63}=d_{26},g_{64}=d_{28},g_{65}=d_{21},g_{66}=d_{23},g_{67}=d_{25},g_{68}=d_{27},g_{71}=d_{32},g_{72}=d_{34},$$

$$g_{73}=d_{36},g_{74}=d_{38},g_{75}=d_{31},g_{76}=d_{33},g_{77}=d_{35},g_{78}=d_{37},g_{81}=0,g_{82}=d_{44},g_{83}=d_{46},g_{84}=d_{48},$$

$$g_{85}=d_{41},g_{86}=g_{87}=g_{88}=0,f_{11}=g_{51}+\lambda g_{55}-\lambda^2,f_{12}=g_{52}+\lambda g_{56},f_{13}=g_{53}+\lambda g_{57},f_{14}=g_{54}+\lambda g_{58},$$

$$f_{21}=g_{61}+\lambda g_{65},f_{22}=g_{62}+\lambda g_{66}-\lambda^2,f_{23}=g_{63}+\lambda g_{67},f_{24}=g_{64}+\lambda g_{68},f_{31}=g_{71}+\lambda g_{65},$$

$$f_{32}=g_{72}+\lambda g_{76},f_{33}=g_{73}+\lambda g_{77}-\lambda^2,f_{34}=g_{74}+\lambda g_{78},f_{41}=\lambda g_{85},f_{42}=g_{82},f_{43}=g_{83},f_{44}=g_{84},$$

$$L_{21}=\begin{bmatrix}g_{51}&g_{52}&g_{53}&g_{54}\\g_{61}&g_{62}&g_{63}&g_{64}\\g_{71}&g_{72}&g_{73}&g_{74}\\g_{81}&g_{82}&g_{83}&g_{84}\end{bmatrix}L_{22}=\begin{bmatrix}g_{55}&g_{56}&g_{57}&g_{58}\\g_{65}&g_{66}&g_{67}&g_{68}\\g_{75}&g_{76}&g_{77}&g_{78}\\g_{85}&g_{86}&g_{87}&g_{88}\end{bmatrix}$$

APPENDIX B

$$R_{11}(x_1)=[(h_{14}-\lambda_1)\{(\delta_1)_{\lambda=-\lambda_1}+(h_{15}-\lambda_1h_{12})\{(\delta_2)_{\lambda=-\lambda_1}\}+(h_{16}-\lambda_1h_{13})\{(\delta_3)_{\lambda=-\lambda_1}\}-\{(\delta_4)_{\lambda=-\lambda_1}\}]e^{-\lambda_1x_1}$$

$$R_{12}(x_1)=[(h_{14}-\lambda_2)\{(\delta_1)_{\lambda=-\lambda_2}+(h_{15}-\lambda_2h_{12})\{(\delta_2)_{\lambda=-\lambda_2}\}+(h_{16}-\lambda_2h_{13})\{(\delta_3)_{\lambda=-\lambda_2}\}-\{(\delta_4)_{\lambda=-\lambda_2}\}]e^{-\lambda_2x_1}$$

$$R_{13}(x_1)=[(h_{14}-\lambda_3)\{(\delta_1)_{\lambda=-\lambda_3}+(h_{15}-\lambda_3h_{12})\{(\delta_2)_{\lambda=-\lambda_3}\}+(h_{16}-\lambda_3h_{13})\{(\delta_3)_{\lambda=-\lambda_3}\}-\{(\delta_4)_{\lambda=-\lambda_3}\}]e^{-\lambda_3x_1}$$

$$R_{14}(x_1)=[(h_{14}-\lambda_4)\{(\delta_1)_{\lambda=-\lambda_4}+(h_{15}-\lambda_4h_{12})\{(\delta_2)_{\lambda=-\lambda_4}\}+(h_{16}-\lambda_4h_{13})\{(\delta_3)_{\lambda=-\lambda_4}\}-\{(\delta_4)_{\lambda=-\lambda_4}\}]e^{-\lambda_4x_1}$$

$$R_{21}(x_1)=[(h_{24}-\lambda_1h_{21})\{(\delta_1)_{\lambda=-\lambda_1}+(h_{25}-\lambda_1h_{22})\{(\delta_2)_{\lambda=-\lambda_1}\}+(h_{26}-\lambda_1h_{23})\{(\delta_3)_{\lambda=-\lambda_1}\}-\beta_2\{(\delta_4)_{\lambda=-\lambda_1}\}]e^{-\lambda_1x_1}$$

