Journal of Solid Mechanics Vol. 8, No. 1 (2016) pp. 175-183

Consolidation Around a Heat Source in an Isotropic Fully Saturated Rock with Porous Structure in Quasi-Static State

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Received 25 December 2015; accepted 28 February 2016

ABSTRACT

The titled problem of coupled thermoelasticity for porous structure has been solved with an instantaneous heat source acting on a plane area in an unbounded medium. The basic equations of thermoelasticity, after being converted into a one-dimensional form, have been written in the form of a vector-matrix differential equation and solved by the eigenvalue approach for the field variables in the Laplace transform domain in closed form. The deformation, temperature and pore pressure have been determined for the space time domain by numerical inversion from the Laplace transform domain. Finally the results are analyzed by depicting several graphs for the field variables.

Keywords : Consolidation; Porous; Isotropic; Thermoelasticity; Quasi-static.

1 INTRODUCTION

THE study in the theory of thermoelastic interactions with voids were first initiated by Nunziato and Cowin[1] and Iesan [2] to develop a non-linear theory of elastic materials. The linear theory of elastic materials with voids has been developed by Cowin and Nunziato [3]. In this linear theory of elastic material with voids, the change in void volume fraction and strain are taken as independent kinematic variables. Iesan [4] presented a linear theory of thermoelastic material with voids where he derived the basic equations for the system and proved the uniqueness of solution, reciprocity relation and variational characterization of solution in the dynamical theory. Dhaliwal and Wang [5] formulated the heat flux dependent thermoelasticity theory for an elastic material with voids. Puri and Cowin [6] analyzed the behavior of plane harmonic waves in a linear elastic material with voids. Ciarlatta and Chirita [7] have pointed out that the basic concept underline this theory is that of a material for which the bulk density is written as the product of two fields: the density field of the matrix material and the volume fraction field. Cicco and Diacco [8] presented a theory of thermoelastic material with voids without energy dissipation. Other relevant work in this field are Cherita and Scalia [9] and Scalia, Pompei and Chirita [10] who enriched the theory under the assumption that the constitutive coefficients are positive definite. Chirita and Ciarletta [11] discussed the structural stability of thermoelastic model of porous media. The problem on consolidation around a volumic spherical decaying heat source has been studied by Giraud and Rousset [12]. Booker and Savvidou [13] presented solutions for temperature, pressure and stress fields arising from a spherical heat source buried in a thermally consolidating material of infinite extent.

Thermoelastic vibration analysis of Mems/Nems plate resonators with voids has been studied in Sharma and Grover [14]. Kumar and Leena rani [15] investigated the temperature and other field variables in a homogeneous,

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isotropic, generalized thermoelastic half space with voids due to normal, tangential force and thermal source. Kumar and Devi [16] obtained a general solution for the field equations of thermoelastic material with voids of one relaxation time parameter (Lord and Shulman theory [17]) depending on modulus of elasticity and thermal conductivity on reference temperature.The intended applications of the theory of elastic material with voids are to geological materials such as rock and soils and to manufacture porous materials.

In this paper, the titled problem of coupled thermoelasticity for porous structure has been solved in Laplace transform domain when an instantaneous heat source is acting on a plane area in an unbounded medium. The deformation, temperature and pore pressure have been determined in the space-time domain by numerical method. Finally the results are analyzed by depicting several graphs for the field variables.

2 FORMULATION AND SOLUTION OF THE PROBLEM

We consider a quasi-static, infinitesimal deformation of isotropic fully saturated rocks with porous structure. If an instantaneous heat source is acting on a plane area in an unbounded medium in which the solid and fluid phases are chemically inert and where the inertial forces are negligible, then the complete set of differential equations for linear thermoporoelasticity (vide Rice and Cleary [18] , Coussy [19]) are given by

$$\left(K_0+\frac{4}{3}G\right)\text{grad div}\,\vec{u}-G\ \text{curl curl}\,\vec{u}-b\ \text{grad}\,P-3\alpha_0 K_0\ \text{grad}\,\theta=0 \tag{1}$$

$$k\nabla^2 P=b\,\frac{\partial\varepsilon_v}{\partial x}+\frac{1}{M}\frac{\partial P}{\partial t}-3\alpha_m\,\frac{\partial\theta}{\partial t} \tag{2}$$

$$\lambda_T\nabla^2\theta=c\,\frac{\partial\theta}{\partial t}-Q_v+3\alpha_0 K_0\theta_0\dot{u}_{k,k} \tag{3}$$

where $\vec{u}$ is the displacement vector of matrix particles, ε_v denotes the volume strain, P is the excess pore pressure, θ is the excess temperature, G is the shear modulus, K_0 is the drained bulk modulus, b is the Biot dimensionless coefficient of effective stress, M is the Biot bulk modulus of the fluid, k is the Darcy conductivity, α_0 is the drained expansion coefficient of matrix particles, α_m is a differential expansion coefficient between the matrix and the fluid, λ_T is thermal conductivity, c is the volumetric specific heat and Q_v denotes the intensity of any distributed heat source. The poroelastic parameters b and M were introduced by Biot [20].

