Journal of Solid Mechanics Vol. 2, No. 4 (2010) pp. 393-402

Wave Propagation in Sandwich Panel with Auxetic Core

D. Qing-Tian[1,*], Y. Zhi-Chun[2]

[1]*School of Science, Chang'an University, Xi'an, China*
[2]*School of Aeronautics, Northwestern Polytechnical University, Xi'an, China*

Received 21 December 2010; accepted 30 December 2010

ABSTRACT

Auxetic cellular solids in the forms of honeycombs and foams have great potential in a diverse range of applications, including as core material in curved sandwich panel composite components, radome applications, directional pass band filters, adaptive and deployable structures, filters and sieves, seat cushion material, energy absorption components, viscoelastic damping materials and fastening devices etc. In this paper, the characteristic of wave propagation in sandwich panel with auxetic core is analyzed. A three-layer sandwich panel is considered which is discretized in the thickness direction by using semi-analytical finite element method. Wave propagation equations are obtained through some algebraic manipulation and applying standard finite element assembling procedures. The mechanical properties of auxetic core can be described by the geometric parameters of the unit cell and mechanical properties of the virgin core material. The characteristics of wave propagation in sandwich panel with conventional hexagonal honeycomb core and re-entrant auxetic core are discussed, and effects of panel thickness, geometric properties of unit cell on dispersive curves are also discussed. Variations of Poisson's ratio and core density with inclined angle are presented.

Keywords: Sandwich panel; Auxetic material; Negative Poisson's ratio; Elastic wave; Semi-analytical finite element method.

1 INTRODUCTION

THE Poisson's ratio of a material is defined as the ratio of the lateral contractile strain to the longitudinal tensile strain for a material undergoing tension in the longitudinal direction. Most materials have positive Poisson's ratio, which means the materials contract transversely under unaxial extension, and expand laterally when compressed in one direction. Materials with negative Poisson's ratio exhibit the unusual property of becoming fatter when stretched and thinner when compressed. Therefore, the negative Poisson's ratio has been treated as an abnormally elastic parameter for a long time. Evans etc. [1] termed materials and structures with negative Poisson's ratio as auxetics or auxetic materials.

Compared with other conventional materials, auxetic materials exhibit various enhanced physical and mechanical characteristics from increased indentation, impact and failure resistance to improved acoustic damping properties, and have attracted considerable attention in both functional and structural materials applications in recent years. Alderson and Alderson [2] presented a review covered aspects of auxetic materials that are considered most relevant to aerospace engineering applications, and discussed the potential of auxetics as strain amplifiers, piezoelectric devices, and structural health monitoring components. Hadjigeorgiou and Stavroulakis [3] introduced auxetic materials into the design of smart structure, and the problem of the shape control of sandwich beams is

[*] Corresponding author.
E-mail address: deng_qingtian@yahoo.com.cn (D. Qing-Tian).

analyzed under loading conditions by using auxetic materials as core and piezoelectric actuators as face layers to provide control forces. Yu and Cleghorn [4] investigated the free flexural vibration problem of symmetric rectangular honeycomb panels with simple support boundary conditions by using three different plate theories, in which the honeycomb core of hexagonal cells is modeled as a thick layer of orthotropic material. Remillat et al. [5] analyzed the problem of Lamb wave propagation in cross-ply laminate composites with through-the-thickness negative Poisson's ratio by using classical laminate theory. Tee et al. [6] investigated the flexural wave propagation properties of a novel class of negative Poisson's ratio honeycombs with tetrachiral topology numerically and experimentally.