$$R_{22}(x_1)=[(h_{24}-\lambda_2h_{21})\{(\delta_1)_{\lambda=-\lambda_2}+(h_{25}-\lambda_2h_{22})\{(\delta_2)_{\lambda=-\lambda_2}\}+(h_{26}-\lambda_2h_{23})\{(\delta_3)_{\lambda=-\lambda_2}\}-\beta_2\{(\delta_4)_{\lambda=-\lambda_2}\}]e^{-\lambda_2x_1}$$

$$R_{23}(x_1)=[(h_{24}-\lambda_3h_{21})\{(\delta_1)_{\lambda=-\lambda_3}+(h_{25}-\lambda_3h_{22})\{(\delta_2)_{\lambda=-\lambda_3}\}+(h_{26}-\lambda_3h_{23})\{(\delta_3)_{\lambda=-\lambda_3}\}-\beta_2\{(\delta_4)_{\lambda=-\lambda_3}\}]e^{-\lambda_3x_1}$$

$$R_{24}(x_1)=[(h_{24}-\lambda_4h_{21})\{(\delta_1)_{\lambda=-\lambda_4}+(h_{25}-\lambda_4h_{22})\{(\delta_2)_{\lambda=-\lambda_4}\}+(h_{26}-\lambda_4h_{23})\{(\delta_3)_{\lambda=-\lambda_4}\}-\beta_2\{(\delta_4)_{\lambda=-\lambda_4}\}]e^{-\lambda_4x_1}$$

$$R_{31}(x_1)=[(h_{34}-\lambda_1h_{31})\{(\delta_1)_{\lambda=-\lambda_1}+(h_{35}-\lambda_1h_{32})\{(\delta_2)_{\lambda=-\lambda_1}\}+(h_{36}-\lambda_1h_{33})\{(\delta_3)_{\lambda=-\lambda_1}\}-\beta_3\{(\delta_4)_{\lambda=-\lambda_1}\}]e^{-\lambda_1x_1}$$

$$R_{32}(x_1)=[(h_{34}-\lambda_2h_{31})\{(\delta_1)_{\lambda=-\lambda_2}+(h_{35}-\lambda_2h_{32})\{(\delta_2)_{\lambda=-\lambda_2}\}+(h_{36}-\lambda_2h_{33})\{(\delta_3)_{\lambda=-\lambda_2}\}-\beta_3\{(\delta_4)_{\lambda=-\lambda_2}\}]e^{-\lambda_2x_1}$$

$$R_{33}(x_1)=[(h_{34}-\lambda_3h_{31})\{(\delta_1)_{\lambda=-\lambda_3}+(h_{35}-\lambda_3h_{32})\{(\delta_2)_{\lambda=-\lambda_3}\}+(h_{36}-\lambda_3h_{33})\{(\delta_3)_{\lambda=-\lambda_3}\}-\beta_3\{(\delta_4)_{\lambda=-\lambda_3}\}]e^{-\lambda_3x_1}$$

$$R_{34}(x_1)=[(h_{34}-\lambda_4h_{31})\{(\delta_1)_{\lambda=-\lambda_4}+(h_{35}-\lambda_4h_{32})\{(\delta_2)_{\lambda=-\lambda_4}\}+(h_{36}-\lambda_4h_{33})\{(\delta_3)_{\lambda=-\lambda_4}\}-\beta_3\{(\delta_4)_{\lambda=-\lambda_4}\}]e^{-\lambda_4x_1}$$

$$R_{41}(x_1)=[(h_{44}-\lambda_1h_{41})\{(\delta_1)_{\lambda=-\lambda_1}+(h_{45}-\lambda_1h_{42})\{(\delta_2)_{\lambda=-\lambda_1}\}+(h_{46}-\lambda_1h_{43})\{(\delta_3)_{\lambda=-\lambda_1}\}]e^{-\lambda_1x_1}$$

$$R_{42}(x_1)=[(h_{44}-\lambda_2h_{41})\{(\delta_1)_{\lambda=-\lambda_2}+(h_{45}-\lambda_2h_{42})\{(\delta_2)_{\lambda=-\lambda_2}\}+(h_{46}-\lambda_2h_{43})\{(\delta_3)_{\lambda=-\lambda_2}\}]e^{-\lambda_2x_1}$$

