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Magneto-Electro-Thermo-Mechanical Response of a Multiferroic Doubly-Curved Nano-Shell

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ABSTRACT

Free vibration of a simply-supported magneto-electro-elastic doubly-curved nano-shell is studied based on the first-order shear deformation theory in the presence of the rotary inertia effect. To model the electric and magnetic behaviors of the nano-shell, Gauss's laws for electrostatics and magneto statics are used. By using Navier's method, the partial differential equations of motion are reduced to a single ordinary differential equation. Then, an analytical relation is obtained for the natural frequency of magneto-electro-elastic doubly-curved nano-shell. Some examples are presented to validate the proposed model. Moreover, the effects of the electric and magnetic potentials, temperature rise, nonlocal parameter, parameters of Pasternak foundation, and the geometry of the nano-shell on the natural frequencies of magneto-electro-elastic doubly-curved nano-shells are investigated. It is found that natural frequency of magneto-electro-elastic doubly-curved nano-shell decreases with increasing the temperature, increasing the electric potential, or decreasing the magnetic potential.

Keywords: Magneto-electro-elastic; Nano-shell; Doubly-curved; First-order theory.

1 INTRODUCTION

MAGNETO-ELECTRO-ELASTIC (MME) materials are smart materials exhibiting magneto-electric (ME) coupling which enables them to convert mechanical, electrical and magnetic energies to each other. Understanding the vibration behavior of nano-structures is the key step in designing nano-sized devices like sensors. So, it is important to study the vibration response of MEE nano-structures before using them as sensors, actuators, energy harvesters, etc.

Arash and Wang [1] introduced and reviewed several nonlocal continuum models to model carbon nano-tubes and graphene sheets. Pradhan and Kumar [2] and Babaei and Shahidi [3], respectively, used differential quadrature (DQ) and Galerkin methods to determine the vibration response of graphene sheets. Three-dimensional (3D) theory [4], Mindlin plate theory [5], higher-order shear deformation theory (HSDT) [6], and refined plate theory (RPT) [7] in conjunction with nonlocal continuum theory are used to study the vibration behavior of nano-plates. Analooei et al. [8] used finite strip method to study the vibration of isotropic and orthotropic nano-plates. Aksencer and Aydogdu [9] used Navier-type solution for vibration analysis of simply-supported nano-plate. They also used Levy-type solution for vibration analysis of nano-plates with two opposite edges simply-supported and the others

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arbitrary. Tadi Beni et al. [10] and Rouhi et al. [11] presented shear deformable models for free vibration analysis of cylindrical nano-shells. Pouresmaeeli et al. [12] studied the vibration behavior of simply-supported viscoelastic nano-plate based on nonlocal Kirchhoff theory. They found that the frequency decreases with increasing the structural damping coefficient. Ghorbanpour Arani et al. [13] studied the free vibration of a magnetostrictive nano-plate based on Reddy's third-order shear deformation theory (TSDT) and Eringen's nonlocal continuum model. Vibration of nano-tube reinforced fluid-conveying micro-tubes supported by visco-Pasternak foundation has also been investigated by Arani et al. [14]. Liu et al. [15] and Ke et al. [16] investigated vibration of piezoelectric nano-plates based on nonlocal Kirchhoff and nonlocal Mindlin theories, respectively. Ke et al. [17] studied thermo-electro-mechanical vibration of piezoelectric cylindrical nano-shells using nonlocal Love's thin shell theory. Vaezi et al. [18] investigated the effects of electric and magnetic potentials on the free vibration of a MEE micro-beam based on Euler-Bernoulli beam theory. Amiri et al. [19] used a MEE beam model to study the vibration and instability of fluid conveying smart micro-tubes. Ebrahimi and Barati [20] examined vibrational behavior of a magneto-electro-viscoelastic nano-beam based on HSDT. Li et al. [21] studied the bending, buckling and free vibration of MEE nano-beams based on nonlocal Timoshenko theory. They determined the effects of electric and magnetic potentials on the vibration response of these nano-beams. Nonlocal TSDT [22] and nonlocal Timoshenko theory [23] have been used by Ansari et al. to study the nonlinear forced vibration of a MEE nano-beam. Ke and Wang [24] used nonlocal Timoshenko theory in conjunction with DQ method to obtain the free vibration response of a MEE nano-beam. Ke et al. [25] studied the free vibration of a MEE nano-plate based on the nonlocal Kirchhoff plate. They found that the natural frequency of the MEE nano-plate is sensitive to mechanical, electrical and magnetic loadings, while it is insensitive to the thermal loading. By considering surface and nonlocal effects, Wang et al. [26] introduced a two-dimensional theory to study the response of MEE nano-plates. Li et al. [27] investigated buckling and free vibration of a MEE nano-plate resting on Pasternak foundation based on nonlocal Mindlin theory. Pan and Waksmanski [28] presented exact closed-form solution for the 3D deformation of a layered MEE plate with nonlocal effect. Ansari and Gholami [29] studied the free vibration of MEE nano-plates in pre- and post-buckled states based on nonlocal Mindlin theory in conjunction with pseudo-arc length continuation approach. Farajpour et al. [30] used nonlocal Kirchhoff theory in conjunction with a perturbation method to obtain closed-form expression for nonlinear frequency of a MEE nano-plate with movable and immovable simply-supported boundary conditions. Ke et al. [31] presented a model based on nonlocal Love's shell theory to investigate the vibration response of a MEE cylindrical nano-shell. Ghadiri and Safarpour [32] investigated the free vibration response of MEE cylindrical nano-shells based on first-order shear deformation theory (FSDT) and Navier-type method. Mohammadimehr et al. [33] studied the free vibration of MEE cylindrical panels reinforced by carbon nano-tubes.

