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Wave Propagation in Micropolar Thermoelastic Diffusion Medium

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ABSTRACT

The present investigation is concerned with the reflection of plane waves from a free surface of a micropolar thermoelastic diffusion half space in the context of coupled theory of thermoelastic diffusion. The amplitude ratios of various reflected waves are obtained in a closed form. The dependence of these amplitude ratios with an angle of propagation as well as other material parameter are shown graphically. Effects of micropolarity and diffusion are observed on these amplitude ratios. Some special cases of interest are also deduced from the present investigation.

Keywords: Micropolar; Diffusion; Amplitude ratios; Reflection; Free plane

1 INTRODUCTION

SOME important differences between the classical theory and experiment are found in those problems where considerable stress gradient occurs. One should mention here the stress concentration in the neighborhood of holes, notches, cracks, which is very important from the point of view of safety problems in engineering structures.

As a result, the linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effects by Eringen [1,2] and Nowacki [3], and is known as micropolar coupled thermoelasticity.Touchert et. al [4] also derived the basic equations of linear theory of micropolar coupled thermoelasticity.

The study of phenomenon of diffusion is of great interest due to its various applications in geophysics and industrial application. Also these days, oil companies are interested in the process of thermoelastic diffusion for more efficient extraction of oil from deposits. Thermoelastic diffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion and that of strain. Thermodiffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion and that of strain.

Nowacki [5-8] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves. Sherief et.al [9] developed the theory of generalized thermoelastic diffusion with one relaxation time , which allows the finite speed of propagation of waves. Singh [10, 11] investigated the reflection of P and SV waves from free surface of an elastic solid on generalized thermodiffusion elastic medium.

Aouadi [12,13] investigated the different types of problems in thermoelastic diffusion. Sharma et.al [14], Sherief and El Maghraby [15] discussed various types of problems in thermoelastic diffusion. Othman et.al [16] studied source problem in generalized thermoelastic diffusion under the dependence of the modulus of elasticity on the reference temperature. Othman et. al [17] investigated disturbance in elastic medium with the help of normal mode

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analysis and studied the effect of diffusion in the context of Green and Naghdi thermoelasticity. Recently Kumar et.al [18] discussed source problem in micropolar thermoelastic diffusion medium.

Till now, reflection from free plane boundary in micropolar and thermodiffusion medium are obtained seprately by other researchers, but in this research paper reflection from free surface in micropolar thermodiffusion medium is considered to analyze the effect of micropolarity and diffusion on the amplitude ratio's of various reflected waves for a particular model.

2 BASIC EQUATIONS

Following Eringen [1] and Nowaki [3], the governing Eqns. for isotropic, homogeneous micropolar thermodiffusion elastic solid in absence of body forces, body couples, heat sources and diffusive mass sources are:

The constitutive relations:

$$t_{kl} = \lambda u_{r,r}\delta_{kl} + \mu(u_{k,l} + u_{l,k}) + K(u_{l,k} - \varepsilon_{klm}\phi_m) - \beta_1 T\delta_{kl} - \beta_2 C\delta_{kl}, \tag{1}$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k}, \tag{2}$$

$$P = -\beta_2 e_{jj} + bC - aT, \tag{3}$$

Stress eqns of motion:

$$(\lambda + \mu)u_{l,kl} + (\mu + K)u_{k,ll} + K\varepsilon_{klm}\phi_{m,l} - \beta_1 T_{,k} - \beta_2 C_{,k} = \rho\ddot{u}_k, \tag{4}$$

Couple stress eqns of motion:

$$(\alpha + \beta)\phi_{l,lk} + \gamma\phi_{k,ll} + K\varepsilon_{klm}u_{m,l} - 2K\phi_k = \rho j\ddot{\phi}_k, \tag{5}$$

Equation of heat conduction:

$$\rho C_E \frac{\partial T}{\partial t} + \beta_1 T_0 \frac{\partial e}{\partial t} + aT_0 \frac{\partial C}{\partial t} = K^* T_{,ii}, \tag{6}$$

Equation of mass diffusion:

$$D\beta_2 e_{,ii} + DaT_{,ii} + \frac{\partial C}{\partial t} - DbC_{,ii} = 0, \tag{7}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad (i = 1,2,3)$$

λ, μ - Lame's constants, α_t - coefficient of linear thermal expansion, α_c - coefficient of diffusion expansion, ρ - density, K^* - thermal conductivity, C_E - specific heat, t_{ij} -components of stress tensor, m_{ij} -components of couple stress tensor, e_{ij} - components of strain tensor, $e = e_{kk}$, δ_{ij} -kronecker delta, u_i -displacement components, ϕ_i -microrotational components, C -concentration, j -microrotation interia, $K, \alpha, \beta, \gamma, a, b$ -material constant, t -time, T -absolute temperature, T_0 -temperature of medium in its natural state assumed to be such that $|T/T_0| < 1$, D - thermoelastic diffusion constant, P -chemical potential per unit mass.

