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Three-dimensional Free Vibration Analysis of a Transversely Isotropic Thermoelastic Diffusive Cylindrical Panel

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ABSTRACT

The present paper is aimed to study an exact analysis of the free vibrations of a simply supported, homogeneous, transversely isotropic, cylindrical panel based on three-dimensional generalized theories of thermoelastic diffusion. After applying the displacement potential functions in the basic governing equations of generalized thermoelastic diffusion, it is noticed that a purely transverse mode is independent of thermal and concentration fields and gets decoupled from the rest of motion. The equations for free vibration problem are reduced to four equations of second-order and one fourth-order ordinary differential equation after expanding the displacement potential, temperature change and concentration functions with an orthogonal series. The formal solution of this system of equations can be expressed by using modified Bessel function with complex arguments. The numerical results for lowest frequency have been obtained and presented graphically. The effect of diffusion on lowest frequency has also been presented graphically. Some special cases of secular equation are also discussed.

Keywords: Cylindrical panel; Thermoelastic diffusion; Circumferential wave number; Secular equations; Free vibrations; Lowest frequency.

1 INTRODUCTION

THE Classical theory of thermoelasticity is based on the Fourier's heat conduction equation. The Fourier's heat conduction theory assumes that the thermal disturbances propagate at infinite speed which is unrealistic from the physical point of view. Two different generalizations of the classical theory of thermoelasticity have been developed which predict only finite velocity of propagation for heat and displacement fields. The first one is given by Lord and Shulman [14] which incorporates a flux rate term into the Fourier's law of heat conduction and formulates a generalized theory admitting finite speed for thermal signals. The second is given by Green and Lindsay [9] which develops a temperature rate dependent thermoelasticity by including temperature rate among the constitutive variables, which does not violate the classical Fourier's law of heat conduction.

The spontaneous movement of the particles from a high concentration region to the low concentration region is defined as diffusion. The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment. Nowacki [16-19] developed the theory of thermoelastic diffusion using coupled thermoelastic model. Gawinecki and Szymaniec [8] proved a theorem about global existence of the solution for a nonlinear parabolic thermoelastic diffusion problem. Gawinecki et al. [7] proved a theorem about existence, uniqueness and regularity of the solution for the same problem. Uniqueness and reciprocity theorems for the equations of generalized thermoelastic diffusion problem, in isotropic media, was proved by Sherief et al. [21] on the basis of the variational principle equations, under restrictive

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assumptions on the elastic coefficients. Due to the inherit complexity of the derivation of the variational principle equations, Aouadi [1] proved this theorem in the Laplace transform domain, under the assumption that the functions of the problem are continuous and the inverse Laplace transform of each is also unique. Recently, Aouadi [2] derived the uniqueness and reciprocity theorems for the generalized problem in anisotropic media, under the restriction that the elastic, thermal conductivity and diffusion tensors are positive definite. Sherief and Saleh [22] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Kumar and Kansal [13] developed the basic equations of anisotropic thermoelastic diffusion based upon Green-Lindsay model.

The cylindrical panels are used as structural components, and their vibration characteristics are important for practical design. For the last 40-50 years, anisotropic materials have been widely used in many areas because of their excellent static and dynamic behavior and low strength-to-weight ratio. Many researches [3, 12, 24, 25] have devoted themselves to the vibration analysis of anisotropic cylindrical shells and/or panels, basing their works mainly on two-dimensional approximate theories or numerical methods. Soldatos and Hadhgeorgious [26]used an iterative approach to predict the frequencies of isotropic cylindrical shells and panels based on the governing equations of three-dimensional elasticity. So and Leissa [23] studied free vibrations of thick hollow cylinders by the Ritz method. Jiang [11] employed the perturbation method to study three-dimensional vibrations of fiber reinforced composite laminated cylindrical shells, while Fan and Ding [6] and Ye and Soldatos [27] used a state space method to analyze laminated orthotropic cylindrical shells and cross-ply cylindrical panels, respectively. Mirsky [15] obtained a Bessel function solution for simply supported transversely isotropic cylindrical shells. Chau [5] employed a similar method to study the free vibration problems of simply supported transversely isotropic cylinders. Ip et al. [10] presented an exact analysis of the free vibrations of a simply supported, transversely isotropic cylindrical panel by using a modified Bessel function solution with complex arguments for the sake of complex eigen values, and clarified the correctness and effectiveness of their method by presenting numerical examples and comparing their results with existing papers. Sharma [20] presented an exact analysis of the free vibrations of a simply supported homogeneous, transversely isotropic, cylindrical panel, in the context of various generalized theories of thermoelasticity.

In this paper, the free vibration analysis of a simply supported, homogeneous, transversely isotropic, cylindrical panel, based on three-dimensional generalized theories of the thermoelastic diffusion, have been studied. To solve the equations of motion, heat conduction and mass diffusion, three displacement potential functions [Buchwald [4]] are used. The outstanding advantage of displacement function approach is that it can be used to obtain solution to problems under any boundary conditions prescribed in terms of either stress or displacement or any combination of these. After solving the equations, it is observed that the purely transverse wave gets decoupled from rest of motion and remains independent of thermal and concentration fields. The model of the free vibration problem is reduced to a system of four equations of second-order and one fourth-order ordinary differential equation after expanding the displacement potentials, temperature and concentration functions with an orthogonal series. The Bessel functions with complex arguments are used to express the solution of this system of differential equations. The numerical solution of secular equation has been carried out to compute the lowest frequency in order to illustrate the analytical results. This type of work is generally applicable to circular cylindrical panels of arbitrary thickness, from thin shell to extremely thick ones and in particularly this can be used in applications involving aerospace, offshore, submarine structures, pressure vessels, civil engineering structures, chemical pipes, and even automotive suspension components. It can also be used to check the applicability of various kinds of two-dimensional shell theory in elastokinetics.

2 FORMULATION AND SOLUTION OF THE PROBLEM

Let us consider a homogeneous, transversely isotropic, thermoelastic diffusive cylindrical panel of length L at uniform temperature T_0 in the undisturbed state initially. Let Γ_1 and Γ_2 the inner and outer radii of cylindrical panel respectively and ς be the central angle (see Fig.1). Following Sherief et al. [21] and Kumar and Kansal [13], the basic equations for homogeneous anisotropic generalized thermoelastic diffusion in the absence of body forces, heat and mass diffusion sources are:

(i) Stress-strain-temperature-concentration relation

$$\sigma_{ij} = c_{ijkm}e_{km} + a_{ij}(T+\tau_1\dot{T}) + b_{ij}(C+\tau^1\dot{C}) \tag{1}$$

(ii) Entropy relation

$$\rho T_0 S = k + \rho C_E(T+\alpha\dot{T}) - a_{ij}T_0 e_{ij} + aT_0(C+\beta\dot{C}) \tag{2}$$

(iii) Chemical potential relation

$$P = b_{km}e_{km} + b(C+\tau^1\dot{C}) - a(T+\tau_1\dot{T}) \tag{3}$$

(iv) Equations of motion

$$\sigma_{ij,j} = \rho\ddot{u}_i \tag{4}$$

(v) Equation of heat conduction

$$\rho C_E(\dot{T}+\tau_0\ddot{T}) - a_{ij}T_0(\dot{e}_{ij}+\varepsilon\tau_0\ddot{e}_{ij}) + aT_0(\dot{C}+\gamma\ddot{C}) = K_{ij}T_{,ij} \tag{5}$$

(vi) Equation of mass diffusion

$$\frac{\partial(\alpha^*_{ij}P_{,j})}{\partial x_i} = \dot{C} + \varepsilon\tau^0\ddot{C} \tag{6}$$

where $c_{ijkm}(c_{ijkm} = c_{kmij} = c_{jikm} = c_{ijmk})$ are elastic parameters. $a_{ij}(=a_{ji})$, $b_{ij}(=b_{ji})$ are tensors of thermal and diffusion moduli, respectively, ρ and C_E are, respectively, the density and specific heat at constant strain; a and b are, respectively, coefficients describing the measure of thermodiffusion effects and of diffusion effects, and T_0 is the reference temperature assumed to be such that $|T/T_0| << 1, T(x_1,x_2,x_3,t)$ is the temperature change, and C is the concentration, τ^0 and τ^1 are diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$ and τ_0 and τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$, $\sigma_{ij}(=\sigma_{ji})$, $K_{ij}(=K_{ji})$, $e_{ij}(=e_{ji}) = (u_{i,j}+u_{j,i})/2$ are components of stress, thermal conductivity, and strain tensor, respectively, u_i are the components of displacement vector $\vec{u}$. $\alpha^*_{ij}(=\alpha^*_{ji})$ are diffusion parameters, P and S are the chemical potential and entropy per unit mass, respectively, k is a material constant. Here, $\alpha = \beta = \varepsilon = \gamma = k = \tau_0 = \tau^0 = \tau_1 = \tau^1 = 0$ for Conventional Theory (CT) model, $\alpha = \beta = k = \tau_1 = \tau^1 = 0$, $\varepsilon = 1$, $\gamma = \tau_0$ for Lord-Shulman (L-S) model and $\alpha = \tau_0$, $\beta = \tau^0$, $\varepsilon = 0$, $\gamma = \tau_0$ for Green-Lindsay (G-L) model. The symbols "," and "." correspond to partial and time derivatives, respectively.