To the best of the author's knowledge, the free vibration of MEE doubly-curved nano-shells has not been studied. So, free vibration of a simply-supported magneto-electro-elastic doubly-curved nano-shell is studied based on FSDT and Gauss's laws for electrostatics and magneto statics. After obtaining a closed-form relation for the natural frequency, some examples are presented to validate the proposed method. Finally, the effects of several parameters on the natural frequencies of MEE doubly-curved nano-shells are investigated.

2 PROBLEM MODELING

Based on FSDT, the displacement field of a shallow doubly-curved shell is expressed by [34]:

$$u = u_0 + z\,\theta_x, \quad v = v_0 + z\,\theta_y, \quad w = w_0 \tag{1}$$

where u_0, v_0, and w_0 are the displacements of the mid-surface along x, y, and z directions, respectively, and θ_x and θ_y are the rotations of a transverse normal about the y and x directions, respectively. Using this displacement field, one can obtain the strain–displacement relations [34]:

$$\varepsilon_x = u_{0,x} + w_0/R_x + z\,\theta_{x,x}, \quad \varepsilon_y = v_{0,y} + w_0/R_y + z\,\theta_{y,y} \tag{2}$$

$$\gamma_{yz} = w_{0,y} + \theta_y, \quad \gamma_{xz} = w_{0,x} + \theta_x, \quad \gamma_{xy} = u_{0,y} + v_{0,x} + z\left(\theta_{x,y} + \theta_{y,x}\right) \tag{3}$$

For a transversely-isotropic MEE nano-material, the constitutive relations can be written as [27]:

$$\left(1-\eta\nabla^2\right)\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}+\begin{bmatrix}0 & 0 & e_{31}\\ 0 & 0 & e_{32}\\ 0 & e_{24} & 0\\ e_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}0\\ 0\\ \phi_{,z}\end{Bmatrix}+\begin{bmatrix}0 & 0 & q_{31}\\ 0 & 0 & q_{32}\\ 0 & q_{24} & 0\\ q_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}0\\ 0\\ \psi_{,z}\end{Bmatrix}-\begin{Bmatrix}\beta_{11}\\ \beta_{22}\\ 0\\ 0\\ 0\end{Bmatrix}\Delta T \tag{4}$$

$$\left(1-\eta\nabla^2\right)\begin{Bmatrix}D_x\\ D_y\\ D_z\end{Bmatrix}=\begin{bmatrix}0 & 0 & 0 & e_{15} & 0\\ 0 & 0 & e_{24} & 0 & 0\\ e_{31} & e_{32} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}-\begin{bmatrix}\eta_{11} & 0 & 0\\ 0 & \eta_{22} & 0\\ 0 & 0 & \eta_{33}\end{bmatrix}\begin{Bmatrix}0\\ 0\\ \phi_{,z}\end{Bmatrix}-\begin{bmatrix}d_{11} & 0 & 0\\ 0 & d_{22} & 0\\ 0 & 0 & d_{33}\end{bmatrix}\begin{Bmatrix}0\\ 0\\ \psi_{,z}\end{Bmatrix}+\begin{Bmatrix}0\\ 0\\ p_z\end{Bmatrix}\Delta T \tag{5}$$

$$\left(1-\eta\nabla^2\right)\begin{Bmatrix}B_x\\ B_y\\ B_z\end{Bmatrix}=\begin{bmatrix}0 & 0 & 0 & q_{15} & 0\\ 0 & 0 & q_{24} & 0 & 0\\ q_{31} & q_{32} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}-\begin{bmatrix}d_{11} & 0 & 0\\ 0 & d_{22} & 0\\ 0 & 0 & d_{33}\end{bmatrix}\begin{Bmatrix}0\\ 0\\ \phi_{,z}\end{Bmatrix}-\begin{bmatrix}\mu_{11} & 0 & 0\\ 0 & \mu_{22} & 0\\ 0 & 0 & \mu_{33}\end{bmatrix}\begin{Bmatrix}0\\ 0\\ \psi_{,z}\end{Bmatrix}+\begin{Bmatrix}0\\ 0\\ m_z\end{Bmatrix}\Delta T \tag{6}$$