3 FORMULATION AND SOLUTION THE PROBLEM

We consider a homogeneous, isotropic micropolar thermodiffusion elastic solid in undeformed state at temperature T_0, which we designate as the plane $x_3 \geq 0$ of rectangular cartesian co-ordinate $Ox_1x_2x_3$. We consider thermoelastic plane wave in x_1x_3-plane with wave front parallel to x_2-axis and all the fields variables depends only on x_1, x_3 and t.

For two dimensional problem, we take

$$\vec{u} = (u_1, 0, u_3), \quad \vec{\phi} = (0, \phi_2, 0). \tag{8}$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x_i' = \frac{\omega_1}{c_1} x_i, \quad t_{3i}' = \frac{t_{3i}}{\beta_1 T_0}, \quad u_i' = \frac{\rho c_1 \omega_1}{\beta_1 T_0} u_i, \quad C' = \frac{\beta_2}{\rho c_1^2} C, \quad T' = \frac{\beta_1}{\rho c_1^2} T,$$
$$\phi_2' = \frac{\rho c_1^2}{\beta_1 T_0} \phi_2, \quad t' = \omega_1 t, \quad m_{32}' = \frac{\omega_1}{c_1 \beta_1 T_0} m_{32}, \qquad i = 1, 3 \tag{9}$$

where

$$c_1^2 = \left(\frac{\lambda + 2\mu + K}{\rho} \right) \quad and \quad \omega_1 = \frac{\rho C_E c_1^2}{K^*}.$$

The expression relating displacement components $u_1(x_1, x_3, t)$ and $u_3(x_1, x_3, t)$ to the scalar potential functions $q(x_1, x_3, s)$ and $\psi(x_1, x_3, s)$ in dimensionless form are given by :

$$u_1 = \frac{\partial q}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial q}{\partial x_3} + \frac{\partial \psi}{\partial x_1}, \tag{10}$$

Making use of Eqs. (8)-(10) in Eqs (4)- (7) (suppressing the primes), we obtain:

$$\left((a_1 + a_2)\nabla^2 - \frac{\partial^2}{\partial t^2} \right) q - a_4 T - a_4 C = 0, \tag{11}$$

$$a_8 \frac{\partial}{\partial t} (\nabla^2 q) + \left(\frac{\partial}{\partial t} - \nabla^2 \right) T + a_9 \frac{\partial C}{\partial t} = 0, \tag{12}$$

$$a_{10} \nabla^4 q + a_{11} \nabla^2 T + \left(a_{12} \frac{\partial}{\partial t} - a_{13} \nabla^2 \right) C = 0, \tag{13}$$

$$\left(\frac{\partial^2}{\partial t^2} - a_2 \nabla^2 \right) \psi = a_3 \phi_2, \tag{14}$$

$$\left(a_5 \nabla^2 - a_7 - \frac{\partial^2}{\partial t^2} \right) \phi_2 = a_6 \nabla^2 \psi \tag{15}$$

where

$$a_1 = \frac{(\lambda+\mu)}{\rho c_1^2}, \quad a_2 = \frac{(\mu+K)}{\rho c_1^2}, \quad a_3 = \frac{K}{\beta_1 T_0}, \quad a_4 = \frac{\rho c_1^2}{\beta_1 T_0}, \quad a_5 = \frac{\gamma}{\rho j c_1^2}, \quad a_6 = \frac{K\beta_1 T_0}{j\rho^2 c_1^2 \omega_1^2},$$

$$a_7 = \frac{2K}{j\rho\omega_1^2}, \quad a_8 = \frac{\beta_1^3 T_0^2}{K^*\omega_1^2\rho^2 c_1^2}, \quad a_9 = \frac{\beta_1 T_0 c_1^2}{K^*\omega_1\beta_2}, \quad a_{10} = \frac{D\beta_1\beta_2 T_0}{\rho c_1^4},$$

$$a_{11} = \frac{Da\rho}{\beta_1}, \quad a_{12} = \frac{\rho c_1^2}{\beta_2\omega_1}, \quad a_{13} = \frac{Db\rho}{\beta_2}, \quad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$$

4 REFLECTION AT THE FREE SURFACE

We consider a plane wave (Longitudinal displacement (LD) wave, Thermal wave (T) and Mass diffusive wave(MD-wave) propagating through the micropolar thermoelastic diffusion solid half space ($x_3 > 0$) and making an angle of incidence θ_0 with the x_3-axis, at the free surface ($x_3 = 0$). Corresponding to each incident wave, we get waves as reflected Longitudinal displacement(LD)wave, Transverse (T) Wave, Mass diffusive (MD) wave, set of coupled transverse waves (CD-I and CD-II wave) as shown in Fig. 1.

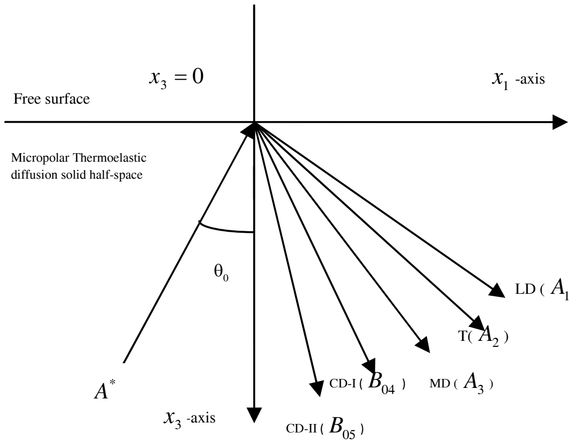

Fig.1
Geometry of the Problem.

