Analytical Solutions of the FG Thick Plates with In-Plane Stiffness Variation and Porous Substances Using Higher Order Shear Deformation Theory

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ABSTRACT

This paper presents the governing equations on the rectangular plate with the variation of material stiffness through their thick using higher order shear deformation theory (HSDT). The governing equations are obtained by using Hamilton's principle with regard to variation of Young's modulus in through their thick with regard sinusoidal variation of the displacement field across the thickness. In addition, the effects of the substances in FG-porous plate are investigated.   © 2018 IAU, Arak Branch. All rights reserved.

Keywords: Functionally graded materials; Navier solution; Porous material; Rectangular plate.

1 INTRODUCTION

ONE of the deep understanding consequences of mechanical behavior existing material in nature such as bone and sea plate is using it to form the different material. Functionally graded materials (FGMs) are the material, which produces by using developed technology and forming process. One of the functions is effect tension concentration and its continuation in surface regard to monotonous changes of synthesis volume deduction [1]. Nowadays, FGMs have substituted by material in biomechanics, navel, machinery industry, and other functions. Although, FGM material are very heterogeneous but material function change is very monotonous [2]. The material, which their structure has constituted of two solid phase and liquid phase called porous material. Mineral, solid levels, stones, plants wooden structure and etc. are the porous material, which exists richly in the ambience. Nowadays, using the porous material in reason of low weight, high flexibility resistance against hairy cracks, heat insufficiently and vocal insufficiently have gained high popularity in different industries has been developed. In the present study, one FG plate response under influence of monetary load is analyzed. Also, different mathematical volume fraction for example in exponent rule of some of the scholarly uses exponential rule and same others use power distribution rules [3-6]. More solution methods in shear deformation based on classic theory which could be resulted in appropriated outputs. This theory has been used by some researchers [7-9]. Several studies have been carried out using FSDT [10-14]. For eluding the use of shear correction factors, several HSDT, such as, the third-order shear deformation theory (TSDT) [15-18], the sinusoidal shear deformation theory (SSDT) [19-21] and the hyperbolic shear deformation theory [22-23] have been proposed. In addition, all two-dimensional plate theories ignore the thickness stretching effect. Indeed, a constant transverse displacement through the thickness was considered [24-26].

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In this theory the extension effect in the line of thickness slight is ignored, and rather than the use of ordinate removal constant. In this paper, the higher order theory with elasticity extension effect in the line of thickness has been used, and also sinusoidal shear deformation and extension across the thickness removal layers include bending, shear and thickness stretching, and resulted in the motion equations of Hamilton principle and been attained stress and removal formulas for bending.

2   SOLUTION METHOD

One rectangular FG plate is shown in Fig. 1, where the porous material is considered as thickness *h*, length *a*, and width *b*. The coordinate origin is on the middle surface and considered elasticity and heterogeneous plate by changing the elasticity model in the thickness axis. One plate volume fraction distribution follows of exponent distribution law and ordered in follow method.

$$v_m(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{1}$$

which *p* is the parameter, which depends on features changes in the thickness axis. Poisson effect changes remained constant and considered young module changes for porous FG plate as follow [20].

$$E(z) = (E_m - E_c)\left(\frac{z}{h} + \frac{1}{2}\right)^p + E_c - (E_m - E_c)\frac{\alpha}{2} \tag{2}$$

which in this relation E_c is ceramic elasticity module, and E_m is metal elasticity module, and α is porous biot coefficient.
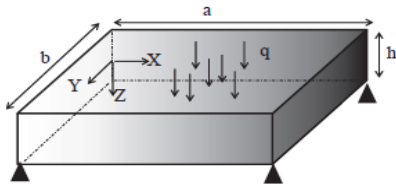

Fig. 1
Schematic representation of the geometry of the plate.

