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Vibration Analysis of FG Nanoplate Based on Third-Order Shear Deformation Theory (TSDT) and Nonlocal Elasticity

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ABSTRACT

In present study, the third-order shear deformation theory has been developed to investigate vibration analysis of FG Nano-plates based on Eringen nonlocal elasticity theory. The materials distribution regarding to the thickness of Nano-plate has been considered based on two different models of power function and exponential function. All equations governing on the vibration of FG Nano-plate have been derived from Hamilton's principle. It has been also obtained the analytical solution for natural frequencies and corresponding mode shapes of simply supported FG Nano-plates. In addition, the general form of stiffness and mass matrix elements has been expressed based on this theory. The effect of different parameters such as power and exponential indexes of targeted function , nonlocal parameter of Nano-plate, aspect ratio and thickness to length ratio of Nano-plate on non-dimensional natural frequencies of free vibration responses have been investigated. The obtained analytical results show an excellent agreement with other available solutions of previous studies. The formulation and analytical results obtained from proposed method can be used as a benchmark for further studies to develop this area of research.

Keywords : Nano-plate; Functionally graded material (FGM); Nonlocal elasticity; Third order of shear deformation theory (TSDT); Natural frequency.

1 INTRODUCTION

IN recent years, the Nano-scale electromechanical systems (NEMS) such as high frequency Nano-actuators, Nano-sensors, Nano-super capacitors and Nano-semiconductor have been paid much attentions due to developments of engineering sciences. In the other sides, FG materials are a class of non-homogeneous materials obtained from a combination of the two materials to create a combination with specific functions. In FG materials, the possibility of delamination is decreased due to the stress concentration at the interface, with gradual change in volume fraction of compounds compere to their sudden change in multilayer composite materials [1]. Generally, Power law functions [2, 3] and exponential functions [4, 5] are used to describe the changes in properties of FG material. An important part of the studies have been conducted on the behavior of bending, vibration and buckling of one-dimensional nanostructures emphasized on nonlocal elasticity theory (Aydogdu [6], Civalk and Demir [7], Reddy [8, 10], Reddy and Pang [9], Roque et al [11] and Wang et al [12]), These nanostructures include Nano-beams, Nano-rods and carbonic nanotubes. In recent years, the application of FG materials has been highly developed in Nano-scale devices and systems such as thin films [13, 14 and 15], atomic force microscopy [16] and

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bio-mass sensor applications [17]. Salehipour et al have analytically investigated the free vibration FG Micro/Nano-plates with different combination of plain bearings and fixed bearings and free boundary conditions using Eringen nonlocal elasticity theory [18]. Nami and Janghorban have conducted a study on resonance behavior of rectangular FG Nano-plates with plain bearing boundary condition [19]. They utilized from theories independent from nonlocal elasticity scale and strain gradient. Salehipour et al developed exact Solution of the free vibration to FG Nano-plates using three-dimensional theories of elasticity [20, 21]. Natarajan et al have also analyzed vibration behavior of FG Nano-plates using first-order shear deformation theory [22].

In present study, the third-order shear deformation theory has been developed to investigate vibration analysis of FG Nano-plates based on Eringen nonlocal elasticity theory. All equations governing on the vibration of FG Nano-plate have been derived from Hamilton's principle. It has been also obtained the analytical solution for natural frequencies and corresponding mode shapes of simply supported FG Nano-plates. Finally, the effect of different parameters such as power and exponential indexes of targeted function, nonlocal parameter of Nano-plate, aspect ratio and thickness to length ratio of Nano-plate on non-dimensional natural frequencies of free vibration responses have been investigated.

2 MODELING AND DESCRIPTION OF RELATIONS GOVERNING ON THE PROBLEM

2.1 A brief history of nonlocal elasticity theory

The nonlocal elasticity theory was firstly introduced by Eringen to take into account the effect of small scale parameter in modeling continuum mechanics in non-classical problems [23]. In nonlocal theory despite of classical elasticity theory, the elasticity is modeled at a single point of continuous physical model which depends on the strain of all its parts. In the other words, strain in a single point depends on the elasticity and its partial derivatives in mentioned point.

