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Thermal Creep Analysis of Functionally Graded Thick-Walled Cylinder Subjected to Torsion and Internal and External Pressure

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ABSTRACT

Safety analysis has been done for the torsion of a functionally graded thick-walled circular cylinder under internal and external pressure subjected to thermal loading. In order to determine stresses the concept of Seth's transition theory based on generalized principal strain measure has been used. This theory simplifies the set of mechanical equations by mentioning the order of the measure of deformation. This theory helps to achieve better agreement between the theoretical and experimental results. Results have been analyzed with or without thermal effects for functionally graded and homogeneous cylinder with linear and nonlinear strain measure. From the analysis, it has been concluded that in creep torsion cylinder made up of less functionally graded material (FGM) under pressure is better choice for designing point of view as compared to homogeneous cylinder. This is due to shear stresses which are maximum for cylinder made up of functionally graded material as compared to homogeneous material.

Keywords : Thermal; Creep; Torsion; Strain measure; Functionally graded material.

1 INTRODUCTION

HOLLOW cylinder subjected to creep torsion finds its application in aerospace, chemical plants, nuclear power plants etc. Therefore, the highest quality is necessary to manufacture these systems for safe, long term stability and reliable operations. Functionally graded materials are new generation non-homogeneous engineering materials whose composition changes over volume fraction so that a certain variety of the local material properties can be achieved [1-3]. Functionally graded materials are used as thermal barriers for designing structural component in aerospace applications and nuclear reactors. Due to better thermal resistance and mechanical performance of functionally graded materials their applications found in structural components which operate under extremely high-temperature environment to reduce the possibility of fracture. The classical theory of deformation considered the jump conditions, yield criterion and linear strain measure to determine the stresses using the concept of infinitesimal strain theory [4, 5]. However, transition theory given by Seth's [6] does not require any of the above assumptions and thus solves a more general problem using the concept of generalized strain measure [7, 8]. This generalized strain measure approach can be used to find the stresses in plasticity and creep problems by determining the asymptotic solution at the transition points of the governing differential equation. The investigation of torsion in a cylinder made up of homogeneous materials has been analyzed by some researchers using transition theory [9, 10].

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Shear stresses for a circular cylinder made up of transversely isotropic material subjected to torsion have been determined by Gupta and Rana [11]. They found that the value of shear stresses for a cylinder made up of transversely isotropic material is more than that of cylinder made up of isotropic incompressible material. Lekhnitskii's [12] elaborated exact solution for the stress function in a hollow cylinder made up of anisotropic material. Rooney et al. [13] studied cylindrical bar made up of inhomogeneous material with constant shear modulus and discussed the impact of material inhomogeneity on the torsion response. Ting [14, 15] solved the problem of cylindrically elastic tube made up of anisotropic material subjected to pressure, torsion and extension. Horgan et al. [1] investigated torsion in a solid cylindrical bar made up of isotropic material whose shear modulus is varying with position and concluded that maximum shear stress does not occur on the boundary of the rod. Chen et al. [16] solved the same problem, including the influence of a uniform temperature change, using Lekhnitskii's stress field approach. Tarn [17] solved the problem of circular tube or bar subjected to torsion, pressure, uniform electric loading and temperature change. Batra [18] has analyzed analytically the torsion of circular cylindrical bar made of isotropic linear elastic material with varying material moduli in the longitudinal direction. Uscilowska [19] studied the torsion problem of a hollow rod made up of functionally graded material using the method of fundamental solution. Bayata et al. [20] gave a general solution for the torsion of hollow cylinders made up of functionally graded materials and determine the angle of twist and shear stress for material whose Young's modulus and Poisson's ratio varying in the radial direction. Sharma et al. [21] have studied elastic-plastic torsion in a functionally graded cylinder under external pressure using Seth's transition theory. They observed that the shear stresses are less for less compressible non-homogeneous cylinder as compared to highly compressible non-homogeneous cylinder.

In this work, creep behavior of a functionally graded cylinder in torsion under internal and external pressure which is subjected to thermal loading is analyzed. The material properties i.e. Young's modulus of the cylinder varying using power law in the radial direction. The influence of various paramters such as pressure, temperature and strain measure on creep stresses has also been studied. The results have been discussed numerically and depicted graphically.

2 MATERIAL AND GEOMETRIC PROPERTIES OF THE CYLINDER

Consider a thick-walled functionally graded circular cylinder shown in Fig. 1. In cylinder, a and b are the internal and external radii, respectively, and r is the radial distance $(a \le r \le b)$. Cylinder is subjected to internal pressure p_1 and external pressure p_2 with thermal loading as can be seen in Fig. 1. Creep analysis is more significant at high temperature as compare to atmospheric temperature. Therefore, thermal analysis of torsion in creep help engineers in the designing of structural components like steam generators, nuclear power plants and other structures according to industrial demand.

The components of displacement (u, v, w) in cylindrical polar coordinates (r, θ, z) [8] are given by

$$u = r(1-\beta) \ , \quad v = \eta r z \ \text{ and } \quad w = d.z \ , \tag{1}$$

where β is a function of $r = \sqrt{x^2+y^2}$, d is a constant and η is the angle of twist per unit length.

The compressibility of functionally graded cylinder is taken as:

$$C = C_0 r^k , \tag{2}$$

where $a \le r \le b$, C_0 is material constant and k (≥ 0) is geometric parameter.

The generalized components of strain [9] are expressed as:

$$e_{rr} = \frac{1}{n^m}\left[1-(r\beta'+\beta)^n\right]^m, e_{\theta\theta} = \frac{1}{n^m}\left[1-\beta^n\right]^m, e_{zz} = \left(\frac{D^n}{n}\right)^m\left[1-\left(\frac{\eta r\beta}{D}\right)^n\right]^m, e_{\theta z} = \frac{1}{n^m}\left[\eta^{n/2} r^{n/2}\beta^n\right]^m, e_{r\theta} = e_{zr} = 0, \tag{3}$$

where $D^n = \left[1-(1-d)^n\right]$, n is the strain measure and $\beta' = \dfrac{d\beta}{dr}$, m is geometric parameter.

In secondary creep state, creep strain rate is minimum and retains its ability to experience deformation [22]. Therefore, in this work, we assumed that $m = 1$, which holds for secondary creep.

According to theory of elasticity, the thermal stress-strain relationship for isotropic material is given by

$$T_{ij} = \lambda\delta_{ij} I_1 + 2\mu e_{ij} - \xi\theta\delta_{ij}, \ (i, j = 1,2,3) \tag{4}$$

where T_{ij}, e_{ij} are stress and strain tensors, respectively, $I_1 = e_{kk}$ is strain invariants, λ, μ are Lame's constants, δ_{ij} is Kronecker's delta, $\xi = \alpha(3\lambda + 2\mu)$, α is the coefficient of thermal expansion and θ is temperature.

The temperature function θ has to satisfy the Laplace equation i.e.

$$\theta,_{ii} = 0 \tag{5}$$

The temperature field satisfying Eq. (5) is $\theta = \theta_0$ at $r = a, \theta = 0$ at $r = b$, where θ_0 is a constant, given by $\theta = \left(\theta_0 \log\dfrac{r}{b}\right) \Big/ \left(\log\dfrac{a}{b}\right)$.

Eq. (4) using Eq. (3) with $m = 1$ can be rewritten as:

$$\begin{aligned}
T_{rr} &= \left(\frac{\lambda+2\mu}{n}\right)\left(1-(r\beta'+\beta)^n\right) + \frac{\lambda}{n}\left(1-\beta^n\right) + \frac{\lambda}{n}\left[D^n - (\eta r\beta)^n\right] - \xi\theta, \\
T_{\theta\theta} &= \left(\frac{\lambda}{n}\right)\left(1-(r\beta'+\beta)^n\right) + \left(\frac{\lambda+2\mu}{n}\right)\left(1-\beta^n\right) + \frac{\lambda}{n}\left[D^n - (\eta r\beta)^n\right] - \xi\theta, \\
T_{zz} &= \left(\frac{\lambda}{n}\right)\left(1-(r\beta'+\beta)^n\right) + \left(\frac{\lambda}{n}\right)\left(1-\beta^n\right) + \left(\frac{\lambda+2\mu}{n}\right)\left[D^n - (\eta r\beta)^n\right] - \xi\theta, \\
T_{\theta z} &= \frac{2\mu}{n}\left[\eta^{n/2} r^{n/2} \beta^n\right], T_{zr} = T_{r\theta} = 0.
\end{aligned} \tag{6}$$

The equation of equilibrium in the absence of body forces is defined as:

$$\frac{d}{dr}(T_{rr}) + \frac{(T_{rr} - T_{\theta\theta})}{r} = 0, \tag{7}$$

where T_{rr} and $T_{\theta\theta}$ are radial and circumferential stresses, respectively.