Solutions of the Eqs (11)-(15) are sought in the form of harmonic traveling wave

$$(q, T, C, \phi_2, \psi) = (q_0, T_0, C_0, \phi_{2o}, \psi_0)e^{i\kappa(x_1\sin\theta - x_3\cos\theta + \upsilon t)}, \tag{16}$$

where υ is the phase velocity, k is the wave number.

Making use of Eqs. (16) in Eqs .(11)-(15), we obtain two Eqs as follows:

$$\upsilon^6 - R\upsilon^4 + Q\upsilon^2 - S = 0, \tag{17}$$

$$\upsilon^4 - A\upsilon^2 + B = 0, \tag{18}$$

where $\upsilon = \dfrac{\omega}{k}$ is the velocity of the coupled waves; $\upsilon_1, \upsilon_2, \upsilon_3$ are the velocities of the coupled waves namely LD-wave, T -Wave, MD-Wave and υ_4, υ_5 are the velocities of CD-I and CD-II waves given by Eqs .(17) and (18) respectively where

$$R = \frac{(a_1 + a_2)a_{12} + \iota\omega a_{13} + \iota\omega a_9 a_{11} + \iota\omega a_4 a_8 a_{12} + a_{12}}{a_{12}},$$
$$Q = \frac{(a_1 + a_2)(a_{12} + \iota\omega a_{13} + \iota\omega a_9 a_{11}) + \iota\omega a_{13}(1 + \iota\omega a_4 a_8) + Q_1}{a_{12}},$$
$$S = \frac{(a_1 + a_2)\iota\omega a_{13} - \iota\omega a_4 a_{10}}{a_{12}}, \quad A = \frac{(a_2 + a_5)\omega^2 + a_3 a_6 - a_2 a_7}{\omega^2 - a_7},$$
$$B = \frac{a_2 a_5 \omega^2}{\omega^2 - a_7}, \quad Q_1 = \iota\omega a_4 a_9 a_{10} + \iota\omega a_4(\iota\omega a_8 a_{11} - a_{10}).$$

5 BOUNDARY CONDITIONS

The boundary conditions at $x_3 = 0$ in this case are

$$t_{33} = 0, \quad t_{31} = 0, \quad m_{32} = 0, \quad \frac{\partial T}{\partial x_3} = 0, \quad \frac{\partial C}{\partial x_3} = 0 \tag{19}$$

In view of Eq.(19) , we assume the values of q, T, C, ϕ_2 and ψ satisfying the boundary conditions (19) as:

$$\{q, T, C\} = \sum (1, d_i, f_i)(A_{0i} e^{\iota k_i (x_1 \sin\theta_0 - x_3 \cos\theta_0) + \iota\omega t} + A_i e^{\iota k_i (x_1 \sin\theta_i + x_3 \cos\theta_i) + \iota\omega t}), \tag{20}$$

$$\{\psi, \phi_2\} = \sum (1, e_n)(B_{0j} e^{\iota k_j (x_1 \sin\theta_0 - x_3 \cos\theta_0) + \iota\omega t} + B_j e^{\iota k_j (x_1 \sin\theta_j + x_3 \cos\theta_j) + \iota\omega t}), \tag{21}$$

where

$$d_i = \frac{(a_4 a_{10} - a_{13})k_i^4 + (a_{13}\omega^2 - \iota\omega a_{12})k_i^2 + \iota\omega^2 a_{12}}{(a_4 a_{11} - a_{13})k_i^2 + a_4 a_{12}},$$
$$f_i = \frac{(a_4 a_{10} - a_{11})k_i^4 + \omega^2 a_{11} k_i^2}{(a_4 a_{11} - a_{13})k_i^2 + a_4 a_{12}}, \; e_n = \frac{a_2 k_j - \omega^2}{a_3}, \quad (i = 1,2,3, \quad n = 1,2 \; \& \quad j = 4,5)$$

where A_{0i} and A_i are the amplitude of the incident and reflected LD-wave, T-wave, MD-wave, B_{0j} and B_j are the amplitude of the incident(CD-I, CD-II) waves respectively.

