Nonlinear Vibration Analysis of the Fluid–Filled Single Walled Carbon Nanotube with the Shell Model Based on the Nonlocal Elacticity Theory

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ABSTRACT

Nonlinear vibration of a fluid-filled single walled carbon nanotube (SWCNT) with simply supported ends is investigated in this paper based on Von-Karman's geometric nonlinearity and the simplified Donnell's shell theory. The effects of the small scales are considered by using the nonlocal theory and the Galerkin's procedure is used to discretize partial differential equations of the governing into the ordinary differential equations of motion. To achieve an analytical solution, the method of averaging is successfully applied to the nonlinear governing equation of motion. The SWCNT is assumed to be filled by the fluid (water) and the fluid is presumed to be an ideal non compression, non rotation and in viscid type. The fluid-structure interaction is described by the linear potential flow theory. An analytical formula was obtained for the nonlinear model and the effects of an internal fluid on the coupling vibration of the SWCNT-fluid system with the different aspect ratios and the different nonlinear parameters are discussed in detail. Furthermore, the influence of the different nonlocal parameters on the nonlinear vibration frequencies is investigated according to the nonlocal Eringen's elasticity theory.

Keywords: Nonlinear vibration; Fluid-filled SWCNT; Donnell's shell model; Nonlocal parameter

1 INTRODUCTION

CARBON nanotubes (CNTs) have always attracted scientists and engineers because of their wide range of applications and the superior mechanical properties. The fluid-filled CNTs may be used as gas storage tanks or as nanopipes for conveying medicines to a person's blood stream. With the perfect hollow cylindrical geometry and the superior mechanical properties, these tubes can be used in a variety of technological and biomedical applications to hold fluid such as gas storage tanks[1]or drug-delivery devices[2, 3]. Hence, the CNTs transport properties could be sensitive to their vibration modes and frequencies, it is essential to consider the mechanical properties of fluid-filled CNTs. There are two major categories for simulating the mechanical properties of the CNTs: The molecular dynamics approaches (MD) and the continuum mechanics. The molecular simulations are computationally expensive and limited to study the small systems. Hence, the continuum modeling is an efficient method for considering the CNTs characterization. There are two different modeling which are used in continuum mechanics: the beam theories and the shell theories.

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In the last two decades many continuum structural models have been proposed for considering the CNTs characterization. For the first time, Yokobson et al., (1996) [4]used a traditional continuum shell model to predict the buckling of a SWCNTs and compared it with the MD simulation. Most of the researches which have been done on the nanotubes containing fluid are those which have been done on the dynamic characteristics of CNTs conveying fluid Yoon et al. [5] studied the vibration and instability of CNTs conveying fluid with the method of beam model. Yan et al.[6, 7] ,Wang et al. [8] and Khosravian.et al [9] discussed the dynamical stability behaviors of fluid-conveyed CNTs, and found that the natural resonant frequencies depend on the fluid flow velocity and the instability of the CNTs occurs at a critical flow velocity. Rasekh and Khadem. [10] studied the relationship of nonlinear amplitude and frequency for SWCNTs in the presence of an internal fluid flow is expressed using the multiple scales perturbation method. Ghavanloo et al. [11]used Euler-Bernoulli'beam theory to study the vibration and instability of CNTs conveying fluid and resting on a linear viscoelastic Winkler foundation by using the finite element model. Although many researches have been done on the dynamic characteristics of CNTs conveying fluid, there are few reports on fluid-filled CNTs vibration and the dynamic behavior of fluid-filled carbon nanotubes still remain many unexplored in the literatures. Dong et al. (2008) [12]studied and calculated characteristics of wave propagation in fluid-filled multi-walled carbon nanotube. Yan et al. (2010) [13]studied the noncoaxial vibration in CNTs and found that the resonant frequencies are decreased due to the effect of the flow. Also, they[14] have studied the nonlinear vibration of the double walled fluid-filled CNTs based on the harmonic balance method.