Wan et al. [7] developed a theoretical approach to predict negative Poisson's ratios of auxetic honeycombs based on the large deflection model, which is also suitable to small flexure and elastic buckling. They found that the Poisson's ratio of auxetic honeycombs are not a constant at large deformation and vary significantly with the strain. Scarpa and Tomlinson [8] analyzed vibration of sandwich plates with in-plane negative Poisson's ratio values based on the first order sandwich plate theory, in which the anisotropic mechanical properties are described by using the cellular material theory. They found that the dynamic performance of a sandwich structure could be significantly improved with a proper design of the unit cell shape of the honeycomb. Ruzzene et al [9, 10], Ruzzene and Scarpa [11] studied the wave propagation problem in sandwich beam and plate with auxetic core, and two dimensional cellular structures. Their results presented some valuable application of auxetic materials. For auxetic honeycombs based on the deformation of the honeycomb cells influenced by flexure, stretching and hinging, the theoretical model has been developed to predict the elastic constants of honeycombs and derive expressions for the tensile moduli, shear moduli [12] and Poisson's ratios.

This paper discusses elastic wave propagation characteristics in sandwich panel comprised top and bottom outer skin layers bonded to an internal core material by adhesive interface layer. And both conventional hexagonal honeycomb with positive Poisson's ratio and re-entrant auxetic honeycomb with negative Poisson's ratio are considered to be the core materials. Effects of configuration of unit cell to dispersive property are discussed.

2 SANDWICH PANEL MODEL

A typical component used in aerospace applications-the sandwich panel composite, is considered, which comprises top and bottom outer skin layers bonded to an internal core material by adhesive interface layer. The outer skins are typically made out of aluminum or fibre-reinforced composite laminate material and provide in-plane strength and stiffness as well as protection to the internal materials. The core material provides out-of-plane strength and stiffness for low weight, which is typically a foam or a honeycomb, and can be metal, polymer, fibre-reinforced polymer etc. [2]. An infinite sandwich panel is considered in this paper, as shown in Fig.1 which includes top and bottom outer skin and internal core material. The bottom outer skin thickness is H_1, internal honeycomb core material thickness is H_2 and top outer skin thickness is H_3, and the total thickness of the sandwich panel is $H=H_1+H_2+H_3$.

The displacement, stress and strain fields are

$$\mathbf{u}=\begin{bmatrix}u & v & w\end{bmatrix}^{\mathrm{T}} \tag{1}$$

$$\boldsymbol{\sigma}=\begin{bmatrix}\sigma_x & \sigma_y & \sigma_z & \sigma_{yz} & \sigma_{xz} & \sigma_{xy}\end{bmatrix}^{\mathrm{T}} \tag{2}$$

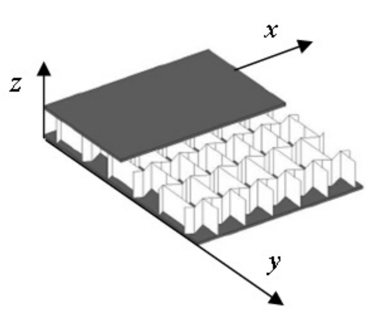

Fig. 1
Model of sandwich panel with auxetic core.

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \varepsilon_z & \varepsilon_{yz} & \varepsilon_{xz} & \varepsilon_{xy} \end{bmatrix}^{\mathrm{T}} \tag{3}$$

where u, v, w are displacement components in x, y, z directions, σ_{ij} is the stress components and ε_{ij} is the strain components. The constitutive relations of the material are

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon} \tag{4}$$

where $\mathbf{C}$ is the stiffness matrix. According to Hamilton principle [13],

$$\delta H = \int_{t_1}^{t_2} \delta(\Phi - K)\ \mathrm{d}t \tag{5}$$

where Φ is the strain energy, K is the kinetic energy and

$$\begin{aligned} \Phi &= \frac{1}{2}\int_V \boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{C}\boldsymbol{\varepsilon}\ \mathrm{d}V \\ K &= \frac{1}{2}\int_V \dot{\mathbf{u}}^{\mathrm{T}}\rho\dot{\mathbf{u}}\ \mathrm{d}V \end{aligned} \tag{6}$$

Substituting the expressions of strain energy and kinetic energy in to the Hamilton equation

$$\int_{t_1}^{t_2}\left[\int_V \delta(\boldsymbol{\varepsilon}^{\mathrm{T}})\mathbf{C}\boldsymbol{\varepsilon}\ \mathrm{d}V + \int_V \delta(\mathbf{u}^{\mathrm{T}})\rho\ddot{\mathbf{u}}\ \mathrm{d}V\right]\mathrm{d}t = 0 \tag{7}$$