Eringen has expressed the differential equation of this theory in a way that Non-local elasticity tensor is signed with σ_{ij} and local elasticity tensor with t_{ij} .

$$\left(1-\mu\nabla^2\right)\sigma_{ij}=t_{ij} \tag{1}$$

2.2 The model of functionally graded material

2.2.1 The model of exponential functionally graded material

The amount and how a change in properties of the ceramic material on the z axis is shown with exponential index of FG material i.e. ζ parameter. Following exponential functions are used to describe the properties distribution in the materials:

$$P(z)=P_c\,e^{\zeta\left(z+\frac{h}{2}\right)} \tag{2}$$

where, z is the thickness of the plate and $P(z)$ indicate a general property of material such as Young's modulus. The coefficients P_c and P_m are high-level properties (full ceramic) and low-level properties (full metal) of the material, respectively.

2.2.2 The model of exponential functionally graded material

The amount of volume distribution fraction of ceramic material on the z axis is shown with power index of FG material i.e. ζ and is defined using following relation:

$$V_c=\left(\frac{z}{h}+\frac{1}{2}\right)^{\zeta} \tag{3}$$

where V_c is the ceramic material parameter on its distribution profile. After defining volume distribution fraction for this material, it should be noted that the numerical parameters or power index of FG material is a positive number; in a way that if ζ equals to zero, then the beam is quite ceramic and the metallicity increased with its increasing toward infinity rate. Different properties of material vary along the thickness to form:

$$P(z) = P_c V_c + P_m V_m \tag{4}$$

2.3 Equilibrium equations with third-order shear deformation theory (TSDT) and nonlocal elasticity

Fig. 1 represents the geometric model of rectangular plate system with sides *A* and *B* and thickness *h* made up of FG materials.

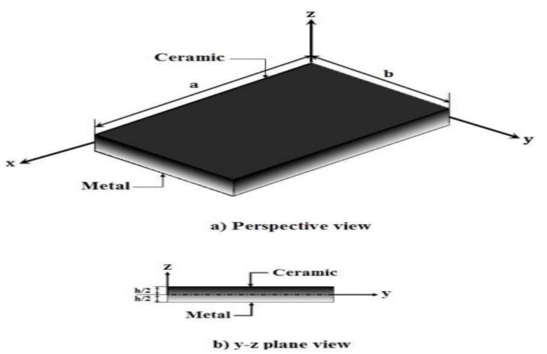

Fig.1
Geometric model of system.

2.3.1 The relations of third-order shear deformation theory

According to the Reddy shear theory for thick plates, the deformation field in plate can be rewritten as follow:

$$\begin{aligned}
u(x,y,z) &= u_0(x,y) + z\phi_x - \frac{4z^3}{3h^2}\left(\phi_x + \frac{\partial w_0}{\partial x}\right) \\
v(x,y,z) &= v_0(x,y) + z\phi_y - \frac{4z^3}{3h^2}\left(\phi_y + \frac{\partial w_0}{\partial y}\right) \\
w(x,y,z) &= w_0(x,y)
\end{aligned} \tag{5}$$

In addition, the strain-displacement relations in third-order shear deformation theory are according to the following equations:

$$\begin{aligned}
&\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + z^3 \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix} \\
&\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + z^2 \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix}
\end{aligned} \tag{6}$$

$$\begin{Bmatrix}\varepsilon_{xx}^{(3)}\\ \varepsilon_{yy}^{(3)}\\ \gamma_{xy}^{(3)}\end{Bmatrix}=-c_1\begin{Bmatrix}\dfrac{\partial\varphi_x}{\partial x}+\dfrac{\partial^2 w_0}{\partial x^2}\\ \dfrac{\partial\varphi_y}{\partial y}+\dfrac{\partial^2 w_0}{\partial y^2}\\ \dfrac{\partial\varphi_x}{\partial y}+\dfrac{\partial\varphi_y}{\partial x}+2\dfrac{\partial^2 w_0}{\partial x\,\partial y}\end{Bmatrix}$$
$$\begin{Bmatrix}\gamma_{yz}^{(0)}\\ \gamma_{xz}^{(0)}\end{Bmatrix}=\begin{Bmatrix}\phi_y+\dfrac{\partial w_0}{\partial y}\\ \phi_x+\dfrac{\partial w_0}{\partial x}\end{Bmatrix} \tag{7}$$
$$\begin{Bmatrix}\gamma_{yz}^{(2)}\\ \gamma_{xz}^{(2)}\end{Bmatrix}=-c_2\begin{Bmatrix}\phi_y+\dfrac{\partial w_0}{\partial y}\\ \phi_x+\dfrac{\partial w_0}{\partial x}\end{Bmatrix}$$