$$R_{43}(x_1)=[(h_{44}-\lambda_3h_{41})\{(\delta_1)_{\lambda=-\lambda_3}+(h_{45}-\lambda_3h_{42})\{(\delta_2)_{\lambda=-\lambda_3}\}+(h_{46}-\lambda_3h_{43})\{(\delta_3)_{\lambda=-\lambda_3}\}]e^{-\lambda_3x_1}$$

$$R_{44}(x_1)=[(h_{44}-\lambda_4h_{41})\{(\delta_1)_{\lambda=-\lambda_4}+(h_{45}-\lambda_4h_{42})\{(\delta_2)_{\lambda=-\lambda_4}\}+(h_{46}-\lambda_4h_{43})\{(\delta_3)_{\lambda=-\lambda_4}\}]e^{-\lambda_4x_1}$$

$$R_{51}(x_1)=[(h_{54}-\lambda_1h_{51})\{(\delta_1)_{\lambda=-\lambda_1}+(h_{55}-\lambda_1h_{52})\{(\delta_2)_{\lambda=-\lambda_1}\}+(h_{56}-\lambda_1h_{53})\{(\delta_3)_{\lambda=-\lambda_1}\}]e^{-\lambda_1x_1}$$

$$R_{52}(x_1)=[(h_{54}-\lambda_2 h_{51})\{(\delta_1)_{\lambda=-\lambda_2}+(h_{55}-\lambda_2 h_{52})\{(\delta_2)_{\lambda=-\lambda_2}\}+(h_{56}-\lambda_2 h_{53})\{(\delta_3)_{\lambda=-\lambda_2}\}]e^{-\lambda_2 x_1}$$

$$R_{53}(x_1)=[(h_{54}-\lambda_3 h_{51})\{(\delta_1)_{\lambda=-\lambda_3}+(h_{55}-\lambda_3 h_{52})\{(\delta_2)_{\lambda=-\lambda_3}\}+(h_{56}-\lambda_3 h_{53})\{(\delta_3)_{\lambda=-\lambda_3}\}]e^{-\lambda_3 x_1}$$

$$R_{54}(x_1)=[(h_{54}-\lambda_4 h_{51})\{(\delta_1)_{\lambda=-\lambda_4}+(h_{55}-\lambda_4 h_{52})\{(\delta_2)_{\lambda=-\lambda_4}\}+(h_{56}-\lambda_4 h_{53})\{(\delta_3)_{\lambda=-\lambda_4}\}]e^{-\lambda_4 x_1}$$

$$R_{61}(x_1)=[(h_{64}-\lambda_1 h_{61})\{(\delta_1)_{\lambda=-\lambda_1}+(h_{65}-\lambda_1 h_{62})\{(\delta_2)_{\lambda=-\lambda_1}\}+(h_{66}-\lambda_1 h_{63})\{(\delta_3)_{\lambda=-\lambda_1}\}]e^{-\lambda_1 x_1}$$

$$R_{62}(x_1)=[(h_{64}-\lambda_2 h_{61})\{(\delta_1)_{\lambda=-\lambda_2}+(h_{65}-\lambda_2 h_{62})\{(\delta_2)_{\lambda=-\lambda_2}\}+(h_{66}-\lambda_2 h_{63})\{(\delta_3)_{\lambda=-\lambda_2}\}]e^{-\lambda_2 x_1}$$

$$R_{63}(x_1)=[(h_{64}-\lambda_3 h_{61})\{(\delta_1)_{\lambda=-\lambda_3}+(h_{65}-\lambda_3 h_{62})\{(\delta_2)_{\lambda=-\lambda_3}\}+(h_{66}-\lambda_3 h_{63})\{(\delta_3)_{\lambda=-\lambda_3}\}]e^{-\lambda_3 x_1}$$

$$R_{64}(x_1)=[(h_{64}-\lambda_4 h_{61})\{(\delta_1)_{\lambda=-\lambda_4}+(h_{65}-\lambda_4 h_{62})\{(\delta_2)_{\lambda=-\lambda_4}\}+(h_{66}-\lambda_4 h_{63})\{(\delta_3)_{\lambda=-\lambda_4}\}]e^{-\lambda_4 x_1}$$