3 SEMI-ANALYTICAL FINITE ELEMENT METHOD

Assuming the elastic wave propagation along the *x*-direction, the discretization procedure is performed in the panel thickness direction z by a set of one-dimensional finite elements with quadratic shape functions and three nodes, with three degrees of freedom per node, as shown in Fig.2 where z_1, z_2, z_3 are coordinates of nodes 1, 2, 3 along *z*-direction. The displacement vector can be approximated over the element domain as [14]

$$\mathbf{u}^{(e)}(x,y,z,t) = \begin{bmatrix} \sum_{j=1}^{3} N_j(z)U_{xj} \\ \sum_{j=1}^{3} N_j(z)U_{yj} \\ \sum_{j=1}^{3} N_j(z)U_{zj} \end{bmatrix}^{(e)} \mathrm{e}^{i(kx-\omega t)} = \mathbf{N}(z)\mathbf{q}^{(e)}\mathrm{e}^{i(kx-\omega t)} \tag{8}$$

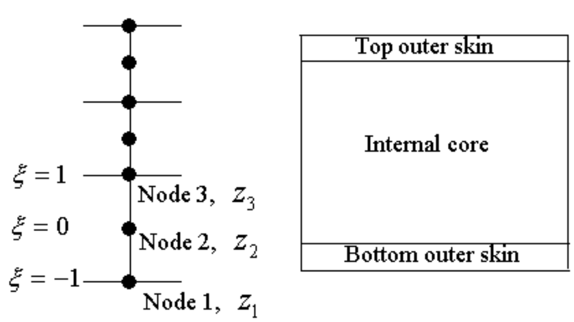

Fig. 2
Discretization of sandwich panel.

where $N_j(z)$ is the shape functions, U_{xj}, U_{yj}, U_{zj} are the unknown nodal displacements in the x_1, x_2, x_3 directions, k is wave vector component in *x*-direction, ω is the circular frequency and

$$\mathbf{N}(z) = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 \end{bmatrix} \tag{9}$$

$$z = \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}, \quad \begin{cases} N_1 = \dfrac{1}{2}(\xi^2 - \xi) \\ N_2 = (1 - \xi^2) \\ N_3 = \dfrac{1}{2}(\xi^2 + \xi) \end{cases} \tag{10a}$$

$$\mathbf{q}^{(e)} = \begin{bmatrix} U_{x1} & U_{y1} & U_{z1} & U_{x2} & U_{y2} & U_{z2} & U_{x3} & U_{y3} & U_{z3} \end{bmatrix}^{\mathrm{T}} \tag{10b}$$

The strain vector in the element can be represented as a function of the nodal displacements [14]

$$\boldsymbol{\varepsilon}^{(e)} = \left[\mathbf{L}_x \frac{\partial}{\partial x} + \mathbf{L}_y \frac{\partial}{\partial y} + \mathbf{L}_z \frac{\partial}{\partial z} \right] \mathbf{N}(z) \mathbf{q}^{(e)} \mathrm{e}^{\mathrm{i}(kx - \omega t)} = (\mathbf{B}_1 + \mathrm{i}k\mathbf{B}_2) \mathbf{q}^{(e)} \mathrm{e}^{\mathrm{i}(kx - \omega t)} \tag{11}$$

where

$$\mathbf{L}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{L}_y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{L}_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B}_1 = \mathbf{L}_z \mathbf{N}_{,z}, \quad \mathbf{B}_2 = \mathbf{L}_x \mathbf{N}, \quad \mathbf{N}_{,z} = \mathbf{N}_{,\xi} \frac{d\xi}{dz} \tag{12}$$