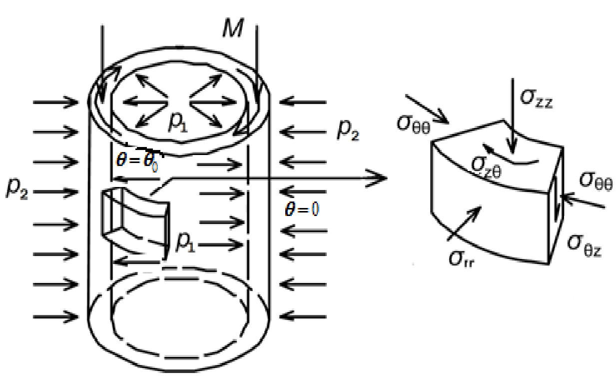


Fig.1
A functionally graded thick-walled cylinder under pressure and thermal loading subjected to torsion.

2.1 Identification of transition point

As there exists a intermediate state, i.e., transition state in between elastic and creep state and at transition, the differential system defining the elastic state should reach some kind of criticality. The non-linear differential equation at transition state is obtained by substituting Eq. (6) in Eq. (7) as,

$$n\beta P(P+1)^{n-1}\frac{dP}{d\beta}=r\left(\frac{\mu'}{\mu}-\frac{C'}{C}\right)\left[\left\{(3-2C)-(1-C)(1-d)^n\right\}\frac{1}{\beta^n}-\left\{(1-C)\left(1+\eta^n r^n\right)+(P+1)^n\right\}\right]$$
$$C\left[1-(P+1)^n\right]+rC'\left[\left(1+\eta^n r^n\right)-\left\{2-(1-d)^n\right\}\frac{1}{\beta^n}\right]-nP\left[(1-C)+(P+1)^n\right]-n\eta^n r^n(1-C)(P+1)$$
$$-n\,r^{n+1}\eta^{n-1}\eta'(1-C)-\frac{\overline{\theta_0}\,nC}{2\mu\beta^n}\left[\xi' r\log\left(\frac{r}{b}\right)+\xi\right], \tag{8}$$

where $r\beta'=\beta P$, $C=\dfrac{2\mu}{(\lambda+2\mu)}$, $\overline{\theta_0}=\dfrac{\theta_0}{\log\frac{a}{b}}$, $\eta=\dfrac{2\eta_1(3-2C)}{(2-C)}$ and η_1 is a constant.

Eq. (8) shows that the transition points of differential equation are $P=-1$ and $P=\pm\infty$. The transition point are critical points of the differential equation where differential equation is asymptotically stable and derivatives are not differentiable. At this physical point, distinction between elastic and creep disappears. The asymptotic solution through $P\to-1$ gives creep stresses [6, 8, 10, 11] depends upon the transition function used.

The boundary conditions are

$$T_{rr}=-p_1 \text{ at } r=a \text{ and } T_{rr}=-p_2 \text{ at } r=b. \tag{9}$$

The resultant axial force in the circular cylinder is given by

$$\int_a^b r\,T_{zz}\,dr=0\,. \tag{10}$$

2.2 Solution through the principal stress difference

For finding the creep stresses at the transition point $P\to-1$, we define the transition function [6, 8-11] through the principal stress difference as follows:

$$R=T_{rr}-T_{\theta\theta}=\frac{2\mu}{n}\beta^n\left[1-(P+1)^n\right]. \tag{11}$$

Taking the logarithmic differentiation of the Eq. (11) with respect to *r*, we have

$$\frac{d}{dr}(\log R)=\frac{nP}{r}+\frac{\mu'}{\mu}-\frac{n\beta P(P+1)^{n-1}\frac{dP}{d\beta}}{r\left[1-(P+1)^n\right]}. \tag{12}$$

Substituting the value of $\dfrac{dP}{d\beta}$ from Eq. (8) in Eq. (12) and taking the asymptotic value $P\to-1$, we get

$$\frac{d}{dr}(\log R)=2\frac{\mu'}{\mu}-\frac{C'}{C}-\frac{2n}{r}+X\,, \tag{13}$$

where

$$X = \frac{(n-1)C}{r} - C\frac{\mu'}{\mu} + \frac{C'}{\beta^n}\left[2-(1-d)^n\right] - \left(\frac{\mu'}{\mu} - \frac{C'}{C}\right)\left[(3-2C)-(1-C)(1-d)^n\right]\frac{1}{\beta^n}$$
$$+(1-C)\, n\, r^n \eta^{n-1}\eta' + \frac{nC\overline{\theta_0}}{2\mu r\beta^n}\left[\xi + \xi' r\log\left(\frac{r}{b}\right)\right].$$

Integration of Eq. (13) yields

$$R = A\frac{\mu^2}{Cr^{2n}}\exp f(r), \tag{14}$$

where $f(r) = \int X\, dr$ and A is a constant of integration.

Using Eq. (11) and Eq. (14), we have

$$R = T_{rr} - T_{\theta\theta} = A\frac{\mu^2}{Cr^{2n}}\exp f(r) = A\, r\, F, \tag{15}$$

where $F = \frac{\mu^2}{Cr^{2n+1}}\exp f(r)$.

The asymptotic value of β from Eqs. (11) and (15) is

$$R = A\frac{\mu^2}{Cr^{2n}}\exp f(r), \beta^n = A\frac{n\mu}{2Cr^{2n}}\exp f(r). \tag{16}$$

Substituting Eq. (15) in Eq. (7) and integrating, we get

$$T_{rr} = B - A\int F\, dr, \tag{17}$$

where B is a constant of integration and asymptotic value of β is $\frac{D}{r}$ as $P \to -1$, D is a constant.

The constants A and B are obtained by using the boundary conditions from Eq.(9) in Eq.(17) as:

$$A = \frac{p_2 - p_1}{\int_a^b F\, dr},\ B = -p_2 + A\int F\, dr\Big|_{r=b}. \tag{18}$$

Substituting the value of B in Eq. (17), we get

$$T_{rr} = -p_2 + A\int_r^b F\, dr. \tag{19}$$

The circumferential, axial and shearing stresses are obtained from Eqs. (15), (18) and Eq. (6) are

$$T_{\theta\theta} = -p_2 + A\left[\int_r^b F\, dr - rF\right], T_{zz} = \frac{(1-C)}{(2-C)}(T_{rr} + T_{\theta\theta}) + 2\mu\frac{(3-2C)}{(2-C)}e_{zz} - 2\mu\frac{(3-2C)}{(2-C)}\alpha\theta,$$
$$T_{\theta z} = A\left(\frac{\mu^2\eta^{n/2}r^{-3n/2}}{C}\exp f(r)\right). \tag{20}$$

The twisting couple M is given by

$$M = 2\pi\int_a^b r^2 T_{\theta z}\, dr = 2\pi\int_a^b A\left(\frac{\mu^2\eta^{n/2} r^{(-3n+4)/2}}{C}\exp f(r)\right)dr, \tag{21}$$

where

$$D^n = \frac{\left[\lambda\int_a^b \frac{rC(3-2C)}{(1-C)(2-C)}\alpha\theta dr - \int_a^b \frac{r(1-C)}{(2-C)}\left[T_{rr}+T_{\theta\theta}\right]dr\right]}{\frac{\lambda}{n}\int_a^b \frac{rC(3-2C)}{(1-C)(2-C)}dr} + A\left(\frac{2\eta_1(3-2C)}{(2-C)}\right)^n \frac{n\mu}{2Cr^n}\exp f(r)$$

and

$$e_{zz} = \frac{\left[\lambda\int_a^b \frac{rC(3-2C)}{(1-C)(2-C)}\alpha\theta dr - \int_a^b \frac{r(1-C)}{(2-C)}\left[T_{rr}+T_{\theta\theta}\right]dr\right]}{\lambda\int_a^b \frac{rC(3-2C)}{(1-C)(2-C)}dr}.$$

