# **Reflection of Waves in a Rotating Transversely Isotropic Thermoelastic Half-space Under Initial Stress**

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#### ABSTRACT

The present paper concerns with the effect of initial stress on the propagation of plane waves in a rotating transversely isotropic medium in the context of thermoelasticity theory of GN theory of type-II and III. After solving the governing equations, three waves propagating in the medium are obtained. The fastest among them is a quasi-longitudinal wave. The slowest of them is a thermal wave. The remaining is called quasi-transverse wave. The prefix 'quasi' refers to their polarizations being nearly, but not exactly, parallel or perpendicular to the direction of propagation. The polarizations of these three waves are not mutually orthogonal. After imposing the appropriate boundary conditions, the amplitudes of reflection coefficients have been obtained. Numerically, simulated results have been plotted graphically with respect to frequency to evince the effect of initial stress and anisotropy.

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#### **1** INTRODUCTION

In the last three decades, widespread attention has been given to thermoelasticity theories, which admit a finite speed for thermal signals (second sound). In contrast to the conventional coupled thermoelasticity theories based on a parabolic heat equation, this theories are referred to as generalized theories. Among these generalized theories, the first generalization is proposed by Lord-Shulman [20], which involves one relaxation time. The second generalization to the coupled thermoelasticity theory is developed by Green and Lindsay [11], which involves two relaxation times.

Experimental studies indicated that the relaxation times can be of relevance in the cases involving a rapidly propagating crack tip, shock wave propagations, laser techniques, etc. because of the experimental evidence in support of fitness of heat propagation speed, the generalized thermoelasticity theories are considered to be more realistic than the conventional theory in dealing with the practical problems involving very large heat fluxes at short intervals like those in laser units and energy channels. The third generalization is known as low-temperature thermoelasticity introduced by Hetnarski and Ignazack [19]. Most engineering materials such as metals possess a relatively high rate of thermal damping and thus are not suitable for use in experiments concerning second sound propagation. But, given the state of recent advances in material science, it may be possible in the foreseeable future to identify an idealized material for the purpose of studying the propagation of thermal waves at finite speed.

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In the 1990s, Green and Naghdi [12-13-14] proposed three new thermoelastic theories based on entropy equality rather than the usual entropy inequality. The constitutive assumptions for the heat flux vector are different in each theory. Thus, they obtained three theories that they called thermoelasticity of type I, type II and type III. When the theory of type I is linearized, we obtain the classical system of thermoelasticity. The theory of type II (a limiting case of type III) does not admit energy dissipation. In the context of the linearized version of this theory, theorems on uniqueness of solutions have been established by Hetnarski and Ignazack [19] and Green and Naghdi [14]. Boundary-initiated waves in a half-space and in an unbounded body with cylindrical cavity have been studied by Green and Naghdi [12] and Chandrasekharaiah and Srinath [3-4]. Also plane waves thermal shock problems have been studied by Othman et al. [26] and Othman and Song [25] . Gupta [15-16] analyzed the problem of wave propagation in the transversely isotropic thermoelastic and reflection in the transversely isotropic thermo-visco-elastic half-space under GN theory of type II and type III.

The development of initial stresses in the medium is due to many reasons, for example resulting from differences of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external forces, gravity variations etc. The earth is assumed to be under high initial stresses. It is therefore of much interest to study the influence of these stresses on the propagation of stress waves. Biot [1] showed the acoustic propagation under initial stress which is fundamentally different from that under stress-free state. He has obtained the velocities of longitudinal and transverse waves along the co-ordinate axis only.

The wave propagation in solids under initial stresses has been studied by many authors for various models. The study of reflection and refraction phenomena of plane waves in an unbounded medium under initial stresses is due to Chattopadhyay et al. [6], Sidhu and Singh [30] and Dey et al. [9]. Montanaro [22] investigated the isotropic linear thermoelasticity with a hydrostatic initial stress. Singh et al. [32], Singh [31] and Othman and Song [23] studied the reflection of thermoelastic waves from a free surface under a hydrostatic initial stress in the context of different theories of generalized thermoelasticity. Gupta and Gupta [17-18] investigated the wave motion and then discussed the reflection of waves in an anisotropic initially stressed fiber-reinforced thermoelastic medium.