Snell's law is given as :

$$\frac{\sin\theta_0}{\upsilon_0} = \frac{\sin\theta_1}{\upsilon_1} = \frac{\sin\theta_2}{\upsilon_2} = \frac{\sin\theta_3}{\upsilon_3} = \frac{\sin\theta_4}{\upsilon_4} = \frac{\sin\theta_5}{\upsilon_5}, \tag{22}$$

where

$$k_1\upsilon_1 = k_2\upsilon_2 = k_3\upsilon_3 = k_4\upsilon_4 = k_5\upsilon_5 = \omega \quad at \quad x_3 = 0\,,$$

$$\upsilon_0 = \begin{cases} \upsilon_1, & \text{for} \quad \text{incident} \quad \text{LD}-\text{wave} \\ \upsilon_2, & \text{for} \quad \text{incident} \quad \text{T}-\text{wave} \\ \upsilon_3, & \text{for} \quad \text{incident} \quad \text{MD}-\text{wave} \end{cases}$$

Making use of the potential given by Eqs.(20)-(21) in boundary conditions (19) and using Eq. (22), we get a system of homogeneous Eqs which can be written as:

$$\sum a_{ij} Z_j = Y_i, \quad (j = 1,2...,5), \tag{23}$$

where

$$a_{1i} = a_1 k_i^2 \cos^2\theta_i + h_1 k_i^2 \sin^2\theta_i + h_2 k_i^2 \cos_i^2 + a_4(d_i + f_i),$$
$$a_{1j} = (a_1 - h_1 + h_2)\sin\theta_j \cos\theta_j k_j^2, \quad a_{2i} = (2h_3 + h_2)k_i^2 \sin\theta_i \cos\theta_i,$$
$$a_{2j} = -[(h_2 + h_3)\cos^2\theta_j - h_3 \sin^2\theta_j]k_j^2, \quad a_{3i} = \iota f_i \cos\theta_i k_i, \quad a_{3j} = 0$$
$$a_{4i} = \iota d_i \cos\theta_i k_i, \quad a_{4j} = 0, \quad a_{5i} = 0, \quad a_{5j} = e_n \iota k_j \cos\theta_j,$$
$$h_1 = \frac{\lambda}{\rho c_1^2}, \quad h_2 = \frac{K}{\rho c_1^2}, \quad h_3 = \frac{\mu}{\rho c_1^2}, \quad Z_i = \frac{A_i}{A^*}, \quad Z_j = \frac{A_j}{A^*},$$
$$(i = 1,2,3, \quad j = 4,5, \quad \& \, n = 1,2)$$

For Incident LD-Wave, $A^* = A_{01}$ and

$$Y_1 = -a_{11}, \quad Y_2 = a_{21}, \quad Y_3 = a_{31}, \quad Y_4 = a_{41}, \quad Y_5 = a_{51}\,,$$

For Incident T-Wave, $A^* = A_{02}$ and

$$Y_1 = -a_{12}, \quad Y_2 = a_{22}, \quad Y_3 = a_{32}, \quad Y_4 = a_{42}, \quad Y_5 = a_{52}\,,$$

For Incident MD-Wave: $A^* = A_{03}$

$$Y_1 = -a_{13}, \quad Y_2 = a_{23}, \quad Y_3 = a_{33}, \quad Y_4 = a_{43}, \quad Y_5 = a_{53}\,.$$

6 PARTICULAR CASES

6.1 Neglecting diffusion effect $(\beta_2 = b = a = 0)$

we obtain the corresponding expression for micropolar thermoelastic half space with some of its changed values as:

$$\sum a_{ij} Z_j = Y_i, \quad (j = 1,2...,4), \tag{24}$$

where

$$a_{1i} = (a_1 + h_2)k_i^2 \cos^2\theta_i + h_1 k_i^2 \sin^2\theta_i + a_4 d_i,$$
$$a_{1j} = (a_1 - h_1 + h_2)\sin\theta_j \cos\theta_j k_j^2, \quad a_{2i} = (2h_3 + h_2)k_i^2 \sin\theta_i \cos\theta_i,$$
$$a_{2j} = -[(h_2 + h_3)\cos^2\theta_j - h_3 \sin^2\theta_j]k_j^2, \quad a_{4i} = \iota d_i \cos\theta_i k_i, \quad a_{4j} = 0,$$
$$a_{5i} = 0, \quad a_{5j} = e_n \iota k_j \cos\theta_j, \quad Z_i = \frac{A_i}{A^*}, \quad Z_j = \frac{A_j}{A^*}, \qquad (i = 1,2 \quad j = 4,5)$$

and the values of υ_1, υ_2 are obtained from Eq.

$$(\upsilon^4 + M\upsilon^2 + N)(\tilde{q}, \tilde{T}) = 0,$$
$$M = \frac{\omega^2 + 1}{a_4 a_8 + \omega^2}, \quad N = \frac{\omega^2}{a_4 a_8 + \omega^2}. \tag{25}$$

The above results tally with those obtain by Kumar and Singh [19]:

6.2 Neglecting Micropolarity effect

we obtain the corresponding expression for thermodiffusion elastic solid half space with some of its changed values as :

$$\sum a_{ij} Z_j = Y_i, \quad (j = 1,2...,4), \tag{26}$$

where

$$a_{1i} = a_1 k_i^2 \cos^2\theta_i + h_1 k_i^2 \sin^2\theta_i + a_4(d_i - f_i),$$
$$a_{14} = (a_1 - h_1)\sin\theta_4 \cos\theta_4 k_4^2, \quad a_{2i} = 2h_3 k_i^2 \sin\theta_i \cos\theta_i,$$
$$a_{24} = -h_3[\cos^2\theta_4 - \sin^2\theta_4]k_4^2, \quad a_{3i} = \iota f_i \cos\theta_i k_i, \quad a_{34} = 0,$$
$$a_{4i} = \iota d_i \cos\theta_i k_i, \quad a_{44} = 0, \qquad Z_i = \frac{A_i}{A^*}, \quad Z_4 = \frac{A_4}{A^*}, \qquad (i = 1,2,3)$$

and the values of υ_4 are obtained from Eq.