Stresses for functionally graded thick-walled cylinder whose compressibility varying in the radial direction are given as:

$$\begin{aligned}
T_{rr} &= -p_2 + A_1\int_r^b F_1\, dr,\ T_{\theta\theta} = -p_2 + A_1\left[\int_r^b F_1\, dr - rF_1\right],\\
T_{zz} &= \frac{\left(1-C_0r^k\right)}{\left(2-C_0r^k\right)}\left[-2p_2 + A_1\left(\int_r^b 2F_1\, dr - rF_1\right)\right] + \frac{\lambda C_0 r^k\left(3-2C_0r^k\right)}{\left(1-C_0r^k\right)\left(2-C_0r^k\right)}\left(e_{zz}-\alpha\theta\right),\\
T_{\theta z} &= A_1 r^{-k-2n}\frac{E^2\left(2-C_0r^k\right)^2}{4C_0(3-2C_0r^k)^2}\left[\frac{2\eta_1(3-2C_0r^k)r}{\left(2-C_0r^k\right)}\right]^{n/2}\exp f_1,\\
M &= 2\pi\int_a^b\left(A_1 r^{-k-2n+2}\frac{E^2\left(2-C_0r^k\right)^2}{4C_0(3-2C_0r^k)^2}\left[\frac{2\eta_1(3-2C_0r^k)r}{\left(2-C_0r^k\right)}\right]^{n/2}\exp f_1\right)dr,
\end{aligned} \tag{22}$$

where

$$A_1 = \frac{p_2-p_1}{\int_a^b F_1\, dr},\ F_1 = \frac{E^2\left(2-C_0r^k\right)^2}{4C_0(3-2C_0r^k)^2} r^{-k-2n-1}\exp f_1,$$

$$e_{zz} = \frac{\left[\lambda\int_a^b \frac{C_0r^{k+1}(3-2C_0r^k)}{(1-C_0r^k)(2-C_0r^k)}\alpha\theta dr - \int_a^b \frac{r(1-C_0r^k)}{(2-C_0r^k)}\left[T_{rr}+T_{\theta\theta}\right]dr\right]}{\lambda\int_a^b \frac{C_0r^{k+1}(3-2C_0r^k)}{(1-C_0r^k)(2-C_0r^k)}dr}$$

and

$$f_1 = \frac{(n-1)C_0 r^k}{k} + \frac{2kC_0 r^{n+k}}{D^n(n+k)} - \frac{C_0 k}{D^n}\left[\int \frac{r^{n+k-1}\left(3-2C_0 r^k\right)dr}{\left(1-C_0 r^k\right)}\right] + \log\left(1-C_0 r^k\right)$$
$$-2\eta_1 C_0 nk\left[\int \frac{r^{n+k-1}\left(1-C_0 r^k\right)}{\left(2-C_0 r^k\right)^2}\left\{\frac{2\eta_1\left(3-2C_0 r^k\right)}{\left(2-C_0 r^k\right)}\right\}^{n-1} dr\right] + \frac{nC_0\overline{\theta_0}}{ED^n}\left[\int \frac{r^{n+k-1}\left(3-2C_0 r^k\right)}{\left(2-C_0 r^k\right)} \left\{\xi + \xi' r\log\left(\frac{r}{b}\right)\right\}dr\right].$$

Now we introduce the components in non-dimensional form as:

$$R = \frac{r}{b},\ R_0 = \frac{a}{b},\ P_2 - P_1 = \frac{p_2 - p_1}{E},\ \sigma_{rr} = \frac{T_{rr}}{E},\ \sigma_{\theta\theta} = \frac{T_{\theta\theta}}{E},\ \sigma_{zz} = \frac{T_{zz}}{E},\ \sigma_{\theta z} = \frac{T_{\theta z}}{E}.$$

Now thermal creep principal and shear stresses under internal and external pressure in non-dimensional form from Eq. (22) are expressed as:

$$\sigma_{rr} = -P_2 + A_2\int_R^1 bF_2\,dR,\ \sigma_{\theta\theta} = -P_2 + bA_2\left[\int_R^1 F_2\,dR - RF_2\right],$$
$$\sigma_{zz} = \frac{\left(1-C_0 b^k R^k\right)}{\left(2-C_0 b^k R^k\right)}\left[-2P_2 + bA_2\left(\int_R^1 2F_2\,dR - RF_2\right)\right] + \left(e_{zz} - \theta_1\frac{\log R}{\log R_0}\right),$$
$$\sigma_{\theta z} = bRA_2F_2\left(2\eta_1 bR\right)^{\frac{n}{2}}\left[\frac{(3-2C_0 b^k R^k)}{(2-C_0 b^k R^k)}\right]^{\frac{n}{2}}, \tag{23}$$
$$M = 2\pi\int_{R_0}^1\left(A_2 b^{-k-2n+3}R^{-k-2n+2}\frac{E^2\left(2-C_0 b^k R^k\right)^2}{4C_0(3-2C_0 b^k R^k)^2}\left[\frac{2\eta_1(3-2C_0 b^k R^k)bR}{\left(2-C_0 b^k R^k\right)}\right]^{n/2}\exp f_2\right)dR,$$

where

$$\alpha\theta_0 = \theta_1,\ A_2 = \frac{P_2 - P_1}{\int_a^b bF_2\,dR},\ F_2 = \frac{E^2\left(2-C_0 b^k R^k\right)^2}{4C_0(3-2C_0 b^k R^k)^2}b^{-k-2n-1}R^{-k-2n-1}\exp f_2,$$
$$f_2 = \frac{(n-1)C_0 b^k R^k}{k} + \frac{2kC_0 b^{n+k}R^{n+k}}{D^n(n+k)} - \frac{C_0 k}{D^n}\left[\int\frac{b^{n+k}R^{n+k-1}\left(3-2C_0 b^k R^k\right)dR}{\left(1-C_0 b^k R^k\right)}\right] + \log\left(1-C_0 b^k R^k\right) + \frac{n\theta_1 b^n}{\log R_0 D^n}\times$$
$$\left[\int\left\{\left(3-2C_0 b^k R^k\right) - \frac{2k\left(3-2C_0 b^k R^k\right)\log R}{\left(2-C_0 b^k R^k\right)}\right\}R^{n-1}dR\right] - 2\eta_1 C_0 nk\left[\int\frac{b^{n+k}R^{n+k-1}\left(1-C_0 b^k R^k\right)}{\left(2-C_0 b^k R^k\right)^2}\left\{\frac{2\eta_1\left(3-2C_0 b^k R^k\right)}{\left(2-C_0 b^k R^k\right)}\right\}^{n-1}dR\right]$$

and

$$e_{zz} = \frac{\left[\int_{R_0}^1\frac{b^2 R\,E\,\theta_1\log R}{\log R_0}dR - \int_{R_0}^1\frac{b^2 R(1-C_0 b^k R^k)}{(2-C_0 b^k R^k)}\left[\sigma_{rr} + \sigma_{\theta\theta}\right]dr\right]}{\int_{R_0}^1 b^2 R\,E\,dR}.$$

Eq. (23) represents thermal radial, circumferential, axial, shear stresses and twisting couple in a non-dimensional form for secondary creep state.

3 RESULTS AND DISCUSSION

The material properties of the cylinder made up of functionally graded material are defined as: thermal expansion coefficient $\alpha = 17.3\times10^{-6}\left[{}^{o}C^{-1}\right]$ (Stainless steel), compressibility coefficient $C_0 = 0.5$, Poisson's ratio $\nu = 0.3$. The inner and outer radii of the cylinder are taken as $a = 1\,[m]$ and $b = 2\,[m]$, respectively. The geometric parameters of the compressibility are $k = 0, -0.5, -1, -1.5$.

To observe the effect of temperature and pressure with different parameters of strain measure and compressibility, Figs. 2 and 3 and Table 1. have been drawn between radii ratio and stresses. The angle of twist is considered as $\eta_1 = 50$ with internal pressure greater than that of external pressure.

Influence of strain measure and non-homogeneity on circumferential stresses without temperature: It is observed from Fig. 2 and Table 1. that without thermal effects, circumferential stresses are tensile in nature while radial stresses are compressive in nature and are maximum at internal surface with linear strain measure. It has been noticed that circumferential stresses are maximum for cylinder made up of less functionally graded material as compared to highly functionally graded material or homogeneous material. It is noticed that with the change in measure from linear to nonlinear, circumferential stresses are maximum at external surface. Also, circumferential stress decreases significantly with the change in measure from linear to nonlinear which further decreases with the increase in nonlinearity of measure. Also, it has been noticed that circumferential stresses are high for highly functionally graded cylinder as compared to homogeneous and less functionally graded cylinder.

Influence of strain measure and non-homogeneity on circumferential stresses in the presence of temperature: From Fig. 3 and Table 1. it has been noticed that with the introduction of thermal effects, circumferential stresses are tensile in nature. Circumferential stress also increases for homogeneous cylinder while decreases for functionally graded cylinder with linear and nonlinear strain measure.