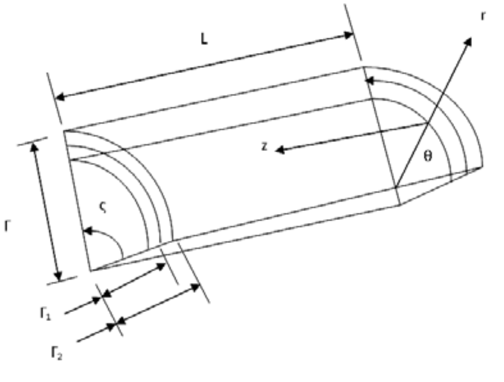

Fig. 1
Geometry of the problem.

Applying the transformation:

$$x' = x\cos\phi + y\sin\phi, \qquad y' = -x\sin\phi + y\cos\phi, \qquad z' = z \tag{7}$$

where ϕ is the angle of rotation in the *x-y* plane and using the relation $x = r\cos\theta,\ y = r\sin\theta$ and *z=z*, in the Eqs. (4)-(6) , we obtain the basic equations for homogeneous, transversely isotropic, generalized thermodifffusive elastic solid in cylindrical coordinates as:

$$\begin{aligned}
&\sigma_{rr,r} + \frac{1}{r}\sigma_{r\theta,\theta} + \sigma_{rz,z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho\ddot{u},\\
&\sigma_{r\theta,r} + \frac{1}{r}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + \frac{2}{r}\sigma_{r\theta} = \rho\ddot{v},\\
&\sigma_{rz,z} + \frac{1}{r}\sigma_{\theta z,\theta} + \sigma_{zz,z} + \frac{\sigma_{rz}}{r} = \rho\ddot{w}
\end{aligned} \tag{8}$$

$$K_1\nabla_1^2 T + K_3 T_{,zz} - \rho C_E(\dot{T} + \tau_0\ddot{T}) = T_0\left(\frac{\partial}{\partial t} + \varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)(a_1(e_{rr} + e_{\theta\theta}) + a_3 e_{zz}) + aT_0(\dot{C} + \gamma\ddot{C}) \tag{9}$$

$$\begin{aligned}
&\alpha_1^*[b_1\nabla_1^2(e_{rr} + e_{\theta\theta}) + b_3\nabla_1^2 e_{zz}] + \alpha_3^*[b_1(e_{rr,zz} + e_{\theta\theta,zz}) + b_3 e_{zz,zz}] + \alpha_1^* a[(1 + \tau_1\frac{\partial}{\partial t})\nabla_1^2 T] +\\
&\qquad \alpha_3^* a[(1 + \tau_1\frac{\partial}{\partial t})T_{,zz}] - \alpha_1^* b[(1 + \tau^1\frac{\partial}{\partial t})\nabla_1^2 C] - \alpha_3^* b[(1 + \tau^1\frac{\partial}{\partial t})C_{,zz}] + \dot{C} + \varepsilon\tau^0\ddot{C} = 0,
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
&\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - a_1(T + \tau_1\dot{T}) - b_1(C + \tau^1\dot{C}),\ \sigma_{r\theta} = c_{66}e_{r\theta},\\
&\sigma_{\theta\theta} = c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - a_1(T + \tau_1\dot{T}) - b_1(C + \tau^1\dot{C}),\ \sigma_{\theta z} = c_{44}e_{\theta z},\\
&\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - a_3(T + \tau_1\dot{T}) - b_3(C + \tau^1\dot{C}),\ \sigma_{rz} = c_{44}e_{rz}
\end{aligned} \tag{11}$$

$$\begin{aligned}
&e_{rr} = \frac{\partial u_r}{\partial r}, \qquad e_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r}{r}, \qquad e_{zz} = \frac{\partial u_z}{\partial z}, \qquad e_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right),\\
&e_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \qquad e_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial\theta}\right), \qquad \Delta_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2},\\
&a_{ij} = -a_i\delta_{ij}, \qquad b_{ij} = -b_i\delta_{ij}, \qquad \alpha_{ij}^* = \alpha_i^*\delta_{ij}, \qquad K_{ij} = K_i\delta_{ij}, \qquad c_{66} = (c_{11} - c_{12})/2,\\
&a_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3, \qquad a_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3,\\
&b_1 = (c_{11} + c_{12})\alpha_{1c} + c_{13}\alpha_{3c}, \qquad b_3 = 2c_{13}\alpha_{1c} + c_{33}\alpha_{3c}
\end{aligned} \tag{12}$$

Here, $\alpha_t,\ \alpha_{tc}\ (t = 1,\ 2,\ 3)$ are the coefficients of linear thermal expansion and diffusion expansion respectively. In the above, Eqs. (11)-(12), we use the contracting subscript notations $1 \to 11,\ 2 \to 22,\ 3 \to 33,\ 4 \to 23,\ 5 \to 13,\ 6 \to 12$ to relate c_{ijkm} to $c_{in}(i, j, k, m = 1,2,3$ and $l, n = 1,2,3,4,5,6)$. We define the dimensionless quantities:

$$\begin{aligned}
&r' = \frac{\omega_1^* r}{v_1}, \qquad z' = \frac{\omega_1^* z}{v_1}, \qquad t' = \omega_1^* t, \qquad u_i' = \frac{\omega_1^* u_i}{v_1}, \qquad T' = \frac{a_1 T}{\rho v_1^2},\\
&C' = \frac{b_1 C}{\rho v_1^2}, \tau_0' = \omega_1^*\tau_0, \qquad \tau_1' = \omega_1^*\tau_1, \qquad \tau^{0'} = \omega_1^*\tau^0, \qquad \tau^{1'} = \omega_1^*\tau^1,\\
&\sigma_{ij}' = \frac{\sigma_{ij}}{a_1 T_0}, \qquad v_1^2 = \frac{c_{11}}{\rho}, \qquad \omega_1^* = \frac{\rho C_E v_1^2}{K_1}, \qquad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2},
\end{aligned} \tag{13}$$

$$\Gamma' = \frac{\omega_1^* \Gamma}{v_1}, \qquad L' = \frac{\omega_1^* L}{v_1}, \qquad \Gamma_1' = \frac{\omega_1^* \Gamma_1}{v_1}, \qquad \Gamma_2' = \frac{\omega_1^* \Gamma_2}{v_1}, \qquad \delta_1 = \frac{c_{44}}{c_{11}},$$
$$\delta_2 = \frac{c_{13}+c_{44}}{c_{11}}, \qquad \delta_3 = \frac{c_{11}-c_{12}}{2c_{11}}, \qquad \delta_4 = \frac{c_{33}}{c_{11}}, \qquad p_1 = \frac{a_3}{a_1}, \qquad p_2 = \frac{b_3}{b_1},$$
$$p_3 = \frac{K_3}{K_1}, \qquad \zeta_1 = \frac{a T_0 v_1^2 a_1}{\omega_1^* K_1 b_1}, \qquad \zeta_2 = \frac{a_1^2 T_0}{\rho K_1 \omega_1^*}, \qquad q_1^* = \frac{\alpha_1^* \omega_1^* b_1^2}{\rho v_1^4},$$
$$q_2^* = \frac{\alpha_3^* \omega_1^* b_1 b_3}{\rho v_1^4}, \qquad q_3^* = \frac{\alpha_1^* \omega_1^* b_1 a}{a_1 v_1^2}, \qquad q_4^* = \frac{\alpha_3^* \omega_1^* b_1 a}{a_1 v_1^2}, \qquad q_5^* = \frac{\alpha_1^* \omega_1^* b}{v_1^2},$$
$$q_6^* = \frac{\alpha_3^* \omega_1^* b}{v_1^2}, \qquad q_7^* = \frac{\alpha_3^* \omega_1^* b_1{}^2}{\rho v_1^4}, \qquad q_8^* = \frac{\alpha_1^* \omega_1^* b_1 b_3}{\rho v_1^4}$$

Here, ω_1^* is the characteristic frequency of the panel. Following Buchwald [4], we introduce the displacement potential functions as

$$u_r = -\frac{\partial \phi}{\partial r} + \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \qquad u_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} - \frac{\partial \psi}{\partial r}, \qquad u_z = \frac{\partial \varphi}{\partial z} \tag{14}$$