Some researchers in past have investigated different problem of rotating media. Chand et al. [5] presented an investigation on the distribution of deformation, stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half-space. Many authors (Schoenberg and Censor [28]; Clarke and Burdness [7]; Destrade [8] studied the effect of rotation on elastic waves. Sharma [29] discussed effect of rotation on waves propagating in a thermoelastic medium. Othman [24] investigated plane waves in generalized thermoelasticity with two relaxation times under the effect of rotation. Othman and Song [26-27] presented the effect of rotation in magneto-thermoelastic medium. Mahmoud [21] discussed the effect of Rotation, Gravity Field and Initial Stress on Generalized Magneto-Thermoelastic Rayleigh Waves in a Granular Medium.

In this article, effect of initial stress on the plane wave reflection in rotating transversely isotropic medium in the context of thermoelasticity with GN theory of type II and III has been investigated. A cubic equation resulting in the three values of phase velocities and attenuation quality factor has been obtained. Furthermore, the expressions for the amplitude ratios of the reflected wave corresponding to the three incident waves have been obtained. These expressions are then evaluated numerically and plotted graphically to manifest the effect of initial stress and anisotropy.

#### 2 FORMULATION OF THE PROBLEM

In the context of thermoelasticity based on Green-Naghdi theory of type II and type III, the equation of motion for the initially stressed transversely isotropic medium, taking the rotation term about y-axis as a body force is

$$t_{ij,j} - P\omega_{ij,j} = \rho[\vec{u}_i + (\vec{\Omega} \times \vec{\Omega} \times \vec{u})_i + (2\vec{\Omega} \times \vec{u})_i]$$
<sup>(1)</sup>

where  $\hat{\Omega}$  is the uniform angular velocity and  $\rho$  is the density of the medium. The generalized energy equation can be expressed as:

$$K_{ij}\vec{T}_{,ij} + K^{*}_{ij}T_{,ij} = (T_{0}\beta_{ij}\vec{u}_{i,j} + \rho c^{*}\vec{T}), \quad i, j = 1, 2, 3,$$
<sup>(2)</sup>

The constitutive equations have the form

$$t_{ij} = C_{ijkl} e_{kl} - \beta_{ij} T , \qquad (3)$$

where the deformation tensor is defined by  $e_{ij} = (u_{i,j} + u_{j,i})/2$ ,  $t_{ij}$  are components of stress tensor,  $u_i$  the mechanical displacement,  $e_{ij}$  are components of infinitesimal strain, P is the normal initial stress,  $\omega_{ij} = (u_{j,i} - u_{i,j})/2$ , T the temperature change of a material particle,  $T_0$  the reference uniform temperature of the body,  $K_{ij}$  is the thermal conductivity,  $K_{ij}^*$  are the characteristic constants of the theory,  $\beta_{ij} = C_{ijkl} \alpha_{kl}$  are the thermal elastic coupling tensor,  $\alpha_{kl}$  are the coefficient of linear thermal expansion, c the specific heat at constant strain,  $C_{ijkl}$  are characteristic constants of material following the symmetry properties  $C_{ijkl} = C_{klij} = C_{jikl}$ ,  $K_{ij}^* = K_{ji}^*$ ,  $K_{ij} = K_{ji}$ ,  $\beta_{ij} = \beta_{ji}$ . The comma notation is used for spatial derivatives and superimposed dot represents time differentiation.

Following Slaughter [33], the appropriate transformations have been used on the set of Eq.(3), to derive equations for transversely isotropic medium. We restrict our analysis for two dimensions, in which we consider the component of the displacement vector in the form

$$\vec{u} = (u_1, 0, u_3)$$
 (4)

Here, we consider plane waves propagating in plane such that all particles on a line parallel to  $x_2$ -axis are equally displaced. Therefore, all the field quantities will be independent of  $x_2$  coordinate, i.e.  $\partial/\partial x_2 \equiv 0$ .