$$\upsilon^2 - a_2 = 0,$$

with changed values of

$$a_2 = \frac{\mu}{\rho c_1^2}.$$

The above results are similar as obtained by Singh [10]:

7 NUMERICAL RESULT AND DISCUSSION

We now present some numerical results. The material parameter chosen for this purpose are Micropolar parameter Eringen [20]:

$$\lambda = 9.4\times10^{10} N \quad m^{-2}, \quad \mu = 4.0\times10^{10} N \quad m^{-2}, \quad K = 1.0\times10^{10} N \quad m^{-2}$$
$$\gamma = 0.779\times10^{-9} N, \quad \rho = 1.74\times10^{3} Kg \quad m^{-3},$$

Thermoelastic diffusion parameter Thomas [21]:

$$C_E = 1.0 J \quad kg^{-1} \quad deg^{-1}, \quad K^* = 1.7\times10^{2} J \quad m^{-1} \quad sec^{-1} \quad deg^{-1}, \quad T_0 = 298 \quad K,$$
$$\alpha_t = 1.78\times10^{-5} K^{-1}, \quad \alpha_c = 1.98\times10^{-4} m^3 \quad kg^{-1}, \quad a = 1.2\times10^{4} m^2 \quad s^{-2} \quad K^{-1},$$
$$b = 0.9\times10^{6} m^5 \quad kg^{-1} \quad s^{-2}, \quad j = 0.2\times10^{-19} m^2, \quad D = 0.85\times10^{-8} kg \quad s \quad m^{-3}$$

The variations of amplitude ratios for micropolar thermoelastic diffusion (MTD), thermoelastic diffusion (TD) and micropolar thermoelasticity (MT) medium are shown graphically in Figs. (2)-(16) with angle of incidence $0 \le \theta \le 90$. The solid line corresponds for MTD, dashed line indicates TD and small dashed line represents MT.

7.1 Incident LD-Wave

Fig. 2, depicts the variation of amplitude ratio $|Z_1|$. It is noticed that the values of $|Z_1|$ for MTD increase initially in range $0^0 \le \theta \le 30^0$, then decrease in remaining range, whereas in case of MT values increase in whole range, but with small magnitude. Also it is evident that values of $|Z_1|$ for TD shows small increase in range $0^0 \le \theta \le 30^0$ and then converges towards origin.

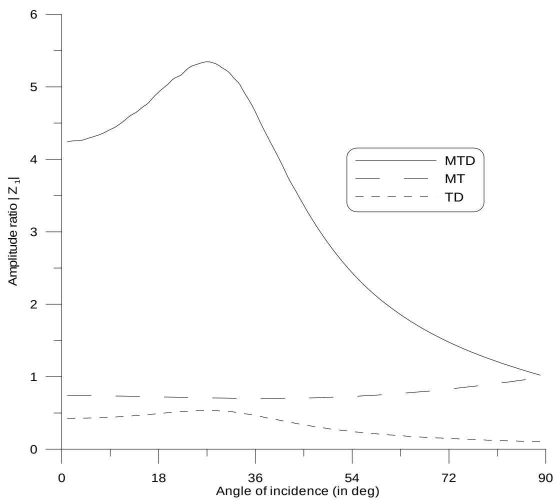

Fig.2
shows variation of amplitude ratio $|Z_1|$ with angle of incidence for LD-wave.

The variations of amplitude ratio $|Z_2|$ is shown in Fig 3. It is noticed that, as angle of incidence increases, the values of $|z_2|$ for MT decrease, whereas values of $|Z_2|$ for MTD and MT increase in range $0^0 \le \theta \le 30^0$, which reveals the impact of diffusion, then with further increase in θ, values for MT and MTD decrease abruptly.

Fig. 4 shows the variation of $|Z_3|$ with θ. It is noticed that values of $|Z_3|$ for MT increases with sharp magnitude, while values for MTD increases steadily in range $0^0 \le \theta \le 36^0$. In remaining range value of $|Z_3|$ for MT decreases abruptly as compared to MTD, which shows the impact of diffusion.

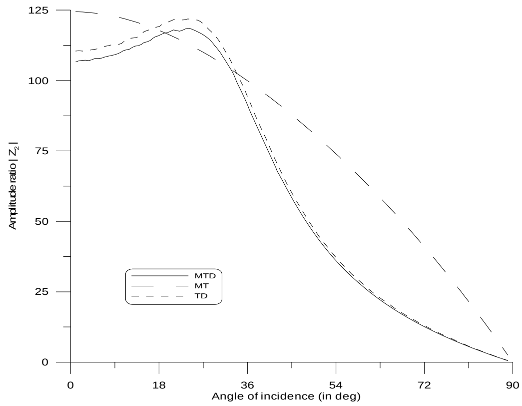

Fig.3
shows variation of amplitude ratio $|Z_2|$ with angle of incidence for LD-wave.