Using Eq.(13), into Eqs. (8)-(10), after suppressing the primes and then applying Eq . (14), in the resulting Eqs. we obtain

$$(\nabla_1^2 + \delta_1 \frac{\Gamma^2}{L^2}\frac{\partial^2}{\partial z^2} - \Gamma^2 \frac{\partial^2}{\partial t^2})\phi - \delta_2 \frac{\Gamma^2}{L^2}\frac{\partial^2 \varphi}{\partial z^2} + \Gamma^2 (T + \tau_1 \dot{T}) + \Gamma^2 (C + \tau^1 \dot{C}) = 0 \tag{15}$$

$$(\delta_3 \nabla_1^2 + \delta_1 \frac{\Gamma^2}{L^2}\frac{\partial^2}{\partial z^2} - \Gamma^2 \frac{\partial^2}{\partial t^2})\psi = 0 \tag{16}$$

$$-\delta_2 \nabla_1^2 \phi + (\delta_1 \nabla_1^2 + \delta_4 \frac{\Gamma^2}{L^2}\frac{\partial^2}{\partial z^2} - \Gamma^2 \frac{\partial^2}{\partial t^2})\varphi - p_1 \Gamma^2 (T + \tau_1 \dot{T}) - p_2 \Gamma^2 (C + \tau^1 \dot{C}) = 0 \tag{17}$$

$$\nabla_1^2 T + p_3 \frac{\Gamma^2}{L^2}\frac{\partial^2 T}{\partial z^2} = \Gamma^2 (\dot{T} + \tau_0 \ddot{T}) + \varsigma_1 \Gamma^2 (\dot{C} + \gamma \ddot{C}) + \varsigma_2 (\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2})(-\nabla_1^2 \phi + p_1 \frac{\Gamma^2}{L^2}\frac{\partial^2 \varphi}{\partial z^2}) \tag{18}$$

$$\begin{aligned} &-\nabla_1^2 (q_1^* \nabla_1^2 + q_7^* \frac{\Gamma^2}{L^2}\frac{\partial^2}{\partial z^2})\phi + \frac{\Gamma^2}{L^2}\frac{\partial^2}{\partial z^2}(q_8^* \nabla_1^2 + q_2^* \frac{\Gamma^2}{L^2}\frac{\partial^2}{\partial z^2})\varphi + \Gamma^2 (q_3^* \nabla_1^2 + q_4^* \frac{\Gamma^2}{L^2}\frac{\partial^2}{\partial z^2})(T + \tau_1 \dot{T}) \\ &-\Gamma^2 (q_5^* \nabla_1^2 + q_6^* \frac{\Gamma^2}{L^2}\frac{\partial^2}{\partial z^2})(C + \tau^1 \dot{C}) + \Gamma^4 (\dot{C} + \varepsilon \tau^0 \ddot{C}) = 0 \end{aligned} \tag{19}$$

where $r'' = r'/\Gamma'$, $z'' = z'/L'$. Here and in the following analysis, primes have been suppressed for convenience unless stated otherwise. Eq. (16) corresponds to purely transverse waves, which are decoupled from rest of motion and are not affected by the temperature change and concentration fields. To solve Eqs. (15)-(19), we assume the displacement potential, temperature change and concentration functions as

$$\begin{aligned} \phi(r,\theta,z,t) &= \bar{\phi}(r)\sin(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \\ \varphi(r,\theta,z,t) &= \bar{\varphi}(r)\sin(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \\ T(r,\theta,z,t) &= \bar{T}(r)\sin(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \\ C(r,\theta,z,t) &= \bar{C}(r)\sin(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \\ \psi(r,\theta,z,t) &= \bar{\psi}(r)\sin(q\pi z)\cos(\kappa\theta)\exp(\iota\omega t) \end{aligned} \tag{20}$$

where $\kappa = n\pi / \varsigma$. Here, n and q are the circumferential and axial wave numbers. Eqs . (15)-(19) ,with the aid of Eq. (20) yield

$$(\nabla_2^2 + l_1)\overline{\phi} + l_2\overline{\varphi} + \Gamma^2\tau_t^1\overline{T} + \Gamma^2\tau_c^1\overline{C} = 0 \tag{21}$$

$$(\nabla_2^2 + g_1^2)\overline{\psi} = 0 \tag{22}$$

$$-\delta_2\nabla_2^2\overline{\phi} + \delta_1(\nabla_2^2 + l_3)\overline{\varphi} - p_1\Gamma^2\tau_t^1\overline{T} - p_2\Gamma^2\tau_c^1\overline{C} = 0 \tag{23}$$

$$(\nabla_2^2 - l_4)\overline{T} = \iota\omega(\varsigma_1\tau_c^0\Gamma^2\overline{C} - \varsigma_2\tau_e^0(\nabla_2^2\overline{\phi} + p_1t_L^2\overline{\varphi})) \tag{24}$$

$$-\nabla_2^2(q_1^*\nabla_2^2 - q_7^*t_L^2)\overline{\phi} - (q_8^*\nabla_2^2 - q_2^*t_L^2)t_L^2\overline{\varphi} + \Gamma^2\tau_t^1(q_3^*\nabla_2^2 - q_4^*t_L^2)\overline{T} - \Gamma^2\tau_c^1(q_5^*\nabla_2^2 - q_6^*t_L^2)\overline{C} + \iota\omega\tau_f^0\Gamma^4\overline{C} = 0 \tag{25}$$

where

$$\nabla_2^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{\kappa^2}{r^2}, l_1 = \delta_1(\Omega^2 - t_L^2), \qquad l_2 = \delta_2 t_L^2, \qquad l_3 = \Omega^2 - \frac{\delta_4}{\delta_1}t_L^2, \qquad l_4 = \iota\omega^{-1}\tau_t^0(\Omega^2\delta_1 - \frac{\iota\omega p_3 t_L^2}{\tau_t^0}),$$

$$g_1^2 = \frac{\delta_1}{\delta_3}(\Omega^2 - t_L^2), \qquad \Omega = \frac{\omega\Gamma}{\sqrt{\delta_1}}, \qquad t_L = \frac{\Gamma q\pi}{L}, \qquad \tau_t^1 = 1 + \iota\omega\tau_1, \qquad \tau_c^1 = 1 + \iota\omega\tau^1, \qquad \tau_t^0 = 1 + \iota\omega\tau_0,$$

$$\tau_c^0 = 1 + \iota\omega\tau^0, \qquad \tau_e^0 = 1 + \iota\omega\varepsilon\tau_0, \qquad \tau_f^0 = 1 + \iota\omega\varepsilon\tau^0$$

Eq. (22) is a Bessel equation with its possible solution as

$$\overline{\psi} = \begin{cases} A_5 J_\kappa(g_1 r) + B_5 Y_\kappa(g_1 r), & g_1^2 > 0 \\ A_5 r^\kappa + B_5 r^{-\kappa}, & g_1^2 = 0 \\ A_5 I_\kappa(g_1 r) + B_5 K_\kappa(g_1 r), & g_1^2 < 0 \end{cases} \tag{26}$$

where $g_1'^2 = -g_1^2$, $J_\kappa()$ and $Y_\kappa()$ are, respectively, Bessel functions of the first and second kinds, while $I_\kappa()$ and $K_\kappa()$ are, respectively, modified Bessel functions of first and second kinds, A_5 and B_5 are two arbitrary constants. Generally, $g_1^2 \neq 0$ and hence the case corresponding to $g_1^2 = 0$ will not be discussed further. We investigate the further analysis by taking the form of ψ for $g_1^2 < 0$; the case for $g_1^2 > 0$ is similarly follows. From Eqs. (21), and Eqs. (23)-(25), we obtain

$$(\nabla_2^2 + m_1^2)(\nabla_2^2 + m_2^2)(\nabla_2^2 + m_3^2)(\nabla_2^2 + m_4^2)\overline{H} = 0 \tag{27}$$

where $\overline{H}$ may be any of the functions $\overline{\phi}, \overline{\varphi}, \overline{T}$ or $\overline{C}$ and $m_i^2, i = 1,\ 2,\ 3$ are the roots of the equation

$$m^8 + A^* m^6 + B^* m^4 + C^* m^2 + D^* = 0 \tag{28}$$

where the coefficients A^*, B^*, C^* and D^* are given in appendix A. The functions $\overline{\phi}, \overline{\varphi}, \overline{T}$ and $\overline{C}$ are obtained as

$$(\overline{\phi}, \overline{\varphi}, \overline{T}, \overline{C}) = \sum_{i=1}^{4}(1, W_i, \frac{S_i}{\Gamma^2}, \frac{E_i}{\Gamma^2})(A_i I_\kappa(m_i r) + B_i K_\kappa(m_i r)) \tag{29}$$