Thus the field equations and constitutive relations for such a medium reduces to:

$$C_{11}u_{1,11} + \frac{C_{55}}{2}u_{1,33} + (C_{13} + \frac{C_{55}}{2})u_{3,13} - \frac{P}{2}(u_{3,13} - u_{1,33}) - \beta_{1}T_{,1} = \rho \Big[\ddot{u}_{1} - \Omega^{2}u_{1} + 2\Omega\dot{u}_{3}\Big],$$
(5)

$$\frac{C_{55}}{2}u_{3,11} + C_{33}u_{3,33} + (C_{13} + \frac{C_{55}}{2})u_{1,13} - \frac{P}{2}(u_{1,31} - u_{3,11}) - \beta_3 T_{,3} = \rho \Big[\ddot{u}_3 - \Omega^2 u_3 + 2\Omega \dot{u}_1\Big],$$
(6)

$$K_{1}\dot{T}_{,11} + K_{3}\dot{T}_{,33} + K_{1}^{*}T_{,11} + K_{3}^{*}T_{,33} = \rho c^{*}\ddot{T} + T_{o}(\beta_{1}\ddot{u}_{1,3} + \beta_{3}\ddot{u}_{3,1}),$$
(7)

where  $\beta_1 = C_{11}\alpha_1 + C_{13}\alpha_3$ ,  $\beta_3 = C_{31}\alpha_1 + C_{33}\alpha_3$  and we have used the notations  $11 \rightarrow 1, 13 \rightarrow 5, 33 \rightarrow 3$ , for the material constants.

It is convenient to change the preceding equations into the dimensionless forms. To do this, the non-dimensional parameters are introduced as:

$$x_{i}^{'} = \frac{x_{i}}{L}, u_{i}^{'} = \frac{u_{i}}{L}, t_{ij}^{'} = \frac{t_{ij}}{C_{11}}, t^{'} = \frac{t}{t_{o}}, T^{'} = \frac{T}{T_{o}},$$
(8)

where  $L_{,t_o}, T_o$  are parameters having dimension of length, time and temperature respectively.

## **3 PLANE WAVE PROPAGATION AND REFLECTION OF WAVES**

Let  $\vec{p} = (p_1, 0, p_3)$  denote the unit propagation vector, *c* and  $\xi$  are respectively the phase velocity and the wave number of the plane waves propagating in  $x_1 - x_3$  plane. For plane wave solution of the equations of motion of the form

$$(u_1, u_3, T) = (\overline{u_1}, \overline{u_3}, \overline{T}) e^{i\xi(p_1x_1 + p_3x_3 - ct)}.$$
(9)

with the help of Eqs.(8) and (9) in Eqs.(5) and (7), three homogeneous equations in three unknowns are obtained. Solving the resulting system of equations for non-trivial solution results in

$$Ac^{6} + Bc^{4} + Cc^{2} + D = 0$$
,

where

$$\begin{split} A &= f_{10}, B = -f_{5}f_{10} - f_{1}f_{10} + f_{6}f_{8} + f_{3}f_{7} + f_{9}, D = f_{1}f_{5}f_{9} - f_{2}f_{4}f_{9}, \\ C &= -f_{5}(f_{9} + f_{1}f_{10} - f_{3}f_{7}) - f_{1}(f_{9} - f_{6}f_{8}) - f_{2}(f_{4}f_{10} + f_{6}f_{7}) + f_{3}f_{4}f_{8}, \\ f_{1} &= p_{1}^{2}d_{1} + p_{3}^{2}d_{3} - d_{13}Pp_{3}^{2}/2, f_{2} = p_{1}p_{3}d_{3} - d_{13}Pp_{1}p_{3}/2, f_{3} = ip_{1}d_{4}, f_{4} = p_{1}p_{3}d_{2} - d_{13}Pp_{1}p_{3}/2, \\ f_{5} &= p_{1}^{2}d_{3} + p_{3}^{2}d_{5} + d_{13}Pp_{1}^{2}/2, f_{6} = ip_{3}d_{6}, f_{7} = ip_{7}d_{11}, f_{8} = ip_{3}d_{12}, f_{9} = i\omega p_{1}^{2} - d_{8}p_{1}^{2} + i\omega \bar{k}p_{3}^{2} - d_{9}p_{3}^{2} \\ f_{10} &= \varepsilon_{1}, d_{1} = \frac{C_{11}t_{o}^{2}}{\rho L^{2}}, d_{2} = \frac{(C_{13} + C_{55}/2)t_{o}^{2}}{\rho L^{2}}, d_{3} = \frac{C_{55}t_{o}^{2}}{2\rho L^{2}}, d_{4} = \frac{\beta_{1}T_{o}t_{o}^{2}}{\rho L^{2}}, d_{5} = \frac{C_{33}t_{o}^{2}}{\rho L^{2}}, d_{6} = \frac{\beta_{3}T_{o}t_{o}^{2}}{\rho L^{2}}, \\ \bar{k} = d_{7} = \frac{k_{3}}{k_{1}}, d_{8} = \frac{k_{1}^{*}t_{o}}{k_{1}}, d_{9} = \frac{k_{3}^{*}t_{o}}{k_{1}}, \varepsilon_{1} = d_{10} = \frac{\rho C^{*}L^{2}}{k_{1}t_{o}}, d_{11} = \frac{\beta_{1}L^{2}}{k_{1}t_{0}}, d_{12} = \frac{\beta_{3}L^{2}}{k_{1}t_{0}}, d_{13} = \frac{t_{0}^{2}}{\rho L^{2}}. \end{split}$$