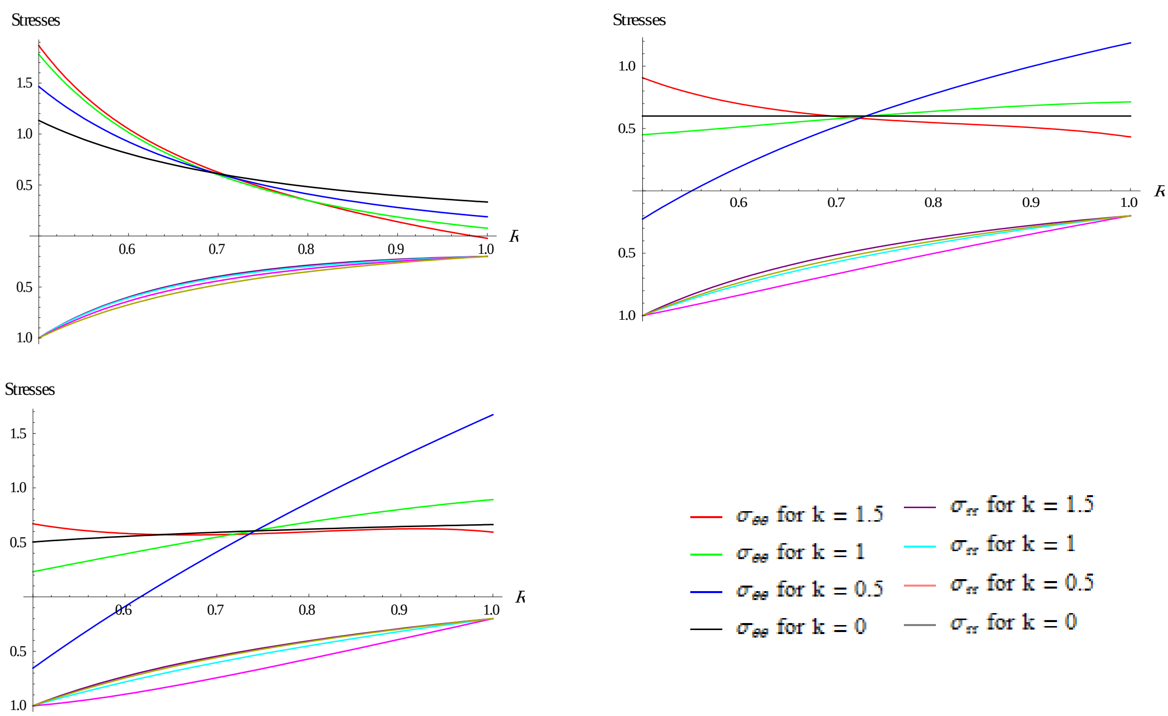


Fig.2
Creep stresses in thick-walled FGM cylinder for $P_1 = 1$ and $P_2 = 0.2$ with $N = 1, 3, 5$.

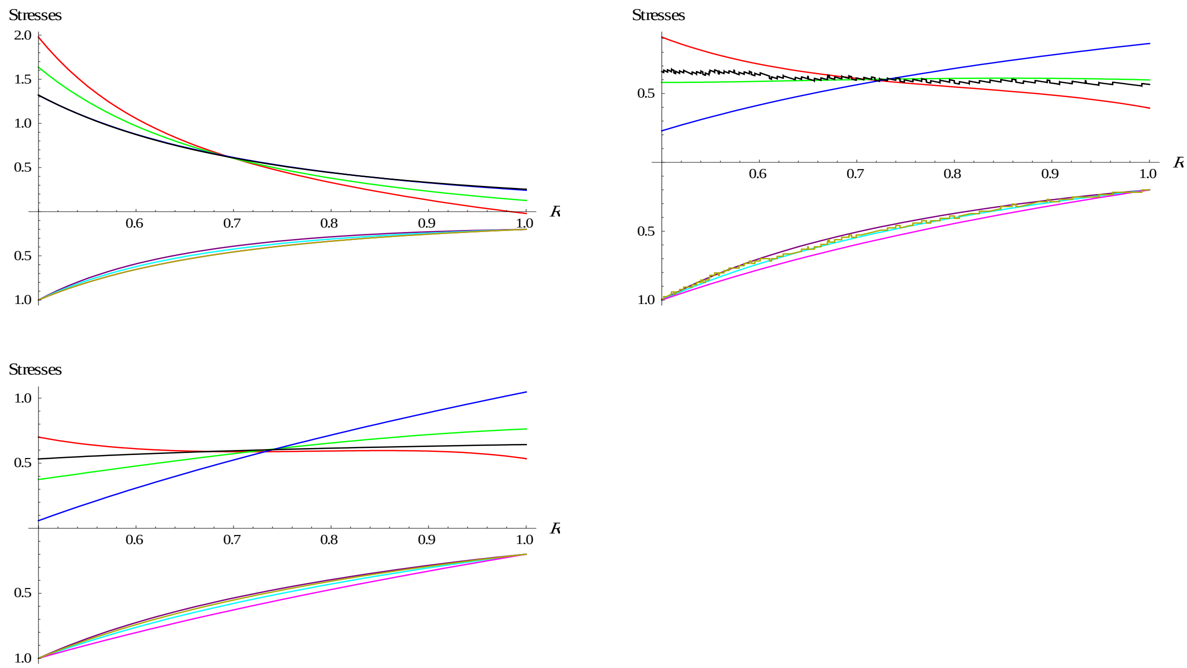
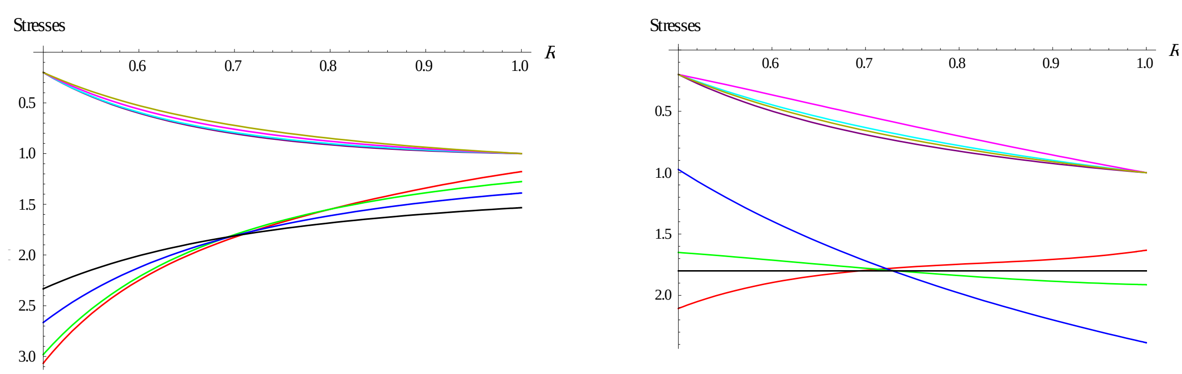


Fig.3
Thermal creep stresses in thick-walled FGM cylinder for $P_1 = 1$ and $P_2 = 0.2$ at temperature $\theta_1 = 0.1$ with $N = 1, 3, 5$.

Figs. 4, 5 and Table 1. describe the behavior of creep stresses against radii ratio when internal pressure is less than that of external pressure.

Influence of strain measure and non-homogeneity on circumferential stresses without temperature: From Fig. 4 and Table 1. it is observed that circumferential stresses are compressive and maximum at internal surface for linear measure. These stresses are maximum at external surface for nonlinear measure. However, these stresses are maximum for less functionally graded cylinder with linear measure and maximum for highly functionally graded cylinder with nonlinear measure.

Influence of strain measure and non-homogeneity on circumferential stresses with temperature: from Fig. 5 and Table 1. it is observed that the introduction of temperature creep stress increases for homogeneous and functionally graded cylinder with nonlinear measure. For linear measure, these stress increases for homogeneous and less functionally graded cylinder while decreases for highly functionally graded cylinder. Moreover, circumferential stresses are maximum at internal surface with linear measure. However, for highly functionally graded cylinder these circumferential stresses are maximum at external surface when strain measure is nonlinear.



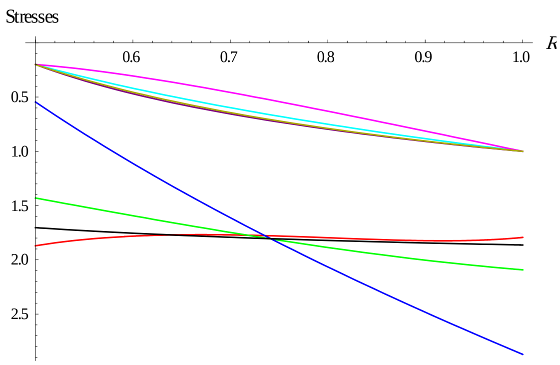


Fig.4
Creep stresses in thick-walled FGM cylinder for $P_1 = 0.2$ and $P_2 = 1$ with $N = 1, 3, 5$.