3   FIELD EQUATIONS

Removing field is being based on following theories by Belabed and coworker [21]. (1) Ordinate removal divides into three section: bending, shear, and stretching components. (2) The shear component in plane removal is sinus consequences of traction process. In the basis of these theories removal field relation attained as:

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_b(x,y)}{\partial x} - \xi(z)\upsilon_1$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_b(x,y)}{\partial x} - \xi(z)\upsilon_2 \tag{3}$$

$$w(x,y,z) = w_b(x,y) + w_s(x,y) + \zeta(z)\varphi(x,y)$$

In this equation, $u_0(x,y)$ and $v_0(x,y)$ are the displacement functions of the middle surface of the plate. $w_b(x,y)$, $w_s(x,y)$ are the bending and shear components of the transverse displacement, respectively. In addition, the additional displacement $\varphi(x,y)$ accounts for the stretching effect, and υ_1, υ_2 are rotations of *yz* and *xz* planes [22]. The sinusoidal features described as the following model:

$$\xi(z) = \frac{h}{\pi} \sin\left(\frac{\pi}{h} z\right)$$
(4)

And

$$\zeta(z) = 1 - \xi'(z) = 1 - \cos\left(\frac{\pi}{h} z\right)$$
(5)

which h is the thickness of the plate. Displacements and rotations are assumed to be small and obey Hooke's law. The linear strains associated with the above displacement field are:

$$\varepsilon_x = \frac{\partial}{\partial x} u(x,y,z) = \frac{\partial}{\partial x} u_0 - z \frac{\partial^2}{\partial x^2} w_b - \xi(z) \frac{\partial^2}{\partial x^2} w_s$$
(6a)

$$\varepsilon_y = \frac{\partial}{\partial x} v(x,y,z) = \frac{\partial}{\partial x} v_0 - z \frac{\partial^2}{\partial y^2} w_b - \xi(z) \frac{\partial^2}{\partial y^2} w_s$$
(6b)

$$\varepsilon_z = \frac{\partial}{\partial x} w(x,y,z) = \zeta'(z) \varphi$$
(6c)

$$\gamma_{xy} = \frac{\partial}{\partial y} u(x,y,z) + \frac{\partial}{\partial x} v(x,y,z) = \frac{\partial}{\partial x} v_0 + \frac{\partial}{\partial y} u_0 - 2z \frac{\partial^2}{\partial x \partial y} w_b - 2\xi(z) \frac{\partial^2}{\partial x \partial y} w_s$$
(6d)

$$\gamma_{xz} = \frac{\partial}{\partial z} u(x,y,z) + \frac{\partial}{\partial x} w(x,y,z) = \zeta(z) \left(\frac{\partial}{\partial x} w_s + \frac{\partial}{\partial x} \varphi \right)$$
(6e)

$$\gamma_{yz} = \frac{\partial}{\partial z} v(x,y,z) + \frac{\partial}{\partial y} w(x,y,z) = \zeta(z) \left(\frac{\partial}{\partial y} w_s + \frac{\partial}{\partial y} \varphi \right)$$
(6f)

To simple relations writing way consider derivational operators as well as follow:

$$d_1 = \frac{\partial}{\partial x}, d_2 = \frac{\partial}{\partial y}, d_{11} = \frac{\partial^2}{\partial x \partial x}, d_{22} = \frac{\partial^2}{\partial y \partial y}, d_{12} = \frac{\partial^2}{\partial x \partial y}$$
(7)

4   STRUCTURAL EQUATIONS

Young model mentioned in relation (2), so the structural equations for FG-porous plate can be written as follow:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}$$
(8)

which the elastic constants is determined with regard to relation (2):

$$Q_{11} = Q_{22} = Q_{33} = \frac{1-\upsilon}{(1+\upsilon)(1-2\upsilon)} E(Z)$$

$$Q_{12} = Q_{13} = Q_{23} = \frac{\upsilon}{(1+\upsilon)(1-2\upsilon)} E(Z) \tag{9}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{1}{2(1+\upsilon)} E(Z)$$

The axial forces, shear forces, bending momentum and shear momentum are determined by integrating stresses along the thickness of the FG plate as follows:

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz = A_{11}d_1u_0 + A_{12}d_2v_0 - B_{11}d_{11}w_b - B_{12}d_{22}w_b - B_{11}^s d_{11}w_s - B_{12}^s d_{22}w_s + L^1\varphi$$

$$N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_Y dz = A_{12}d_1u_0 + A_{22}d_2v_0 - B_{12}d_{11}w_b - B_{22}d_{22}w_b - B_{12}^s d_{11}w_s - B_{22}^s d_{22}w_s + L^1\varphi$$

$$N_z = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_Z \zeta'(Z) dz = L^1(d_1u_0 + d_2v_0) - L^2(d_{11}w_b + d_{22}w_b) - L^3(d_{11}w_s + d_{22}w_s) + L^4\varphi \tag{10a}$$

$$N_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} dz = A_{66}d_1v_0 + A_{66}d_2u_0 - 2B_{66}d_{12}w_b - 2B_{66}^s d_{12}w_s$$

$$M_x^b = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_x dz = B_{11}d_1u_0 + B_{12}d_2v_0 - D_{11}d_{11}w_b - D_{12}d_{22}w_b - D_{11}^s d_{11}w_s - D_{12}^s d_{22}w_s + L^2\varphi$$

$$M_y^b = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_y dz = B_{12}d_1u_0 + B_{22}d_2v_0 - D_{12}d_{11}w_b - D_{22}d_{22}w_b - D_{12}^s d_{11}w_s - D_{22}^s d_{22}w_s + L^2\varphi \tag{10b}$$

$$M_{xy}^b = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\tau_{xy} dz = B_{66}d_2u_0 + B_{66}d_1v_0 - 2D_{66}d_{12}w_b - 2D_{66}^s d_{12}w_s$$

$$M_x^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \xi(z)\sigma_x dz = B_{11}^s d_1u_0 + B_{12}^s d_2v_0 - D_{11}^s d_{11}w_b - D_{12}^s d_{22}w_b - H_{11}^s d_{11}w_s - H_{12}^s d_{22}w_s + L^3\varphi$$

$$M_y^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \xi(z)\sigma_y dz = B_{12}^s d_1u_0 + B_{22}^s d_2v_0 - D_{12}^s d_{11}w_b - D_{22}^s d_{22}w_b - H_{12}^s d_{11}w_s - H_{22}^s d_{22}w_s + L^3\varphi \tag{10c}$$

$$M_{xy}^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \xi(z)\tau_{xy} dz = B_{66}^s d_2u_0 + B_{66}^s d_1v_0 - 2D_{66}^s d_{12}w_b - 2H_{66}^s d_{12}w_s$$

$$S_{xz} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xz}\zeta(z) dz = A_{55}^S d_1w_s + A_{55}^S d_1\varphi \tag{10d}$$

$$S_{yz} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{yz}\zeta(z) dz = A_{44}^S d_2w_s + A_{44}^S d_2\varphi \tag{10e}$$

The coefficients are calculated as follow:

$$A_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}dz \, , B_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}zdz \, , B_{ij}^s = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}\xi(z)dz \, , D_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}z^2dz \, , D_{ij}^s = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}z\,\xi(z)dz \, ,$$

$$H_{ij}^s = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}\xi^2(z)dz \, , L^1 = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{13}\zeta'(z)dz \, , L^2 = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{13}z\,\zeta'(z)dz \, , L^3 = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{13}\xi(z)\zeta'(z)dz \tag{11}$$

$$L^4 = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{33}\zeta'(z)\zeta'(z)dz$$

4   GOVERNING EQUATIONS

Governing equations in considered theory derived from Hamilton's principle which its relation is as well as follow:

$$\int_0^T (\delta U + \delta V - \delta K)dt = 0 \tag{12}$$

In this relation $\delta U, \delta V, \delta K$ are the variations of the strain energy, the potential energy, and the kinetic energy, respectively.