where $\{\boldsymbol{\sigma}\}$ and $\{\boldsymbol{\varepsilon}\}$ are stress and strain vectors, respectively; $\{\boldsymbol{D}\}$ and $\{\boldsymbol{B}\}$ are the electric displacement and magnetic flux density vectors, respectively; $\{\boldsymbol{E}\} = \{0 \quad 0 \quad -\phi_{,z}\}^T$ and $\{\boldsymbol{H}\} = \{0 \quad 0 \quad -\psi_{,z}\}^T$ are electric field and magnetic field vectors, respectively, where ϕ and ψ denote electric and magnetic potentials; $[C_{ij}]$, $[\eta_{ij}]$ and $[\mu_{ij}]$ are the elastic, dielectric and magnetic permeability coefficient matrices, respectively; $[e_{ij}]$, $[q_{ij}]$ and $[d_{ij}]$ are the piezoelectric, piezo magnetic, and ME coefficient matrices, respectively; p_z, m_z and β_{ii} are pyroelectric, pyro magnetic and thermal moduli, respectively; ΔT denotes the temperature change, ∇^2 is the Laplace operator, and η is the nonlocal parameter revealing the size effect on the response of the nano-shell.

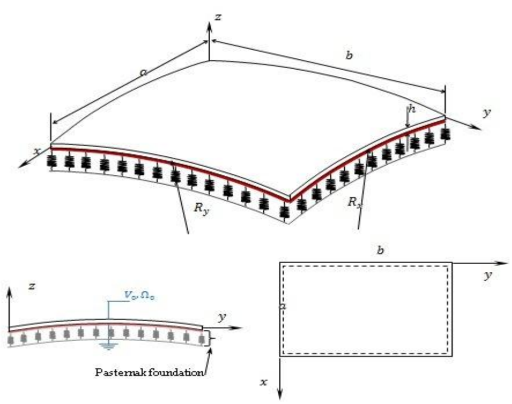

Fig.1
Schematic of the studied nano-shell.

Using Hamilton's principle and based on FSDT, the equations of motion of the nano-shell resting on a Pasternak foundation (Fig.1) can be expressed by [34]:

$$N_{x,x}+N_{xy,y}=0 \tag{7}$$

$$N_{xy,x}+N_{y,y}=0 \tag{8}$$

$$Q_{x,x}+Q_{y,y}+\left(N_x w_{0,x}+N_{xy}w_{0,y}\right)_{,x}+\left(N_{xy}w_{0,x}+N_y w_{0,y}\right)_{,y}-\frac{N_x}{R_x}-\frac{N_y}{R_y}-k_w w_0+k_s\nabla^2 w_0=I_0 w_{0,tt} \tag{9}$$

$$M_{x,x} + M_{xy,y} - Q_x = I_2 \theta_{x,tt} \tag{10}$$

$$M_{xy,x} + M_{y,y} - Q_y = I_2 \theta_{y,tt} \tag{11}$$

where $\{N_x \quad N_y \quad N_{xy}\} = \int_{-h/2}^{h/2} \{\sigma_x \quad \sigma_y \quad \sigma_{xy}\} dz$ are the in-plane force resultants, $\{Q_x \quad Q_y\} = K \int_{-h/2}^{h/2} \{\sigma_{xz} \quad \sigma_{yz}\} dz$ are the transverse force resultants, $\{M_x \quad M_y \quad M_{xy}\} = \int_{-h/2}^{h/2} \{\sigma_x \quad \sigma_y \quad \sigma_{xy}\} zdz$ are the moments resultants, and $\{I_0 \quad I_2\} = \int_{-h/2}^{h/2} \{1 \quad z^2\} \rho_0 dz$ are the mass moments of inertia, in which ρ_0 is the material density and K is the shear correction factor. Moreover, k_w and k_s are spring and shear coefficients of the Pasternak foundation, respectively. To determine theses resultants, the stress components given in Eq. (4) must be used. However, potentials ϕ and ψ are unknown parameters. To determine them, Gauss' laws for electrostatics and magneto statics are used:

$$D_{x,x} + D_{y,y} + D_{z,z} = 0 \qquad\qquad B_{x,x} + B_{y,y} + B_{z,z} = 0 \tag{12}$$

