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Effect of Stress Triaxiality on Yielding of Anisotropic Materials under Plane Stress Condition

Shailendra Singh Bhadauria[1,*], M.S. Hora[1], K.K. Pathak[2]

[1]*Department of Applied Mechanics, Maulana Azad National Institute of Technology, Bhopal, 462051 India*
[2]*Scientist, Advanced Materials Processes Research Institute (CSIR), Bhopal 462026, India*

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ABSTRACT
The triaxiality of the stress state is known to greatly influence the amount of plastic strain which a material may undergo before ductile failure occurs. It is defined as the ratio of hydrostatic pressure, or mean stress, to the von Mises equivalent stress. This paper discusses the effects of stress triaxiality on yielding behavior of anisotropic materials. Hill-von Mises's criteria for anisotropic material have been used with triaxiality factor (TF). Mathematical model that combines the yield stress and anisotropic ratio R (ratio of width strain to thickness strain) along with triaxiality have been formulated. This model is considered as an objective function subjected to inequality constraint. Constrained optimization is solved using genetic algorithm. The results obtained give the set of principal stresses along with corresponding critical triaxiality which is the maximum value at which the material can sustain without failure. If triaxiality extends further more the material will go to plastic deformation and may prone to failure. In this way, the critical triaxiality of materials can be determined to avoid fracture and failure of materials. This article is important from the industrial application point of view by considering triaxiality as a design parameter while designing the component.

Keywords: Stress triaxiality; Hill-von Mises stress; Anisotropy ratio; Yield stress

1 INTRODUCTION

APPLICATION of anisotropic sheets can be found in various components. As such sheets fall in the category of plane stress problems, effect of stress triaxiality on the yielding needs to be studied. Literature review suggested that a lot of work have been done in the area of anisotropic materials. Hill [1] suggested a theory, on a macroscopic scale, that describes the yielding and plastic flow of an anisotropic metal. Chaudhary Sushil and Jiro [2] also studied anisotropy and mobility of the subyield surfaces in the p'-constant shear plane. When the initial stress state was at the origin of the shear plane (isotropic stress state), the shape of the yield surface was approximately circular with the center shifted toward compression in the vertical direction. Anisotropy increased with the progress of shearing. Y^1 and Y^2 surfaces were mobile and moved with the current stress point in the p'-constant shear plane, though Y^3 surface was comparatively immobile. The shapes of yield surfaces also changed with the movement of current stress point. In a study by Yasushi Kurosaki et al. [3], the strain distribution and the fracture position were found to be sensitive to the shape of the yield locus, that is, anisotropic yielding. It is concluded that the Bassani function is much more useful for the simulation than Hill's old criterion. Harvey et al. [4] expressed a state of anisotropy in terms of the uni-axial and shear yield stress of the material. It is suggested that this form of the yield or loading surface can be used to determine the plastic strains for a range of multi-axial and cyclic plasticity problems and the results were presented for the particular case of a tube subjected to cyclic plastic torsion with an axial stress. Banabicd [5] developed plastic anisotropy criteria which can successfully describe the

* Corresponding author. Tel.: +91 909 804 6514.
E-mail address: raju_arizona@yahoo.co.in (Shailendra Singh Bhadauria).