where $c_1=\dfrac{4}{3h^2};c_2=3c_1$.

Therefore, Hooke's law can be written in the following form:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix}=E(z)\begin{bmatrix}\dfrac{1}{1-\nu^2} & \dfrac{\nu}{1-\nu^2} & 0\\ \dfrac{\nu}{1-\nu^2} & \dfrac{1}{1-\nu^2} & 0\\ 0 & 0 & \dfrac{1}{2(1+\nu)}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}$$
$$\begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\frac{E(z)}{2(1+\nu)}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{8}$$

The final strain-displacement relations for the palate can be written as following form:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix}=\frac{E(z)}{1-\upsilon^2}\left(\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}+\upsilon\dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial v_0}{\partial x}+\upsilon\dfrac{\partial u_0}{\partial x}\\ \dfrac{1-\upsilon}{2}\left(\dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\right)\\ \dfrac{1-\upsilon}{2}\left(\phi_y+\dfrac{\partial w_0}{\partial y}\right)\\ \dfrac{1-\upsilon}{2}\left(\phi_x+\dfrac{\partial w_0}{\partial x}\right)\end{Bmatrix}\right)+z\begin{Bmatrix}\dfrac{\partial\phi_x}{\partial x}+\upsilon\dfrac{\partial\phi_y}{\partial y}\\ \dfrac{\partial\phi_y}{\partial y}+\upsilon\dfrac{\partial\phi_x}{\partial x}\\ \dfrac{1-\upsilon}{2}\left(\dfrac{\partial\phi_x}{\partial y}+\upsilon\dfrac{\partial\phi_y}{\partial x}\right)\\ 0\\ 0\end{Bmatrix}-\frac{4z^2}{h^2}\begin{Bmatrix}0\\ 0\\ 0\\ \dfrac{1-\upsilon}{2}\left(\phi_y+\dfrac{\partial w_0}{\partial y}\right)\\ \dfrac{1-\upsilon}{2}\left(\phi_x+\dfrac{\partial w_0}{\partial x}\right)\end{Bmatrix}-\frac{4z^3}{h^2}\begin{Bmatrix}\left(\dfrac{\partial\phi_x}{\partial x}+\dfrac{\partial^2 w_0}{\partial x^2}\right)+\upsilon\left(\dfrac{\partial\phi_y}{\partial y}+\dfrac{\partial^2 w_0}{\partial y^2}\right)\\ \left(\dfrac{\partial\phi_y}{\partial y}+\dfrac{\partial^2 w_0}{\partial y^2}\right)+\upsilon\left(\dfrac{\partial\phi_x}{\partial x}+\dfrac{\partial^2 w_0}{\partial x^2}\right)\\ \dfrac{1-\upsilon}{2}\left(\dfrac{\partial\phi_x}{\partial y}+\dfrac{\partial\phi_y}{\partial x}+2\dfrac{\partial w_0}{\partial x\partial y}\right)\\ 0\\ 0\end{Bmatrix} \tag{9}$$