$$\delta U = \int_{\frac{-h}{2}}^{\frac{h}{2}} \int_A \begin{pmatrix} \sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_z \delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} \\ +\tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz} \end{pmatrix} = 0 \tag{13a}$$

$$\delta V = -\int_A q\,\delta(w_b + w_s + w_{st})dA = 0 \tag{13b}$$

In this relation, the kinetic Energy is ignored duo to the $\delta K = 0$, Substituting the expressions for stresses and strains from Eqs. (6) and (8) into Eqs. (12) and (13) integral then adding efficiency $\delta u_0, \delta v_0, \delta w_b, \delta w_s, \delta\varphi$ attain dominate equation on the plate.

$$\delta u_0 : \frac{\partial N_X}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{14a}$$

$$\delta v_0 : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \tag{14b}$$

$$\delta w_b : \frac{\partial^2 M_x^b}{\partial x^2} + \frac{\partial^2 M_y^b}{\partial y^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\,\partial y} + q = 0 \tag{14c}$$

$$\delta w_s : \frac{\partial^2 M_x^s}{\partial x^2} + \frac{\partial^2 M_y^s}{\partial y^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x\,\partial y} + \frac{\partial S_{XZ}^S}{\partial x} + \frac{\partial S_{yZ}^S}{\partial y} + q = 0 \tag{14d}$$

$$\delta\varphi : \frac{\partial S_{XZ}^S}{\partial x} + \frac{\partial S_{yZ}^S}{\partial y} + q\varsigma(z) - N_Z = 0 \tag{14e}$$

By substituting Eq. (11) in (14), relations can be stated balance equations in the basis of removal filed terms as follow:

$$d_1 A_{11} d_1 u_0 + A_{11} d_{11} u_0 + d_1 A_{12} d_2 v_0 + A_{12} d_{12} v_0 + A_{66} d_{12} v_0 + A_{66} d_{22} u_0 = 0 \tag{15a}$$

$$A_{12} d_{12} u_0 + A_{22} d_{22} v_0 + d_1 A_{66} d_1 v_0 + A_{66} d_{11} v_0 + d_1 A_{66} d_2 u_0 + A_{66} d_{12} u_0 = 0 \tag{15b}$$

$$d_{11} D_{11} d_1 w_b - 2 d_1 D_{11} d_{111} w_b - D_{11} d_{1111} w_b - d_{11} D_{12} d_{22} w_b - 2 d_1 D_{12} d_{122} w_b - D_{12} d_{1122} w_b -$$
$$d_{11} D_{11}^s d_1 w_s - 2 d_1 D_{11}^s d_{1111} w_s - D_{12}^s d_{1111} w_s - d_{11} D_{12}^s {}_{22} w_s - 2 d_1 D_{12}^s d_{122} w_s - D_{12}^s d_{1122} w_s +$$
$$d_{11} L^2 \varphi + 2 d_1 L^2 d_1 \varphi + L^2 d_{11} \varphi - D_{12} d_{1122} w_b - D_{22} d_{2222} w_b - D_{12}^s d_{1122} w_s - D_{22}^s d_{2222} w_s +$$
$$L^2 d_{22} \varphi - 4 d_1 D_{66} d_{122} w_b - 4 D_{66} d_{1122} w_b - 4 d_1 D_{66}^s d_{122} w_s - 4 D_{66}^S d_{1122} w_s + q = 0 \tag{15c}$$

$$d_{11} D_{11}^s d_1 w_b - 2 d_1 D_{11}^s d_{111} w_b - D_{11}^s d_{1111} w_b - d_{11} D_{12}^s d_{22} w_b - 2 d_1 D_{12}^s d_{122} w_b - D_{12}^s d_{1122} w_b - d_{11} H_{11}^s d_1 w_s -$$
$$2 d_1 H_{11}^s d_{1111} w_s - H_{11}^s d_{1111} w_s - d_{11} H_{12}^s d_{22} w_s - 2 d_1 H_{12}^s d_{122} w_s - H_{12}^s d_{1122} w_s + d_{11} L^3 \varphi + 2 d_1 L^3 d_1 \varphi +$$
$$L^3 d_{11} \varphi - D_{12}^s d_{1122} w_b - D_{22}^s d_{2222} w_b - H_{12}^s d_{1122} w_s - H_{22}^s d_{2222} w_s + L^3 d_{22} \varphi - 4 d_1 D_{66}^s d_{122} w_b -$$
$$4 D_{66}^s d_{1122} w_b - 4 d_1 H_{66}^s d_{122} w_s - 4 H_{66}^S d_{1122} w_s + d_1 A_{55}^s d_1 w_s + A_{55}^s d_{11} w_s + d_1 A_{55}^s d_1 \varphi + A_{55}^s d_{11} \varphi +$$
$$A_{44}^s d_{22} w_s + A_{44}^s d_{22} \varphi + q = 0 \tag{15d}$$