K_0 and α_0 can be related to the undrained bulk modulus K and the undrained coefficient of linear thermal expansion α by the relations

$$K=K_0+b^2M \tag{4}$$

$$\alpha K=\alpha_0 K_0+\alpha_m bM \tag{5}$$

The basic equations of thermoelasticity after being converted in one-dimensional form are given by

$$\left(K_0+\frac{4}{3}G\right)\frac{\partial u}{\partial x}-bP-3\alpha_0 K_0\theta=0 \tag{6}$$

$$k\,\frac{\partial^2 P}{\partial x^2}=b\,\frac{\partial^2 u}{\partial x\,\partial t}+\frac{1}{M}\frac{\partial P}{\partial t}-3\alpha_m\,\frac{\partial\theta}{\partial t} \tag{7}$$

$$\lambda_T \nabla^2\theta = c\frac{\partial\theta}{\partial t} - Q_v + 3\alpha_0 K_0\theta_0 \frac{\partial^2 u}{\partial x\,\partial t} \tag{8}$$

In order to non-dimensionalize the system of Eqs. (6)-(8), we define the following quantities

$$x' = \frac{x}{l},\; u' = \frac{u}{l},\; t' = \frac{a_0 t}{l},\; \theta' = \frac{\theta}{\theta_0},\; Q_v' = \frac{l^2}{\lambda_T\theta_0}Q_v,\; P' = \frac{P}{K_1},\; a_0 = \frac{\lambda_T}{cl},\; K_1 = K_0 + \frac{4}{3}G\;. \tag{9}$$

Introducing (9) into Eqs. (6)-(8) and suppressing the primes, we obtain

$$\frac{\partial u}{\partial x} = bP + \frac{3\alpha_0 K_0\theta_0}{K_1}\theta \tag{10}$$

$$\frac{\partial^2 P}{\partial x^2} = b\frac{\lambda_T}{kcK_1}\frac{\partial^2 u}{\partial x\,\partial t} + \frac{\lambda_T}{kcM}\frac{\partial P}{\partial t} - \frac{3\alpha_m\theta_0\lambda_T}{kcK_1}\frac{\partial\theta}{\partial t} \tag{11}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial\theta}{\partial t} - Q_v + \frac{3\alpha_0 K_0}{c}\frac{\partial^2 u}{\partial x\,\partial t} \tag{12}$$

Eqs. (10), (11) and (12) can be written as:

$$\frac{\partial u}{\partial x} = bP + a_1\theta \tag{13}$$

$$\frac{\partial^2 P}{\partial x^2} = ba_2\frac{\partial^2 u}{\partial x\,\partial t} + a_3\frac{\partial P}{\partial t} - a_4\frac{\partial\theta}{\partial t} \tag{14}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial\theta}{\partial t} - Q_v + a_5\frac{\partial^2 u}{\partial x\,\partial t} \tag{15}$$

where

$$a_1 = \frac{3\alpha_0 K_0\theta_0}{K_1},\quad a_2 = \frac{\lambda_T}{kcK_1},\quad a_3 = \frac{\lambda_T}{kcM},\quad a_4 = \frac{3\alpha_m\theta_0\lambda_T}{kcK_1},\quad a_5 = \frac{3\alpha_0 K_0}{c}\;. \tag{16}$$

Let $x = 0$ be the plane area over which the instantaneous heat source $Q_v(x,t)$ of the form

$$Q_v = Q_0\delta(x)H(t)$$

Acts. Q_0 is the strength of heat source and $\delta(x)$ and $H(t)$ are respectively the Dirac delta function and Heaviside unit step function. For the solution of these equations, we shall use the Laplace transform of parameter p of the function $f(t)$ defined by

$$\bar{f}(p) = \int_0^\infty e^{-pt} f(t)\,dt \tag{17}$$

Taking the Laplace transform on Eqs. (13), (14) and (15), we get

$$\frac{du}{dx}=b\bar{P}+a_1\bar{\theta} \tag{18}$$

$$\frac{d^2\bar{P}}{dx^2}=ba_2p\frac{d\bar{u}}{dx}+a_3p\bar{P}-a_4p\bar{\theta} \tag{19}$$

$$\frac{d^2\bar{\theta}}{dx^2}=p\bar{\theta}-\frac{\delta(x)}{p}Q_0+a_5p\frac{d\bar{u}}{dx} \tag{20}$$