The resultants elasticity in nonlocal theory for Nano-plate is:

$$\left(1-\mu\nabla^2\right)\left(\begin{Bmatrix}N_{xx}^{nl}\\ N_{yy}^{nl}\\ N_{xy}^{nl}\end{Bmatrix},\begin{Bmatrix}M_{xx}^{nl}\\ M_{yy}^{nl}\\ M_{xy}^{nl}\end{Bmatrix},\begin{Bmatrix}P_{xx}^{nl}\\ P_{yy}^{nl}\\ P_{xy}^{nl}\end{Bmatrix}\right)=\left(\begin{Bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{Bmatrix},\begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix},\begin{Bmatrix}P_{xx}\\ P_{yy}\\ P_{xy}\end{Bmatrix}\right) \tag{10}$$

which, the equations related to $\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}, \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}$ and $\begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix}$ have been represented in the appendix

2.3.2 Equilibrium equations of the system

The virtual work method and Hamilton's principle have been used to achieve motion equation of the system.

$$\int_0^t \left(\int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_0^b \int_0^a \left(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + 2\sigma_{xy}\delta\varepsilon_{xy} + 2\sigma_{xz}\delta\varepsilon_{xz} + 2\sigma_{yz}\delta\varepsilon_{yz} - 0.5\,\rho(z)\left(\dot{u}_{sys}^2 + \dot{v}_{sys}^2 + \dot{w}_{sys}^2\right)\right) \mathrm{du\,dvdw} \right) \mathrm{dt} = 0 \tag{11}$$

According to the Hamilton's principle, the equilibrium equations of system in nonlocal space can be expressed as following five forms:

$$\frac{\partial N_{xx}^{nl}}{\partial x} + \frac{\partial N_{xy}^{nl}}{\partial y} - \left(1-\mu\nabla^2\right)\left(I_0 \ddot{u}_0 + J_1 \ddot{\phi}_x - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial x} \right) = 0 \tag{12}$$

$$\frac{\partial N_{xy}^{nl}}{\partial x} + \frac{\partial N_{yy}^{nl}}{\partial y} - \left(1-\mu\nabla^2\right)\left(I_0 \ddot{v}_0 + J_1 \ddot{\phi}_y - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial y} \right) = 0 \tag{13}$$

$$\frac{\partial \bar{M}_{xx}^{nl}}{\partial x} + \frac{\partial \bar{M}_{xy}^{nl}}{\partial y} - \bar{Q}_x^{nl} - \left(1-\mu\nabla^2\right)\left(J_1 \ddot{u}_0 + K_2 \ddot{\phi}_x - c_1 J_4 \frac{\partial \ddot{w}_0}{\partial x} \right) = 0 \tag{14}$$

$$\frac{\partial \bar{M}_{xy}^{nl}}{\partial x} + \frac{\partial \bar{M}_{yy}^{nl}}{\partial y} - \bar{Q}_y^{nl} - \left(1-\mu\nabla^2\right)\left(J_1 \ddot{v}_0 + K_2 \ddot{\phi}_y - c_1 J_4 \frac{\partial \ddot{w}_0}{\partial y} \right) = 0 \tag{15}$$

$$\begin{aligned} &\frac{\partial Q_x^{nl}}{\partial x} + \frac{\partial Q_y^{nl}}{\partial y} + \frac{\partial}{\partial x}\left(N_{xx}^{nl}\frac{\partial w_0}{\partial x} + N_{yy}^{nl}\frac{\partial w_0}{\partial y} \right) + \frac{\partial}{\partial y}\left(N_{xy}^{nl}\frac{\partial w_0}{\partial x} + N_{yy}^{nl}\frac{\partial w_0}{\partial y} \right) + c_1\left(\frac{\partial^2 P_{xx}^{nl}}{\partial x^2} + 2\frac{\partial^2 P_{xy}^{nl}}{\partial x\,\partial y} + \frac{\partial^2 P_{yy}^{nl}}{\partial y^2} \right) \\ &= \left(1-\mu\nabla^2\right)\left[I_0 \ddot{w}_0 - c_1^2 I_6\left(\frac{\partial^2 \ddot{w}_0}{\partial x^2} + \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right) \right] + \left(1-\mu\nabla^2\right)\left\{ c_1\left[I_3\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) + J_4\left(\frac{\partial \ddot{\phi}_x}{\partial x} + \frac{\partial \ddot{\phi}_y}{\partial y} \right) \right]\right\} \end{aligned} \tag{16}$$