$$d_1 A_{55}^s d_1 w_s + A_{55}^s d_{11} w_s + d_1 A_{55}^s d_1 \varphi + A_{55}^s d_{11} \varphi + A_{44}^s d_{22} w_s + A_{44}^s d_{22} \varphi + L^2 (d_{11} w_b + d_{22} w_b) + L^3 (d_{11} w_s + d_{22} w_s) - L^4 \varphi = 0 \tag{15e}$$

In considering porous FG plate with length ordinate respectively *a, b, h* determine removal field as well as following with the Navier type solution.

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \\ \varphi \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos\left(\dfrac{m\pi}{a}\right) x \sin\left(\dfrac{n\pi}{b}\right) y \\ V_{mn} \sin\left(\dfrac{m\pi}{a}\right) x \cos\left(\dfrac{n\pi}{b}\right) y \\ W_{bmn} \sin\left(\dfrac{m\pi}{a}\right) x \sin\left(\dfrac{n\pi}{b}\right) y \\ W_{smn} \sin\left(\dfrac{m\pi}{a}\right) x \sin\left(\dfrac{n\pi}{b}\right) y \\ \varphi_{mn} \sin\left(\dfrac{m\pi}{a}\right) x \sin\left(\dfrac{n\pi}{b}\right) y \end{Bmatrix} \tag{16}$$

And

$$q = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn} \sin\left(\frac{m\pi}{a}\right) \sin\left(\frac{n\pi}{b}\right) \tag{17}$$

which Q_{mn} to distribute monotonous extensive load is as follow:

$$Q_{mn} = \begin{cases} \dfrac{16 q_0}{\pi^2 mn} m, n = 1, 3, 5 \dots \\ 0 \, m, n = 2, 4, 6 \end{cases} \tag{18}$$

By substitution (16), (17) relations in relation (14) result one group of algebra equation which can be arranged them as follow:

$$\begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \\ \varphi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \dfrac{16q_0}{\pi^2 mn} \\ \dfrac{16q_0}{\pi^2 mn} \\ 0 \end{Bmatrix} \tag{19}$$

where the coefficients of the above matrix are given as follows:

$$J_{11} = -\left(\frac{m\pi}{a}\right)^2 A_{11} - \left(\frac{n\pi}{b}\right)^2 A_{66}, J_{12} = -\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)A_{12} - \left(\frac{n\pi}{b}\right)\left(\frac{m\pi}{a}\right)A_{66}, J_{13} = J_{14} = J_{15} = 0 \tag{20a}$$

$$J_{21} = -\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)A_{12} - \left(\frac{n\pi}{b}\right)\left(\frac{m\pi}{a}\right)A_{66}, J_{22} = -\left(\frac{n\pi}{b}\right)^2 A_{22} - \left(\frac{m\pi}{a}\right)^2 A_{66}, J_{23} = J_{24} = J_{25} = 0 \tag{20b}$$

$$J_{33} = \left(\frac{m\pi}{a}\right)^2 d_{11}D_{11} + \left(\frac{n\pi}{b}\right)^2 d_{11}D_{12} - \left(\frac{m\pi}{a}\right)^4 D_{11} - \left(\frac{n\pi}{b}\right)^4 D_{22} - 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 D_{12} - 4\left(\frac{n\pi}{b}\right)^2\left(\frac{m\pi}{a}\right)^2 D_{66}$$