$$R_{71}(0)=(\gamma-\lambda_1)\{(\delta_4)_{\lambda=-\lambda_1}\}]e^{-\lambda_1} \quad ,R_{72}(0)=(\gamma-\lambda_2)\{(\delta_4)_{\lambda=-\lambda_2}\}]e^{-\lambda_2}$$

$$R_{73}(0)=(\gamma-\lambda_3)\{(\delta_4)_{\lambda=-\lambda_3}\}]e^{-\lambda_3} \quad ,R_{74}(0)=(\gamma-\lambda_4)\{(\delta_4)_{\lambda=-\lambda_4}\}]e^{-\lambda_4}$$

$$D_1=\begin{vmatrix}0 & R_{12}(0) & R_{13}(0) & R_{14}(0)\\ 0 & R_{22}(0) & R_{23}(0) & R_{24}(0)\\ 0 & R_{32}(0) & R_{33}(0) & R_{34}(0)\\ r^* & R_{72}(0) & R_{73}(0) & R_{74}(0)\end{vmatrix} D_{2=}\begin{vmatrix}R_{11}(0) & 0 & R_{13}(0) & R_{14}(0)\\ R_{21}(0) & 0 & R_{23}(0) & R_{24}(0)\\ R_{31}(0) & 0 & R_{33}(0) & R_{34}(0)\\ R_{71}(0) & r^* & R_{73}(0) & R_{74}(0)\end{vmatrix}$$

$$D_{3=}\begin{vmatrix}R_{11}(0) & R_{12}(0) & 0 & R_{14}(0)\\ R_{21}(0) & R_{22}(0) & 0 & R_{24}(0)\\ R_{31}(0) & R_{32}(0) & 0 & R_{34}(0)\\ R_{71}(0) & R_{72}(0) & r^* & R_{74}(0)\end{vmatrix} D_{4=}\begin{vmatrix}R_{11}(0) & R_{12}(0) & R_{13}(0) & 0\\ R_{21}(0) & R_{22}(0) & R_{23}(0) & 0\\ R_{31}(0) & R_{32}(0) & R_{33}(0) & 0\\ R_{71}(0) & R_{72}(0) & R_{73}(0) & r^*\end{vmatrix}$$

$$\Delta=\begin{vmatrix}R_{11}(0) & R_{12}(0) & R_{13}(0) & R_{14}(0)\\ R_{21}(0) & R_{22}(0) & R_{23}(0) & R_{24}(0)\\ R_{31}(0) & R_{32}(0) & R_{33}(0) & R_{34}(0)\\ R_{71}(0) & R_{72}(0) & R_{73}(0) & R_{74}(0)\end{vmatrix}$$

$$N_{1i}=\frac{1}{\lambda_i}\sum_{i=1}^{8}(h_{14}x_{1i}+h_{15}x_{2i}+h_{16}x_{3i}-x_{4i}),\ N_{2i}=\frac{1}{\lambda_i}\sum_{i=1}^{8}(h_{24}x_{1i}+h_{25}x_{2i}+h_{26}x_{3i}-\beta_2 x_{4i})$$

$$N_{3i}=\frac{1}{\lambda_i}\sum_{i=1}^{8}(h_{34}x_{1i}+h_{35}x_{2i}+h_{36}x_{3i}-\beta_3 x_{4i})$$

APPENDIX C

Phase velocity

The phase velocities of $qP1, qS1, qS2$ and $qP2$ and $V_i\ (i=1,2,3,4)$ are defined by

$$V_i=\frac{\omega}{Re(\xi)}$$

Attenuation

$Q_i\ (i=1,2,3,4)$ are the attenuation coefficients of $qP1, qS1, qS2$ and $qP2$ which are defined by

$$Q_i=Img(\xi_i)$$

Penetration depth

The penetration depth is defined as $B_i = \dfrac{1}{Img(\xi_i)}, (i = 1,2,3,4)$

Specific loss

The specific loss is defined by $W_i = (\dfrac{\Delta W}{W})_i = 4\pi \left| \dfrac{Img(\xi_i)}{Re(\xi_i)} \right|, i - 1,2,3,4$.

where specific loss is the ratio of energy (ΔW) to elastic energy (W). The specific loss is the most dirrect method of defining internal friction for amaterial.