In above relations we have:

$$\begin{aligned} &\bar{M}_{\alpha\beta}^{nl} = M_{\alpha\beta}^{nl} - c_1 P_{\alpha\beta}^{nl} \quad (\alpha,\beta = 1, 2, 6); \\ &\bar{Q}_\alpha = Q_\alpha^{nl} - c_2 R_\alpha^{nl} \quad (\alpha = 4, 5) \end{aligned} \tag{17}$$

In addition, the phrases related to the high order of FG Nano-plate have been used in the equilibrium equations of system are in following form:

$$I_i = \int_{-h/2}^{h/2} \rho(z)^i\, dz \qquad (i = 0,1,2,\ldots,6) \qquad J_i = I_i - c_1 I_{i+2}\ , \quad K_2 = I_2 - 2c_1 I_4 + c_1^2 I_6 \tag{18}$$

3 THE ANALYTICAL SOLUTION OF EQUATION

3.1 The boundary conditions governing on the problem

According to the model, the boundary conditions system of the plate has been considered in the form of four sides simply supported. The geometrical and mechanical conditions at the borders of the plate can be shown using following relations:

$$\begin{aligned}
&u_0(x,0,t)=0, u_0(x,b,t)=0\\
&v_0(0,y,t)=0, v_0(a,y,t)=0\\
&\phi_x(x,0,t)=0, \phi_x(x,b,t)=0\\
&\phi_y(0,y,t)=0, \phi_y(a,y,t)=0\\
&w_0(x,0,t)=0, w_0(x,b,t)=0\\
&w_0(0,y,t)=0, w_0(a,y,t)=0\\
&N_{yy}(x,0,t)=0, N_{yy}(x,b,t)=0\\
&N_{xx}(0,y,t)=0, N_{xx}(a,y,t)=0\\
&M_{yy}(x,0,t)=0, M_{yy}(x,b,t)=0\\
&M_{xx}(0,y,t)=0, M_{xx}(a,y,t)=0
\end{aligned} \tag{19}$$

Therefore, following series which satisfy above conditions are considered as linear motion and angular functions of the plate.

$$\begin{aligned}
&u_0(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}U_{mn}(t).Cos(\alpha x).Sin(\beta y)\\
&v_0(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}V_{mn}(t).Sin(\alpha x).Cos(\beta y)\\
&w_0(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}W_{mn}(t).Sin(\alpha x).Sin(\beta y)\\
&\phi_x(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}X_{mn}(t).Cos(\alpha x).Sin(\beta y)\\
&\phi_y(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}Y_{mn}(t).Sin(\alpha x).Cos(\beta y)
\end{aligned} \tag{20}$$

where, $\beta=\frac{n\pi}{b}$, $\alpha=\frac{m\pi}{a}$ and $U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}$ are unknown parameters.

3.2 Analyzing free vibrations of the system

To analyze of system free vibration frequencies, it has been assumed that the time response is in harmonic form and it is possible to consider the time displacement vector $\vec{\Delta}=\left[U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}\right]^T$ in the form of $e^{i\omega t}\vec{\Delta}=\vec{\Delta}_0$. The matrix form of vibration`s equation of the system is as follow:

$$\left([K]-\omega^2[M]\right)\vec{\Delta}=0 \tag{21}$$

$[k]$ and $[m]$ are thickness and mass matrixes of the system, respectively are in square form with five rows and columns based on the third-order shear deformation theory. The frequency of free vibrations of the system has been signed with ω which is based on radians per second. The determinant of the coefficient`s matrix should be zero to achieve unique solutions form the system and the natural frequencies are the roots of the characteristic equation of system.

4 NUMERICAL RESULTS AND DISCUSSION OF THEM

4.1 Validation

To validate this analytical solution, its results have been comprised with previous studies. Since, no study have been yet conducted on the free vibration of thick FG Nano-plate (with power or exponential) based on the third-order shear deformation theory (TSDT) and nonlocal elasticity, so the results obtained from this analysis have been comprised from following references, respectively.