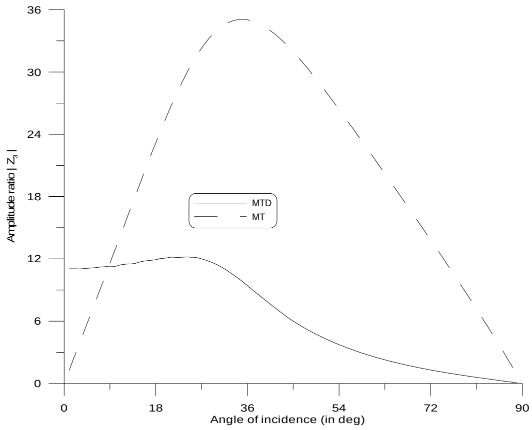

Fig.4
shows variation of amplitude ratio $|Z_3|$ with angle of incidence for LD-wave.

Fig. 5 shows the variation for amplitude ratio $|\,Z_4\,|$ with θ. It is noticed that values of $|\,Z_4\,|$ for MTD and MT shows similar behaviour in entire range i.e. their value increases in $0^0 \le \theta \le 30^0$, with further increase in θ, values for both MTD and MT decrease and approaches to zero value. Also TD shows steady state initial range and with increase in θ, values of TD decrease and converges towards zero value.

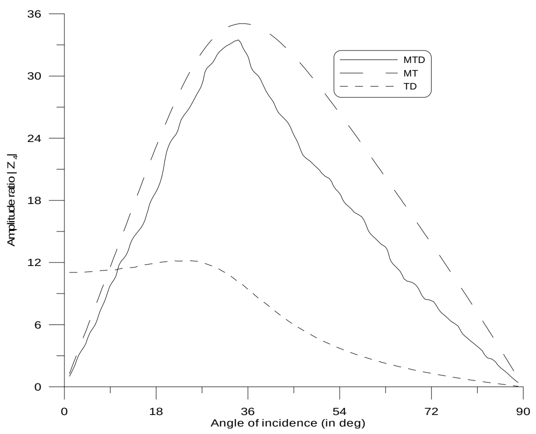

Fig.5
shows variation of amplitude ratio $|Z_4|$ with angle of incidence for LD-wave.

Fig. 6 depicts the variation of amplitude ratio $|\,Z_5\,|$ with θ. It is evident that the values for MTD increase with greater magnitude as compared to TD in ranges $0^0 \le \theta \le 30^0$, which reveals the impact of micropolarity and in remaining range values for both MTD and TD decrease

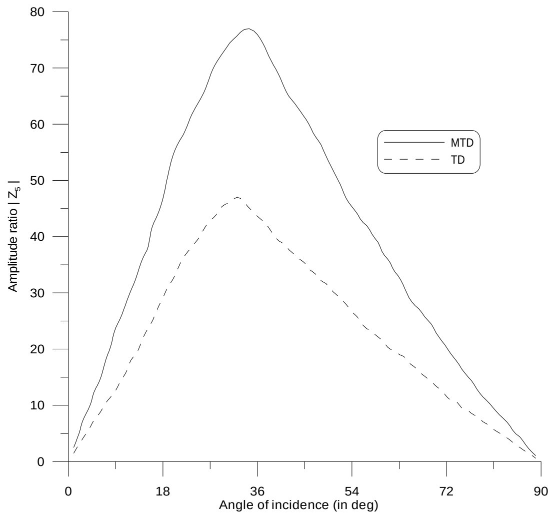

Fig.6
shows variation of amplitude ratio $|Z_5|$ with angle of incidence for LD-wave.

7.2 Incident T-Wave

The variations of amplitude ratio $|Z_1|$ is depicted in Fig 7. It is noticed that values for MTD show steady state, then with increase in angle, values decrease abruptly. Also values of $|Z_1|$ for MTD and MT show small variations in first half of interval and in remaining range, values for TD and MT approaches towards origin.

Fig. 8 depicts the values of $|Z_2|$ with θ. It is noticed that behaviour of variation of $|Z_2|$ for MTD and TD are similar in entire range with different magnitude of variations i.e. their values decrease in range $36^0 \le \theta \le 90^0$, while for MT values shows opposite behaviour i.e. their value increases in entire range but magnitude of increment being very small, which is due to absence of diffusion effect.

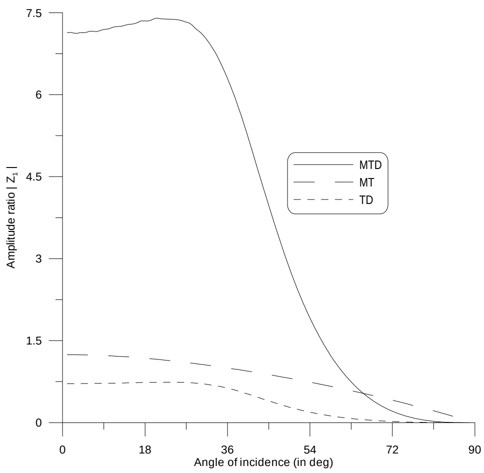

Fig.7
shows variation of amplitude ratio $|Z_1|$ with angle of incidence for T-wave.