Substituting (18) in Eqs. (19) and (20), we get

$$\frac{d^2\bar{P}}{dx^2}=p\left(b^2a_2+a_3\right)\bar{P}+p\left(ba_1a_2-a_4\right)\bar{\theta} \tag{21}$$

$$\frac{d^2\bar{\theta}}{dx^2}=bpa_5\bar{P}+p\left(a_1a_5+1\right)\bar{\theta}-\frac{\delta(x)}{p}Q_0 \tag{22}$$

In order to apply eigenvalue method of Lahiri et.al [21], Eqs. (21) and (22) can be written in the form of vector-matrix differential equation as follows

$$\frac{d}{dx}\begin{pmatrix}\bar{\theta}\\ \bar{P}\\ \bar{\theta}'\\ \bar{P}'\end{pmatrix}=\begin{pmatrix}0&0&1&0\\0&0&0&1\\c_{31}&c_{32}&0&0\\c_{41}&c_{42}&0&0\end{pmatrix}\begin{pmatrix}\bar{\theta}\\ \bar{P}\\ \bar{\theta}'\\ \bar{P}'\end{pmatrix}+\begin{pmatrix}0\\0\\-\dfrac{\delta(x)}{p}Q_0\\0\end{pmatrix} \tag{23}$$

where

$$c_{31}=p\left(a_1a_5+1\right),\quad c_{32}=pba_5,\quad c_{41}=p\left(ba_1a_2-a_4\right),\quad c_{42}=p\left(b^2a_2+a_3\right)$$

The prime indicates differentiation with respect to x.
Eq. (23) can be compactly written as:

$$\frac{d\tilde{v}}{dx}=A\tilde{v}+\tilde{f} \tag{24}$$

where

$$\tilde{v}=\begin{pmatrix}\bar{\theta}\\ \bar{P}\\ \bar{\theta}'\\ \bar{P}'\end{pmatrix},\quad \tilde{f}=\begin{pmatrix}0\\0\\-\dfrac{\delta(x)}{p}Q_0\\0\end{pmatrix},\quad \text{and}\quad A=\begin{pmatrix}0&0&1&0\\0&0&0&1\\c_{31}&c_{32}&0&0\\c_{41}&c_{42}&0&0\end{pmatrix} \tag{25}$$

The characteristic equation of the matrix A takes the form

$$\lambda^4-\left(c_{31}+c_{42}\right)\lambda^2+\left(c_{31}c_{42}-c_{41}c_{32}\right)=0 \tag{26}$$

The roots of this equation which are also the eigenvalues of the matrix *A* are of the form:

$$\lambda = \lambda_1, -\lambda_1,\ \lambda_2, -\lambda_2$$

where

$$\lambda_1^{\,2} + \lambda_2^{\,2} = p\left(b^2 a_2 + a_3 + a_1 a_5 + 1\right),$$
$$\lambda_1^{\,2}\lambda_2^{\,2} = p^2\left(b^2 a_2 + a_3 + a_1 a_3 a_5 + b a_4 a_5\right) \tag{27}$$

The eigenvector *X* corresponding to the eigenvalue λ can be calculated as:

$$X = \left[\left(c_{42} - \lambda^2\right), -c_{41}, \lambda\left(c_{42} - \lambda^2\right), -\lambda c_{41}\right]^T \tag{28}$$

Let

$$V_1 = \left[X\right]_{\lambda=\lambda_1},\quad V_2 = \left[X\right]_{\lambda=-\lambda_1},\quad V_3 = \left[X\right]_{\lambda=\lambda_2},\quad V_4 = \left[X\right]_{\lambda=-\lambda_2},\quad V = \left[V_1\ \ V_2\ \ V_3\ \ V_4\right] \tag{29}$$

Now we consider

$$A = V\Lambda V^{-1} \tag{30}$$

where $\Lambda = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$ is a diagonal matrix whose elements $\lambda_1, \lambda_2, ..., \lambda_n$ are the distinct eigenvalues of *A*.