By considering the total elements in the thickness, Hamilton formulation becomes to

$$\int_{t_1}^{t_2} \left\{ \bigcup_{e=1}^{3} \left[\int_V \delta(\boldsymbol{\varepsilon}^{(e)\mathrm{T}} \mathbf{C}^{(e)} \boldsymbol{\varepsilon}^{(e)} \mathrm{d}V + \int_V \delta(\mathbf{u}^{(e)\mathrm{T}}) \rho^{(e)} \ddot{\mathbf{u}}^{(e)} \mathrm{d}V \right] \right\} \mathrm{d}t = 0 \tag{13}$$

where $\mathbf{C}^{(e)}$ and $\rho^{(e)}$ are the element stiffness matrix and mass density, respectively. By substituting Eqs. (8) and (11) into Eq. (6) and some algebraic manipulation, the element strain energy and kinetic energy can be expressed as follow

$$\int_V \delta(\boldsymbol{\varepsilon}^{(e)\mathrm{T}}) \mathbf{C}^{(e)} \boldsymbol{\varepsilon}^{(e)} \mathrm{d}V = \delta \mathbf{q}^{(e)\mathrm{T}} \int_z \left[\mathbf{B}_1^{\mathrm{T}} \mathbf{C}^{(e)} \mathbf{B}_1 - ik\mathbf{B}_2^{\mathrm{T}} \mathbf{C}^{(e)} \mathbf{B}_1 + ik\mathbf{B}_1^{\mathrm{T}} \mathbf{C}^{(e)} \mathbf{B}_2 + k^2 \mathbf{B}_2^{\mathrm{T}} \mathbf{C}^{(e)} \mathbf{B}_2 \right] \mathbf{q}^{(e)} \ \mathrm{d}z \tag{14}$$

$$\int_V \delta(\mathbf{u}^{(e)\mathrm{T}}) \rho^{(e)} \ddot{\mathbf{u}}^{(e)} \ \mathrm{d}V = \int_z \int_x \delta(\mathbf{u}^{(e)\mathrm{T}}) \rho^{(e)} \ddot{\mathbf{u}}^{(e)} \ \mathrm{d}x \ \mathrm{d}z = -\omega^2 \delta \mathbf{q}^{(e)\mathrm{T}} \int_z \mathbf{N}^{\mathrm{T}} \rho^{(e)} \mathbf{N} \mathbf{q}^{(e)} \ \mathrm{d}z \tag{15}$$

Then, substituting Eqs. (14)-(15) into Eq. (13) yields

$$\int_{t_1}^{t_2} \left\{ \bigcup_{e=1}^{3} \delta \mathbf{q}^{(e)} \left[\mathbf{k}_1^{(e)} + \mathrm{i}k\mathbf{k}_2^{(e)} + k^2 \mathbf{k}_3^{(e)} - \omega^2 \mathbf{m}^{(e)} \right] \mathbf{q}^{(e)} \right\} \mathrm{d}t = 0 \tag{16}$$

where

$$\begin{aligned}
\mathbf{k}_1^{(e)} &= \int_z \left[\mathbf{B}_1^{\mathrm{T}}\mathbf{C}^{(e)}\mathbf{B}_1\right] \mathrm{d}z \\
\mathbf{k}_2^{(e)} &= \int_z \left[\mathbf{B}_1^{\mathrm{T}}\mathbf{C}^{(e)}\mathbf{B}_2 - \mathbf{B}_2^{\mathrm{T}}\mathbf{C}^{(e)}\mathbf{B}_1\right] \mathrm{d}z \\
\mathbf{k}_3^{(e)} &= \int_z \left[\mathbf{B}_2^{\mathrm{T}}\mathbf{C}^{(e)}\mathbf{B}_2\right] \mathrm{d}z \\
\mathbf{m}^{(e)} &= \int_z \left[\mathbf{N}^{\mathrm{T}}\rho^{(e)}\mathbf{N}\right] \mathrm{d}z
\end{aligned} \tag{17}$$