The studies of Aghababaei and Reddy [25] and Hosseini-Hashemi et al [26] on investigation of free vibration of thick homogeneous Nano-plate using third-order shear deformation theory and nonlocal elasticity.

The study of Natarajan et al [22] and Zare et al [27] on investigation of free vibration of thin FG Nano-plate (with power function) using classical theory of plates and nonlocal elasticity.

The study of Salehipour et al [21] on investigation of free vibration of FG Nano-plate (with exponential function) using nonlocal elasticity theory, first order shear deformation theory and three-dimensional elasticity theory.

4.1.1 Comprising the results with references [25] and [26]

The following results were obtained for homogeneous Nano-plate with properties of $E=300GPa, a=10nm$ and dimensionless frequency in the form of $\Omega=\omega a^2\sqrt{\frac{\rho^* h}{D}}$.

The ratio of nonlocal frequency to local frequency (a number between zero and one) in different modes and for different dimensionless nonlocal parameters of $\mathrm{nl}=\frac{\sqrt{\mu}}{\mathrm{a}}$ has been expressed in Table 1.

Table 1
Comparison between nonlocal frequencies ratio for square FG plate with dimensionless parameters according to reference [26].

ModeNum	Refs	nl=0	nl=0.2	nl=0.4	nl=0.6
(1,1)	[26]	1	0.7475	0.4904	0.3512
	Present Study		0.7475	0.4904	0.3512
(1,2)	[26]	1	0.5799	0.3553	0.2308
	Present Study		0.5799	0.3353	0.2308
(1,3)	[26]	1	0.4497	0.244	0.1655
	Present Study		0.4496	0.244	0.1655

Table 2. represents a comparison among dimensionless fundamental frequencies for a homogeneous rectangular FG Nano-plate with $a/b=0.5$ for different dimensionless parameters as well as different ratios of height to length of Nano-plate in different references.

Table 2
Comparison among dimensionless fundamental frequencies for FG Nano-plate with dimensionless parameters by Refs [25] and [26].

h/a	$(nm^2)\mu$	Refs	Ω
0.1	0	[25]	12.1157
		[26]	12.0675
		Present Study	12.0675
	1	[25]	11.4187
		[26]	11.3856
		Present Study	11.3856
	4	[25]	9.9016
		[26]	9.8745
		Present Study	9.8746

4.1.2 Comprising the results with references [22] and [27]

In this reference, the dimensionless frequency of FG plates was obtained based on the classical theory and the theory of nonlocal which its relation is in the form of $\Omega_2 = \omega h\sqrt{\frac{\rho_c}{G_c}}$. The comparison was carried out on a Nano-plate with following properties. Other geometrical and mechanical parameters have been presented as follow:

$$E_c = 348.43GPa, \quad E_m = 201.04GPa, \quad G_c = \frac{E_c}{2(1+\nu)}, \quad \rho_c = 2370, \quad \rho_m = 8166, \quad \nu = 0.3, \quad a = 10nm$$

Firstly, Table 3. was provided to comprise dimensionless frequencies in different dimensional ratios of FG Nano-plate as well as its different nonlocal parameters by fixing exponential index of FG material, its thickness to length ratio equal to 0.05 and power index parameter equal to 5.

Table 3
Comparison between first three peak dimensionless fundamental frequencies for FG plate with dimensionless parameters according to reference [22] and [27].

a/b	Nonlocal	Refs	mode1	mode2	mode3
1	0	[22]	0.0113	0.0278	0.0279
		[27]	0.0114	0.0281	0.0281
		Present Study	0.01133	0.02794	0.02794
	2	[22]	0.0085	0.0161	0.0162
		[27]	0.0085	0.0165	0.0165
		Present Study	0.00847	0.0162	0.0162
2	0	[22]	0.0279	0.044	0.0701
		[27]	0.0281	0.0443	0.0704
		Present Study	0.02794	0.0441	0.07014
	2	[22]	0.0162	0.0216	0.0283
		[27]	0.0165	0.0218	0.0286
		Present Study	0.0162	0.02163	0.02833

The dimensionless frequencies were also in good agreement for another FG Nano-plate with following properties:

$$E_c = 380GPa, \quad E_m = 70GPa, \quad \rho_c = 2702, \quad \rho_m = 3800, \quad \nu = 0.3$$

By fixing all dimensions of Nano-plate, the dimensionless frequency changes of $\Omega_1 = \frac{a^2}{h}\sqrt{\frac{\rho_c}{E_c}}$; were comprised with nonlocal parameter, as well as power index of FG material for two theories in different modes, which its results have been represented in Table 4.