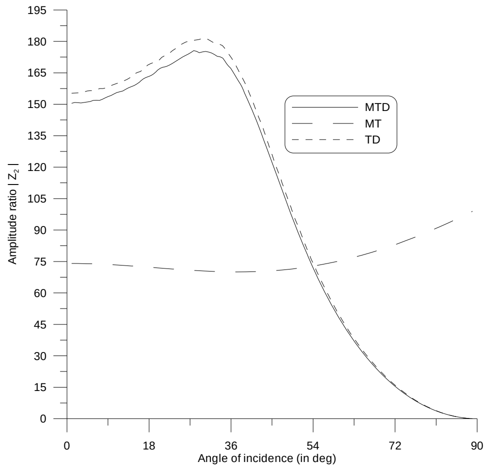

Fig.8
shows variation of amplitude ratio $|Z_2|$ with angle of incidence for T-wave.

As indicated by Fig. 9, which is plot of amplitude ratio $|Z_3|$ for θ. It is noticed that values for MT increase with greater magnitude as compared to MTD in range $0^0 \le \theta \le 36^0$ and in succeeding interval, values for MT decrease abruptly as compared to MTD, which shows significant impact of diffusion.

Fig. 10 depicts the variations of $|Z_4|$ that the values of MTD and MT increases in first half of interval then decreases in remaining range, whereas values of TD shows steady state till $\theta = 45^0$ with small increase in values which is accounted as absence of micropolarity and then approaches towards origin as θ increases.

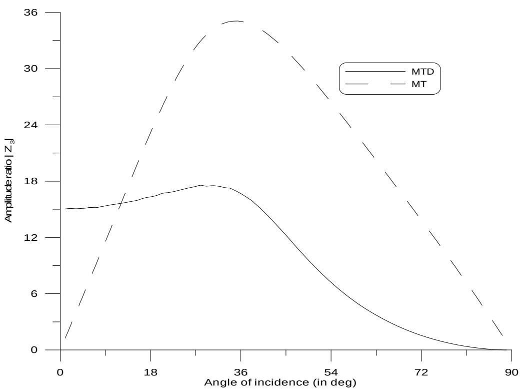

Fig.9
shows variation of amplitude ratio $|Z_3|$ with angle of incidence for T-wave.

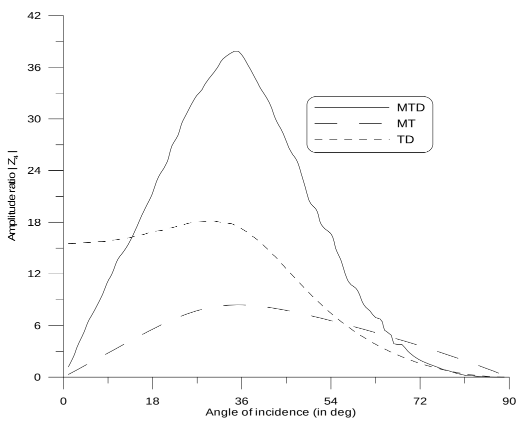

Fig.10
shows variation of amplitude ratio $|Z_4|$ with angle of incidence for T-wave.

The behaviour of variation of $|Z_5|$ is depicted in Fig. 11. It is noticed that values for MTD increase more sharply as compared to TD in range $0^0 \le \theta \le 30^0$, then values for MTD decrease with greater magnitude as compared to TD, which reveals the impact of micropolarity.

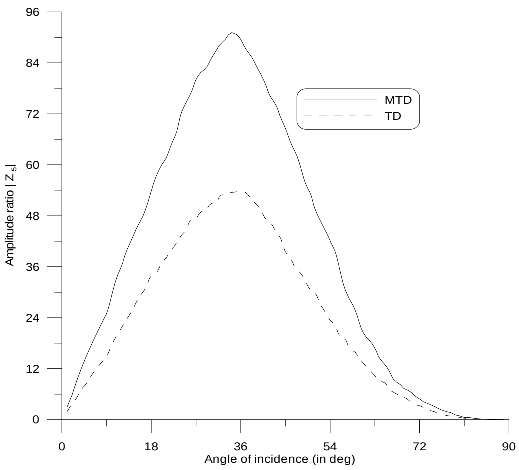

Fig.11
shows variation of amplitude ratio $|Z_5|$ with angle of incidence for T-wave.

7.3 Incident MD-Wave

Figs. 12-14 depicts the variation of $|Z_1|$ and $|Z_2|$ with θ. It is noticed that values of $|Z_1|$ and $|Z_2|$ for MTD shows similar trends i.e. their values at $0^0 \le \theta \le 36^0$, increase and then decreases monotonically. Also variations of $|Z_2|$ for TD are similar as noticed for MTD, while values of $|z_1|$ for TD shows steady state and approaches towards origin.

It is noticed from plot 13, which shows the variation of $|Z_3|$ with θ. Th trends of $|Z_3|$ for MTD is similar to that noticed for MTD in case of $|Z_1|$ with significant difference in their magnitude.