This study focuses on the nonlinear vibration of an empty and a fluid-filled SWCNT with simply supported ends using Donnell's cylindrical shell model considering the effects of an internal fluid on the coupling vibration of the SWCNT-fluid system with the different aspect ratios and the different nonlinear parameters. Moreover, the method of averaging is applied to analyze the nonlinear vibration of the SWCNTs in the analytical calculations. Though the classical (local) continuum models are applicable to some extent, the length scales related with nano technology are often sufficiently small to be used in the local model. But, small length scales such as lattice spacing between individual atoms, becomes more important and its effect can no longer be ignored. It is a possible solution to extend the classic continuum approach to smaller length scales by incorporating information regarding the behavior of nano scales. It is possible by the use of the nonlocal continuum mechanics.

While the local continuum mechanics assumes that the stress state at a given point is dependent uniquely on the strain state at that same point, the nonlocal continuum mechanics regards the stress state at a given point as a function of the strain states of all points in the body. Thus, the theory of nonlocal continuum mechanics contains information about the long-range forces between atoms, and the internal length scale is introduced into the constitutive equations simply as a material parameter. Moreover, the influence of the different nonlocal parameters on the nonlinear vibration frequencies is considered according to the nonlocal elasticity theory proposed by Eringen[15-17]. In addition, the related material and geometric parameters were used from Gupta et al. [18]

2   DONNELL SHELL MODEL

Consider a thin-walled simply supported cylindrical shell with radius R, thickness h and length L, as shown in Fig.1. Where O is the origin placed at the centre of one end of the shell, x is the axial and r is the radial coordinate. The displacement field of the middle surface of the shell is given by the following components: u, v and w; in the axial, circumferential and radial directions, respectively. Donnell's nonlinear shallow-shell theory is used and the nonlinear equation of motion for cylindrical shell is derived by means of the variational techniques. [19-21].
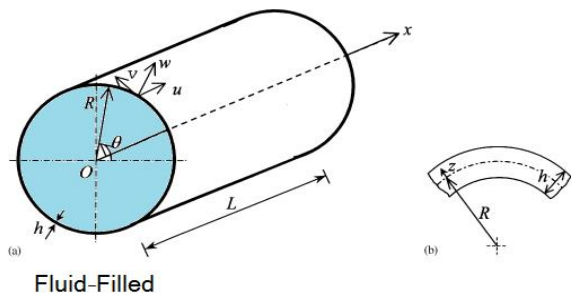

Fig. 1
Cylindrical shell representation of a SWCNT with the coordinate system used. (a) Complete shell;(b) Cross section of the shell surface.

The elastic strain energy U of a circular cylindrical shell, neglecting stress σ_z according to Love's first approximation assumptions, is given by: [19]

$$U = \frac{1}{2} \iiint (\bar{\sigma}_x \bar{\varepsilon}_x + \bar{\sigma}_\theta \bar{\varepsilon}_\theta + \bar{\tau}_{x\theta} \bar{\gamma}_{x\theta}) dx R d\theta dz. \tag{1}$$

According to the Hook's law:

$$\bar{\sigma}_x = \frac{E}{(1-\upsilon^2)}(\bar{\varepsilon}_x + \upsilon \bar{\varepsilon}_\theta),$$

$$\bar{\sigma}_\theta = \frac{E}{(1-\upsilon^2)}(\bar{\varepsilon}_\theta + \upsilon \bar{\varepsilon}_x), \tag{2}$$

$$\bar{\tau}_{x\theta} = \frac{E}{2(1+\upsilon)} \bar{\gamma}_{x\theta}.$$

By using Eqs. (2), the following expression is obtained:

$$U = \frac{1}{2} \iiint \frac{E}{(1-\upsilon^2)}(\bar{\varepsilon}_x^2 + 2\upsilon \bar{\varepsilon}_x \bar{\varepsilon}_\theta + \bar{\varepsilon}_\theta^2 + \frac{(1-\upsilon)}{2} \gamma_{x\theta}^2) dx R d\theta dz. \tag{3}$$