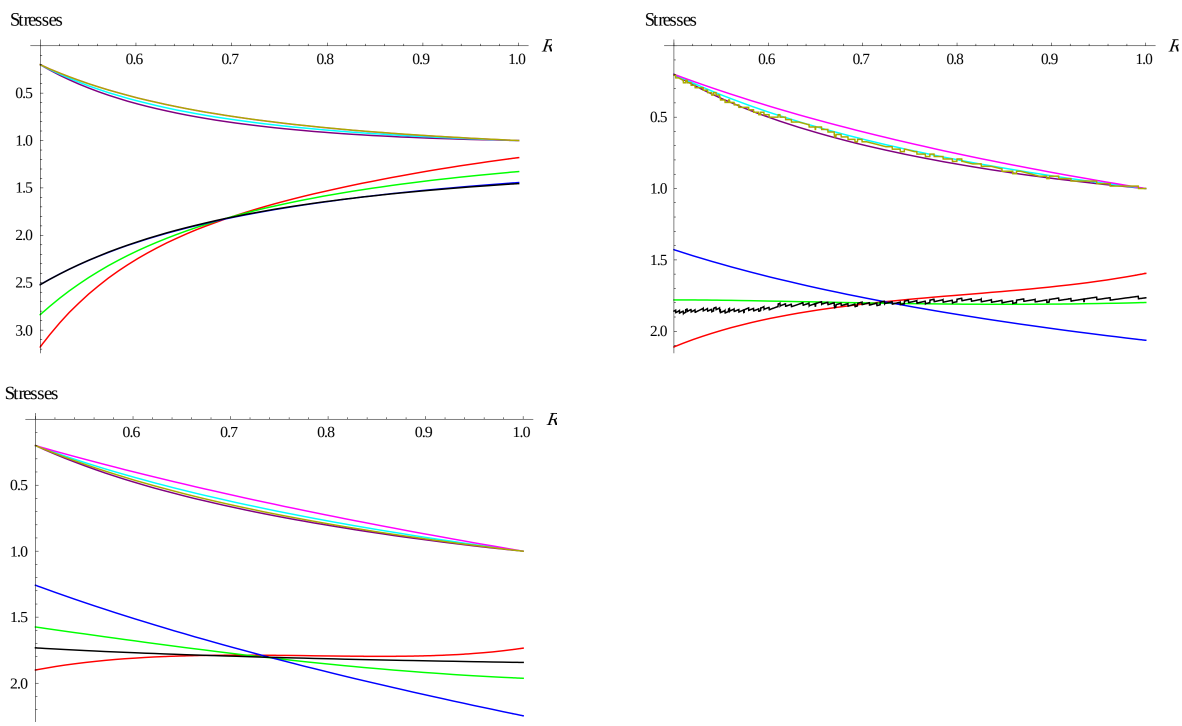


Fig.5
Thermal creep stresses in thick-walled FGM cylinder for $P_1 = 0.2$ and $P_2 = 1$ at temperature $\theta_1 = 0.1$ with $N = 1, 3, 5$.

A shear stress generated when a structural component is twisted. Due to torsion, shear stress generated in addition to principal stresses. The localized parallel shear forces will be highest when the normal forces are high. Due to combined loading i.e. axial and torsional loading, stresses generated in functionally graded material intersecting stresses generating in homogeneous material. As cylinder is subjected to different normal forces on each side and thus when internal pressure is greater than that of external pressure, forces on outer side are small and inner side are high. Non-uniform distribution of pressure and twist creates shear stress that tries to distort the cube. Shear stress act as support force and support force are not evenly distributed because of non-uniform distribution of pressure. Figs. 6-13 and Table 2. have been drawn for shear stresses against radii ratio subjected to pressure which justified above mentioned results. Figs. 6-9 and Table 2. have been drawn for shear stresses against radii ratio when internal pressure is greater than that of external pressure.

Influence of strain measure and non-homogeneity on shear stresses without temperature: At room temperature, creep shear stresses with angle of twist (say, 50) are compressive in nature. These stresses are maximum at internal surface for functionally graded and homogeneous cylinder with linear measure. However, shear stresses for cylinder made up of highly functionally graded material with nonlinear strain measure are maximum at external surface as can be seen from Fig. 6 and Table 2. It is also found that shear stresses are maximum for less functionally graded cylinder as compared to homogeneous and highly functionally graded cylinder.

Influence of strain measure and non-homogeneity on shear stresses with temperature: from Fig. 7 and Table 2. it is noticed that, with the introduction of thermal effects, shear stress increases at internal surface while decreases at external surface. With linear measure, these stresses increases for highly functionally graded cylinder. However, with nonlinear measure shear stress decreases for highly functionally graded cylinder. It is also noticed that shear stresses are more for linear measure as compared to nonlinear measure. From Figs. 8 and 9, it is noticed that, with the increase in internal and external pressure, shear stresses increase significantly. It is also found that these shear stresses are maximum for less functionally graded cylinder as compared to cylinder made up of highly functionally graded material.

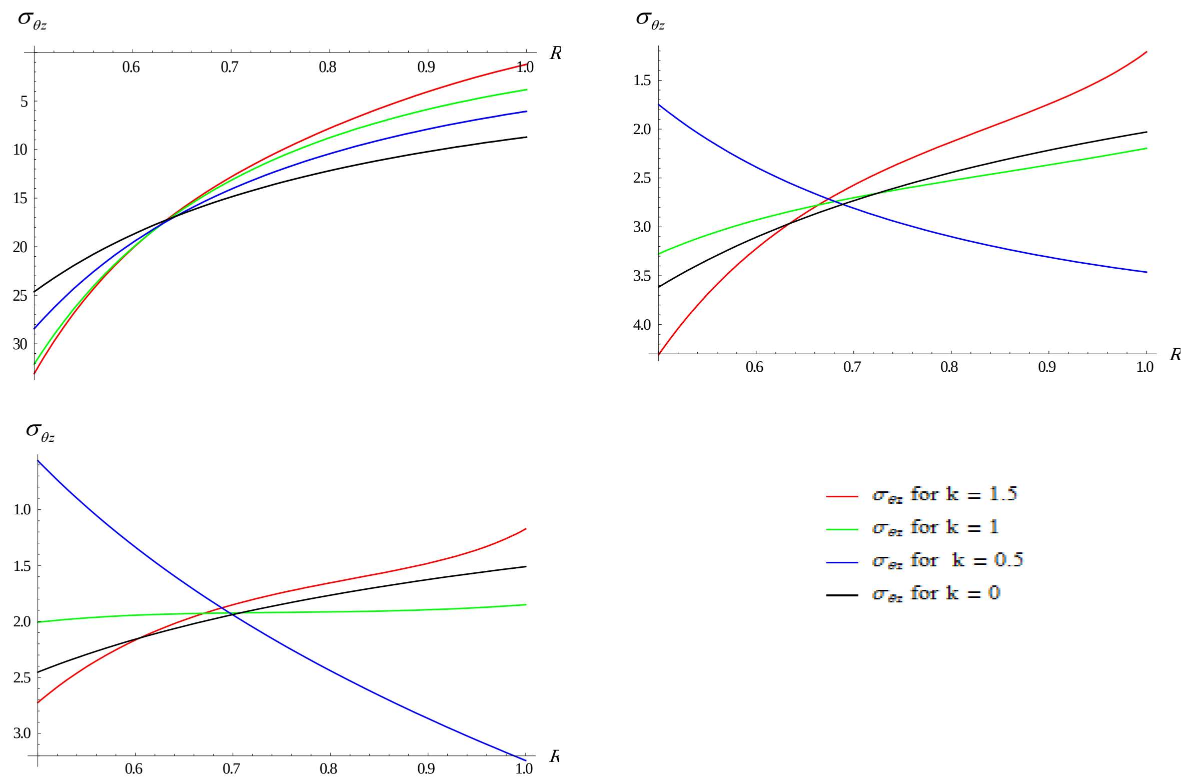
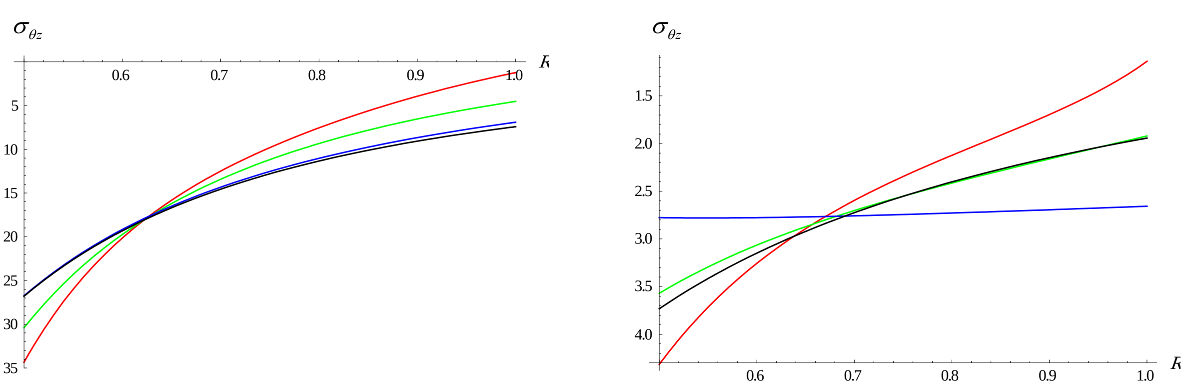


Fig.6
Creep shear stresses in thick-walled FGM cylinder for $P_1 = 1$ and $P_2 = 0.2$ with $N = 1, 3, 5$.