The roots of this equation give three values of  $c^2$ . Three positive values of c will be the velocities of propagation of three possible waves. The waves with velocities  $c_1, c_2, c_3$  correspond to three types of quasi waves. We name these waves as quasi-longitudinal displacement (qLD) wave, quasi thermal wave (qT) and quasi transverse displacement (qTD) wave.

# **4 REFLECTION OF WAVES**

Consider a homogeneous transversely isotropic half-space in the context of thermoelasticity (as shown in Fig. 1) with GN theory of type II and III, rotating with angular velocity  $\vec{\Omega}$  occupying the region  $x_3 > 0$ . Incident qLD or qT or qTD wave at the interface will generate reflected qLD, qT and qTD waves in the half space  $x_3 > 0$ .



The total displacements and temperature distribution are given by:

$$(u_1, u_3, T) = \sum_{j=1}^{6} A_j (1, r_j, s_j) e^{iB_j},$$
(11)

where

$$B_{j} = \begin{cases} \omega(t - x_{1} \sin e_{j} - x_{3} \cos e_{j})/c_{j}, \ j = 1, 2, 3, \\ \omega(t - x_{1} \sin e_{j} + x_{3} \cos e_{j})/c_{j}, \ j = 4, 5, 6, \end{cases}$$
(12)

 $\omega$  is the angular frequency. Here subscripts 1,2,3 respectively denote the quantities corresponding to incident qLD, qT and qTD wave whereas the subscripts 4,5 and 6 respectively denote the corresponding reflected waves and

(10)

$$\begin{split} r_{j} &= \frac{\wedge_{1j}}{\wedge_{j}}, s_{j} = \frac{\wedge_{2j}}{\wedge_{j}}, \wedge_{j} = \begin{vmatrix} \xi^{2}(f_{5} - c_{j}^{2}) & \xi f_{6} \\ c_{j}^{2}\xi^{3}f_{8} & \xi^{2}(f_{9} + f_{10}c_{j}^{2}) \end{vmatrix}, \\ & \wedge_{1j} = \begin{vmatrix} \xi^{2}f_{4} & \xi f_{6} \\ c_{j}^{2}\xi^{3}f_{7} & \xi^{2}(f_{9} + f_{10}c_{j}^{2}) \end{vmatrix}, \wedge_{2j} = \begin{vmatrix} \xi^{2}f_{4} & \xi^{2}(f_{5} - c_{j}^{2}) \\ c_{j}^{2}\xi^{3}f_{7} & c_{j}^{2}\xi^{3}f_{8} \end{vmatrix}. \end{split}$$

For incident:

qLD-wave:  $p_1 = \sin e_1, p_3 = \cos e_1,$ qT-wave:  $p_1 = \sin e_2, p_3 = \cos e_2,$ qTD-wave:  $p_1 = \sin e_3, p_3 = \cos e_3,$ 

For reflected:

qLD-wave:  $p_1 = \sin e_4, p_3 = \cos e_4,$ qT-wave:  $p_1 = \sin e_5, p_3 = \cos e_5,$ qTD-wave:  $p_1 = \sin e_6, p_3 = \cos e_6.$ 

Here  $e_1 = e_4, e_2 = e_5, e_3 = e_6$ , i.e. the angle of incidence is equal to the angle of reflection in generalized thermoelastic transversely isotropic, so that the velocities of reflected waves are equal to their corresponding to their corresponding incident wave's i.e.  $c_1 = c_4, c_2 = c_5, c_3 = c_6$ .