anisotropy of both the plastic strain ratio and yield of AA3103-0 aluminum thin sheets. Hu [6] proposed an enhanced method and an associated yield criterion in order for existing anisotropic yield criteria to fit experimental data in all the orientations. Existing anisotropic yield criteria, the method proposed can also be used to modify various anisotropic yield criteria in order to fit more material test data. Sivakumar et al. [7] investigated that the anisotropy developed during deposition of soil disappears when the sample is loaded to a stress level at least twice the stress generated during the original deposition process. The methods developed have also been applied to test results reported previously on Winnipeg clay, and good agreement was obtained. Padmanabham [8] observed from the numerical study that the optimal blank shape for a part is significantly influenced by the initial anisotropy. The numerical method responds to the material anisotropy and is capable of producing an optimal initial blank shape within few iterations. Liao [9] observed that the yield function plays important roles on the strain distribution and the strain localization as well. Yield functions of the matrix rather influence the earing phenomenon under the cup drawing procedure even similar displacement profiles of the outer boundary could be observed. Lee [10] has shown that a total of four simple tension and compression tests are sufficient to describe the yield surface of the highly anisotropic material. The experi-mental data obtained with a directionally solidified (DS), nickel-based, tantalum carbide reinforced eutectic alloy is shown to be in excellent agreement with the theory. Masanobu Oda [11] investigated that the anisotropic shear strength is well fitted by the extended Drucker–Prager yield function. Based on this, it is concluded that this study provides a step to link the material science approach of soils, in which the spatial arrangement of particles and associated voids plays an important role, to the continuum theory of plasticity. Plunkett [12] developed an anisotropic yield function of hexagonal materials taking into account texture development and anisotropic hardening that captures the influence of evolving texture on the plastic response of hexagonal close packed (hcp) metals. To describe the change of the shape of the yield surface during monotonic loading, the evolution of the anisotropic coefficients involved in the expression of the yield function is considered. Wakashima and Courtney [13] developed a method for predicting the initial yield surface together with the active slip systems which is developed and applied to lamellar eutectics with particular reference to the Pb-Sn eutectic. Anisotropy of yielding behaviour, which depends strongly upon geometric and crystallographic orientation relationships between the phases have been developed. Cazacu Oana [14] proposed generalizations to anisotropic conditions of the invariants of the deviatoric stress. Using these generalized invariants, any isotropic yield criterion can be extended such as to describe any type of material symmetry. Hosford [15] developed a continuum anisotropic yield criteria to simulate the results of calculations based on the crystallographic nature of slip. Masanobu Oda [11] introduced a fabric tensor F_{ij} for three-dimensional assemblies of granular soils as an index showing the anisotropy due to the preferred orientation of constituent particles and is actually determined by using data derived from a material science approach of soils. Davis and Khaleel [16] determined that the cold pilger metal forming technique is known to produce round titanium alloy tubing with mechanical properties that may be significantly anisotropic which is the properties of interest to both the manufacturers and consumers for defining initial manufacturing limitations and defining the final product design limitations.

For the present analysis, a mathematical model is formulated for which Hill-von Mises criteria of yielding for anisotropic material is considered with the triaxiality factor (ratio of hydrostatic stress to equivalent von Mises stress). The value of constant R which is the ratio of width strain to thickness strain is varied with yield stress and the corresponding value of critical triaxiality is obtained after optimization (maximization) of objective function subjected to inequality constraints using the genetic algorithm. A contour map is developed between anisotropy ratio and yield stress that provides the critical value of stress triaxiality.

2 GENETIC ALGORITHMS

Genetic algorithms were computerized search and optimization algorithms based on the mechanics of natural genetics and natural selection. The operation of GA's begins with a population of random strings or decision variables. Thereafter, each string is evaluated to the fitness value. Three main operators viz. reproduction, crossover, and mutation were used to create a new population of points then operate the population. The population is further evaluated and tested for termination. If the termination criterion is not met, the population is iteratively operated by the above three operators and evaluated. This procedure is continued until the termination criterion is met. One cycle of these operations and the subsequent evaluation procedure is known as a generation in GA's terminology.

The basic difference of GA's with most of the traditional optimization methods are that GA uses a coding of variables instead of variables directly, a population of points instead of a single point, and stochastic operators instead of deterministic operators. All these features make GA search robust, allowing them to be applied to a wide variety of problems. The advantage of using GA over other gradient based methods is that the latter can be mapped on local optimum whereas GA predicts global optima. In real-world problems, the objective function usually contains a number of optima of which one or more is the global optimum. Other optima have worse function values compared to the one at the global optimum. Therefore, as a designer or a decision-maker, one may be interested in finding the global optimum point which corresponds to the best function value. Fig. 1 gives the schematic representation of global and local optima.

3 MATHEMATICAL MODELLING

Stress triaxiality factor is given as the ratio of hydrostatic or mean stress to the equivalent von Mises stress, mathematically

$$\frac{\sigma_h}{\sigma_{eqv}} = TF = \frac{1/3\left(\sigma_1+\sigma_2+\sigma_3\right)}{\frac{1}{\sqrt{2}}\sqrt{\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2}} \tag{1}$$

where σ_1, σ_2 and σ_3 are the first, second and third principal stresses and

$\frac{\sigma_1+\sigma_2+\sigma_3}{3}$ = Hydrostatic stress

$\frac{1}{\sqrt{2}}\sqrt{\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2}$ = Equivalent von Mises stress