Then, using Eqs. (5), (6) and (12) gives:

$$\phi_{,z} = \phi_1 z + \phi_0 \tag{13}$$

$$\psi_{,z} = \psi_1 z + \psi_0 \tag{14}$$

where ϕ_0 and ψ_0 are constants of integration, and ϕ_1 and ψ_1 are obtained by:

$$\begin{aligned}
\phi_1 = \frac{1}{d_{33}^2 - \mu_{33}\eta_{33}} \Big\{ & (d_{33}q_{15} - \mu_{33}e_{15}) w_{0,xx} + (d_{33}q_{24} - \mu_{33}e_{24}) w_{0,yy} + \left[d_{33}(q_{15} + q_{31}) - \mu_{33}(e_{15} + e_{31}) \right] \theta_{x,x} + \\
& \left[d_{33}(q_{24} + q_{31}) - \mu_{33}(e_{24} + e_{31}) \right] \theta_{y,y} \Big\} \\
\psi_1 = \frac{1}{d_{33}^2 - \mu_{33}\eta_{33}} \Big\{ & (d_{33}e_{15} - \eta_{33}q_{15}) w_{0,xx} + (d_{33}e_{24} - \eta_{33}q_{24}) w_{0,yy} + \left[d_{33}(e_{15} + e_{31}) - \eta_{33}(q_{15} + q_{31}) \right] \theta_{x,x} + \\
& \left[d_{33}(e_{24} + e_{31}) - \eta_{33}(q_{24} + q_{31}) \right] \theta_{y,y} \Big\}
\end{aligned} \tag{15}$$

To determine ϕ_0 and ψ_0, the ME boundary condition on top and bottom surfaces of the nano-shell is needed. If the nano-shell is poled along the z direction and subjected to electric potential V_0 and magnetic potential Ω_0 between the upper and lower surfaces, the ME boundary condition can be expressed as below:

$$\begin{aligned}
&\phi\left(x, y, z = -\frac{h}{2}\right) = 0, \quad \phi\left(x, y, z = +\frac{h}{2}\right) = V_0 \\
&\psi\left(x, y, z = -\frac{h}{2}\right) = 0, \quad \psi\left(x, y, z = +\frac{h}{2}\right) = \Omega_0
\end{aligned} \tag{16}$$

Eqs. (13) – (16) yield $\phi_0 = V_0/h$ and $\psi_0 = \Omega_0/h$. Then, the resultants are obtained:

$$\left(1 - \eta\nabla^2\right) N_x = C_{11} h u_{0,x} + C_{12} h v_{0,y} + \left(\frac{C_{11}}{R_x} + \frac{C_{12}}{R_y}\right) h w_0 + e_{31} V_0 + q_{31}\Omega_0 - \beta_{11} h \Delta T \tag{17}$$

$$\left(1 - \eta\nabla^2\right) N_y = C_{12} h u_{0,x} + C_{22} h v_{0,y} + \left(\frac{C_{12}}{R_x} + \frac{C_{22}}{R_y}\right) h w_0 + e_{32} V_0 + q_{32}\Omega_0 - \beta_{22} h \Delta T \tag{18}$$

$$\left(1-\eta\nabla^2\right)N_{xy}=C_{66}h\left(u_{0,y}+v_{0,x}\right) \tag{19}$$

$$\left(1-\eta\nabla^2\right)M_x=M_1w_{0,xx}+M_2w_{0,yy}+M_3\theta_{x,x}+M_4\theta_{y,y} \tag{20}$$

$$\left(1-\eta\nabla^2\right)M_y=M_1w_{0,xx}+M_2w_{0,yy}+M_5\theta_{x,x}+M_6\theta_{y,y} \tag{21}$$

$$\left(1-\eta\nabla^2\right)M_{xy}=\frac{h^3}{12}C_{66}\left(\theta_{x,y}+\theta_{y,x}\right) \tag{22}$$

$$\left(1-\eta\nabla^2\right)Q_x=KhC_{55}\left(w_{0,x}+\theta_x\right) \tag{23}$$

$$\left(1-\eta\nabla^2\right)Q_y=KhC_{44}\left(w_{0,y}+\theta_y\right) \tag{24}$$

where

$$\begin{aligned}
M_1&=\frac{e_{31}h^3\left(d_{33}q_{15}-\mu_{33}e_{15}\right)}{12\left(d_{33}^2-\mu_{33}\eta_{33}\right)}+\frac{q_{31}h^3\left(d_{33}e_{15}-\eta_{33}q_{15}\right)}{12\left(d_{33}^2-\mu_{33}\eta_{33}\right)}\\
M_2&=\frac{e_{31}h^3\left(d_{33}q_{24}-\mu_{33}e_{24}\right)}{12\left(d_{33}^2-\mu_{33}\eta_{33}\right)}+\frac{q_{31}h^3\left(d_{33}e_{24}-\eta_{33}q_{24}\right)}{12\left(d_{33}^2-\mu_{33}\eta_{33}\right)}\\
M_3&=\frac{h^3}{12}C_{11}+\frac{e_{31}h^3\left[d_{33}\left(q_{15}+q_{31}\right)-\mu_{33}\left(e_{15}+e_{31}\right)\right]}{12\left(d_{33}^2-\mu_{33}\eta_{33}\right)}+\frac{q_{31}h^3\left[d_{33}\left(e_{15}+e_{31}\right)-\eta_{33}\left(q_{15}+q_{31}\right)\right]}{12\left(d_{33}^2-\mu_{33}\eta_{33}\right)}\\
M_4&=\frac{h^3}{12}C_{12}+\frac{e_{31}h^3\left[d_{33}\left(q_{24}+q_{31}\right)-\mu_{33}\left(e_{24}+e_{31}\right)\right]}{12\left(d_{33}^2-\mu_{33}\eta_{33}\right)}+\frac{q_{31}h^3\left[d_{33}\left(e_{24}+e_{31}\right)-\eta_{33}\left(q_{24}+q_{31}\right)\right]}{12\left(d_{33}^2-\mu_{33}\eta_{33}\right)}\\
M_5&=M_3-\frac{h^3}{12}C_{11}+\frac{h^3}{12}C_{12}\qquad\qquad M_6=M_4-\frac{h^3}{12}C_{12}+\frac{h^3}{12}C_{22}
\end{aligned} \tag{25}$$