Applying standard finite element assembling procedures to Eq. (16)

$$\int_{t_1}^{t_2} \left\{\delta\mathbf{U}^{\mathrm{T}}\left[\mathbf{K}_1 + \mathrm{i}k\mathbf{K}_2 + k^2\mathbf{K}_3 - \omega^2\mathbf{M}\right]\mathbf{U}\right\} \mathrm{d}t = 0 \tag{18}$$

where U is the global vector of unknown nodal displacements and

$$\mathbf{K}_1 = \bigcup_{e=1}^{m}\mathbf{k}_1^{(e)},\ \ \mathbf{K}_2 = \bigcup_{e=1}^{m}\mathbf{k}_2^{(e)},\ \ \mathbf{K}_3 = \bigcup_{e=1}^{m}\mathbf{k}_3^{(e)},\ \ \mathbf{M} = \bigcup_{e=1}^{m}\mathbf{m}^{(e)} \tag{19}$$

Due to the arbitrariness of δU in Eq. (18), the following wave equation is obtained

$$\left[\mathbf{K}_1 + \mathrm{i}k\mathbf{K}_2 + k^2\mathbf{K}_3 - \omega^2\mathbf{M}\right]\mathbf{U} = 0 \tag{20}$$

where U is the global vector of nodal displacement components. When the waves propagate in the panel, there are the non-zero solutions of nodal displacement in Eq. (20) which needs the determinations of the coefficients matrices of Eq. (20) .equal to zero

$$\det\left|\mathbf{K}_1 + \mathrm{i}k\mathbf{K}_2 + k^2\mathbf{K}_3 - \omega^2\mathbf{M}\right| = 0 \tag{21}$$

The eigenvalue problem in Eq. (21) relates the wavenumber k to the frequency ω, one of them being given and the other being the eigenvalue to be solved. If k is given, Eq. (21) is a eigenvalue problem with m real eigenvalues ω^2. If instead ω is given, Eq. (21) is a eigenvalue problem with $2m$ eigenvalues k. The relation between wave length and wave number is $\lambda = 2\pi/k$.

4 HONEYCOMB CORE MATERIALS

Two different honeycomb core materials, as shown in Fig.3 are introduced in this paper, the conventional hexagonal honeycomb with positive Poisson's ratio and re-entrant auxetic honeycomb with negative Poisson's ratio. The deformation configurations of the two materials under unaxial extension are also presented. The two honeycomb materials belong to cellular structure. According to Cellular Material Theory [16], mechanical property of cellular structure can be determined by geometric property of unit cell and mechanical behavior of origin material. Unit cells of two core materials discussed in the paper are shown in Fig.4 where l is the length of the inclined cell rib, h is the length of the vertical cell rib, θ is the thickness of the cell rib, θ is the inclined angle, α and β define the relative cell wall length and the wall's slenderness ratio, respectively, which are important parameters in honeycomb property. Formulas in reference [16] are adopted for calculation of honeycomb core material property.

$$\rho_c = \rho\frac{t/l(h/l+2)}{2\cos\theta(h/l+\sin\theta)} \tag{22}$$

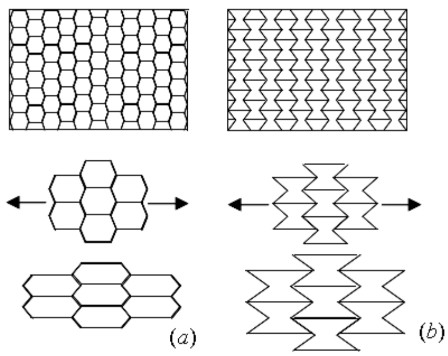

Fig. 3
The honeycomb core in the sandwich plate (*a*) The conventional hexagonal honeycomb model (*b*) The re-entrant hexagonal honeycomb model.

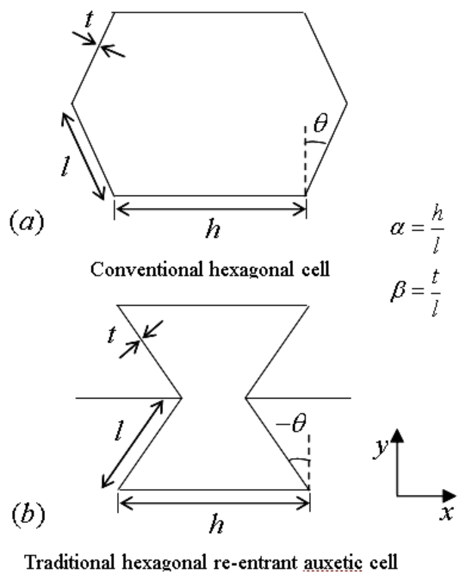

Fig. 4
Geometric of the cell of honeycomb core.

