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A New Eight Nodes Brick Finite Element Based on the Strain Approach

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ABSTRACT

In this paper, a new three dimensional brick finite element based on the strain approach is presented with the purpose of identifying the most effective to analyze linear thick and thin plate bending problems. The developed element which has the three essential external degrees of freedom (*U, V* and *W*) at each of the eight corner nodes, is used with a modified elasticity matrix in order to satisfy the basic hypotheses of the theory of plates. The displacements field of the developed element is based on assumed functions for the various strains satisfying the compatibility and the equilibrium equations. New and efficient formulations of this element is discussed in detail, and the results of several examples related to thick and thin plate bending in linear analysis are used to demonstrate the effectiveness of the proposed element. The linear analyses using this developed element exhibit an excellent performance over a set of problems.

Keywords: Strain approach; Plate bending; Brick element; Finite element.

1 INTRODUCTION

THE plate elements are of such common occurrence in engineering practice, these have attracted a considerable amount of attention over a very long period, providing much valuable information. The formulation of a simple and efficient plate element, which has a good accuracy in numerical analysis, is highly desirable. There have been developed many finite elements for plate bending analysis based on the classical Kirchhoff -Love theory, transverse shear deformation is neglected, the accuracy decreases and its truthfulness is lost with growing thickness. To improve this situation, a considerable interest to develop the plate elements based on the Reissner/Mindlin theory [32, 30], which takes into account the shear effect, and very effective locking-free procedures have been provided.
Most engineering problems in solids and structures are naturally in three dimensions. Many research works have been oriented to the three-dimensional elements, [20, 47] for the thick plates in bending. However, when solving these problems with standard Galerkin finite element methods, some bad behaviours may occur such as the locking phenomenon due to over constraining when dealing with thin plates. By using higher order elements, these locking phenomena are reduced, but the computational effort becomes larger. To overcome this difficulty, the reduced integration technique was introduced as one of the simplest and the most effective ways [46]. Other formulations have been established to develop robust three-dimensional elements, [14, 44, 4, 26, 28, 39, 15, 31, 19, 42 and 43] which allow preventing shear locking when dealing with thin structures. Despite their high number of degrees of freedom, they have many advantages: faster formulation for the stiffness matrixes can easily be taken into account

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variations in thickness of thin structures. In addition, they can be connected without any changes to three-dimensional structures. The main quality of these elements, in addition to its volumetric aspect, is that it relies solely on the degrees of freedom of translation. No degree of freedom of rotation is introduced. Therefore, what is needed is a formulation that allows the development of 3D high order elements with the minimum required degrees of freedom that are simple to implement but efficient to use. The finite element method has been extensively developed applied to many problems; several different approaches for finite element analysis have been applied with success. In this aspect, the strain based approach was found to lead to high order elements with the minimum nodes and degrees of freedom. Many have investigated strain based finite element approach for structural analysis [2, 33, 34, 35, 36, 13, 7, 16, 18, 9, 10, 21, 22, 23 and 24]. With the continued development of the strain approach, many elements were developed by other Researchers, three dimensional elasticity elements [8 and 12], Reissner/Mindlin plate elements [11].

In this paper we present a new brick finite element based on the strain approach and with a modified elasticity matrix [8, 14, 1, and 42] for the linear analysis of either thin or thick plate bending. This new element named SBBEE (Strain Based Brick Equilibrium Element) possesses eight corner nodes with the three essential external degrees of freedom (*u, v* and *w*) at each node. In developing, the present element satisfies equilibrium equations as additional condition as many elements [35, 10]. This element is examined and compared with other elements through a deep numerical evaluation which confirms its good performance.

2 FORMULATION OF THE DEVELOPED ELEMENT

For three dimensional linear analysis problems, the strain in terms of the displacements are given by:

$$\begin{aligned} &\varepsilon_x = U_{,x} \quad \varepsilon_y = V_{,y} \quad \varepsilon_z = W_{,z} \\ &\gamma_{xy} = U_{,y} + V_{,x} \quad \gamma_{yz} = V_{,z} + W_{,y} \quad \gamma_{zx} = W_{,x} + U_{,z} \end{aligned} \tag{1}$$

where *U, V* and *W* are the displacement in the (*x, y, z*)axes , (ε_x, ε_y , ε_z) are the normal strains and (γ_{xy}, γ_{yz} , γ_{zx}) are the shear strains.