Substituting (30) in (24) and premultiplying the resulting equation by V^{-1}, we get

$$\frac{d}{dx}\left(V^{-1}\tilde{v}\right) = \Lambda\left(V^{-1}\tilde{v}\right) + V^{-1}\tilde{f} \quad \text{or,} \quad \frac{d\tilde{y}}{dx} = \Lambda\tilde{y} + V^{-1}\tilde{f}, \quad \text{where } y = V^{-1}\tilde{v} \tag{31}$$

A typical r^th equation of (31) can be written as:

$$\frac{dy_r}{dx} = \lambda_r y_r + Q_r,\quad Q_r = V_r^{-1} f \tag{32}$$

We assume the elements of V^{-1} as:

$$V^{-1} = \left(\omega_{ij}\right);\quad i, j = 1,2,3,4$$

Hence using the expression for $\tilde{f}$ in (25), we calculate from the relation

$$Q_r = \sum_{i=1}^{4} \omega_{ri} f_i$$

The expression for Q_r as:

$$Q_r = \omega_{r3} f_3, \quad r = 1,2,3,4 \qquad \text{i.e. } Q_1 = \omega_{13} f_3, \quad Q_2 = \omega_{23} f_3, \quad Q_3 = \omega_{33} f_3, \quad Q_4 = \omega_{43} f_3 \tag{33}$$

where

$$f_3 = -\frac{Q_0 \delta(x)}{p}$$

The solution of Eq. (32) is given by

$$y_r = e^{\lambda_r x}\left[y_r e^{-\lambda_r x} \right]_{x=-\infty} + e^{\lambda_r x} \int_{-\infty}^{x} Q_r e^{-\lambda_r x} dx, \quad x > 0 \tag{34}$$

Since $y = V^{-1}\tilde{v}$ and the field variables in $\tilde{v}$ vanish at infinity we neglect the first term on the right hand side of (34). Thus from (34)

$$y_r = -e^{\lambda_r x} \int_{-\infty}^{x} \frac{Q_0 \delta(x)}{p} e^{-\lambda_r x} \omega_{r3} dx = -\frac{Q_0 \omega_{r3}}{p} e^{\lambda_r x}, \quad r = 1,2,3,4 \tag{35}$$

The solution for v can be written as:

$$\tilde{v} = \sum_{r=1}^{4} V_r y_r = V_2 y_2 + V_4 y_4 \tag{36}$$

Since y_1 and y_3 are neglected from the physical considerations of the problem. Thus considering first two elements of $\tilde{v}$ and using (29) we can write from (36),

$$\begin{bmatrix} \bar{\theta} \\ \bar{p} \end{bmatrix} = -\begin{bmatrix} c_{42} - \lambda_1^2 \\ -c_{41} \end{bmatrix} \frac{Q_0 \omega_{23}}{p} e^{-\lambda_1 x} - \begin{bmatrix} c_{42} - \lambda_2^2 \\ -c_{41} \end{bmatrix} \frac{Q_0 \omega_{43}}{p} e^{-\lambda_2 x} \tag{37}$$

The elements ω_{23} and ω_{43} can easily be calculated as:

$$\omega_{23} = \frac{1}{\lambda_1 \left(\lambda_1^2 - \lambda_2^2 \right)}, \quad \omega_{43} = -\frac{1}{\lambda_2 \left(\lambda_1^2 - \lambda_2^2 \right)} \tag{38}$$

Substituting these values in (37), we get the complete solution for the temperature $\bar{\theta}$ and pressure $\bar{P}$ in the Laplace transform domain as:

$$\bar{\theta}(x,p) = -\frac{Q_0}{p \lambda_1 \lambda_2 \left(\lambda_1^2 - \lambda_2^2 \right)} \left[\lambda_2 \left(c_{42} - \lambda_1^2 \right) e^{-\lambda_1 x} - \lambda_1 \left(c_{42} - \lambda_2^2 \right) e^{-\lambda_2 x} \right] \tag{39}$$

$$\bar{P}(x,p) = \frac{Q_0 c_{41}}{p \lambda_1 \lambda_2 \left(\lambda_1^2 - \lambda_2^2 \right)} \left[\lambda_2 e^{-\lambda_1 x} - \lambda_1 e^{-\lambda_2 x} \right] \tag{40}$$

Using (39) and (40) in (18) and solving the resulting equation with appropriate condition, we get the displacement component in the Laplace transform domain as:

$$\bar{u}(x,p)=\frac{Q_0}{p\lambda_1^2\lambda_2^2\left(\lambda_1^2-\lambda_2^2\right)}\left[bc_{41}\left(\lambda_1^2e^{-\lambda_2 x}-\lambda_2^2e^{-\lambda_1 x}\right)-a_1\left\{\lambda_1^2\left(c_{42}-\lambda_2^2\right)e^{-\lambda_2 x}-\lambda_2^2\left(c_{42}-\lambda_1^2\right)e^{-\lambda_1 x}\right\}\right] \quad (41)$$

In order to invert the field variables $\bar{\theta}(x,p), \bar{P}(x,p)$ and $\bar{u}(x,p)$ from Laplace transform domain to the space time domain, we use Zakian algorithm [22].