Table 4
Comparison between dimensionless fundamental frequencies for FG plate with dimensionless parameters according to reference [27].

Nonlocal	ζ	Theory	(1,1)	(1,2)	(2,2)	(1,3)
0	0.1	CPT [27]	5.81138	14.3717	22.8824	28.5292
		TSDT	5.52018	13.6247	21.5513	26.6988
	1	CPT [27]	4.62678	11.4209	18.1769	22.6604
		TSDT	3.81261	9.41587	14.8886	18.4687
	10	CPT [27]	3.87362	9.53273	15.1478	18.8672
		TSDT	2.76058	6.78287	10.6742	13.2012
0.1	0.1	CPT [27]	5.32051	11.8058	17.2163	20.4084
		TSDT	5.04469	11.1488	16.0952	18.9407
	1	CPT [27]	4.23696	9.38116	13.6704	16.1995
		TSDT	3.48421	7.70479	11.1296	13.1021
	10	CPT [27]	3.54899	7.83929	11.4151	13.5229
		TSDT	2.52281	5.55027	7.97921	9.36521

4.1.3 Comprising the results with reference [21]

For an exponential FG material, the results of this frequency analysis were also comprised with three-dimensional theory of the plates, as well as with first order of shear deformation in two-dimensional theory of plates. Table 5. represents the dimensionless fundamental frequencies of FG Nano-plate's free vibration in the two primary modes.

Table 5
Comparison between dimensionless fundamental frequencies for FG plate with various power indexes and height to length ratio parameters by reference [21].

Nonlocal	Exp. Index	a/h	Method	First	Second
0	2	5	3D	5.2476	13.777
			FSDT	5.2306	13.777
			TSDT	5.2283	13.777
		10	3D	5.7108	27.554
			FSDT	5.7054	27.554
			TSDT	5.7043	27.554
	3	5	3D	5.1654	13.777
			FSDT	5.1575	13.777
			TSDT	5.1504	13.777
		10	3D	5.6142	27.554
			FSDT	5.6115	27.554
			TSDT	5.6089	27.554
0.3	2	5	3D	4.6081	12.388
			FSDT	4.7032	12.388
			TSDT	4.7011	12.388
		10	3D	5.4922	26.772
			FSDT	5.5436	26.772
			TSDT	5.5426	26.772
	3	5	3D	4.5313	12.388
			FSDT	4.6374	12.388
			TSDT	4.6311	12.388
		10	3D	5.3885	26.772
			FSDT	5.4524	26.772
			TSDT	5.4499	26.772

In general, the correctness and accuracy of present study was validated and confirmed through analyzing and comprising its results with above mentioned references.

4.2 Parametric study of the results

In this section, the effect of fundamental and variable parameters of present study on the vibration`s frequency has been investigated.

4.2.1 The effect of FG material index and nonlocal parameter

In Fig. 2 by linear increasing of power index value of FG material in an unique nonlinear parameter, the dimensionless frequency of Nano-plate's vibrations is decreased in a nonlinear form (it has trended from dimensionless frequency of pure ceramic material toward dimensionless frequency of pure metal material); in a way whatever the power index of FG material is greater, the curve slope of changes is more decreased and the sensitivity of frequency to index changes is reduced.

In a fixed power index, whatever the value of nonlocal parameter is lower, the dimensionless frequency of base mode vibration is lower. Therefore, the blue curve which indicates the mode of local classical theory always estimates the Nano-plate frequency higher than its actual value. This reduction in frequency is more significant when the value of index is lower than 2.