To obtain the rigid body components of the displacement field, all the strains, as given by Eq. (1), are set to zero and the resulting partial differential equations are integrated. The resulting equations for *U, V*, and *W* are given by:

$$U = a_1 + a_4 y + a_6 z \quad , \quad V = a_2 - a_4 x - a_5 z \quad , \quad W = a_3 + a_5 y - a_6 x \tag{2}$$

The rigid body components of the displacement fields are expressed in terms of the six independent constants α_1, α_2 ... α_6. Since the SBBEE element is a parallelepiped with eight nodes and three degrees of freedom (*U*, *V* and *W*) at each of the eight corner nodes as shown in Fig. 1. The final displacement fields should be in terms of twenty four constants. Having used six for the representation of the rigid body modes, the remaining 18 constants are available for expressing the straining deformation of the element. These are apportioned among the strains as follow:

$$\begin{aligned} &\varepsilon_{xx} = \alpha_7 + \alpha_8 y + \alpha_9 z + \alpha_{10} yz - (\alpha_{14} \nu \mathrm{xz}) \\ &\varepsilon_{yy} = \alpha_{11} - (\alpha_{10} \nu \mathrm{yz}) + \alpha_{12} x + \alpha_{13} z + \alpha_{14} xz \\ &\varepsilon_{ZZ} = \alpha_{15} + \alpha_{16} x + \alpha_{17} y + \alpha_{18} xy \\ &\gamma_{xy} = a_{19} - a_{18} z^2 + \alpha_{20} z - \frac{2\nu}{(1-\nu)} \alpha_8 x - \frac{2\nu}{(1-\nu)} \alpha_{12} y \\ &\gamma_{yz} = \alpha_{21} - \alpha_{10} x^2 + \alpha_{22} x \\ &\gamma_{xZ} = \alpha_{23} - \alpha_{14} y^2 + \alpha_{24} y \end{aligned} \tag{3}$$

The strain functions given above of the present element satisfies both the compatibility Eq. (4) and the terms in brackets are added to satisfy the equilibrium Eq. (5), where *ν* is the Poisson's ratio.

$$\begin{cases} \varepsilon_{ij,ij} = \varepsilon_{ii,jj} + \varepsilon_{jj,ii} \\ 2\varepsilon_{kk,ij} = \left(\varepsilon_{ki,j} + \varepsilon_{kj,i} - \varepsilon_{ij,k} \right)_{,k} \end{cases} \tag{4}$$

$$\sigma_{ij,j} = 0 \tag{5}$$

Eq. (3) are equated to the corresponding expression in terms of *U, V* and *W* from Eq. (1) and the resulting equations are integrated to obtain

$$\begin{aligned} U &= a_7 x + a_8 xy + a_9 xz + a_{10} xyz - \frac{v}{(1-v)} a_{12} y^2 - \frac{1}{2} a_{14} v x^2 z - \frac{1}{2} a_{16} z^2 - \frac{1}{2} a_{18} y z^2 + \frac{1}{2} a_{19} y + \frac{1}{2} a_{20} yz - \frac{1}{2} a_{22} yz + \frac{1}{2} a_{23} z + \frac{1}{2} a_{24} yz \\ V &= -\frac{v}{(1-v)} a_8 x^2 - \frac{1}{2} a_{10} v y^2 z + a_{11} y + a_{12} xy + a_{13} yz + a_{14} xyz - \frac{1}{2} a_{17} z^2 - \frac{1}{2} a_{18} x z^2 + \frac{1}{2} a_{19} x + \frac{1}{2} a_{20} xz + \frac{1}{2} a_{21} z + \frac{1}{2} a_{22} xz - \frac{1}{2} a_{24} xz \\ W &= -\frac{1}{2} a_9 x^2 + \frac{1}{6} a_{10} v y^3 - \frac{1}{2} a_{13} y^2 + \frac{1}{6} a_{14} v x^3 + a_{15} z + a_{16} xz + a_{17} yz + a_{18} xyz - \frac{1}{2} a_{20} xy + \frac{1}{2} a_{21} y + \frac{1}{2} a_{22} xy + \frac{1}{2} a_{23} x + \frac{1}{2} a_{24} yx \end{aligned} \tag{6}$$

The complete displacement functions are the sums of corresponding expressions from Eqs. (6) and (2). These can be written as:

$$\begin{aligned} U &= a_1 + a_4 y + a_6 z + a_7 x + a_8 xy + a_9 xz + a_{10} xyz - \frac{v}{(1-v)} a_{12} y^2 - \frac{1}{2} a_{14} v x^2 z - \frac{1}{2} a_{16} z^2 - \frac{1}{2} a_{18} y z^2 + \frac{1}{2} a_{19} y + \frac{1}{2} a_{20} yz - \frac{1}{2} a_{22} yz + \frac{1}{2} a_{23} z + \frac{1}{2} a_{24} yz \\ V &= a_2 - a_4 x - a_5 z - \frac{v}{(1-v)} a_8 x^2 - \frac{1}{2} a_{10} v y^2 z + a_{11} y + a_{12} xy + a_{13} yz + a_{14} xyz - \frac{1}{2} a_{17} z^2 - \frac{1}{2} a_{18} x z^2 + \frac{1}{2} a_{19} x + \frac{1}{2} a_{20} xz + \frac{1}{2} a_{21} z + \frac{1}{2} a_{22} xz - \frac{1}{2} a_{24} xz \\ W &= a3 + a_5 y - a_6 x - \frac{1}{2} a_9 x^2 + \frac{1}{6} a_{10} v y^3 - \frac{1}{2} a_{13} y^2 + \frac{1}{6} a_{14} v x^3 + a_{15} z + a_{16} xz + a_{17} yz + a_{18} xyz - \frac{1}{2} a_{20} xy + \frac{1}{2} a_{21} y + \frac{1}{2} a_{22} xy + \frac{1}{2} a_{23} x + \frac{1}{2} a_{24} yx \end{aligned} \tag{7}$$