#### **5 BOUNDARY CONDITIONS**

The boundary conditions at the thermally insulated surface  $x_3 = 0$  are given by

$$t_{33} = 0, t_{31} = 0, T_{3} = 0, \tag{13}$$

where

$$t_{33} = C_{13}u_{1,1} + C_{33}u_{3,3} - \beta_3 T, t_{31} = \frac{C_{55}}{2}(u_{1,3} + u_{3,1}).$$
(14)

The wave numbers  $\xi_i$ ,  $j = 1, 2, \dots, 6$  and the apparent velocity  $c_j$ ,  $j = 1, 2, \dots, 6$  are connected by the relation

$$c_1\xi_1 = c_2\xi_2 = \dots = c_6\xi_6 = \omega, \tag{15}$$

at the surface  $x_3 = 0$ . Relation (15) may also be written in order to satisfy the boundary conditions (13) as:

$$\frac{\sin e_1}{c_1} = \frac{\sin e_2}{c_2} = \dots = \frac{\sin e_6}{c_6} = \frac{1}{c}.$$
(16)

Making use of Eqs. (8), (11), (14) and (15) into thermally insulated boundary conditions (13), we obtain

$$\sum_{j=1}^{6} A_{ij} A_j = 0, \ i = 1, 2, 3, \tag{17}$$

where

$$A_{1j} = \begin{cases} a_j^1 + r_j a_j^2 - t_j a_j^3, \quad j = 1, 2, 3, \\ a_j^1 - r_j a_j^2 - t_j a_j^3, \quad j = 4, 5, 6, \end{cases}, A_{2j} = \begin{cases} b_j^1 + r_j b_j^2, \quad j = 1, 2, 3, \\ -b_j^1 + r_j b_j^2, \quad j = 4, 5, 6, \end{cases}, A_{3j} = \begin{cases} t_j c_j^1, \quad j = 1, 2, 3, \\ -t_j c_j^1, \quad j = 4, 5, 6, \end{cases}$$

where

$$a_{j}^{1} = -\frac{i\omega C_{13}}{C_{11}}\frac{\sin e_{j}}{c_{j}}, a_{j}^{2} = \frac{i\omega C_{33}}{C_{11}}\frac{\cos e_{j}}{c_{j}}, a_{j}^{3} = \frac{i\beta_{3}T_{0}s_{j}}{C_{11}}, b_{j}^{1} = \frac{i\omega C_{55}\cos e_{j}}{2C_{11}c_{j}}, b_{j}^{2} = \frac{i\omega C_{55}\sin e_{j}}{2C_{11}c_{j}}, c_{j}^{1} = \frac{\omega\cos e_{j}}{c_{j}}$$

#### Incident qLD-wave:

In case of incident qLD- wave,  $A_2 = A_3 = 0$ . Dividing set of Eq. (17) throughout by  $A_1$ , we obtain a system of three non-homogeneous equations in three unknowns which can be solved by Gauss elimination method and we have

$$Z_{i} = \frac{A_{i+3}}{A_{1}} = \frac{\Delta_{i}^{1}}{\Delta}, i = 1, 2, 3.$$
(18)

#### Incident qT-wave:

In case of incident qT- wave,  $A_1 = A_2 = 0$  and thus we have

$$Z_{i} = \frac{A_{i+3}}{A_{2}} = \frac{\Delta_{i}^{2}}{\Delta}, i = 1, 2, 3.$$
(19)

#### Incident qTD-wave:

In case of incident qTD- wave,  $A_1 = A_2 = 0$  and thus we have

$$Z_{i} = \frac{A_{i+3}}{A_{3}} = \frac{\Delta_{i}^{3}}{\Delta}, i = 1, 2, 3,$$
(20)

where  $\Delta = |A_{i\,i+3}|_{3\times 3}$  and  $\Delta_i^p$  (i = 1, 2, 3, p = 1, 2, 3) can be obtained by replacing, respectively, the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> column of  $\Delta$  by  $\begin{bmatrix} -A_{1p} & -A_{2p} & -A_{3p} \end{bmatrix}^T$ .