$$E_x^c = E\left(\frac{t}{l}\right)^3 \frac{\cos\theta}{(h/l+\sin\theta)\sin^2\theta} \tag{23}$$

$$E_y^c = E\left(\frac{t}{l}\right)^3 \frac{(h/l+\sin\theta)}{\cos^3\theta} \tag{24}$$

$$\nu_{xy}^c = \frac{\cos^2\theta}{(h/l+\sin\theta)\sin\theta} \tag{25}$$

$$G_{xy}^c = E\left(\frac{t}{l}\right)^3 \frac{(h/l+\sin\theta)}{(h/l)^2(1+2h/l)\cos\theta} \tag{26}$$

$$G_{xz}^c = G\left(\frac{t}{l}\right)\frac{\cos\theta}{h/l+\sin\theta} \tag{27}$$

where symbol "*c*" represents core material, E, G and ρ are Young's moduli, shear moduli and mass density of the origin material. As Gibson and Ashby [16] pointed out each cell face of the sheared honeycombs subjected to a non-uniform deformation, in which the initially plane honeycomb may not remain plane during loading. So, the transverse shear modulus G_{yz}^c of the core material is one important mechanical parameter in sandwich constructions, which can be evaluated from theorems of minimum potential and complementary strain energy by upper Voigt and lower Reuss bounds according to procedure in [15, 16].

$$G_{yz}^{c} = G_{yz}^{c-lower} + \frac{K}{H/l}(G_{yz}^{c-upper} - G_{yz}^{c-lower}) \tag{28}$$

where

$$K = \begin{cases} 0.787 & \theta \geq 0 \\ 1.342 & \theta < 0 \end{cases} \tag{29}$$

The actual value of the transverse shear modulus is dependent on the gauge thickness to length ratio H/l of the honeycomb itself. By considering the anisotropic property of honeycomb core material exhibited, stiffness matrix C of internal core is obtained through the relation between stiffness matrix and Young's moduli, shear modulus, Poisson's ratio in reference [17].

5 NUMERICAL CALCULATION

Considering the top and bottom outer skins are isotropic aluminium material, thickness of both outer skins is $H_1 = H_3 = 1$ mm , and material parameters are: Young's moduli is $E = 69$ GPa, shear moduli is $G = 26$ GPa , Poisson's ratio is $\nu = 0.33$, density is $\rho = 2700\,\mathrm{kg/m^3}$. The internal core is produced as honeycomb structure by using the same aluminum material, the thickness of the core is $H_2 = 3\,\mathrm{mm}$. The relevant material property can be obtained from reference [16]. Figs. 5-6 plotted the effects of inclined angle θ of the unit cell to Poisson's ratio and density of the honeycomb structure with different values of α and $\beta = 0.0138571$. It can be seen that accompany with inclined angle turn to be negative values gradually, the Poisson's ratio also come with negative values. And density of auxetic honeycomb can be much higher than conventional hexagonal honeycomb with lower α value.

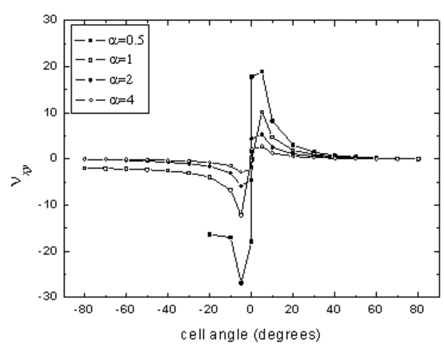

Fig. 5
Poisson's ratio vs. cell inclined angle with different cell aspect ratio α.