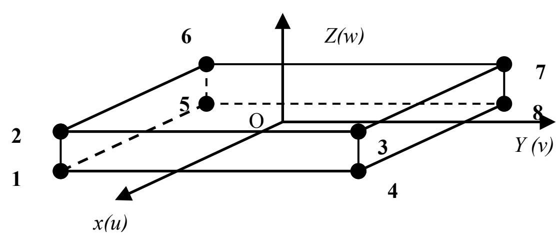

Fig.1
Brick element with *U, V*, and *W* degrees of freedom at each of the eight nodes.

The displacement functions of the developed element SBBEE given by Eq. (7) can be written in matrix form as:

$$\{\delta\} = \left[\varphi(x, y, z) \right] \{\alpha\} \tag{8}$$

The coordinates of each node are then substituted to give:

$$\{\delta_e\} = [C]\{\alpha\} \tag{9}$$

where $\{\delta_e\}$ and $\{\alpha\}$ are respectively the nodal displacement vector and the constant parameters vector. The transformation matrix [*C*] is given in appendix. The constants $\{\alpha\}$ can be obtained as:

$$\{\alpha\} = [C]^{-1} \{\delta_e\} \tag{10}$$

So

$$\{\delta\} = \left[\varphi(x, y, z) \right] [C]^{-1} \{\delta_e\} \tag{11}$$

The present brick element SBBEE has eight nodes and 24 degrees of freedom, and since the matrix [*C*] of the developed element, which relates the 24 nodal displacements to the 24 constants α_1 to α_{24} is not singular, its inverse exists. Following the well-known procedure for displacement type finite elements [47], the stiffness matrix [K^e] for the 3D element can be given by:

$$[K^e] = \int_V [B]^T [D][B]\, dV \tag{12}$$

The strain matrix [*B*] is given as follow:

$$[B] = [Q][C]^{-1} \tag{13}$$

where

$$\iiint_v dv = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \det|J|\, d\varsigma\, d\eta\, d\zeta \tag{14}$$

Hence the stiffness matrix [K^e] for the present element is given by:

$$\left[K^e\right] = [C]^{-T}\left(\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\left(\left[Q(x,y,z)\right]^T [D]\left[Q(x,y,z)\right]\right)\det|J|\, d\varsigma\, d\eta\, d\zeta\right)[C]^{-1} \tag{15}$$

where [*Q*], [*J*] and [*D*] are the strain, the Jacobian and the elasticity matrices respectively, the matrix [K^e] given by Eq. (15) is numerically evaluated and the matrices [*C*] and [*Q*] are given in the appendix.

The 3D elasticity matrix [*D*] is modified. The modification of the constants of the elasticity matrix aims to soften the element stiffness matrix in order to represent the reel behavior of plates in bending, either thick or thin (plane stress formulation with constant transverse shear).

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{bmatrix} = \frac{E}{\left(1-\nu^2\right)}\begin{bmatrix}1 & \nu & 0 & 0 & 0 & 0\\ \nu & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & d_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & d_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & k\,d_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & k\,d_{44}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{zx}\end{bmatrix} \tag{16}$$

$$\{\sigma\} = [D]\{\varepsilon\}$$

where σ_x, σ_y and σ_z are the normal stresses and τ_{xy}, τ_{yz} and τ_{zx} are the shear stresses. The constants are defined as: $d_{33} = \frac{(1-\nu)^2}{(1-2\nu)}$, $d_{44} = \frac{(1-\nu)}{2}$, $k = 5/6$. Where *k* is the shear factor.

3 LINEAR NUMERICAL VALIDATION

3.1 Square plate bending with various conditions

Rectangular plates of length *L* and thickness ratios (*L/h*=10 and 100) under various loading and support conditions (simply supported or clamped plate under uniform or concentrated loading) have been analysed. Only one quarter of the plate is modelled with symmetric boundary conditions imposed on the symmetry lines Fig. 2.

Tables 1. to 4 show the numerical results predicted by the SBBEE element, obtained with different meshes and thickness ratios. As well as by DBB8 element based on the displacement approach, and compared with the reference solution, given in ref [40] for thin plates, in ref [25] for the thick plates under the uniform load and in ref [45] for the thick plates under the concentrated load.

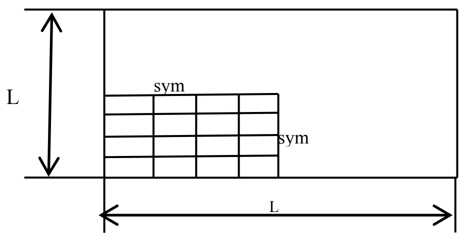

Fig.2
Rectangular plate bending with Various Conditions (E=10.92, ν=0.3 and the shear factor k=5/6).