where

$$W_i = \frac{F_i^*}{F_i}, \qquad S_i = -\frac{F_i^{**}}{F_i}, \qquad E_i = \frac{F_i^{***}}{F_i}$$

$$F_i = \begin{vmatrix} l_2 & \tau_t^1 & \tau_c^1 \\ \delta_1(m_i^2 - l_3) & p_1\tau_t^1 & p_2\tau_c^1 \\ -t_L^2(q_8^* m_i^2 + q_2^* t_L^2) & \tau_t^1(q_3^* m_i^2 + q_4^* t_L^2) & -\tau_c^1(q_5^* m_i^2 + q_6^* t_L^2) - \iota\omega\tau_f^0\,\Gamma^2 \end{vmatrix}$$

$$F_i^* = \begin{vmatrix} m_i^2 - l_1 & \tau_t^1 & \tau_c^1 \\ \delta_2 m_i^2 & p_1\tau_t^1 & p_2\tau_c^1 \\ -m_i^2\,(q_1^* m_i^2 + q_7^* t_L^2) & \tau_t^1(q_3^* m_i^2 + q_4^* t_L^2) & -\tau_c^1(q_5^* m_i^2 + q_6^* t_L^2) - \iota\omega\tau_f^0\,\Gamma^2 \end{vmatrix}$$

$$F_i^{**} = \begin{vmatrix} m_i^2 - l_1 & l_2 & \tau_c^1 \\ \delta_2 m_i^2 & \delta_1(m_i^2 - l_3) & p_2\tau_c^1 \\ -m_i^2\,(q_1^* m_i^2 + q_7^* t_L^2) & -t_L^2(q_8^* m_i^2 + q_2^* t_L^2) & -\tau_c^1(q_5^* m_i^2 + q_6^* t_L^2) - \iota\omega\tau_f^0\,\Gamma^2 \end{vmatrix}$$

$$F_i^{***} = \begin{vmatrix} m_i^2 - l_1 & l_2 & \tau_t^1 \\ \delta_2 m_i^2 & \delta_1(m_i^2 - l_3) & p_1\tau_t^1 \\ -m_i^2\,(q_1^* m_i^2 + q_7^* t_L^2) & -t_L^2(q_8^* m_i^2 + q_2^* t_L^2) & \tau_c^1(q_3^* m_i^2 + q_4^* t_L^2) \end{vmatrix}, \; i = 1,2,3,4.$$

Using Eq. (20) into the Eq. (14) and then in the relations (11), the displacements, temperature change, concentration and stresses are obtained as

$$u_r = -\frac{1}{\Gamma}(\bar{\phi}' + \frac{\kappa}{r}\bar{\psi})\sin(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \tag{30}$$

$$u_\theta = -\frac{1}{\Gamma}(\frac{\kappa}{r}\bar{\phi} + \bar{\psi}')\sin(q\pi z)\cos(\kappa\theta)\exp(\iota\omega t) \tag{31}$$

$$u_z = \frac{t_L}{\Gamma}\bar{\varphi}\cos(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \tag{32}$$

$$\begin{aligned} \sigma_{rr} = {} & \frac{c_{11}}{a_1 T_0 \Gamma^2}\{(-\bar{\phi}'' - \frac{(1-2\delta_3)}{r}\bar{\phi}' + \frac{\kappa^2}{r^2}(1-2\delta_3)\bar{\phi}) - \frac{2\delta_3\kappa}{r}(\bar{\psi}' - \frac{\bar{\psi}}{r}) \\ & -(\delta_2 - \delta_1)t_L^2\bar{\varphi} - \tau_t^1\Gamma^2\bar{T} - \tau_c^1\Gamma^2\bar{C}\}\sin(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t), \end{aligned} \tag{33}$$

$$\sigma_{rz} = \frac{c_{44}t_L}{a_1 T_0 \Gamma^2}(-\bar{\phi}' - \frac{\kappa}{r}\bar{\psi} + \bar{\varphi}')\cos(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \tag{34}$$

$$\sigma_{r\theta} = \frac{c_{11} - c_{12}}{2c_{11}a_1 T_0 \Gamma^2}(-\frac{2\kappa}{r}\bar{\phi}' + \frac{2\kappa}{r^2}\bar{\phi} - \bar{\psi}'' + \frac{\bar{\psi}'}{r} - \frac{\kappa^2}{r^2}\bar{\psi})\sin(q\pi z)\cos(\kappa\theta)\exp(\iota\omega t) \tag{35}$$

$$T = \bar{T}(r)\sin(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \tag{36}$$

$$C = \bar{C}(r)\sin(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \tag{37}$$

Here, prime denotes differentiation with respect to radial co-ordinate r.

3 BOUNDARY CONDITIONS

The boundary conditions at the inner and outer surfaces i.e. at $r = \Gamma_1, \Gamma_2$ of the thermoelastic diffusive cylindrical panel are given by

(i) Mechanical conditions (stress-free surfaces)

$$\sigma_{rr} = 0,\; \sigma_{rz} = 0,\; \sigma_{r\theta} = 0 \tag{38}$$

(ii) Thermal Conditions

$$\frac{\partial T}{\partial r} + h_1 T = 0 \tag{39}$$

(iii) Concentration conditions

$$\frac{\partial C}{\partial r} + h_2 C = 0 \tag{40}$$

Here, $h_1 \to 0$ corresponds to thermally insulated boundaries and $h_1 \to \infty$ refers to isothermal surfaces. Similarly, $h_2 \to 0$ corresponds to impermeable boundaries and $h_2 \to \infty$ refers to isoconcentrated surfaces.

4 **DERIVATION OF SECULAR EQUATION**

Substituting the values of $\sigma_{rr}, \sigma_{rz}, \sigma_{r\theta}, T$ and C from Eqs. (33)-(37) in the boundary conditions Eqs. (38)-(40) at the surfaces $r = \Gamma_1, \Gamma_2$, we obtain a system of ten simultaneous equations and for non-trivial solution of system of equations, the determinant of the coefficients of amplitudes vanishes which gives the free vibration secular equation as

$$\left| X_{ij} \right| = 0, \quad (i, j = 1, 2, \ldots\ldots\ldots, 10) \tag{41}$$

where X_{ij} $(i, j = 1,3,5,7,9)$ are given in appendix B. The coefficients $X_{ij}\,(j = 2,4,6,8,10)$ can be obtained by just replacing modified Bessel functions of first kind in $X_{ij}\,(i = 1,3,5,7,9)$ with that of the second kind, respectively and the coefficients $X_{ij}\,(i = 2,4,6,8,10)$ can be obtained by just replacing k_1 in $X_{ij}\,(j = 1,3,5,7,9)$ with k_2 respectively, where $k_1 = \Gamma_1 / \Gamma = 1 - (s^*/2)$, $k_2 = \Gamma_2 / \Gamma = 1 + (s^*/2)$ and $s^* = (\Gamma_2 - \Gamma_1)/\Gamma$ is the thickness to the mean radius ratio of the panel.

Remark 1: If we assume the displacement potential, temperature change or concentration functions in any of the following form:

$$(\phi, \varphi, T, C) = (\bar{\phi}(r), \bar{\varphi}(r), \bar{T}(r), \bar{C}(r))\cos(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t)\text{and}\psi = \bar{\psi}(r)\cos(q\pi z)\cos(\kappa\theta)\exp(\iota\omega t) \quad \text{or}$$

$$(\phi, \varphi, T, C) = (\bar{\phi}(r), \bar{\varphi}(r), \bar{T}(r), \bar{C}(r))\sin(q\pi z)\cos(\kappa\theta)\exp(\iota\omega t)\text{and}\psi = \bar{\psi}(r)\sin(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t) \quad \text{or}$$

$$(\phi, \varphi, T, C) = (\bar{\phi}(r), \bar{\varphi}(r), \bar{T}(r), \bar{C}(r))\cos(q\pi z)\cos(\kappa\theta)\exp(\iota\omega t)\text{and}\psi = \bar{\psi}(r)\cos(q\pi z)\sin(\kappa\theta)\exp(\iota\omega t)$$

We obtain same free vibration secular Eq. (41).

Remark 2: If we apply the Laplace transform defined by

$$\bar{f}(\varpi) = \int_0^\infty f(t)\, e^{-\varpi t} \mathrm{d}t$$

In the basic equations, we obtain the secular Eq. (41) , in transform domain. To obtain the solution in physical domain, either numerical inversion technique or method of calculus of residues is required for tracking all the singularities present in the problem.

5 SPECIAL CASES

(i) By taking $h_1 \to 0$ and $h_2 \to 0$ in the Eq. (41), we obtain the secular equation corresponding to thermally insulated and impermeable boundaries.