Substituting Eqs. (17) – (24) into Eqs. (7) – (10), and then by using

$$\begin{aligned}
u_0&=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}U_{mn}(t)\cos(m\pi x/a)\sin(n\pi y/b)\\
v_0&=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}V_{mn}(t)\sin(m\pi x/a)\cos(n\pi y/b)\\
w_0&=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}W_{mn}(t)\sin(m\pi x/a)\sin(n\pi y/b)\\
\theta_x&=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}X_{mn}(t)\cos(m\pi x/a)\sin(n\pi y/b)\\
\theta_y&=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}Y_{mn}(t)\sin(m\pi x/a)\cos(n\pi y/b)
\end{aligned} \tag{26}$$

The following ordinary differential equations are obtained:

$$L_1 U + L_2 V + L_3 W = 0 \tag{27}$$

$$L_4 U + L_5 V + L_6 W = 0 \tag{28}$$

$$L_7 \ddot{W} + L_8 W + L_9 U + L_{10} V + L_{11} X + L_{12} Y + L_{13} = 0 \tag{29}$$

$$L_{14} \ddot{X} + L_{15} X + L_{16} Y + L_{17} W = 0 \tag{30}$$

$$L_{18} \ddot{Y} + L_{19} Y + L_{20} X + L_{21} W = 0 \tag{31}$$

where U, V, W, X and Y are unknown functions and (m,n) denotes the mode of vibration. In Eqs. (27) – (31) the subscript '$_{mn}$' has been dropped for brevity.

Eqs. (27) and (28) give U and V in terms of W, while Eqs. (30) and (31) give X and Y:

$$U = \frac{L_3 L_5 - L_2 L_6}{L_2 L_4 - L_1 L_5} W , \qquad V = \frac{L_1 L_6 - L_3 L_4}{L_2 L_4 - L_1 L_5} W \tag{32}$$

$$X = \frac{L_{16} L_{21} - L_2^* L_{17}}{L_1^* L_2^* - L_{16} L_{20}} W , \quad Y = \frac{L_{17} L_{20} - L_1^* L_{21}}{L_1^* L_2^* - L_{16} L_{20}} W \tag{33}$$

where $L_1^* = L_{15} + L_{14} \dfrac{d^2}{dt^2}$ and $L_2^* = L_{19} + L_{18} \dfrac{d^2}{dt^2}$ are ordinary differential operators.

Now, substituting Eqs. (32) and (33) into (29) gives:

$$M_{\mathrm{eq}} \ddot{W} + K_{\mathrm{eq}} W + F_s = 0 \tag{34}$$

In which M_{eq} and K_{eq} are equivalent mass and stiffness of the system and F_s is a constant parameter. These parameters are given in Appendix A. Finally, natural frequency of the nano-shell is simply determined by using $\omega_0 = \sqrt{K_{\mathrm{eq}} / M_{\mathrm{eq}}}$.

3 RESULTS AND DISCUSSION

In this section, first, some examples are presented to validate the proposed model. Then, the effects of several parameters on the natural frequencies of MEE doubly-curved nano-shells are investigated. The MEE nano-shell is doubly-curved and simply-supported. It is subjected to electric (V_0) and magnetic (Ω_0) potentials between its upper and bottom surfaces. For all the numerical examples the shear correction factor (K) is taken to be 5/6.

First, an isotropic spherical shell is considered and its fundamental natural frequency is obtained. The result is shown in Table 1., along with the published results based on Sanders theory (ST) [35], Donnell theory (DT) [36], HSDT [37] and 3D approach [38]. The geometric and material properties of the shell are: a = b = 1.0118, h = 0.0191, R = 1.91, E = 1, ρ_0 = 1 and ν = 0.3. It is seen that the result obtained by present model is in good agreement with previously published ones, and the discrepancy between the results of 3D method and present method is very small.