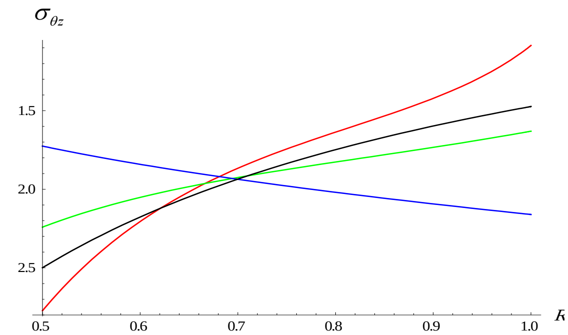


Fig.7
Thermal creep shear stresses in thick-walled FGM cylinder for $P_1 = 1$ and $P_2 = 0.2$ at temperature $\theta_1 = 0.1$ with $N = 1, 3, 5$.

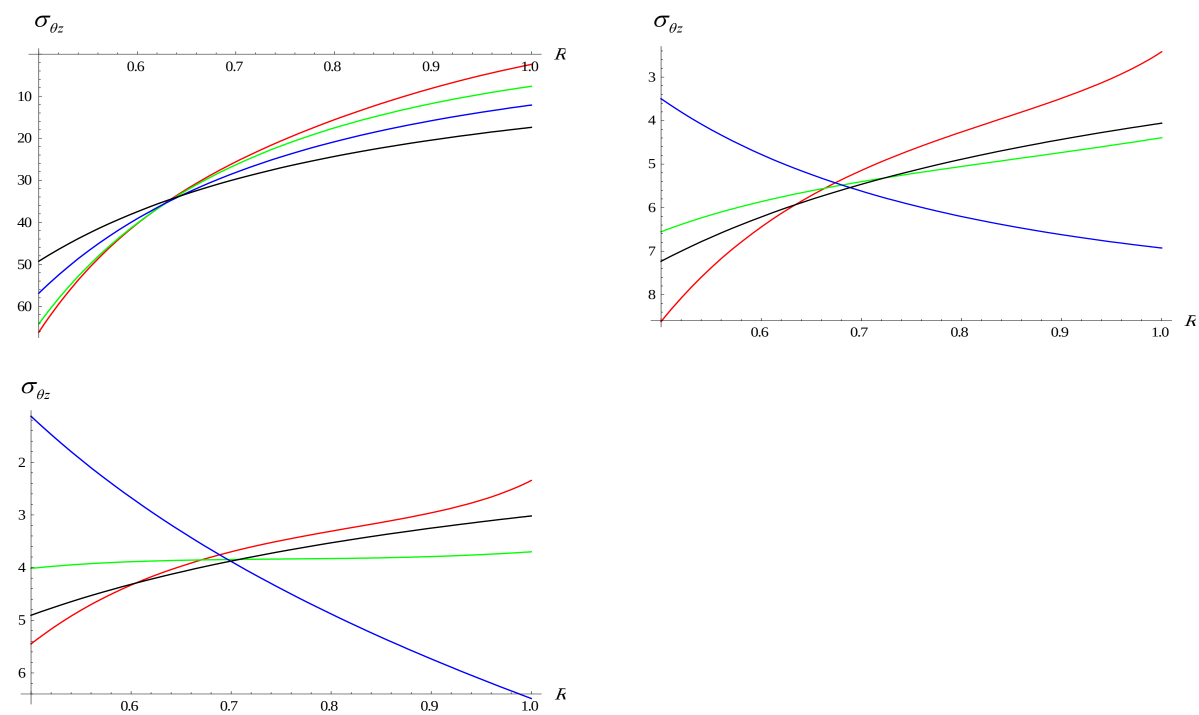
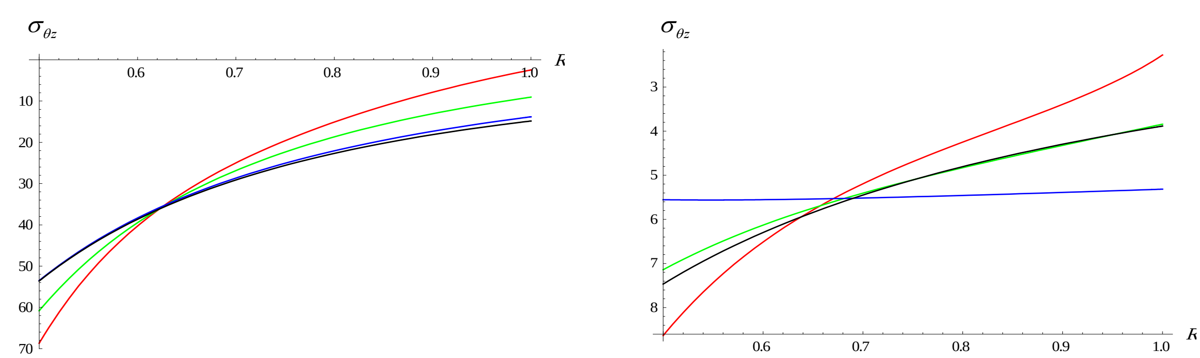


Fig.8
Creep shear stresses in thick-walled FGM cylinder for $P_1 = 2$ and $P_2 = 0.4$ with $N = 1, 3, 5$.



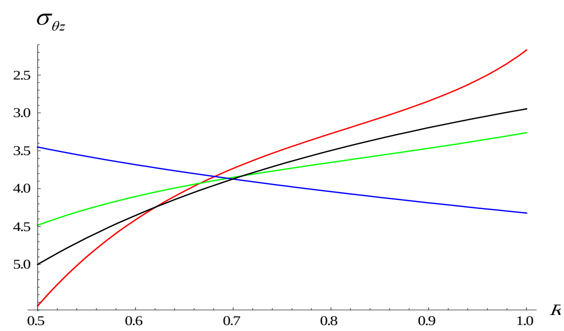


Fig.9
Thermal creep shear stresses in thick-walled FGM cylinder for $P_1 = 2$ and $P_2 = 0.4$ at temperature $\theta_1 = 0.1$ with $N = 1, 3, 5$.

Figs. 10 to 13 and Table 2. have been drawn for shear stresses against radii ratio when internal pressure is less than that of external pressure.

Influence of strain measure and non-homogeneity on shear stresses without temperature: it is observed that creep shear stresses are tensile in nature and are maximum at internal surface with linear and nonlinear measure as can be seen from Fig. 10 and Table 2. it is also noticed that with the increase in nonlinearity shear stress decreases. It is observed that shear stresses are maximum for cylinder made up of less functionally graded material as compared to cylinder made up of highly functionally graded material and homogenous material for linear and nonlinear measure.

Influence of strain measure and non-homogeneity on shear stresses with temperature for linear measure: from Fig. 11 and Table 2. it is observed that with the introduction of temperature, shear stress decreases at external surface and increases at internal surface for homogeneous and less functionally graded cylinder. These shear stresses decrease at internal surface and increases at external surface for highly functionally graded cylinder. With the increase in internal and external pressure, these shear stresses increase significantly as can be seen from Figs. 12 and 13. These shear sterner maximum for less functionally graded cylinder as compared to homogeneous cylinder.