The study shows a good convergence rate for the presented element to the reference solutions for thick and thin plates and is in good agreement with other numerical results. The developed element does not suffer from any shear locking phenomena since it converges to the Kirchhoff solutions for thin plates, contrarily for the displacement brick element DBB8, which suffer from some shear locking for the thin plate case, but they still converge to the correct result competitively fast for tick plate case.

Table 1
Non-dimensional central deflection of a simply supported plate with a uniform load ((WD/ql^4) × 100**)**.

Mesh	l/h=10		l/h=100	
	DBB8	SBBEE	DBB8	SBBEE
2×2	0.2316	0.3526	0.0045	0.0664
4×4	0.3650	0.4289	0.0171	0.3090
6×6	0.4112	0.4452	0.0358	0.3790
8×8	0.4312	0.4518	0.0574	0.3955
10×10	0.4414	0.4551	0.0780	0.4011
12×12	0.4473	0.4571	0.0867	0.4037
16×16	0.4504	0.4589	0.0950	0.4058
Reference solution	0.46169		0.4062	

Table 2
Non-dimensional central deflection of a simply supported plate with a concentrated load ((WD/pl^2) × 100**)**.

Mesh	l/h=10		l/h=100	
	DBB8	SBBEE	DBB8	SBBEE
2×2	0.7358	1.0410	0.0134	0.0388
4×4	1.130	1.2900	0.0479	1.0870
6×6	1.277	1.3620	0.0972	1.1240
8×8	1.346	1.3960	0.1594	1.1400
10×10	1.385	1.4180	0.2175	1.1480
12×12	1.405	1.4290	0.2318	1.1420
16×16	1.435	1.4480	0.2850	1.1520
Reference solution	1.44267		1.16	

Table 3
Non-dimensional central deflection of a clamped plate with a uniform load ((WD/ql^4) × 100).

Mesh	l/h=10		l/h=100	
	DBB8	SBBEE	DBB8	SBBEE
2×2	0.0698	0.0831	0.0010	0.0021
4×4	0.1150	0.1367	0.0037	0.0507
6×6	0.1320	0.1445	0.0078	0.1009
8×8	0.1394	0.1470	0.0132	0.1168
10×10	0.1431	0.1482	0.0191	0.1220
12×12	0.1452	0.1489	0.0246	0.1241
16×16	0.1465	0.1486	0.0346	0.1247
Reference solution	0.15046		0.126	

Table 4
Non-dimensional central deflection of a clamped plate with a concentrated load ((WD/pl^2) $\times 100$).

Mesh	l/h=10		l/h=100	
	DBB8	SBBEE	DBB8	SBBEE
2×2	0.3295	0.3639	0.0046	0.0083
4×4	0.5710	0.6448	0.0169	0.2196
6×6	0.6680	0.7097	0.0359	0.4369
8×8	0.7150	0.7401	0.0602	0.5072
10×10	0.7423	0.7586	0.0875	0.5326
12×12	0.7604	0.7718	0.1132	0.5441
16×16	0.7818	0.7880	0.1624	0.5518
Reference solution	0.77775		0.56	

3.2 The effect of L/h ratio on the central deflection of square plates

Plates with various conditions (loading and boundary conditions) are studied for several values of (L/h) ratio. The results presented in Tables 5. and 6 are given for 10x10 meshes in terms of the central deflection to the reference Kirchhoff solution [40]. The SBBEE element should not lock in thin plate, it is free from any shear locking contrarily for the displacement element DBB8.

Table 5
Influence of (L/h) on the central deflection (W/W_{ref}) of simply supported plates.

l/h	Concentrated load		Distributed load	
	DBB8	SBBEE	DBB8	SBBEE
5	1.8159	1.8158	1.3406	1,3471
10	1.1940	1.2224	1.0866	1,1204
20	0.9273	1.0533	0.9093	1,0358
40	0.6258	1.0023	0.6324	1,0040
50	0.5058	0.9944	0.5142	0,9987
100	01875	0.9784	0.1918	0,9874
W_{ref}	$1.16\times10^{-2}\ pl^2/D$		$0.4062\times10^{-2}\ ql^4/D$	

Table 6
Influence of (L/h) on the central deflection (W_C/W_{ref}) of Clamped plates.

l/h	Concentrated load		Distributed load	
	DBB8	SBBEE	DBB8	SBBEE
5	2.4886	1.6883	1.6857	1.6883
10	1.3255	1.1762	1.1357	1.1762
20	0.9120	1.03472	0.86607	1.03472
40	0.5533	0.99566	0.5390	0.99566
50	0.4327	0.98921	0.4217	0.98921
100	0.1562	0.96825	0.1517	0.96825
W_{ref}	$0.56\times10^{-2}\ pl^2/D$		$0.126\times10^{-2}\ ql^4/D$	

3.3 Single element aspect ratio sensitivity test

The geometrical of this test, the mesh discretization and material parameters are also shown in Fig. 3. This proposed test is used to assess the sensitivity of an element for locking when the aspect ratio (length to depth) is high as 16, the bending moment is applied in the free end of the element. The results in terms of the vertical displacements presented in Table 7. show that the developed element does not lock even for high aspect ratios and these results are exactly equal to theoretical value for all the aspect ratios.