Table 1
Fundamental natural frequency ω_0 (*rad/s*) of an isotropic spherical shell.

ST [35]	DT [36]	HSDT [37]	3D [38]	Present
0.52830	0.52864	0.50223	0.52543	0.53510

As the second comparison, the frequency ratio of an isotropic nano-plate with $a = b = 10$ *nm*, $h = 0.34$ *nm*, $E = 1.06$ *TPa*, $\rho_0 = 2250$ kg/m^3, and $\nu = 0.25$ is obtained for different values of nonlocal parameter. The results are presented in Table 2., where frequency ratios are determined by using the following relation:

$$\text{Frequency ratio} = \frac{\text{frequency ratio calculated using nonlocal theory}}{\text{frequency ratio calculated using local theory}} \tag{35}$$

Table 2
Frequency ratios of a graphene sheet.

η (nm^2)	Method		
	Pradhan and Kumar [2]	Pouresmaeeli et al. [39]	Present study
0	1	1	1
1	0.9139	0.91386	0.91386
2	0.8468	0.84673	0.84673
3	0.7926	0.79251	0.79251

Table 3., presents the dimensionless fundamental frequencies of an isotropic nano-plate for different nonlocal parameters, and a/h and b/a ratios. The results are compared with the ones obtained by methods based on nonlocal Kirchhoff theory (CPT), FSDT, TSDT and a RPT. Again, it is seen that the proposed model predicts the frequencies with good accuracy.

Table 3
Dimensionless fundamental frequency $\omega = \omega_0 h\sqrt{\rho_0 / G}$ of an isotropic Nano plate ($\nu = 0.3$).

b/a	a/h	$\eta(nm^2)$	Method				
			CPT[a]	FSDT[a]	TSDT[a]	RPT [7]	Present
1	10	0	0.0963	0.0930	0.0935	0.09303	0.09371
		1	0.0800	0.0850	0.0854	0.08502	0.08564
		2	0.0816	0.0788	0.0791	0.07877	0.07935
		3	0.0763	0.0737	0.0741	0.07373	0.07427
	20	0	0.0241	0.0239	0.0239	0.02386	0.02391
		1	0.0220	0.0218	0.0218	0.02180	0.02185
		2	0.0240	0.0202	0.0202	0.02021	0.02025
		3	0.0191	0.0189	0.0189	0.01891	0.01895
2	10	0	0.0602	0.0589	0.0591	0.05888	0.05916
		1	0.0568	0.0556	0.0557	0.05555	0.05582
		2	0.0539	0.0527	0.0529	0.05274	0.05299
		3	0.0514	0.0503	0.0505	0.05031	0.05055
	20	0	0.0150	0.0150	0.0150	0.01496	0.01498
		1	0.0142	0.0141	0.0141	0.01411	0.01414
		2	0.0135	0.0134	0.0134	0.01340	0.01342
		3	0.0129	0.0128	0.0128	0.01278	0.01280

[a] reported by Malekzadeh and Shojaee [7]

Next, dimensionless fundamental frequencies of a MEE nano-plate made of piezoelectric $BaTiO_3$ and magnetostrictive $CoFe_2O_4$ are determined. The material properties of the MEE material are given in Table 4. The dimensionless fundamental frequencies are obtained by using $\omega = \omega_0 a\sqrt{I_0/(C_{11}h)}$. Table 5., presents the frequencies for different values of nonlocal parameter. In addition, the effect of temperature rise on the dimensionless fundamental frequency has been investigated and the results are presented in Table 6. As in the case for isotropic shell and nano-plate, the proposed method approximates the frequencies of MEE nano-plate with good accuracy.

Table 4
Material properties of $BiTiO_3$-$CoFe_2O_4$ composite material [29].

Properties	$BaTiO_3$-$CoFe_2O_4$
Elastic [GPa]	$c_{11} = c_{22} = 226$, $c_{12} = 125$, $c_{33} = 216$, $c_{44} = c_{55} = 44.2$, $c_{66} = 50.5$
Piezoelectric [C/m^2]	$e_{31} = e_{32} = -2.2$, $e_{15} = e_{24} = 5.8$
Dielectric [$\times 10^{-9}$ $C/(Vm)$]	$\eta_{33} = 6.35$
Piezo magnetic [$N/(Am)$]	$q_{31} = q_{32} = 290.1$, $q_{15} = q_{24} = 275$
Magnetoelectric [$\times 10^{-12}$ $Ns/(CV)$]	$d_{33} = 2737.5$
Magnetic [$\times 10^{-6}$ Ns^2/C^2]	$\mu_{33} = 83.5$
Thermal Moduli [$\times 10^5$ $N/(Km^2)$]	$\beta_{11} = \beta_{22} = 4.74$
Pyroelectric [$\times 10^{-6}$ C/N]	$p_z = 25$
Pyro magnetic [$\times 10^{-6}$ $N/(AmK)$]	$m_z = 5.19$
Mass density [kg/m^3]	5550

Table 5
The effect of nonlocal parameter (η) on the dimensionless fundamental frequency of a MEE nano-plate ($\Delta T = V_0 = \Omega_0 = 0$).