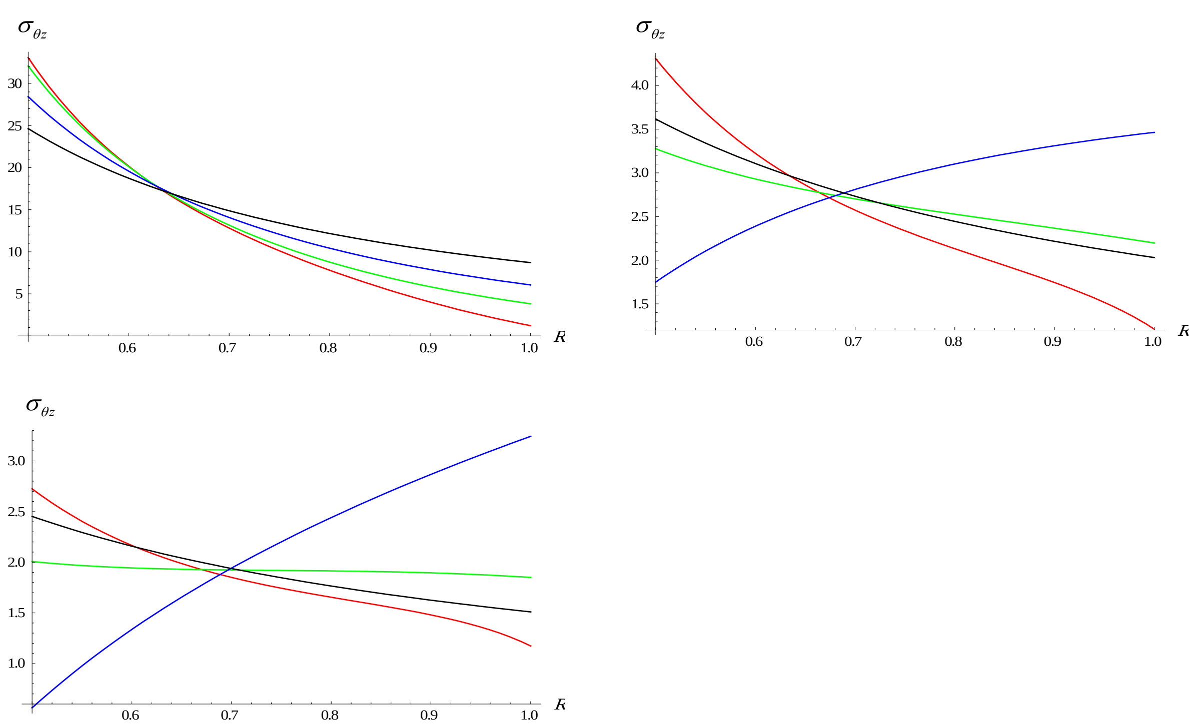


Fig.10
Creep shear stresses in thick-walled FGM cylinder for $P_1 = 0.2$ and $P_2 = 1$ with $N = 1, 3, 5$.

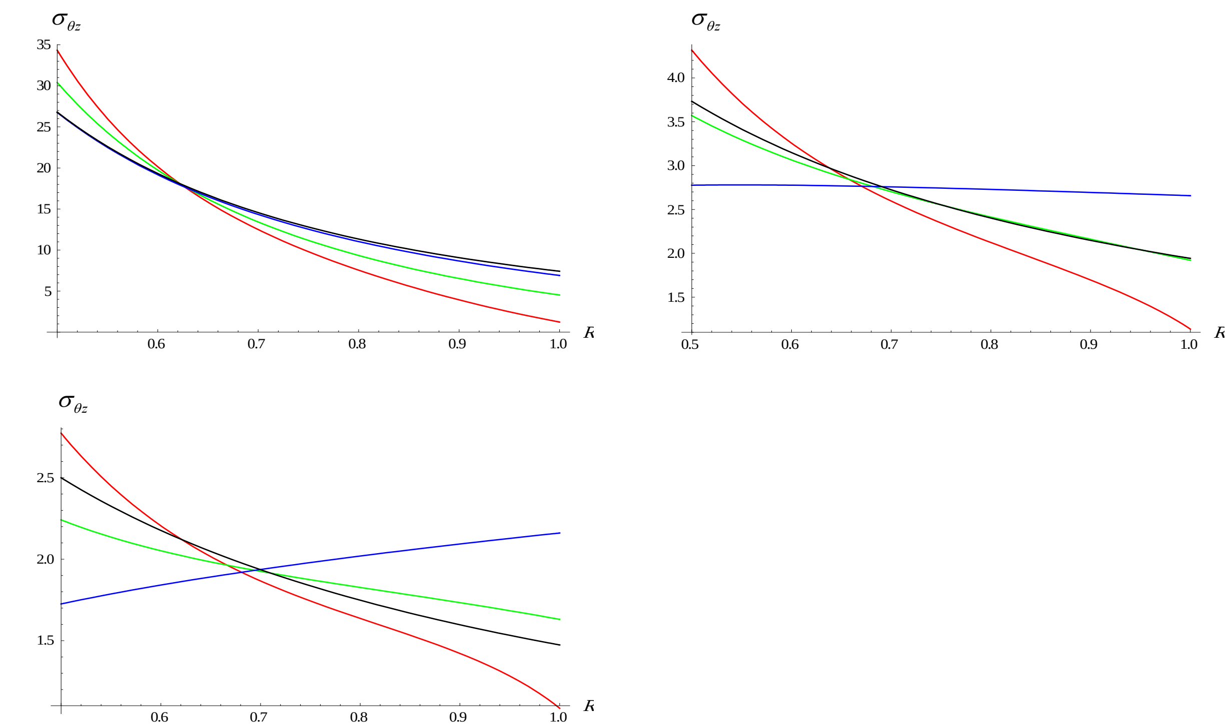


Fig.11
Thermal creep shear stresses in thick-walled FGM cylinder for $P_1 = 0.2$ and $P_2 = 1$ at temperature $\theta_1 = 0.1$ with $N = 1, 3, 5$.

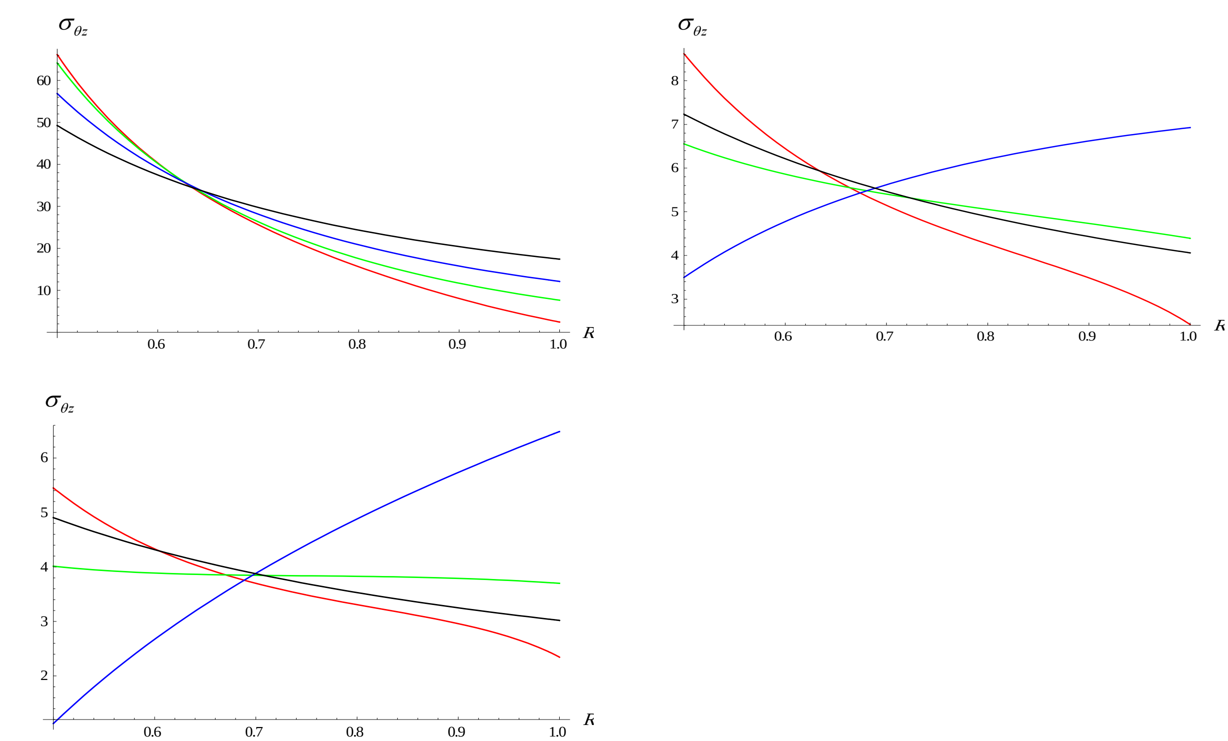


Fig.12
Creep shear stresses in thick-walled FGM cylinder for $P_1 = 0.4$ and $P_2 = 2$ with $N = 1, 3, 5$.