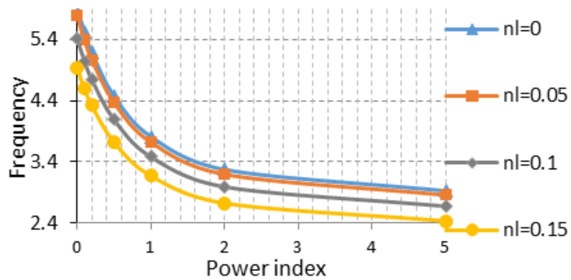

Fig.2
Frequency curves for FG Nano-plate with various power index and nonlocal parameters.

In Fig. 3 according to the horizontal axis which represents the dimensionless parameter value of exponential index, two modes can be considered to the unique nonlocal parameter: in the first mode if the dimensionless index be between zero and one, then by increasing the value from very small amounts, the frequency trends toward a maximum value with a very sharp slope and in a nonlinear form while the exponential index is equal to one. The maximum value varies for any different nonlocal parameters of Nano-plate, but all of them occur in horizontal coordinate of 1. In the second mode for index values of higher than 1, the frequency of base mode is reduced with a slight slope by an increase in the exponential index. Similar to the power mode, nonlocal parameter increasing leads to a nonlinear decrease in dimensionless frequency.

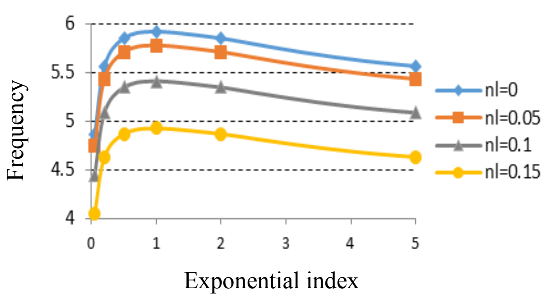

Fig.3
Frequency curves for FG Nano-plate with various exponential index and nonlocal parameters.

5 CONCLUSIONS

In present study, the third-order shear deformation theory has been developed to investigate vibration analysis of FG Nano-plates based on Eringen nonlocal elasticity theory. The materials distribution regarding to the thickness of Nano-plate has been considered based on two different models of power function and exponential function. All equations governing on the vibration of FG Nano-plate have been derived from Hamilton’s principle. It has been also obtained the analytical solution for natural frequencies and corresponding mode shapes of simply supported FG Nano-plates. In addition, the general form of stiffness and mass matrixes' elements has been expressed based on this theory. In addition, the general form of mass matrix elements and thickness of FG Nano-plate have been expressed according to this theory.

The effect of different parameters including power and exponential indexes of FG, nonlocal parameter of Nano-plate, dimensional ratio and thickness to length ratio of Nano-plate on the dimensionless frequency of free vibrations were investigated.

The results showed that an increase in nonlocal parameter, as well as increase in power or exponential indexes of FG Nano-plate leads to a decrease in structure stiffness of Nano-plate-based system which is detectable through reduction in the frequency of system's oscillations.

APPENDIX

The relations related to the elasticity resultants can be written in following form:

$$\left(\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}, \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}, \begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix}\right) = \int_A \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} \left(1, z, z^3\right) dA$$

$$\begin{Bmatrix} Q_{xz} \\ Q_{yz} \\ R_{xz} \\ R_{yz} \end{Bmatrix} = \int_A \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{xz} z^2 \\ \sigma_{yz} z^2 \end{Bmatrix} dA \tag{22}$$

The strains-based elasticity resultants are obtained by replacing related equations:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + \begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + \begin{bmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix} \tag{23}$$

$$\begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + \begin{bmatrix} H_{11} & H_{12} & H_{16} \\ H_{12} & H_{22} & H_{26} \\ H_{16} & H_{26} & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix}$$

where, the high order tensile and flexural stiffness matrixes of FG plate in third-order shear deformation theory are defined in following forms:

$$\left\{A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\right\} = \int_{-h/2}^{h/2} Q_{ij}(z)\left\{1, z, z^2, z^3, z^4, z^6\right\} dz \;, \quad i, j = 1, 2, 6. \tag{24}$$

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