Yield stress as per Hill – von Mises criteria for anisotropic material is given as

$$\sigma_1^2+\sigma_2^2-\left(\frac{2R}{R+1}\right)\sigma_1\sigma_2=\sigma_y^2 \tag{2}$$

where R is the ratio of width strain to thickness strain and is given by the relation

$$R=\frac{\ln\left(w_o/w\right)}{\ln\left(t_o/t\right)} \tag{3}$$

In-plane stress condition, hydrostatic stress can be written as

$$\sigma_h=\frac{\sigma_1+\sigma_2}{3} \tag{4}$$

Since triaxiality factor is given by the equation

$$TF=\frac{\text{Hydrostatic stress}}{\text{Equivalent von Mises stress}} \tag{5}$$

Substituting Eq. (4) into Eq. (1), we have

$$TF=\frac{1/3\left(\sigma_1+\sigma_2\right)}{\sqrt{\sigma_1^2+\sigma_2^2-\left(\frac{2R}{R+1}\right)\sigma_1\sigma_2}} \tag{6}$$

Eq. (6) is considered as an objective function subjected to

$$\sigma_1^2 + \sigma_2^2 - \left(\frac{2R}{R+1}\right)\sigma_1\sigma_2 - \sigma_Y^2 \tag{7}$$

$$\sigma_{1L} \le \sigma_1 \le \sigma_{1U} \tag{8}$$

$$\sigma_{2L} \le \sigma_2 \le \sigma_{2U} \qquad 1 \le R \le 5 \tag{9}$$

where

σ_{1L} = Lower limit of first principal stress
σ_{1U} = Upper limit of first principal stress
σ_{2L} = Lower limit of second principal stress
σ_{2U} = Upper limit of second principal stress
R = Anisotropic ratio or anisotropic hardening parameter

The upper and lower bound for first and second principal stresses considered for optimization is given as:

$$0 \le \sigma_1 \le 1000$$
$$0 \le \sigma_2 \le 1000$$

Solution of the constrained programming problem is carried out using genetic algorithm. Specifications of GA parameters considered for the present analysis are shown in Table 1. Twenty five sets were obtained to make the optimization strategy. The sets consisted of five values of yield stress starting from 200 MPa with an increment of 50 MPa to 400 MPa has been included with every single value of anisotropic ratio (hardening parameter) which is also varied from 1 to 5. For every set of yield stress and anisotropic ratio, the obtained optimized set of values of first and second principal stresses (global maxima) are shown in Table 2 and the corresponding triaxiality is given in Table 3.

4 RESULTS AND DISCUSSION

After constrained optimization, the critical values of set of first and second principal stresses are given in Table 2 and corresponding triaxiality is shown in Table 3. With the help of results obtained, a contour map of critical stress triaxiality is developed between anisotropic ratio and yield stress (Fig. 2).

It is observed that stress triaxiality is independent of yield stress but increases with increase in anisotropic ratio (anisotropic hardening parameter) and is found to be constant for all yield stresses. In Fig. 3, it is shown that triaxiality increases with increase in anisotropic ratio and all the curves follows almost similar path and intersects at R = 3 which means that for R = 3 stress triaxiality is equal for materials for all the yield stress considered. Thus, for particular set of principal stress which is one of the most important parameters considered while designing certain component, higher anisotropic ratio is dangerous as compared to lower anisotropic ratio. For the purpose of more clarity, a contour map is developed indicating behaviour of stress triaxiality with respect to first and second principal stress which is shown in Fig. 4.

Table 1
GA parameters

S. No	GA parameter	Value
1	Population	30
2	Generations	100
3	Reproduction type	2 points Crossover
4	Selection type	Sigma Scaling
5	Mutation probability	0.005
6	Reproduction probability	0.85
7	Selection probability	0.85

Table 2
Set of principal stress at different yield stress and anisotropy ratio

Anisotropy ratio	Yield stress (MPa)									
	200		250		300		350		400	
	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
1	837.78	212.97	252.26	247.66	330.34	255.52	361.38	337.36	405.00	394.77
2	245.60	244.26	313.07	298.36	376.90	356.55	434.73	422.13	503.98	473.44
3	282.83	282.83	358.23	348.44	426.49	421.98	501.93	487.21	599.55	501.89
4	326.46	301.64	479.47	391.21	468.65	398.98	581.41	493.50	638.69	625.56
5	352.46	338.98	440.83	423.14	502.75	306.01	608.26	604.14	695.29	690.25