$$J_{34} = \left(\frac{m\pi}{a}\right)^2 d_{11}D_{11} + \left(\frac{n\pi}{b}\right)^2 d_{11}D_{12} - \left(\frac{m\pi}{a}\right)^4 D_{11} - \left(\frac{n\pi}{b}\right)^4 D_{22} - 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 D_{12} - 4\left(\frac{n\pi}{b}\right)^2\left(\frac{m\pi}{a}\right)^2 D_{66} \tag{20c}$$

$$J_{33} = \left(\frac{m\pi}{a}\right)^2 d_{11}D_{11}^s + \left(\frac{n\pi}{b}\right)^2 d_{11}D_{12}^s - \left(\frac{m\pi}{a}\right)^4 D_{11}^s - \left(\frac{n\pi}{b}\right)^4 D_{22}^s - 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 D_{12}^s - 4\left(\frac{n\pi}{b}\right)^2\left(\frac{m\pi}{a}\right)^2 D_{66}^s$$

$$J_{35} = d_{11}L^2 - L^2\left(\frac{m\pi}{a}\right)^2 - L^2\left(\frac{n\pi}{b}\right)^2, J_{13} = J_{23} = 0$$

$$J_{43} = \left(\frac{m\pi}{a}\right)^2 d_{11}D_{11} + \left(\frac{n\pi}{b}\right)^2 d_{11}D_{12} - \left(\frac{m\pi}{a}\right)^4 D_{11} - \left(\frac{n\pi}{b}\right)^4 D_{22} - 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 D_{12} - 4\left(\frac{n\pi}{b}\right)^2\left(\frac{m\pi}{a}\right)^2 D_{66}$$

$$J_{44} = \left(\frac{m\pi}{a}\right)^2 d_{11}H_{11}^s + \left(\frac{n\pi}{b}\right)^2 d_{11}H_{12}^s - \left(\frac{m\pi}{a}\right)^4 H_{11}^s - \left(\frac{n\pi}{b}\right)^4 H_{22}^s - 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 H_{12}^s - 4\left(\frac{n\pi}{b}\right)^2\left(\frac{m\pi}{a}\right)^2 H_{66}^s - \left(\frac{m\pi}{a}\right)^2 A_{55}^s - \left(\frac{n\pi}{b}\right)^2 A_{44}^s \tag{20d}$$

$$J_{45} = d_{11}L^3 - L^3\left(\frac{m\pi}{a}\right)^2 - L^3\left(\frac{n\pi}{b}\right)^2 - \left(\frac{m\pi}{a}\right)^2 A_{55}^s - \left(\frac{n\pi}{b}\right)^2 A_{44}^s$$

$$J_{41} = J_{42} = 0$$

$$J_{53} = -L^2\left(\frac{m\pi}{a}\right)^2 - L^2\left(\frac{n\pi}{b}\right)^2$$

$$J_{54} = -L^3\left(\frac{m\pi}{a}\right)^2 - L^3\left(\frac{n\pi}{b}\right)^2 - \left(\frac{m\pi}{a}\right)^2 A_{55}^s - \left(\frac{n\pi}{b}\right)^2 A_{44}^s \tag{20e}$$

$$J_{55} = -\left(\frac{m\pi}{a}\right)^2 A_{55}^s - \left(\frac{n\pi}{b}\right)^2 A_{44}^s - L^4$$

$$J_{51} = J_{52} = 0$$

5   CONCLUSIONS AND DISCUSSIONS

A rectangle porous FG plate of metal and ceramic with size of *a=b=5h*, *p=3*, the ratio of *a=b=1*, and the thickness *h=0.2* that are given in Table 1, is considered. Moreover, the various structural technique for FGM is considered so

that is depend on the applied programs such as (a) porous drive, (b) chemical synthesis of one phase materials, (c) the volume fraction exponent of the material phases.

Table 1
The plate materials.