#### 6 NUMERICAL RESULTS AND DISCUSSION

To illustrate the theoretical problem numerical results are presented. The Cobalt material was chosen for the purpose of numerical computation, whose physical data is given in Dhaliwal et al. [10]

$$C_{11} = 3.071 \times 10^{11} Nm^{-2}, C_{12} = 1.650 \times 10^{11} Nm^{-2}, C_{13} = 1.027 \times 10^{11} Nm^{-2}, C_{33} = 3.581 \times 10^{11} Nm^{-2}, C_{55} = 1.51 \times 10^{11} Nm^{-2}, \beta_1 = 7.04 \times 10^6 Nm^{-2} K, \beta_1 = 6.98 \times 10^6 Nm^{-2} K, \rho = 8.836 \times 10^3 Kgm^{-3}, \beta_1 = 7.04 \times 10^6 Nm^{-2} K, \beta_2 = 0.000 Nm^{-2} K$$

 $K_1 = 6.90 \times 10^2 W m^{-1} K$ ,  $K_3 = 7.01 \times 10^2 W m^{-1} K$ ,  $K_1^* = 1.313 \times 10^2 W$  sec,  $K_3^* = 1.54 \times 10^2 W$  sec,  $c^* = 4.27 \times 10^2 J Kg K$ , T = 298K.

The physical quantities displacement, temperature, amplitude ratios depend not only on time 't' and space coordinates but also on the characteristic parameter of the Green-Naghdi theory of type II and type III. Here, all variables are taken in non-dimensional form. Figs. 2-10 exhibits the variations of amplitude ratio of reflected qLD, qTD and qT waves, for incident qLD, qTD and qT waves for rotating transversely isotropic under GN type II and type III (RTI) and rotating isotropic thermoelastic (RI) at three different values of initial stress INT(0, 2, 4). In Figs. 2-4, the graphical representation is given for the variation of amplitude ratios  $|Z_1|, |Z_2|$  and  $|Z_3|$  for incident qLD wave. Figs. 5-7 and 8-10, respectively represent the similar situation, when qTD and qT waves are incident. In these figures the solid curves lines correspond to the case of RTI, while broken curves correspond to the case of RI. The curves without center symbol represent the case without initial stress (i.e., INT=0), curves with center symbol (-0-0-) represents the variation corresponding to INT=2 and curves with center symbol (-x-x-) represents the variation corresponding to INT=4. *Incident qLD-wave:* 



#### Fig. 2

Illustration of the variation of amplitude ratios of  $|Z_i|, i = 1, 2, 3$ , with frequency for incident qLD-wave.

## Fig. 3

Illustration of the variation of amplitude ratios of  $|Z_i|, i = 1, 2, 3$ , with frequency for incident qLD-wave.

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Fig. 2 shows that the values of amplitude ratios for all the cases, initially oscillates and then attain a steady value. From Figs. 3 and 4, it can be depicted that the amplitude of waves gets increased due to anisotropy (can be seen in the inset). *Incident qTD-wave:* 





Illustration of the variation of amplitude ratios of  $|Z_i|, i = 1, 2, 3$ , with frequency for incident qT-wave.

#### Fig. 6

Illustration of the variation of amplitude ratios of  $|Z_i|, i = 1, 2, 3$ , with frequency for incident qT-wave.



Incident qT-wave:





#### Fig. 8

Illustration of the variation of amplitude ratios of  $|Z_i|, i = 1, 2, 3$ , with frequency for incident qTD-wave.

# Fig. 9

Illustration of the variation of amplitude ratios of  $|Z_i|, i = 1, 2, 3$ , with frequency for incident qTD-wave.





#### 7 CONCLUSIONS

Effect of initial stress and anisotropy on the reflection of waves from the free surface of rotating transversely isotropic medium in the context of thermoelasticity with GN theory of type-II and III has been discussed. It is depicted from the graphical results that anisotropy and initial stress play an important role on amplitude ratios of reflected waves. It can be concluded from the graphs that, in most of the cases, the values of amplitude ratio get increased due to the effect of anisotropy.

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