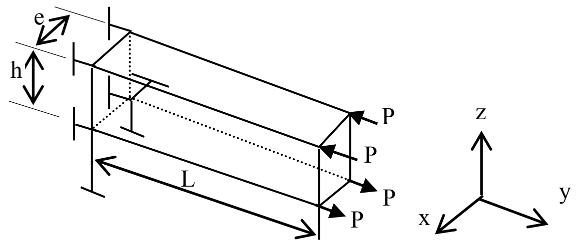

Fig.3
Single element aspect ratio sensitivity test ($E = 207 \times 10^9$ N/m^2, $\nu = 0.25$, $h = e = 0$, $12m$, $P = 6900$ N and $L/h = 1\text{-}16$).

Table 7
Normalized tip deflection for Single element aspect ratio sensitivity test.

Aspect ratio	SBBEE	PN5 × 1[b]	FCCSA/NASTRAN[a]	PN340[a]	Theory × (10^{-6})[a]
1	1.0	1.0	0.938	1.0	3.333
2	1.0	1.0	0.937	1.0	13.33
4	1.0	1.0	0.937	1.0	53.33
8	1.0	1.0	0.937	1.0	213.3
16	1.0	1.0	0.937	-	853.3

[a] Source: Bassayya, bhattacharya and Shriniva [5].
[b] Source: Bassayya and Shriniva [4].

3.4 MacNeal's elongated beam

The problem of the straight cantilever beam is modelled by three different meshes shown in Fig. 4, treated by MacNeal and Harder [29] and by many researches [4, 5].

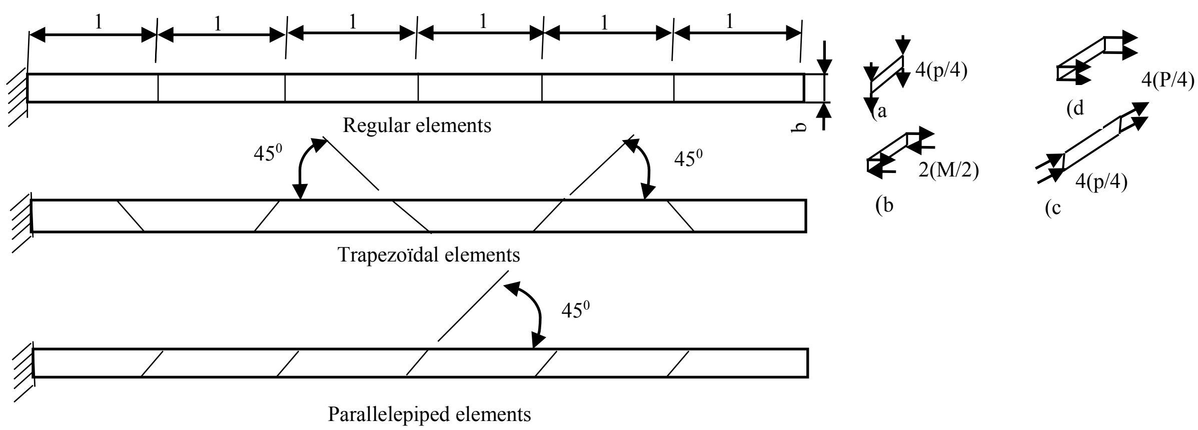

Fig.4
MacNeal's elongated beam (P=1, M=10, L=6, b=0.2, $E = 10^7$, ν=0.3 and h=0.1).

The results of the normalized deflection at the free end presented in Table 8. show that all elements perform well for the rectangular mesh. However, for meshes which contain the distortion, the strain based element SBBEE and PN5 × 1 element have excellent performance which indicates there is insensitivity of these elements to the mesh distortion, contrary for other elements (Ansys, PN 340 and PN34) given in reference [5].

Table 8
Normalized deflection for MacNeal's elongated beam.

Element shape	Load type	Normalized deflection at the free end (*W*)						
		SBBEE	HEX8[a]	HEX20[a]	ANSYS[a]	PN340[a]	PN34[a]	PN5 × 1[b]
Regular	Load type (a)	0.980	0.981	0.970	0.982	0.982	0.981	0.998
	Load type (b)	0.991	-	-	-	-	-	0.999
	Load type (C)	0.980	0.981	0.961	0.980	0.980	0.981	1.000
	Load type (d)	0.995	-	0.994	-	-	-	0.985
trapezoidal	Load type (a)	0.980	0.069	0.886	0.065	0.065	0.982	0.999
	Load type (b)	0.991	-	-	-	-	-	1.000
	Load type (C)	0.980	0.051	0.920	0.370	0.370	0.051	0.996
	Load type (d)	0.998	-	0.994	-	-	-	0.988
parallelepiped	Load type (a)	0.980	0.080	0.967	0.620	0.620	0.980	0.997
	Load type (b)	0.991	-	-	-	-	-	1.000
	Load type (C)	0.980	0.055	0.941	0.547	0.547	0.055	0.998
	Load type (d)	1.013	-	0.994	-	-	-	0.989

[a] Source: Bassayya, bhattacharya and Shriniva [5].
[b] Source: Bassayya and Shriniva [4].