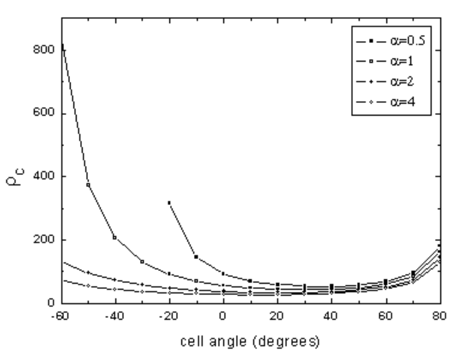

Fig. 6
Density vs. cell inclined angle with different cell aspect ratio α.

Fig. 7 presented the dispersive curves of wave in sandwich panel with conventional honeycomb core material $(\theta = 30^{\circ})$ and auxetic core material $(\theta = -30^{\circ})$. It can be seen that the curves located in two main zones, where both curves are close to each other in one lower zone and curves of regular hexagonal core are much higher than auxetic core in another zone. Fig. 8 presented effects of sandwich panel gauge thickness (H=3mm, 5mm, 7mm) to dispersive curves, which indicated the significant dependence of frequency over the gauge thickness ratio H/l. It can be seen that the curves move upwards with the decrease of gauge thickness H. Fig. 9 presented effects of inclined angle θ to dispersive curves. Considering results in Fig.7, it can be seen that the dispersive curves move upwards the curves with the increase of inclined angle θ from negative values to positive values. And with the increase of negative inclined angle θ, the curves move to the lower zone.

Fig.10 presented effects of length ratio α of vertical rib and inclined rib to dispersive curves with $\theta = -30^{\circ}$ and $\beta = 0.0138571$. According to the results of $\alpha = 1,2,3$ it can be seen that α showed relatively significant effect on dispersive curves. And with the increase of α, the curves move upwards.

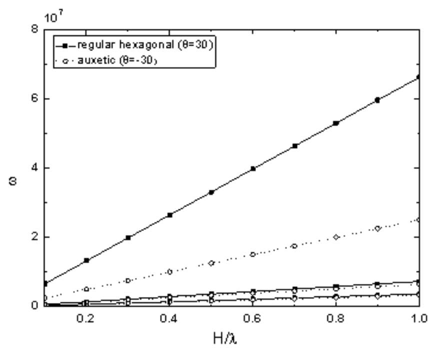

Fig. 7
Dispersive curves of the sandwich panel with auxetic and regular honeycomb cores.

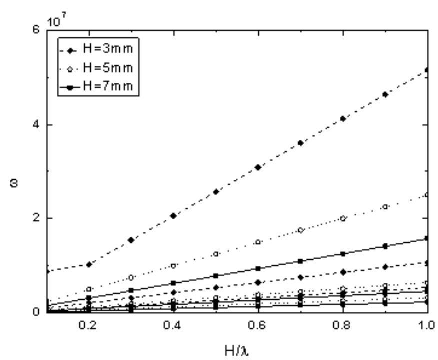

Fig. 8
Effects of sandwich thickness to the spectrum.

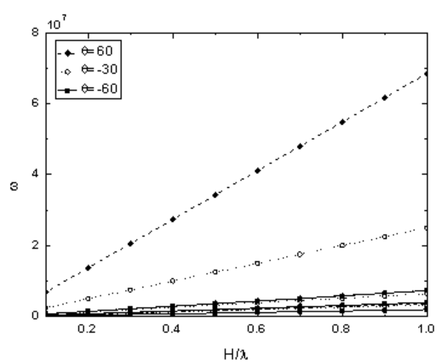

Fig. 9
Effects of inclined angle to the spectrum of the sandwich panel.

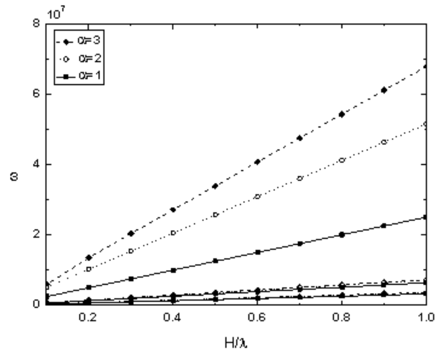

Fig. 10
Effects of cell aspect ratio to the spectrum of the sandwich panel.