Fig. 15, shows the variation of $|Z_4|$ with θ. The values of $|Z_4|$ for MTD increase with sharp magnitude in range $0^0 \le \theta \le 36^0$ and then decrease abruptly. Also values of $|Z_4|$ for TD increase steadily in range $0^0 \le \theta \le 30^0$ and then converges towards origin.

Fig. 16 depicts the variation of $|Z_5|$ with θ. The trends of variation for MTD and TD are similar in entire range, magnitude of values for TD is greater as compared to MTD, which is due to absence of micropolarity effect.

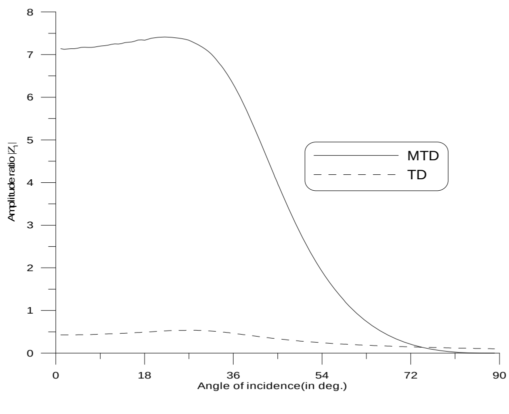

Fig.12
shows variation of amplitude ratio $|Z_1|$ with angle of incidence for MD-wave.

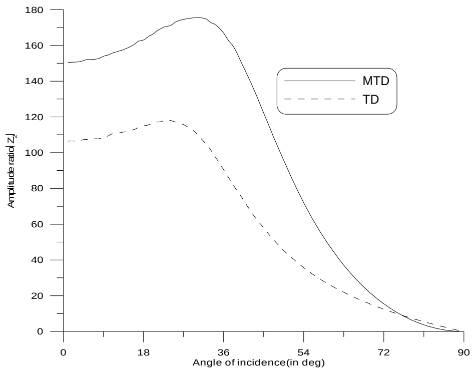

Fig.13
shows variation of amplitude ratio $|Z_2|$ with angle of incidence for MD-wave.

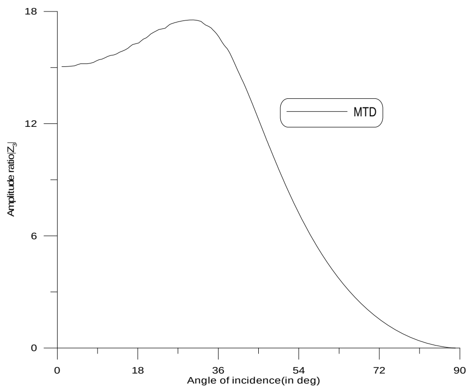

Fig.14
shows variation of amplitude ratio $|Z_3|$ with angle of incidence for MD-wave.

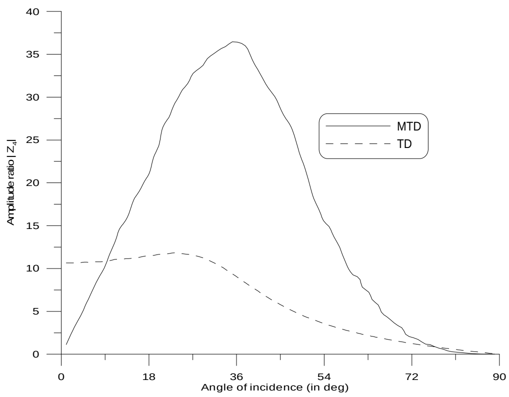

Fig.15
shows variation of amplitude ratio $|Z_4|$ with angle of incidence for MD-wave.

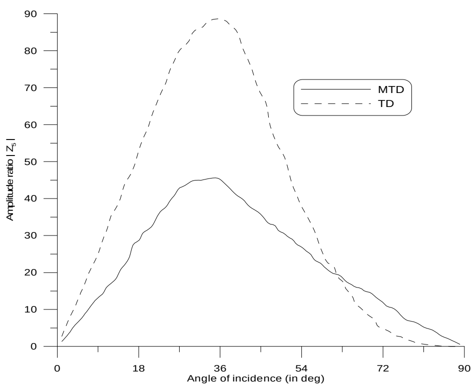

Fig.16
shows variation of amplitude ratio $|Z_5|$ with angle of incidence for MD-wave.

8 CONCLUSION

The most significant conclusion which emerges out from the above numerical discussion is that reflection is influenced by the presence of micropolarity and thermodiffusion effects. It is observed from the Figs .2-16, that the impact of micropolarity on amplitude ratios is more as compared to thermodiffusion i.e. trends for thermodiffusion is similar to that noticed for micropolar thermoelastic diffusion in most of the figures with significant difference in their magnitude. The problem though is theoretical but it can provide useful information for experimental researchers working in the field of geophysics, earthquake engineering and seismologist working in the field of mining tremors and drilling into the crust of the earth. Also this theory has industrial utilities in improving the conditions for extraction of oil due to study of phenomenon of diffusion.

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