3 NUMERICAL RESULTS AND DISCUSSIONS

In order to illustrate the preceding results graphically, we have chosen compressible clay for numerical evaluation. As in [12] the thermoporoelastic parameters are taken as:

$$k_0=100\ \ MPa,\quad G=60\ \ Mpa,\quad b=1.00,\quad M=5500\ \ Mpa,\quad k=4\times10^{-12}ms^{-1},\quad c=2.85\times10^{6}Jm^{-3}K^{-1},$$
$$\lambda_T=1.7\ \ Wm^{-1}K^{-1},\quad \alpha_0=1.0\times10^{-5}K^{-1},\quad \alpha=4.36\times10^{-5}K^{-1}$$

We now present the graphs of pore pressure P, temperature T and displacement u which vary when x vary. Fig.1 depicts the variation of pressure P along the distance x for different values of time t. At $x=0$, P assumes the values of 0.01796, 0.03112, 0.04017, 0.04753 and 0.05389 for $t=1,3,5,7$ and 9 respectively. As x increases, the values gradually decreases like an exponentially decaying curve for each prescribed value of t and ultimately approaches to zero when $x=10$. We may further infer from Fig.1 that in the range $(9\prec x\prec10)$, the pore pressure P approaches zero whatever be the values of time t.

Fig.2 shows the variation of temperature T along the distance x for different prescribed values of t. At $x=0$, the temperature assumes the values of 0.5647, 0.9781, 1.263, 1.494 and 1.694 at $t=1,3,5,7$ and 9 respectively. After that the temperature decreases gradually in all cases and finally tends to zero when $x\approx10$. It may be noted that the variation of pressure and temperature as in Fig.1 and Fig.2 respectively are similar in nature. In this case also, it is predicted that the temperature tends to zero in the range $(9\prec x\prec10)$ for all values of t.

Fig.3 exhibits the variation of displacement u for different values of time t along the distance x. It is observed that initially when $x=0$, displacement u is contractive in nature in the range (-0.17,-0.01889) for prescribed values of t as assumed in earlier cases. Then it gradually increases and approached to zero at about $x\approx10$. When the displacement u varies with respect to time t, we may infer that $u\to0\forall t$ in the range $(9\prec x\prec10)$.

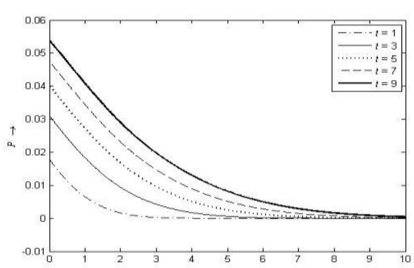

Fig.1
The pressure distribution for fixed *t*.

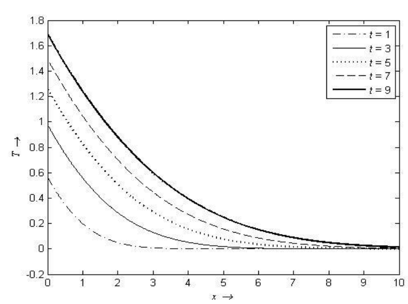

Fig.2
The temperature distribution for fixed *t*.

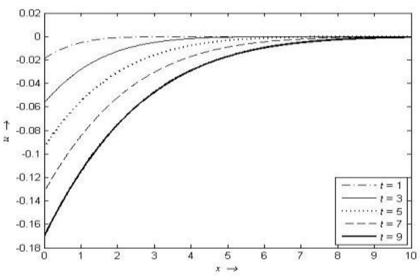

Fig.3
The displacement distribution for fixed t.

4 CONCLUSIONS

We developed a solution scheme of one-dimensional coupled thermoelasticity problem for porous structure to determine the field variables in the space-time domain. From Figs. 1-3, it is observed that the displacement distribution shows the opposite behavior compared to temperature and pressure distribution. As time increases the temperature and pressure increases whereas the displacement decreases and ultimately all the three field variables approach to zero in the range $(9 \prec x \prec 10)$.

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