(ii) By taking $h_1 \to \infty$ and $h_2 \to \infty$ in the Eq. (41), we obtain the secular equation corresponding to isothermal and isoconcentrated surfaces.

(iii) By taking $h_1 \to 0$ and $h_2 \to \infty$ in the Eq. (41), we obtain the secular equation corresponding to thermally insulated and isoconcentrated boundaries.

(iv) By taking $h_1 \to \infty$ and $h_2 \to 0$ in the Eq. (41), we obtain the secular equation corresponding to isothermal and impermeable boundaries.

(v) By taking

$$\begin{aligned} &c_{11} = c_{33} = \lambda + 2\mu, \qquad c_{12} = c_{13} = \lambda, \qquad c_{44} = \mu,\ a_1 = a_3 = \beta_1, \\ &b_1 = b_3 = \beta_2, \qquad K_1 = K_3 = K,\ \alpha_1^* = \alpha_3^* = D \end{aligned} \tag{42}$$

in Eq. (41), the results for homogeneous isotropic thermoelastic diffusive cylindrical panel can be obtained.

(vi) If we neglect diffusion effects, then the above analysis is reduced to one as discussed by Sharma and Sharma [20].

6 NUMERICAL RESULTS AND DISCUSSION

The material chosen for the purpose of numerical calculation is copper which is a transverse isotropic material. The physical data for a single crystal of copper material [13] is given below:

$$\begin{aligned} &c_{11} = 18.78\times10^{10}\ \text{Kgm}^{-1}\text{s}^{-2}, \quad c_{12} = 8.76\times10^{10}\ \text{Kgm}^{-1}\text{s}^{-2}, \quad c_{13} = 8.0\times10^{10}\ \text{Kgm}^{-1}\text{s}^{-2}, \quad c_{33} = 18.2\times10^{10}\ \text{Kgm}^{-1}\text{s}^{-2}, \\ &c_{44} = 5.06\times10^{10}\ \text{Kgm}^{-1}\text{s}^{-2}, \quad T_0 = 0.293\times10^{3}\ \text{K}, \quad C_E = 0.6331\times10^{3}\ \text{JKg}^{-1}\text{K}^{-1}, \quad \alpha_1 = 2.98\times10^{-5}\ \text{K}^{-1}, \\ &\alpha_3 = 2.4\times10^{-5}\ \text{K}^{-1}, \quad \alpha_{1c} = 2.1\times10^{-4}\ \text{Kg}^{-1}\text{m}^{3}, \quad \alpha_{3c} = 2.5\times10^{-4}\ \text{Kg}^{-1}\text{m}^{3}, \quad a = 2.4\times10^{4}\ \text{m}^{2}\text{s}^{-2}\text{K}^{-1}, \\ &b = 13\times10^{5}\ \text{Kg}^{-1}\text{m}^{5}\text{s}^{-2}, \quad \alpha_1^* = 0.95\times10^{-8}\,\text{Kgsm}^{-3}, \quad \alpha_3^* = 0.9\times10^{-8}\,\text{Kgsm}^{-3}, \quad \rho = 8.954\times10^{3}\ \text{Kgm}^{-3}, \\ &K_1 = 0.433\times10^{3}\ Wm^{-1}K^{-1}, \quad K_3 = 0.450\times10^{3}\ \text{Wm}^{-1}\text{K}^{-1} \end{aligned}$$

The values of relaxation times are taken as

$$\tau_0 = 0.02s, \qquad \tau_1 = 0.01s, \qquad \tau^0 = 0.03s, \qquad \tau^1 = 0.04s$$

For the purpose of numerical calculation, we take the case of free vibrations of a closed circular cylindrical shell. For closed cylindrical shells, the central angle $\varsigma = 2\pi$ and the integer n must be even, since the shell vibrates in circumferential full waves. Thus, the frequency equation for closed cylindrical shell can be written by setting $\kappa = 1,\ 2,\ 3, \ldots$.

6.1 Muller's method

Figs. 2 and 3 show a flow chart to compute the lowest frequency by using the Muller's method. The Eq. (41) is a complex polynomial equation. For a given value of t_L, the Eq. (41) ,can be written as $F(\Omega)=0$. Muller's method is used to find an estimated root of $F(\Omega)=0$. The algorithm of Muller's method to find lowest frequency is as follows:

1. Decide initially three approximations say Ω_1, Ω_2 and Ω_3 of the root, number of iterations (maxit) and two error bounds (eps1 and eps2).

2. Put *I*=1.

3. If $I \le$ maxit, then compute $F(\Omega_1)$, $F(\Omega_2)$ and $F(\Omega_3)$, otherwise write "Process fails to converge the root" and go to step 10.

4. Compute q^*, x^*, y^* and z^* by the following relations:

$$q^* = \frac{\Omega_3-\Omega_2}{\Omega_2-\Omega_1}, \qquad x^* = q^*F(\Omega_3) - q^*(1+q^*)F(\Omega_2) + q^{*2}F(\Omega_1),$$

$$y^* = (2q^*+1)F(\Omega_3) - (1+q^*)^2F(\Omega_2) + q^{*2}F(\Omega_1), \qquad z^* = (1+q^*)F(\Omega_3)$$

5. If $x^* \ne 0$, then calculate discriminant $(\text{disc}) = y^{*2} - 4x^*z^*$, $z_1 = y^* + \sqrt{\text{disc}}$, $z_2 = y^* - \sqrt{\text{disc}}$.

6. If $|z_1| \le |z_2|$, then compute

$$\Omega_4 = \Omega_3 - \frac{2z^*(\Omega_3-\Omega_2)}{z_2}$$

Otherwise compute

$$\Omega = |\Omega_4| \ \Omega_4 = \Omega_3 - \frac{2z^*(\Omega_3-\Omega_2)}{z_1}$$

7. If $|\Omega_4-\Omega_3| <$ eps1 and If $|F(\Omega_4)| <$ eps2, then root Ω_4 is obtained and go to step 9. Otherwise put $\Omega 1 = \Omega_2$, $\Omega_2 = \Omega_3$ and $\Omega_3 = \Omega_4$, $I = I+1$ and go to step 3.

8. Otherwise if $x^* = 0$, then check whether $y^* \ne 0$ or $y^* = 0$. If $y^* \ne 0$, then calculate $\Omega_4 = \Omega_3 - (z^*(\Omega_3-\Omega_2)/y^*)$, and go to step 7. If not, write "Muller's method fails to find the root" and go to step 10.

9. Compute lowest frequency (Ω) by the relation $\Omega = |\Omega_4|$.

10. Stop the process.

6.2 Lowest frequency

The values of lowest frequency (Ω) corresponding to different theories of thermoelastic diffusion have been given in Tables 1 and 2 for two values of circumferential wave number $(\kappa = 1,2)$ with respect to the parameter t_L for different values of s^* ($s^* = 0.02$ and 0.2) respectively, whereas the values of lowest frequency (Ω) corresponding to different theories of thermoelasticity have been given in table 3 for $\kappa = 1$ and $s^* = 0.02$.