APPENDIX D

Considering a system of simultaneous differential equations in the form

$$\frac{d\vec{v}}{dx} = \vec{A}\vec{v} + \vec{f}$$

where $\vec{v} = [v_1 v_2 ....v_n]^T$, $\vec{A} = (a_{ij})_{nXn}$, $\vec{f} = [f_1 f_2 ....f_n]^T$

Let us consider the coefficient matrix $\vec{A}$ can be written as:

$$A = V \Lambda V^{-1}$$

where, $\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & \\ 0 & \lambda_2 & 0 & \\ 0 & 0 & \lambda_3 & \end{bmatrix}$ and $V = [V_1, V_2, ..., V_n]$

where $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigen values of the coefficient matrix A. $V_1, V_2, ...V_n$ are the eigen vectors corresponding to the eigen values $\lambda_1, \lambda_2, ..., \lambda_n$ respectively.

Now multiplying the Eq. (1) by V^{-1} we get

$$V^{-1}\frac{d\vec{v}}{dx} = V^{-1}(V \Lambda V^{-1})\vec{v} + V^{-1}\vec{f}$$

$$\frac{d(V^{-1}\vec{v})}{dx} = \Lambda(V^{-1}\vec{v}) + V^{-1}\vec{f}$$

$$\frac{d\vec{y}}{dx} = \Lambda\vec{y} + V^{-1}\vec{f}$$

where $\vec{y} = V^{-1}\vec{v} \Rightarrow \vec{v} = V\vec{y}$

The *r-th* Eq. (3) is

$$\frac{dy_r}{dx} = \lambda_r y_r + Q_r$$

where $Q_r = V_r^{-1}\vec{f}$, $V_r^{-1} = (\omega_{ij})$, $Q_r = \sum_{i=1}^{n} \omega_{ri} f_i$

The solution of (4) is:

$$y_r = c_r e^{\lambda_r x} + e^{\lambda_r x} \int Q_r e^{-\lambda_r x} dx \quad , \quad \vec{v} = \sum_{r=1}^{n} V_r y_r$$

REFERENCES

[1] Chandrasekharaiah D.S., Srinath K.S., 1998, Thermoelastic plane waves without energy dissipation in a rotating body, *Mechanics Research Communications* **24**: 551-560.

[2] Green A.E., Lindsay K.A., 1972, Thermoelasticity, *Journal of Elasticity* **2**: 1-7.

[3] Green A.E., Naghdi P.M., 1991, A re- examination of the basic postulate of thermo-mechanics, *Proceedings of the Royal Society of London* **432**: 171-194.

[4] Green A.E., Naghdi P.M., 1992, An unbounded heat wave in an elastic solid, *Journal of Thermal Stresses* **15**: 253-264.

[5] Green A.E., Naghdi P.M., 1992, Thermoelasticity without energy dissipation, *Journal of Elasticity* **31**: 189-208.

[6] Lord H.W., Shulman Y.A., 1967, Geeneralized dynamical theory of thermoelasticity, *Journal of the Mechanics and Physics of Solids* **15**: 299-309.

[7] Roychoudhuri S.K., Bandyopadhyay N., 2004, Thermoelastic wave propagation in a rotating elastic medium without energy dissipation, *International Journal of Mathematics and Mathematical Sciences* **1**: 99-107.

[8] Roychoudhuri S.K., Dutta P.S., 2005, Thermoelastic interaction without energy dissipation in an infinite solid with distributed periodically varing heat sources, *International Journal of Solid and Structures* **42**: 4192-4203.

[9] Chattopadhyay A., Rogerson G.A., 2001, Wave reflection in slightly compressible finitely deformed elastic media, *Archives of Applied Mathematics* **71**: 307-316.

[10] Sarkar N., Lahiri A., 2013, The effect of gravity field on the plane waves in a fiber- reinforced two temperature magneto thermoelastic medium under Lord- Shulman theory, *Journal of Thermal Stresses* **36**: 895-914.

[11] Sarkar N., Lahiri A., 2012, A three dimensional thermoelastic problem for a half Space without energy dissipation, *International Journal of Engineering Science* **51**: 310-325.

[12] Sharma J.N., Chouhan R.S., 1999, On the problem of body forces and heat sources in thermoelasticity without energy dissipation, *Indian Journal of Pure and Applied Mathematics* **30**: 595-610.