$Ti - 6Al - 4v$ (metal)	Al / ZrO_2 (ceramic)
$E_m = 66.2Gpa$	$E_c = 117Gpa$
$\upsilon = 0.33$	$\upsilon = 0.33$

In Fig. 2, the mid-plate digression is in slight of plate length with regard to the maximum digression in plate center. By comparing the result with implemented research by Amirpoor [23], it can be shown that the Young's modulus is changed in slight of the thickness. In addition, the material volume fraction is varied along the thickness caused the maximum digression in the plate core while the time of Young's modulus is changed along the length maximum digression dose not occurs in the plate core but occur in right side, which the materials concentrations are fallen [23].
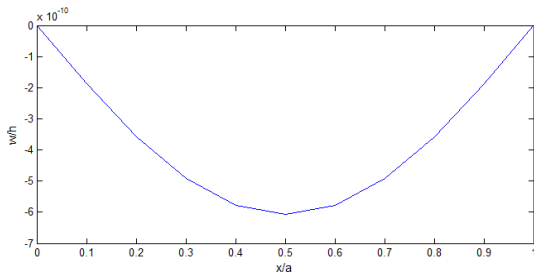

Fig. 2
Deflection of the FG plate with variation of the material stiffness through the thickness.

Fig. 3 shows Young's modulus changes in direction of the plate thicknss. It can be seen that in the mideal plane for *p*=3, the more Young's modulus changes changes changes is occurred. In latter determin contraction and consider their changing process along the thickness.

Fig. 4 shows the strain changes in direction of thickness. It should be noted that in Fig. 4, the natural plate of the porous FG plate is in the mideal plate.
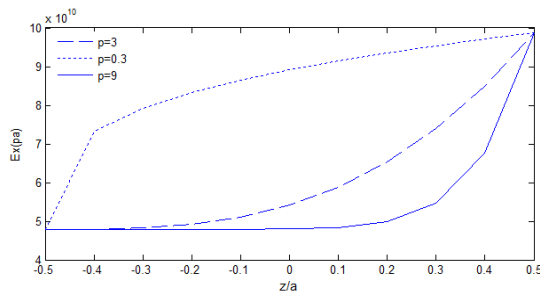

Fig. 3
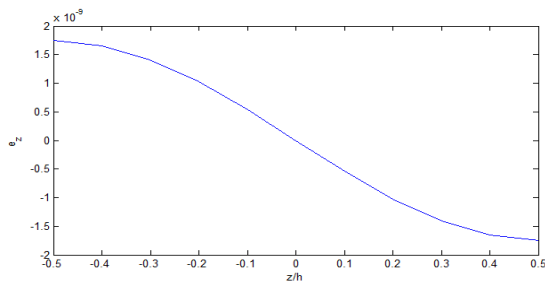Variation of Young's modulus ($E(x)$) through the length with different power-law indices.

Fig. 4
The strain ε_z at the center of the FG plate with variation of material stiffness through the thickness.

Fig.5 shows the resulted responses of analytic soulotion for porous FG plate from relation (8). Here, it must be mentioned that the effect of the sinusoid function $\xi(z), \zeta'(z)$ in relation(6) in σ_x response to comparing with

bending, which is linear function of z mentioned little. In other words, sinusoid terms are shorter than the liner terms because of this difraction is dominated issue.

In Fig. 6 we see that the effect of porous volume fraction (α) in porous FG plate diffraction that inceased maximum digression by increasing α considerably, which shows that this amount decrease by increasing α in response to diffraction.
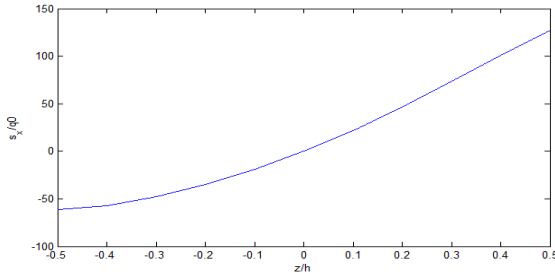

Fig. 5
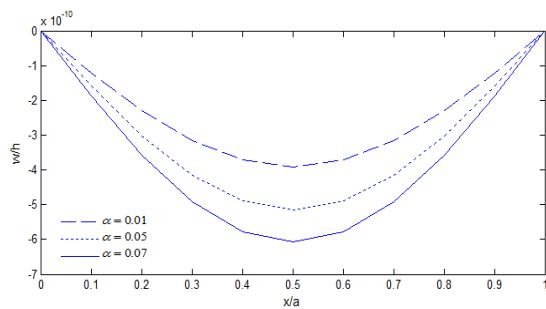The stress S_{xx} at the mid-plane of the FG plate with variation of the material stiffness through the thickness.