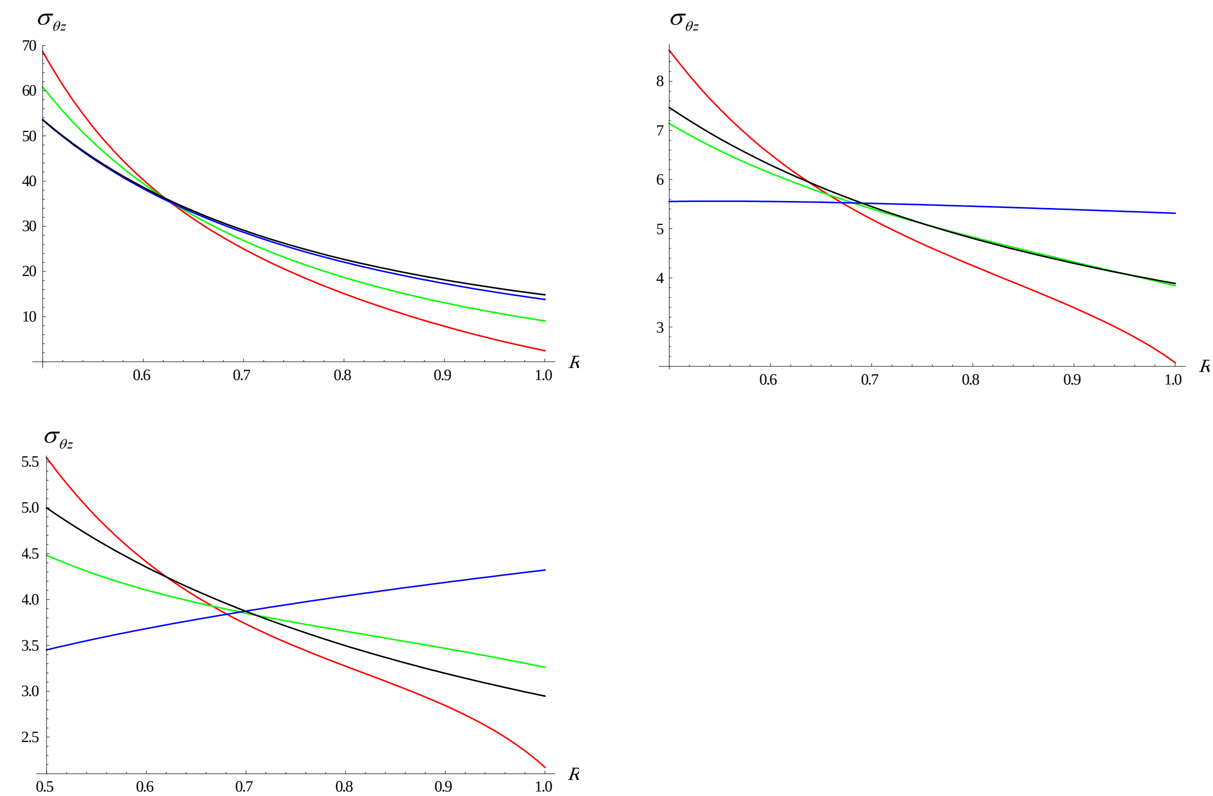


Fig.13
Thermal creep shear stresses in thick-walled FGM cylinder for $P_1 = 0.4$ and $P_2 = 2$ at temperature $\theta_1 = 0.1$ with $N = 1, 3, 5$.

Table 1
Thermal creep stresses for FGM cylinder with $P_1 = 1, P_2 = 0.2$ and $P_1 = 0.2, P_2 = 1$.

$\sigma_{\theta\theta}$		$N = 1$			$N = 3$			$N = 5$		
θ_1	k \ R	0.5	0.75	1	0.5	0.75	1	0.5	0.75	1
0 with $P_1 > P_2$	0	1.133	0.541	0.333	0.600	0.600	0.600	0.504	0.608	0.664
	0.5	1.466	0.500	0.189	-0.226	0.657	1.186	-0.655	0.644	1.671
	-1	1.782	0.460	0.076	0.450	0.610	0.713	0.231	0.619	0.892
	-1.5	1.868	0.475	-0.022	0.907	0.567	0.432	0.671	0.581	0.594
0.1 with $P_1 > P_2$	0	1.321	0.517	0.254	0.659	0.597	0.566	0.532	0.606	0.643
	0.5	1.320	0.520	0.243	0.229	0.626	0.863	0.058	0.623	1.047
	1	1.637	0.480	0.127	0.580	0.606	0.599	0.374	0.616	0.763
	1.5	1.978	0.454	-0.020	0.910	0.576	0.394	0.700	0.589	0.535
0 with $P_1 < P_2$	0	-2.333	-1.741	-1.533	-1.800	-1.800	-1.800	-1.704	-1.808	-1.864
	0.5	-2.666	-1.700	-1.389	-0.974	-1.857	-2.386	-0.545	-1.844	-2.871
	1	-2.982	-1.660	-1.276	-1.650	-1.810	-1.913	-1.431	-1.819	-2.092
	1.5	-3.068	-1.675	-1.178	-2.107	-1.767	-1.632	-1.871	-1.781	-1.794
0.1 with $P_1 < P_2$	0	-2.521	-1.717	-1.454	-1.859	-1.797	-1.766	-1.732	-1.806	-1.843
	0.5	-2.520	-1.720	-1.443	-1.429	-1.826	-2.063	-1.258	-1.823	-2.247
	1	-2.837	-1.680	-1.327	-1.780	-1.806	-1.799	-1.574	-1.816	-1.963
	1.5	-3.178	-1.654	-1.180	-2.110	-1.776	-1.594	-1.900	-1.789	-1.735

Table 2
Thermal creep shear stresses for FGM cylinder with $P_1 = 1, P_2 = 0.2$ and $P_1 = 0.2, P_2 = 1$.

$\sigma_{\theta z}$		$N = 1$			$N = 3$			$N = 5$		
θ_1	k \ R	0.5	0.75	1	0.5	0.75	1	0.5	0.75	1
0 with $P_1 > P_2$	0	-24.634	-13.409	-8.709	-3.616	-2.579	-2.030	-2.453	-1.847	-1.510
	0.5	-28.440	-12.071	-6.056	-1.749	-2.968	-3.464	-0.563	-2.202	-3.243
	-1	-32.084	-10.736	-3.821	-3.277	-2.610	-2.197	-2.007	-1.919	-1.850
	-1.5	-33.072	-10.114	-1.216	-4.308	-2.336	-1.209	-2.725	-1.743	-1.173
0.1 with $P_1 > P_2$	0	-26.800	-12.801	-7.416	-3.733	-2.553	-1.943	-2.500	-1.836	-1.474
	0.5	-26.753	-12.536	-6.905	-2.777	-2.744	-2.657	-1.725	-1.979	-2.161
	1	-30.408	-11.178	-4.525	-3.571	-2.552	-1.921	-2.241	-1.874	-1.631
	1.5	-34.342	-9.801	-1.230	-4.316	-2.346	-1.137	-2.773	-1.744	-1.085
0 with $P_1 < P_2$	0	24.634	13.409	8.709	3.616	2.579	2.030	2.453	1.847	1.510
	0.5	28.440	12.071	6.056	1.749	2.968	3.464	0.563	2.202	3.243
	1	32.084	10.736	3.821	3.277	2.610	2.197	2.007	1.919	1.850
	1.5	33.072	10.114	1.216	4.308	2.336	1.209	2.725	1.743	1.173
0.1 with $P_1 < P_2$	0	26.800	12.801	7.416	3.733	2.553	1.943	2.500	1.836	1.474
	0.5	26.753	12.536	6.905	2.777	2.744	2.657	1.725	1.979	2.161
	1	30.408	11.178	4.525	3.571	2.552	1.921	2.241	1.874	1.631
	1.5	34.342	9.801	1.230	4.316	2.346	1.137	2.773	1.744	1.085

4 CONCLUSIONS

In this paper, the influence of strain measure and temperature for creep behavior on functionally graded cylinder in torsion have been examined under internal and external pressure. It has been observed that with or without thermal effects, cylinder made up of less functionally graded material is on the safer side of design in torsion as compared cylinder made up of highly functionally graded material and homogeneous material for linear and nonlinear strain measures. This is because of the reason that shear stresses are maximum for less functionally graded cylinder as compared to cylinder made up of highly functionally graded material and homogeneous material.

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