Method	η ($\times 10^{-16}$)					
	0	0.36	1.44	3.24	5.76	9
Ke et al. [24]	0.3698	0.3379	0.2764	0.2219	0.1813	0.1518
Ansari and Gholami [28]	0.3684	–	0.2756	–	0.1808	–
Present study	0.3717	0.3397	0.2779	0.2231	0.1823	0.1526

Table 6
The effect of the temperature rise on the dimensionless fundamental frequency of a MEE nano-plate ($\eta = 1.44 \times 10^{-16}$, $V_0 = \Omega_0 = 0$).

Method	ΔT					
	0	30	60	90	120	150
Ke et al. [24]	0.2764	0.2749	0.2734	0.2719	0.2704	0.2689
Present study	0.2779	0.2766	0.2753	0.2741	0.2729	0.2716

The effect of radius of curvature (R_x) on the dimensionless fundamental frequency of nanoscale spherical and cylindrical MEE shells is investigated and the result is shown in Fig. 2. In this example, $a = b = 60$ nm, $h = 4$ nm and $\eta = 0.2$ nm^2. It is seen that for a specific R_x, the spherical shell has higher frequency which decreases with higher rate when R_x increases.

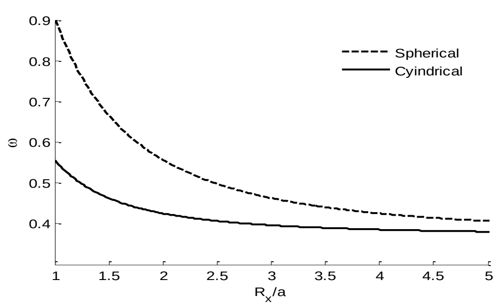

Fig.2
Dimensionless fundamental frequency of spherical and cylindrical MEE nano-shells ($\Delta T = V_0 = \Omega_0 = 0$).

Fig. 3 shows the effect of temperature change on the dimensionless fundamental frequency of two MEE nano-shells (with $a = b = 60$ nm, $h = 4$ nm, and $R = 5a$) and a MEE nano-plate (with $a = b = 60$ nm, $h = 4$ nm). For all of the structures, the dimensionless frequencies decrease with increasing the temperature.

Effects of electric and magnetic potentials on the natural frequency of MEE nano-shells are also investigated and the results are shown in Figs. 4 and 5, respectively. The geometric properties of the nano-structures are the same as previous example. It is observed that increasing the electric potential, decreases the natural frequency of MEE nano-shells. Moreover, for negative electric potentials, higher natural frequencies are resulted. The converse happens when magnetic potential changes. That is, as the magnetic potential increases, the natural frequency of the MEE nano-shell increases.

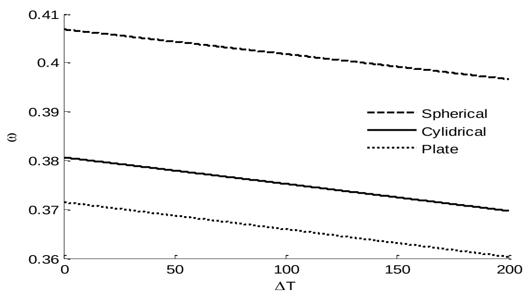

Fig.3
Effect of temperature change on the dimensionless fundamental frequency of MEE nano-structures (η = 0.2 nm^2, $V_0 = \Omega_0 = 0$).

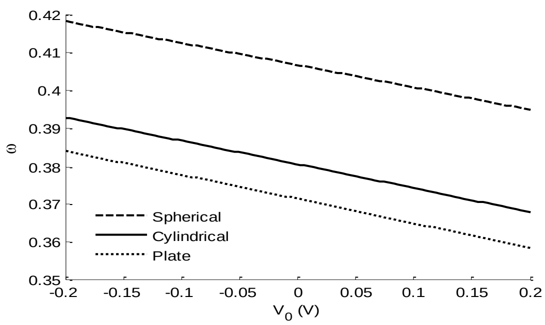

Fig.4
Effect of electric potential on the dimensionless fundamental frequency of MEE nano-structures (η = 0.2 nm^2, $\Delta T = \Omega_0 = 0$).

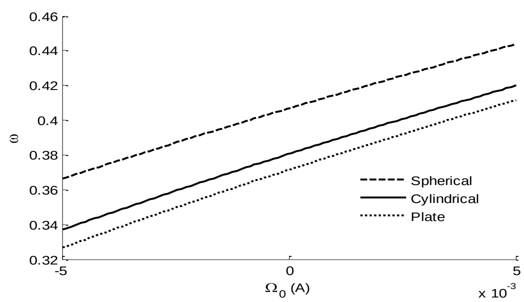

Fig.5
Effect of magnetic potential on the dimensionless fundamental frequency of MEE nano-structures (η = 0.2 nm^2, $\Delta T = V_0 = 0$).