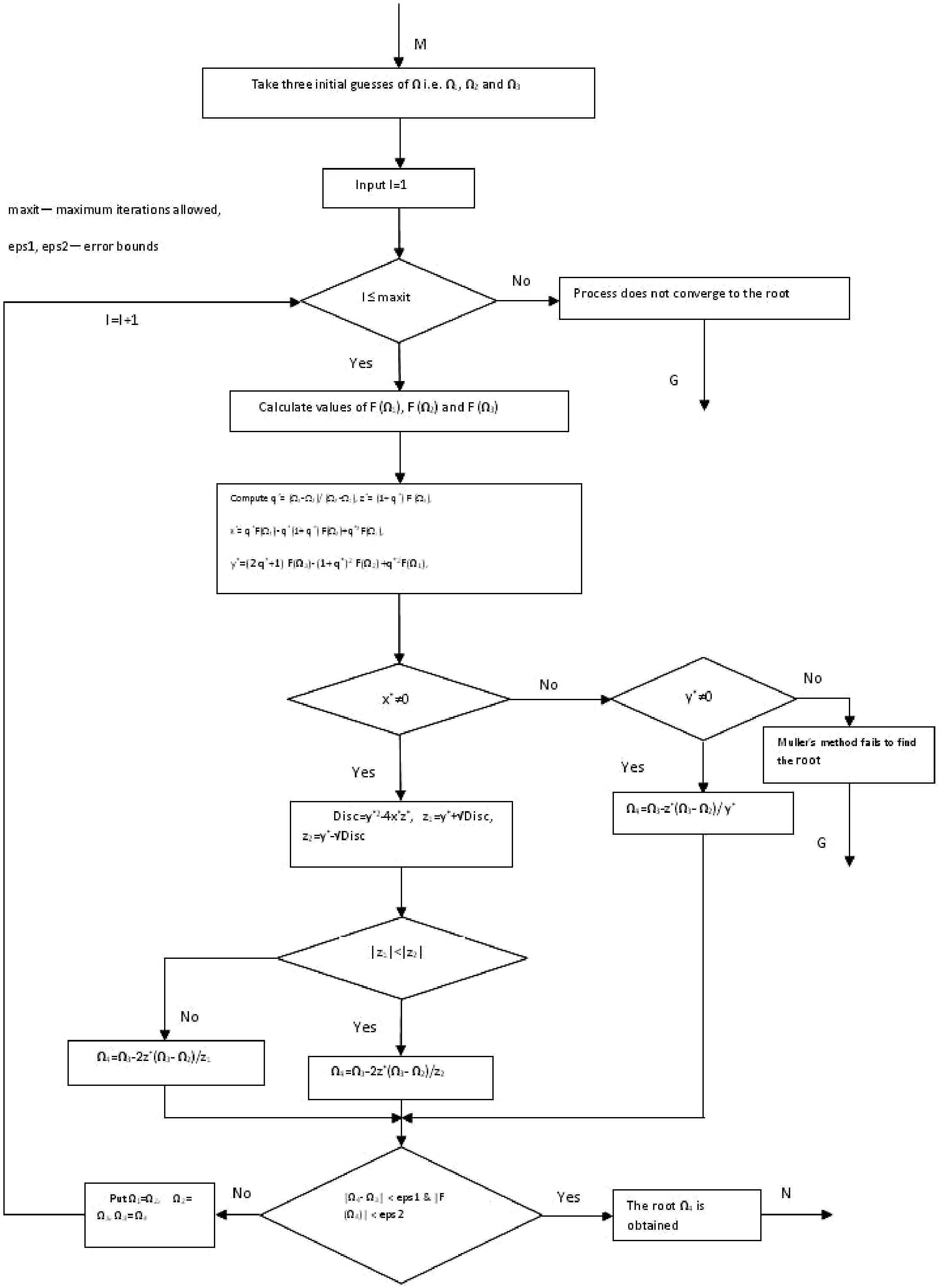

Fig. 2
Flow chart to find an approximate root of *F*.

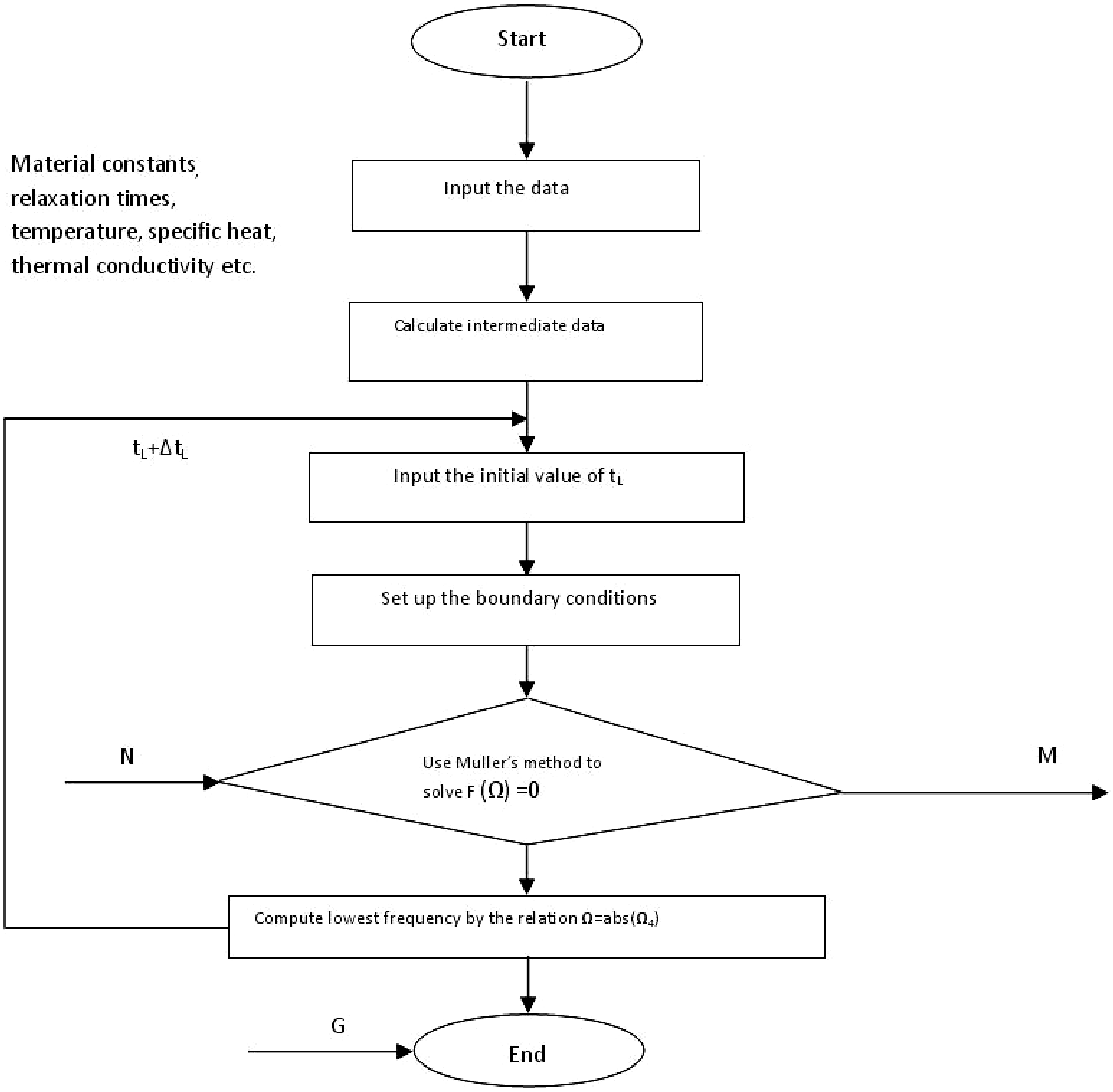

Fig. 3
Flow chart to find an approximate root of *F*.

The variations of lowest frequency (Ω) corresponding to different theories of thermoelastic diffusion have been shown in Figs. 4 and 5 for two values of $s^*(s^*=0.02$ and $0.2)$ with respect to the parameter t_L for the values of circumferential wave number $(\kappa=1,2)$ respectively. The diffusion effect on the lowest frequency (Ω) have been shown in Fig.6 for $\kappa=1$ and $s^*=0.02$. In Figs. 4 and 5 and 6 for $s^*=0.02$, the solid line corresponds to CT theory of thermoelastic diffusion and small dash and long dash lines represent L-S and G-L theories of thermoelastic diffusion respectively and these are represented by CTD, LSD and GLD. In Figures 4 and 5 for $s^*=0.2$, the star, circle and triangle symbols on these lines correspond to CT, L-S and G-L theories of thermoelastic diffusion, whereas in Fig. 6 for $s^*=0.02$, the cross, lower triangle and square symbols on these lines correspond to CT, L-S and G-L theories of thermoelasticity and these are represented by CT, LS and GL.

It is observed from Fig.4 that the values of Ω increase for both values of s^* in different theories of thermoelastic diffusion. The values of Ω are lower with the increase in the values of s^*. Corresponding to G-L theory for both values of s^*, the values of Ω are more in comparison to CT and L-S theories, but if we compare CT and L-S theories of thermoelastic diffusion, we find that corresponding to the value of $s^*=0.02$, the values in

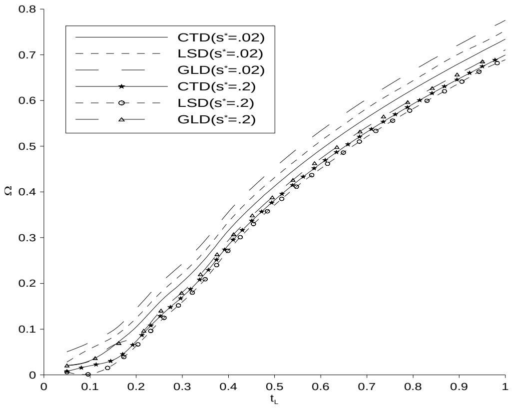

Fig. 4
Variations of lowest frequency (Ω) with t_L for κ=1.