Table 3
Tiaxiality factor at different yield stress and anisotropy ratio

Anisotropy ratio	Yield stress (MPa)				
	200	250	300	350	400
	Triaxiality factor (TF) at yield stress				
1	0.661	0.666	0.650	0.665	0.666
2	0.816	0.816	0.814	0.816	0.814
3	0.942	0.942	0.942	0.942	0.917
4	1.046	1.053	1.053	1.023	1.053
5	1.152	1.152	0.898	1.154	1.154

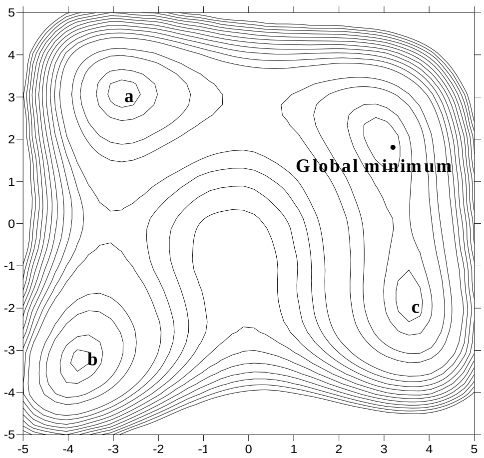

Fig. 1
Local and global minima.

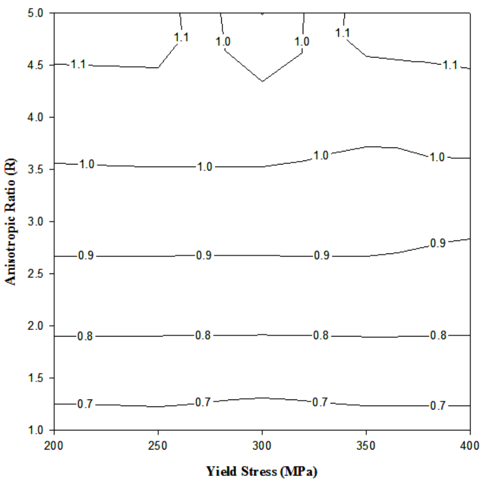

Fig. 2
Triaxiality contour map.

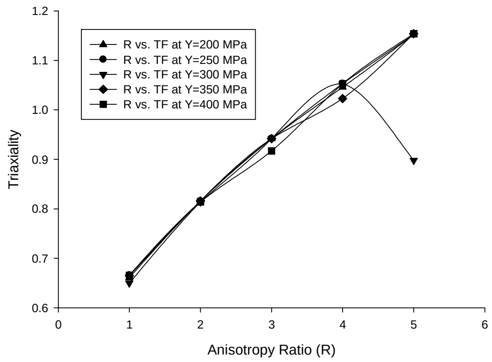

Fig. 3
Stress triaxiality vs. anisotropic ratio.

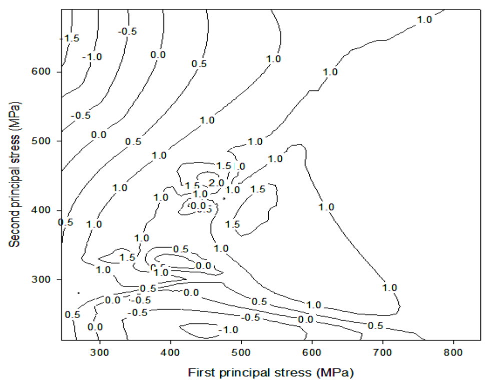

Fig. 4
Stress triaxiality vs. first and second principal stresses.

5 CONCLUSIONS

The main focus of this paper is to develop a general methodology for determination of critical stress triaxiality of anisotropic material and its role on ductile failure. Mathematical model for the stress triaxiality, yield stress, and anisotropic ratio have been formulated for plane stress condition and optimization is carried out using genetic algorithm approach. Thus, it can be concluded that loading of 2-D anisotropic material should be done in such a way so as to avoid the set of such values of set of first and second principal stresses that gives the higher stress triaxiality so as to avoid the failure of materials. Based on the findings of this study, stress triaxiality can be considered as an unavoidable parameter for product design and efficient materials can be planned for specific engineering applications.

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