6 CONCLUSIONS

Auxetic materials have exhibited great potential in aerospace engineering because of their negative Poisson's ratio. Based on semi-analytical finite element method and cellular material theory, elastic wave propagation in sandwich panel with honeycomb core material has been analyzed in this paper. Both conventional hexagonal honeycomb with positive Poisson's ratio and re-entrant auxetic honeycomb with negative Poisson's ratio are chosen to be the core materials. Effects of geometric property of unit cell to dispersive characteristic are discussed. The numerical results showed that sandwich panel gauge thickness, length ratio of horizontal rib to inclined rib, and inclined angle of unit cell have much more effects on dispersive property.

ACKNOWLEDGMENTS

This work is supported by the Ph.D. Programs Foundation of Ministry of Education of China (Grant No. 20060699001) and the Project of New Century Excellent Talents of China (Grant No. NCET-04-0965) and National Aerospace Science Foundation of China (04I53072). The authors thank the anonymous reviewers for their helpful suggestions.

REFERENCES

[1] Evans K.E., Nkansah M.A., Hutchinson I.J., Rogers S.C., 1991, Molecular network design, *Nature* **353**: 12-125.

[2] Alderson A., Alderson K.L., 2007, Auxetic materials, *Proceeding of Institute of Mechanical Engineers, Part G: Journal of Aerospace Engineering* **221**: 565-575.

[3] Hadjigeorgiou E.P., Stavroulakis G.E., 2004, The use of auxetic materials in smart structures, *Computational Methods in Science and Technology* **10**(2): 147-160.

[4] Yu S.D., Cleghorn W.L., 2005, Free flexural vibration analysis of symmetric honeycomb panels, *Journal of Sound and Vibration* **284**: 189-204.

[5] Remillat C., Wilcox P., Scarpa F., 2008, Lamb wave propagation in negative Poisson's ratio composites, *Proceedings of SPIE* 6935.

[6] Tee K.F., Spadoni A., Scarpa F., Ruzzene M., 2010, Wave propagation in auxetic tetrachiral honeycombs, *Journal of Vibration and Acoustics* **132**: 031007.

[7] Wan H., Ohtaki H., Kotosaka S., Hu G.M., 2004, A study of negative Poisson's ratios in auxetic honeycombs based on a large deflection model, *European Journal of Mechanics A/Solids* **23**: 95-106.

[8] Scarpa F., Tomlinson G., 2000, Theoretical characteristics of the vibration of sandwich plates with in-plane negative Poisson's ratio values, *Journal of Sound and Vibration* 2**30**(1): 45-67.

[9] Ruzzene M., Mazzarella L., Tsopelas P., Scarpa F. 2002, Wave propagation in sandwich plates with periodic auxetic core, *Journal of Intelligent Material Systems and Structures* **13**(9): 587-597.

[10] Ruzzene M., Scarpa F., 2003, Control of wave propagation in sandwich beams with auxetic core, *Journal of Intelligent Materials Systems and Structures* **1**(7): 448-468.

[11] Ruzzene M., Scarpa F., Soranna, F. 2003, Wave beaming effects in two dimensional cellular structures, *Smart Materials and Structures* **12**(3): 363-372.

[12] Lira C., Innocenti P., Scarpa F. 2009, Transverse elastic shear of auxetic multi-reentrant honeycombs, *Composite Structures* **90**(3): 314-322.

[13] Reddy J.N., 2002, *Energy Principle and Variational Methods in Applied Mechanics*, Second Edition, John Wiley, New York.

[14] Cook R.D., 2001, *Concepts and Applications of Finite Element Analysis*, John Wiley, New York.

[15] Scarpa F., Tomlin P.J. 2000, On the transverse shear modulus of negative Poisson's ratio honeycomb structures, *Fatigue and Fracture Engineering Materials and Structures* **23**: 717-720.

[16] Gibson L.J., Ashby M.F., 1997, *Cellular Solids: Structure and Properties*, Cambridge University Press, Cambridge, UK, Second Edition.

[17] Decolon C., 2002, *Analysis of Composite Structures*, Hermes Penton Science, London.