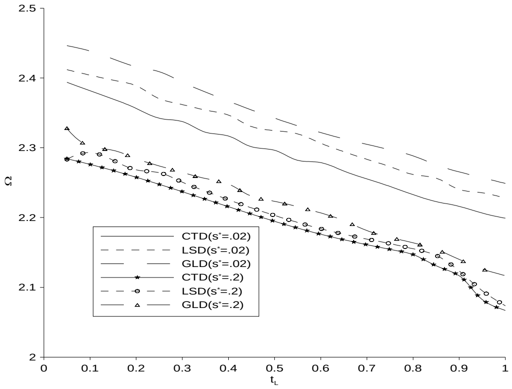

Fig. 5
Variations of lowest frequency (Ω) with t_L for κ=2.

Table 1
Values of lowest frequency (Ω) corresponding to different theories of thermoelastic diffusion for s^*=.02 and s^*=.2 with respect to the values of t_L for κ=1

Values of t_L	Values of Ω for s^*=.02			Values of Ω for s^*=.2		
	CT	L-S	G-L	CT	L-S	G-L
0.05	0.02078	0.02806	0.0505	0.0078	0.00606	0.0205
0.1	0.03099	0.0572	0.0710	0.0199	0.00167	0.0304
0.15	0.0628	0.0826	0.0972	0.0322	0.022	0.0646
0.2	0.1045	0.1239	0.1446	0.0735	0.0629	0.0836
0.25	0.1583	0.1769	0.1985	0.1264	0.115	0.1366
0.3	0.2028	0.2215	0.243	0.1706	0.1593	0.1807
0.35	0.2543	0.272	0.2946	0.2219	0.2097	0.2322
0.4	0.3158	0.3346	0.3564	0.284	0.2726	0.2946
0.45	0.3674	0.3871	0.4081	0.3364	0.326	0.3471
0.5	0.4128	0.4316	0.4535	0.3821	0.3715	0.3929
0.55	0.4544	0.4722	0.4952	0.4238	0.4136	0.4347
0.6	0.4929	0.5117	0.5338	0.4623	0.4511	0.4732
0.65	0.5287	0.5475	0.5697	0.4979	0.4867	0.5089
0.7	0.5623	0.5829	0.6033	0.5312	0.521	0.5422
0.75	0.5941	0.6139	0.6351	0.5625	0.5523	0.5736
0.8	0.6243	0.6432	0.6654	0.5923	0.5821	0.6033
0.85	0.6533	0.6739	0.6944	0.6207	0.6103	0.6318
0.9	0.6813	0.7024	0.7224	0.6481	0.6378	0.6592
0.95	0.7082	0.7262	0.7494	0.6744	0.6667	0.6855
1	0.7340	0.7531	0.7753	0.6996	0.6889	0.7108

Table 2
Values of lowest frequency (Ω) corresponding to different theories of thermoelastic diffusion for s^*=.02 and s^*=.2 with respect to the values of t_L for κ=2

Values of t_L	Values of Ω for s^*=.02			Values of Ω for s^*=.2		
	CT	L-S	G-L	CT	L-S	G-L
0.05	2.3939	2.4119	2.4463	2.2844	2.2834	2.3282
0.1	2.3817	2.4041	2.4386	2.2761	2.2931	2.3019
0.15	2.3698	2.3969	2.4273	2.2675	2.2821	2.2964
0.2	2.3565	2.3889	2.4161	2.2578	2.2683	2.2844
0.25	2.3423	2.3703	2.409	2.2476	2.2643	2.274
0.3	2.3375	2.3619	2.3939	2.2371	2.2507	2.2646
0.35	2.3223	2.354	2.3799	2.2263	2.2378	2.256
0.4	2.3168	2.3468	2.3667	2.2155	2.2256	2.248
0.45	2.3014	2.3303	2.3542	2.2049	2.2139	2.2306
0.5	2.2964	2.3246	2.3424	2.1946	2.203	2.2233
0.55	2.2821	2.3199	2.3315	2.1849	2.193	2.2156
0.6	2.2788	2.3065	2.3217	2.1761	2.184	2.2067
0.65	2.2666	2.2944	2.3128	2.1682	2.1761	2.1957
0.7	2.2554	2.2834	2.3049	2.1611	2.1692	2.1814
0.75	2.2446	2.273	2.2972	2.1545	2.1627	2.1717
0.8	2.2328	2.2617	2.2884	2.147	2.1554	2.1639
0.85	2.2229	2.2565	2.2756	2.1309	2.1458	2.1544
0.9	2.2161	2.2402	2.2654	2.1167	2.1235	2.1401
0.95	2.2064	2.2355	2.2572	2.0824	2.0952	2.1264
1	2.1991	2.2278	2.2489	2.0669	2.0737	2.1169

Table 3
Values of lowest frequency (Ω) corresponding to different theories of thermoelasticity for s^*=.02 with respect to the values of t_L for κ=1

Values of t_L	Values of Ω for s^*=.02		
	CT	L-S	G-L
0.05	0.9703	0.1584	0.1706
0.1	0.9871	0.17	0.1801
0.15	0.9934	0.1686	0.1974
0.2	1.0005	0.2249	0.2437
0.25	1.0064	0.2802	0.3008
0.3	1.0138	0.3359	0.3568
0.35	1.0192	0.3915	0.4136
0.4	1.024	0.4469	0.4654
0.45	1.0311	0.5021	0.525
0.5	1.0352	0.5572	0.5793
0.55	1.0401	0.6121	0.6316
0.6	1.0454	0.6669	0.6978
0.65	1.0535	0.7216	0.75
0.7	1.0764	0.7762	0.7993
0.75	1.1027	0.8308	0.8699
0.8	1.1315	0.8853	0.9223
0.85	1.1626	0.9398	0.9743
0.9	1.1956	0.9943	1.0338
0.95	1.2304	1.0488	1.0872
1	1.2667	1.1033	1.1354

case of L-S theory are higher than that of CT theory, whereas corresponding to the value of $s^*=0.2,$ the values in case of CT theory are higher than that of L-S theory. From Fig. 5 it is evident that for both values of s^* the values of Ω decrease. The values of Ω decrease more as the value of s^* increases. For both values of s^* the values of Ω are more in G-L theory as compared to CT and L-S theories. Therefore, we note from Figs. 4 and 5 that for the value of $\kappa=1$ the values of Ω increase, whereas for $\kappa=2,$ the values of Ω decrease. Therefore, for two different values of κ, the opposite trend is noticed. It is noticed from Fig. 6 that for the value of $s^* = 0.02$ the values of Ω increase in different theories of thermoelasticity. The values corresponding to CT theory of thermoelasticity are more in comparison to LS and GL theories of thermoelasicity. On neglecting the diffusion effect, there is more increase in the values of Ω in all three theories.

7 CONCLUSIONS

In this article, free vibration analysis of homogeneous, transversely isotropic thermoelastic diffusive cylindrical panel, based upon the three-dimensional generalized theories of thermoelastic diffusion, has been investigated. We have used three displacement potential functions for solving the equations of motion, heat conduction and mass diffusion. The purely transverse wave gets decoupled from rest of motion and remains independent of thermal and concentration fields. After expanding the displacement potential, temperature change and concentration functions with an orthogonal series, the equations for free vibration problem are reduced to four equations of second-order and one fourth-order ordinary differential equation. The formal solution of this system of equations can be expressed by using modified Bessel function with complex arguments. The numerical results for lowest frequency have been obtained and presented graphically. From numerical results, we have noticed that the trend of the values of lowest frequency (Ω) is similar for both values of thickness to the mean radius to the panel (s^*), but opposite for different values of circumferential wave number (κ). On neglecting the diffusion effect, we find that the increase is more in the values of lowest frequency (Ω). The work represented here is more general with an application to circular cylindrical panels of arbitrary thickness, from thin shell to extremely thick ones. This can be used in applications involving civil engineering structures, automotive suspension components etc. The solutions obtained are also applicable to both closed hollow cylinders and open ones (panels), depending upon whether $\kappa = n\pi/\varsigma$ is an integer or not. It can also be used to check the applicability of various kinds of two-dimensional shell theory in elastokinetics.