Effects of nonlocal parameter and foundation parameters on the natural frequencies of MEE doubly-curved nano-shells are investigated, too. Fig. 6 shows the effect of nonlocal parameter on the dimensionless fundamental frequencies of spherical and cylindrical MEE nano-shells with $a = b = 60$ nm, $h = 4$ nm, $R = 5a$. It is seen that nonlocal parameter has small effect on the frequency and tends to decrease it. In Fig. 7, effects of foundation parameters on the dimensionless fundamental frequency of spherical MEE nano-shell with $a = b = 60$ nm, $h = 4$ nm, $R = 5a$ are shown. The dimensionless foundation parameters are obtained by $K_w = k_w a^4/(C_{ij\max} h^3)$ and $K_s = k_s a^2/(C_{ij\max} h^3)$ where $C_{ij\max}$ is the maximum value of the stiffness coefficients of the MEE nano-shell. It is observed that both dimensionless spring and shear coefficients increase the stiffness (and subsequently the natural frequency) of the system. However, dimensionless shear coefficient (K_s) has more effect on the natural frequency of MEE nano-shells.

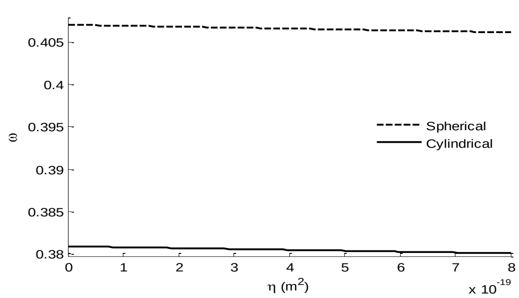

Fig.6
Variation of dimensionless fundamental frequency of MEE nano-shell in terms of nonlocal parameter ($\Delta T = V_0 = \Omega_0 = 0$).

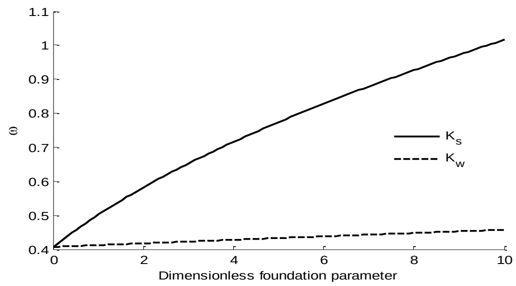

Fig.7
Variation of dimensionless fundamental frequency of spherical MEE nano-shell in terms of foundation parameters (η = 0.2 nm^2, $\Delta T = V_0 = \Omega_0 = 0$).

4 CONCLUSIONS

Free vibration of simply-supported MEE doubly-curved nano-shells is studied analytically based on FSDT. Some examples are presented and it is found that: (a) spherical nano-shell has higher natural frequencies which decrease with higher rate when the curvature of the nano-shell increases, (b) natural frequency decreases with increasing the temperature, (c) increasing the electric potential decreases the natural frequency of MEE nano-shells, (d) when the magnetic potential increases, the natural frequency of the MEE nano-shell increases, (e) nonlocal parameter decreases the natural frequency of MEE nano-shell, and (f) foundation parameters increase the natural frequencies of MEE nano-shells. However, dimensionless shear coefficient has more effect on the natural frequencies.

APPENDIX A

The coefficients of Eq. (34) are:

$$M_{\text{eq}} = \alpha_5 L_7 + \alpha_6 L_8 + \alpha_6 \left(k_1 + k_2\right) - \alpha_2 L_{11} - \alpha_4 L_{12} \tag{A.1}$$

$$K_{\text{eq}} = \alpha_5 L_8 + \alpha_5 \left(k_1 + k_2\right) + L_{11}\left(L_{16} L_{21} - \alpha_1\right) + L_{12}\left(L_{17} L_{20} - \alpha_3\right) \tag{A.2}$$

$$F_s = \alpha_{15} L_{13} \tag{A.3}$$

Where

$$\alpha_1 = L_{17} L_{19}, \quad \alpha_2 = L_{17} L_{18}, \quad \alpha_3 = L_{15} L_{21}, \quad \alpha_1 = L_{14} L_{21}, \quad \alpha_5 = L_{15} L_{19} - L_{16} L_{20}, \quad \alpha_6 = L_{14} L_{19} - L_{16} L_{20} \tag{A.4}$$

$$k_1 = \frac{L_9\left(L_3 L_5 - L_2 L_6\right)}{L_2 L_4 - L_1 L_5}, \qquad k_2 = \frac{L_{10}\left(L_1 L_6 - L_3 L_4\right)}{L_2 L_4 - L_1 L_5} \tag{A.5}$$

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