3.5 Circular plate analysis

A circular plate with simply supported and clamped edges subjected to unit point and uniformly distributed loads is analysed. From symmetry considerations, only a quarter of the plate is modeled indicated in Fig. 5 and meshed with 3, 12, 48 and 192 distorted elements. The thickness of the plate will be discretized by one element, the geometrical and material parameters are also given in Fig. 5. The normalized center deflections are compared to the analytical results, coming from [40].

Numerical results obtained by the SBBEE presented in Figs. 6, 7, 8 and 9 present an overall good convergence behavior to the exact solutions for central displacement compared to analytical results.

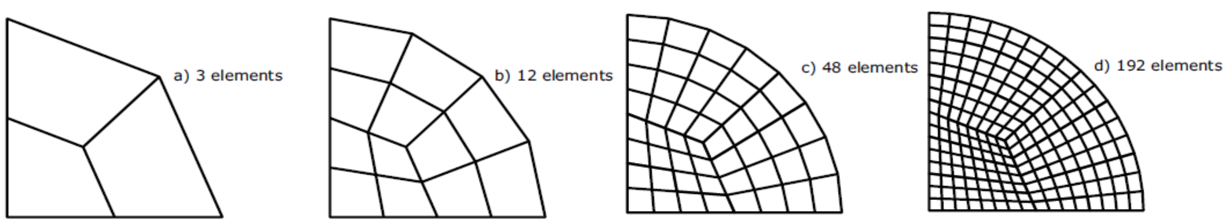

Fig.5
Circular plate problem - Finite elements' meshes employed (Radius *R*=5 *E* =1 0.92; ν= 0.3; Uniform loading: *q*=1; concentrated load *p*=1).

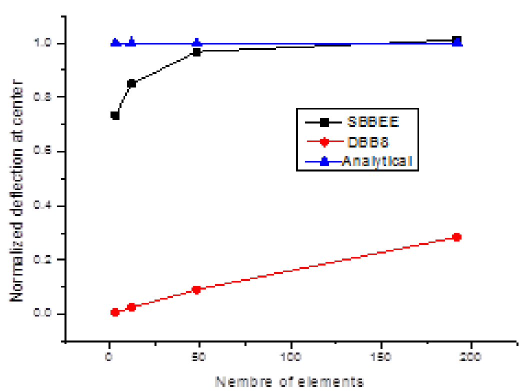

Fig.6
The normalized defection at center for simply supported thin circular plate under a uniform load.

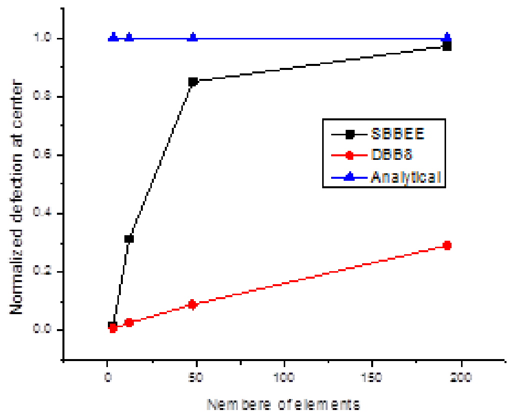

Fig.7
The normalized defection at center for clamped thin circular plate under a uniform load.

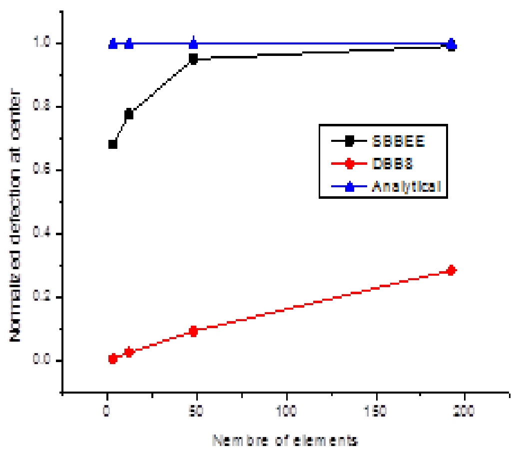

Fig.8
The normalized defection at center for simply supported thin circular plate under a concentrated load.

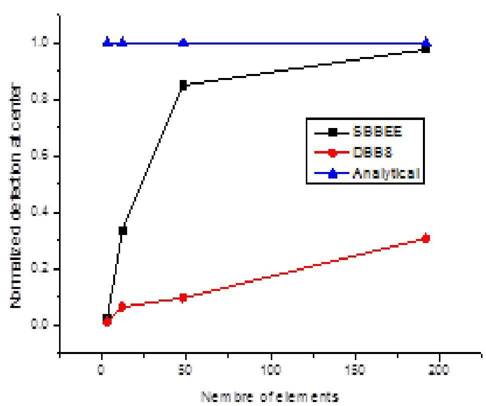

Fig.9
The normalized defection at center for clamped thin circular plate under a concentrated load.