Fig. 6
Effect of porous volume fraction (α) in porous FG plate diffraction.

In Fig.7 stress changes σ_{xx} in middel porous FG plate is shown that results of porous volume fraction effect α, and it is shown that the traction and stressed strain maximum occurs in the plate low and high levels, which in it stress becomes zero in $z=0$.
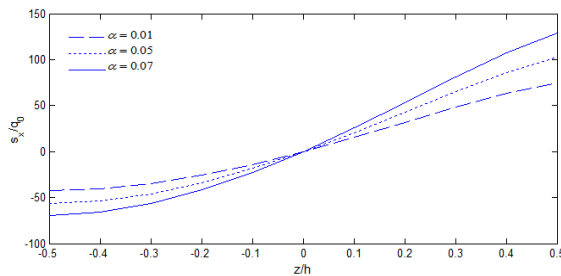

Fig. 7
Stress changes σ_{xx} in mideal porous FG plate have show that results of porous volum fraction effect α.

Fig. 8 show porous volum fraction effect α in resulted strain in slight of plate thickness. It shows that coefficient increasing α occure maximum change in plate lowed high levels and occure $\varepsilon_{zz} = 0$ in mideal level.
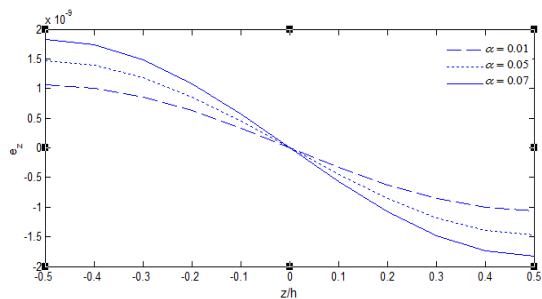

Fig. 8
Porous volum fraction effect α in resulted strain in slight of plate thickness.

In Fig.9 comparison among FG plate digression with young model changes in slight [23] of porous FG plate with young model changes in slight of thickness. It is observed that in FG plate with young model changes in slight of length maximum digression doesnˈt occure in the mid-plate and on the rigth side.while in porous FG plate with young modol changes occur maximum digression in plate core. and also porous coefficient decrease plate severity and increase maximum digression.
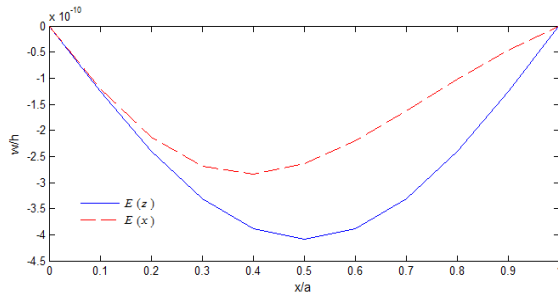

Fig. 9
Comparison among FG plate digression with young model changes in slight of porous FG plate with young model changes in slight of thickness.

6   CONCLUSIONS

In this study, the sinusoidal change is investigated for rectangular thick FG plate by using of power distribution rule of volume fraction along the thickness. The solution is achived using the simple boundary conditions and under monotonous loading. Formulas investigation attained for tension and shear deformation without regard to corrected efficiency which can summarize conclusions in comparison with completed research by amirpoor [23] as follow:

- Maximum digression occurs in plate core.
- Stress distribution σ_{xx} is linear because the sinusoid terms are small in comparing with linear terms.
- General plate severity is more depends on constituent material changes and Young's modol proportion E_c / E_m and the volume porous feraction coefficient (α).
- More digression is not depended on the thickness and many digressions with indicator increasing (p) in slight of thickness.

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