ACKNOWLEDGMENTS

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APPENDIX A

$$
\begin{array}{ll}
l_5 = \tau_c^1 q_6^* t_L^2 + \imath\omega\tau_f^0\Gamma^2, & l_6 = l_5 + l_4\tau_c^1 q_5^* + +\imath\omega\varsigma_1\tau_c^0\Gamma^2\tau_t^1 q_3^*, \\
l_7 = q_5^* p_1 + q_3^* p_2, & l_8 = \imath\omega\varsigma_1\tau_c^0 p_1\tau_t^1 - l_4 p_2\tau_c^1, \\
l_9 = l_5 p_1 + q_4^* p_2\tau_c^1 t_L^2, & l_{10} = \delta_1 q_7^* t_L^2 - \delta_1 l_3 q_1^* - \delta_2 q_8^* t_L^2, \\
l_{11} = \delta_2 q_2^* t_L^2 + l_3\delta_1 q_7^*, & l\,l_{12} = \delta_2 p_1 t_L^2 + \delta_1 l_3, \\
l_{13} = -\delta_1 l_6 + \delta_1 l_3 q_5^*\tau_c^1 + p_2\tau_c^1 q_8^* t_L^2, & l_{14} = l_4 l_5 + \imath\omega\varsigma_1\tau_c^0\tau_t^1 q_4^*\Gamma^2 t_L^2, \\
l_{15} = \varsigma_1\tau_c^0 q_1^* + \varsigma_2\tau_e^0\tau_c^1 q_5^* & l_{16} = l_{10} + \delta_1 l_4 q_1^* - \imath\omega\varsigma_2\tau_e^0\tau_t^1\delta_1 q_3^*\Gamma^2
\end{array}
\tag{A.1}
$$

$$
\begin{aligned}
f_1 &= (q_5^* - q_1^*)\delta_1\tau_c^1, \\
f_2 &= \delta_1\tau_c^1 q_5^* l_1 + l_{13} + l_2\tau_c^1(q_5^*\delta_2 - p_2 q_1^*) - \imath\omega\tau_t^1\delta_1 l_{15}\Gamma^2 + \tau_c^1 l_{16}, \\
f_3 &= l_{13} l_1 + \delta_1 l_4 l_5 - \delta_1 l_3 l_6 + \imath\omega\tau_t^1\Gamma^2 t_L^2(\varsigma_2\tau_c^1\tau_e^0 l_7 p_1 + q_4^*\varsigma_1\tau_c^0\delta_1) + t_L^2(q_8^* l_8 - p_2\tau_c^1 q_2^* t_L^2), \\
f_4 &= l_2(-\delta_2 l_6 - \imath\omega\varsigma_2\tau_e^0\tau_t^1\tau_c^1\Gamma^2 l_7 - l_8 q_1^* + p_2\tau_c^1 q_7^* t_L^2),
\end{aligned}
$$

$$
\begin{aligned}
f_5 &= \iota\omega\varsigma_2\tau_e^0\tau_t^1\tau_c^1\Gamma^2 t_L^2(q_1^* p_1 - q_8^*)(p_2 - p_1) - \iota\omega\tau_t^1\Gamma^2(-\varsigma_1\tau_c^0 l_{10} + \varsigma_2\tau_e^0(-l_5\delta_1 + q_5^* l_{12}\tau_c^1)),\\
f_6 &= \tau_c^1(t_L^2 l_{11} - l_4 l_{10} + \iota\omega\varsigma_2\tau_e^0\tau_t^1\Gamma^2(-l_{12}q_3^* + \delta_1 q_4^* t_L^2),\\
f_7 &= l_1 f_3 - l_{13}l_1^2 + l_{14}(\delta_1 l_3 + \delta_2 l_2) + \iota\omega\varsigma_2\tau_e^0\tau_t^1 l_9\Gamma^2(l_2 - t_L^2 p_1) + l_8 t_L^2(q_7^* l_2 - q_2^* t_L^2),\\
f_8 &= \iota\omega\varsigma_2\tau_e^0\tau_t^1\tau_c^1\Gamma^2 t_L^4(p_2 - p_1)(q_2^* - q_7^* p_1) + \iota\omega\tau_t^1\Gamma^2(l_{11}\varsigma_1\tau_c^0\Gamma^2 t_L^2 + l_5 l_{12}\varsigma_2\tau_e^0) + \tau_c^1(-l_4 l_{11}t_L^2 + \iota\omega\varsigma_2\tau_e^0\tau_t^1\Gamma^2 t_L^2 q_4^* l_{12}),\\
f_9 &= l_1 l_{13} l_{14}\delta_1 - \iota\omega\varsigma_2\tau_e^0 p_1\Gamma^2 t_L^2 l_9 - q_2^* l_8 t_L^4
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
A^* &= -\frac{f_2}{f_1},\\
B^* &= \frac{f_3 + f_4 + f_5 + f_6}{f_1},\\
C^* &= -\frac{f_7 + f_8}{f_1},\\
D^* &= \frac{f_9}{f_1}
\end{aligned}
\tag{A.3}
$$

APPENDIX B

$$
\begin{aligned}
X_{11} &= -m_1^2 I_\kappa''(m_1k_1) - \frac{1-2\delta_3}{k_1} m_1 I_\kappa'(m_1k_1) + \frac{(1-2\delta_3)\kappa^2}{k_1^2} I_\kappa(m_1k_1) - [(\delta_2-\delta_1)t_L^2 W_1 + \tau_t^1 S_1 + \tau_c^1 E_1] I_\kappa(m_1k_1)\\
X_{13} &= -m_2^2 I_\kappa''(m_2k_1) - \frac{1-2\delta_3}{k_1} m_2 I_\kappa'(m_2k_1) + \frac{(1-2\delta_3)\kappa^2}{k_1^2} I_\kappa(m_2k_1) - [(\delta_2-\delta_1)t_L^2 W_2 + \tau_t^1 S_2 + \tau_c^1 E_2] I_\kappa(m_2k_1)\\
X_{15} &= -m_3^2 I_\kappa''(m_3k_1) - \frac{1-2\delta_3}{k_1} m_3 I_\kappa'(m_3k_1) + \frac{(1-2\delta_3)\kappa^2}{k_1^2} I_\kappa(m_3k_1) - [(\delta_2-\delta_1)t_L^2 W_3 + \tau_t^1 S_3 + \tau_c^1 E_3] I_\kappa(m_3k_1)\\
X_{17} &= -m_4^2 I_\kappa''(m_4k_1) - \frac{1-2\delta_3}{k_1} m_4 I_\kappa'(m_4k_1) + \frac{(1-2\delta_3)\kappa^2}{k_1^2} I_\kappa(m_4k_1) - [(\delta_2-\delta_1)t_L^2 W_4 + \tau_t^1 S_4 + \tau_c^1 E_4] I_\kappa(m_4k_1)\\
X_{19} &= -2\delta_3\kappa\left(\frac{1}{k_1} g_1 I_\kappa'(g_1k_1) - \frac{1}{k_1^2} I_\kappa(g_1k_1)\right)
\end{aligned}
$$

$$
\begin{aligned}
X_{31} &= m_1(W_1-1)I_\kappa'(m_1k_1), & X_{33} &= m_2(W_2-1)I_\kappa'(m_2k_1),\\
X_{35} &= m_3(W_3-1)I_\kappa'(m_3k_1), & X_{37} &= m_4(W_4-1)I_\kappa'(m_4k_1),\\
X_{39} &= -\frac{\kappa}{k_1} I_\kappa(g_1k_1), & X_{51} &= -\frac{2\kappa}{k_1} m_1 I_\kappa'(m_1k_1) + \frac{2\kappa}{k_1^2} I_\kappa(m_1k_1),\\
X_{53} &= -\frac{2\kappa}{k_1} m_2 I_\kappa'(m_2k_1) + \frac{2\kappa}{k_1^2} I_\kappa(m_2k_1), & X_{55} &= -\frac{2\kappa}{k_1} m_3 I_\kappa'(m_3k_1) + \frac{2\kappa}{k_1^2} I_\kappa(m_3k_1),\\
X_{57} &= -\frac{2\kappa}{k_1} m_4 I_\kappa'(m_4k_1) + \frac{2\kappa}{k_1^2} I_\kappa(m_4k_1), & X_{59} &= -g_1^2 I_\kappa''(g_1k_1) + \frac{g_1}{k_1} I_\kappa'(g_1k_1) - \frac{\kappa^2}{k_1^2} I_\kappa(g_1k_1),\\
X_{71} &= S_1(m_1 I_\kappa'(m_1k_1) + h_1^* I_\kappa(m_1k_1)), & X_{73} &= S_2(m_2 I_\kappa'(m_2k_1) + h_1 I_\kappa(m_2k_1)),\\
X_{75} &= S_3(m_3 I_\kappa'(m_3k_1) + h_1 I_\kappa(m_3k_1)), & X_{77} &= S_4(m_4 I_\kappa'(m_4k_1) + h_1 I_\kappa(m_4k_1)),\\
X_{79} &= 0, & X_{91} &= E_1(m_1 I_\kappa'(m_1k_1) + h_2 I_\kappa(m_1k_1)),\\
X_{93} &= E_2(m_2 I_\kappa'(m_2k_1) + h_2 I_\kappa(m_2k_1)), & X_{95} &= E_3(m_3 I_\kappa'(m_3k_1) + h_2 I_\kappa(m_3k_1)),\\
X_{97} &= E_4(m_4 I_\kappa'(m_4k_1) + h_2 I_\kappa(m_4k_1)) & X_{99} &= 0
\end{aligned}
\tag{B.1}
$$

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