3.6 Thick-walled cylinder

A Thick-walled cylinder subjected to internal pressure is analysed, the dimensions, mesh discretization (The thickness of the cylinder will be discretized by one element) and material parameters are also shown in Fig. 10. In contrary to other tests, the 3D elasticity matrix used in this problem is not modified.

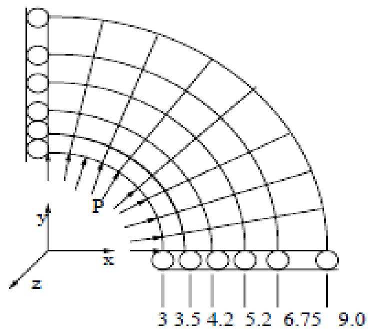

Fig.10
Thick cylinder .thickness = 1; $E = 1 \times 10^3$; mesh: 8×5; v= 0:03; $P = 1$.

The cylinder is of internal radius *a*= 3, outer radius *b*= 9, loading: unit pressure at inner radius units. The analytical solutions [41], can be written as:

$$u_{rr} = p\,\frac{a^2(1+\nu)(b^2+r^2(1-2\nu))}{E\left(b^2-a^2\right)r}$$
$$\sigma_{rr} = p\,\frac{a^2}{b^2-a^2}\left(1-\frac{b^2}{r^2}\right) \qquad (17)$$
$$\sigma_{\theta\theta} = p\,\frac{a^2}{b^2-a^2}\left(1+\frac{b^2}{r^2}\right)$$

The computed results of displacement and stress in Fig. 11 show that the accuracy of this element is quite high. Stress were plotted at the center of the element.

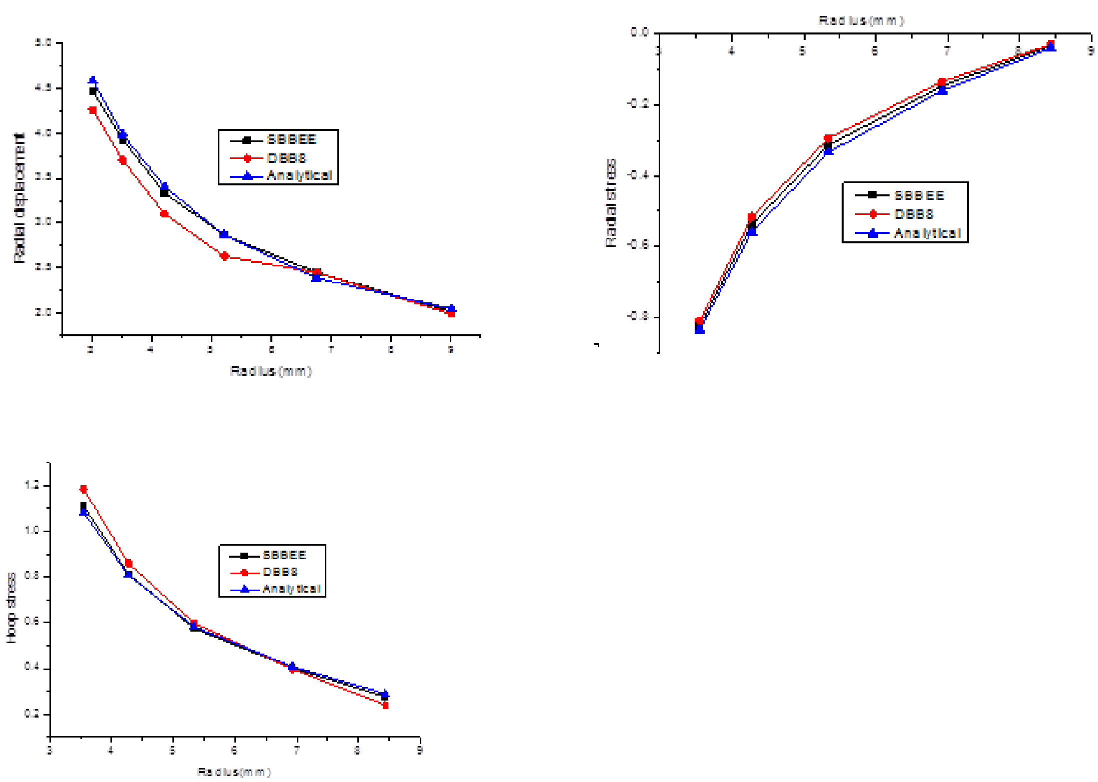

Fig.11
Stress and displacement distribution along the boundary line (y = 0).

3.7 Long cantilever mesh discretization

The cantilever shown in Fig. 12 is subjected to a tip load *P*=100. The regular and two highly distorted elements in thin and very thin plate are used to represent the cantilever (The thickness is discretized by one element). The analytical displacement is solved by: $\delta = (pl^3/3EI) + (pL/AG)$, this problem is analysed in reference [6]

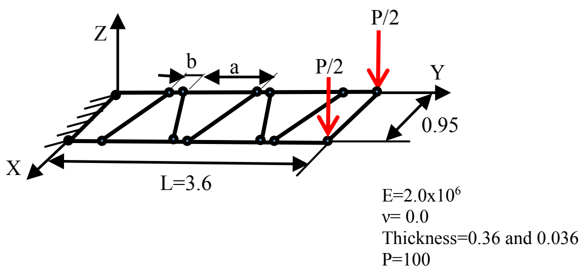

Fig.12
Long cantilever under type load.

The results for both thickness ratios listed in Table 9. show the convergence of the SBBEE element and the non-sensitivity in the effect of element distortions.

Table 9
Normalized Displacement for long cantilever under type load.

a/b	Normalsed Displacement					
	t/L = 1/100			*t/L* = 1/10		
	SBBEE	DBB8	Ref(6)	SBBEE	DBB8	Ref(6)
1.0	0.99317	0.00857	0.9931	0.99726	0.46399	0.9931
5.0	0.97655	0.26988	0.9825	1.0881	0.3235	0.9842
58.0	0.97513	0.00342	0.9704	1.0998	0.00439	0.9743

4 CONCLUSIONS

This study presents an eight brick finite element (SBBEE) based on the strain approach. The developed element containing three translations per node is used for thin and thick plates bending analysis with a change in the law of behavior. The displacements field which contains higher order of polynomial terms satisfies the requirement of the free rigid-body modes and the equilibrium equations as additional conditions. In all the tested problems the new (SBBEE) element is shown to be of high degree of accuracy and can converge rapidly with relatively coarse meshes. It has been found that the element is free from shear locking for plate bending analysis contrarily for the classical element DBB8 based on displacement approach.

APPENDIX

The nodal displacements are given in terms of the constants $\alpha_1, \ldots\ldots \alpha_{24}$ as:

$$\begin{bmatrix} U_1 \\ V_1 \\ W_1 \\ U_2 \\ V_2 \\ W_2 \\ , \\ , \\ U_8 \\ V_8 \\ W_8 \end{bmatrix} = \left[C\right] \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ , \\ , \\ \alpha_{23} \\ \alpha_{24} \end{bmatrix}$$

The matrix $[C]$ (24×24) for the SBBE element is:

$$[C]=\begin{bmatrix}[C_1]\\ [C_2]\\ [C_3]\\ [C_4]\\ [C_5]\\ [C_6]\\ [C_7]\\ [C_8]\end{bmatrix}$$

where

$$[C_i]=\left[\begin{matrix} 1 & 0 & 0 & y_i & 0 & z_i & x_i & x_iy_i & x_iz_i & x_iy_iz_i & 0 & -\frac{v}{(1-v)}(y_i^2) & 0 & -0.5(\nu x_i^2 z_i) & 0 & -0.5z_i^2 & 0 & -0.5y_i z_i^2 \\ 0 & 1 & 0 & -x_i & -z_i & 0 & 0 & -\frac{v}{(1-v)}(x_i^2) & 0 & -0.5(\nu y_i^2 z_i) & y_i & x_iy_i & y_iz_i & x_iy_iz_i & 0 & 0 & -0.5z_i^2 & -0.5x_i z_i^2 \\ 0 & 0 & 1 & 0 & y_i & -x_i & 0 & 0 & -0.5x_i^2 & +\frac{1}{6}(\nu y_i^3) & 0 & 0 & -0.5y_i^2 & +\frac{1}{6}(\nu x_i^3) & z_i & x_iz_i & y_iz_i & x_iy_iz_i \end{matrix}\right.$$
$$\left.\begin{matrix} 0.5y_i & 0.5y_iz_i & 0 & -0.5y_iz_i & 0.5z_i & 0.5y_iz_i \\ 0.5x_i & 0.5x_iz_i & 0.5z_i & 0.5x_iz_i & 0 & -0.5x_iz_i \\ 0 & -0.5x_iy_i & 0.5y_i & 0.5x_iy_i & 0.5x_i & 0.5x_iy_i \end{matrix}\right]$$

And x_i, y_i and z_i are the coordinates of the eight corner nodes i $(i = 1, 8)$.

The strain matrix $[Q(x,y,z)]$ for the SBBE element is:

$$[Q]=\begin{bmatrix} 0&0&0&0&0&0&1&y&z&yz&0&0&0&-\nu xz&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&-\nu yz&1&x&z&xz&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&x&y&xy&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&-\frac{2\nu}{(1-\nu)}x&0&0&0&-\frac{2\nu}{(1-\nu)}y&0&0&0&0&0&-z^2&1&z&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&-x^2&0&0&0&0&0&0&0&0&0&0&1&x&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&-y^2&0&0&0&0&0&0&0&0&1&y \end{bmatrix}$$

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