The strain components $\bar{\varepsilon}_x, \bar{\varepsilon}_\theta$, and $\bar{\gamma}_{x\theta}$ at an arbitrary point of the shell are related to the middle surface strains, $\varepsilon_x, \varepsilon_\theta$, ,and $\gamma_{x\theta}$ and to the changes in the curvature and torsion of the middle surface k_x, k_θ, and $k_{x\theta}$ by the following relationships:[20]

$$\bar{\varepsilon}_x = \varepsilon_x + z k_x, \qquad k_x = -\frac{\partial^2 w}{\partial x^2}$$

$$\bar{\varepsilon}_\theta = \varepsilon_\theta + z k_\theta, \qquad k_x = -\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2} \tag{4}$$

$$\bar{\gamma}_{x\theta} = \gamma_{x\theta} + z k_{x\theta}, \qquad k_{x\theta} = -\frac{2}{R}\frac{\partial^2 w}{\partial \theta \partial x}$$

According to Donnell's nonlinear shell theory, the middle surface strain–displacement relationships for a circular cylindrical shell are given by:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}(\frac{\partial w}{\partial x})^2,$$

$$\varepsilon_\theta = \frac{1}{R}\frac{\partial v}{\partial \theta} + \frac{w}{R} + \frac{1}{2R^2}(\frac{\partial w}{\partial \theta})^2, \tag{5}$$

$$\gamma_{x\theta} = \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} + \frac{1}{R}(\frac{\partial w}{\partial \theta})(\frac{\partial w}{\partial x}).$$

Assuming a positive radial deformation w inwards, the simplified version of Donnell's nonlinear shallow-shell equation is given by:[19]

$$D\nabla^4 w - \frac{1}{R}N_\theta - (N_x w_{,xx} + \frac{2}{R}N_{x\theta}w_{,x\vartheta} + \frac{1}{R^2}N_\theta w_{,\theta\theta}) = -\rho_t . h . \ddot{w}$$

$$\nabla^4 w = (\frac{\partial^2 w}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2})^2 \tag{6}$$

The expressions for the resultants force per unit length in the axial and circumferential directions along with the shear stress resultant are given by:[19, 20]

$$N_x = \frac{1}{R^2}\frac{\partial^2 F}{\partial\theta^2}, N_\theta = \frac{\partial^2 F}{\partial x^2}, N_{x\theta} = -\frac{1}{R}\frac{\partial^2 F}{\partial x \partial\theta}. \tag{7}$$

with substituting Eqs. (7) in to (6):

$$D\nabla^4 w + \rho_t.h.\ddot{w} = \frac{1}{R}\frac{\partial^2 F}{\partial x^2} - (\frac{1}{R^2}\frac{\partial^2 F}{\partial\theta^2}w_{,xx} - \frac{2}{R^2}\frac{\partial^2 F}{\partial x.\partial\theta}w_{,x\vartheta} + \frac{1}{R^2}\frac{\partial^2 F}{\partial x^2}w_{,\theta\theta})$$

$$\frac{1}{E.h}\nabla^4 F = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial x^2}.\frac{\partial^2 w}{\partial\theta^2} + (\frac{\partial^2 w}{\partial x.\partial\theta})^2 \tag{8}$$

where F is the in-plane airy stress function which satisfies the above compatibility equation $D = Eh^3/[12(1-\upsilon^2)]$ is the flexural rigidity of the shell, t is the time, ρ_t is the mass density and E is the elastic modulus and υ the Poisson ratio.

3   FLUID STRUCTURE INTERACTION

The shell is assumed completely filled with a dense fluid. Furthermore, the fluid is assumed to be an ideal non compression, non rotation and in viscid type. Nonlinearities in the dynamic pressure and in the boundary conditions at the fluid–structure interface are neglected, because fluid movements of the order of the shell thickness may be considered to be small; and hence a linear formulation is valid. Indeed, these nonlinear effects have been found to be negligible by Gonc-alves and Batista [22]. In addition, pre-stress in the shell due to fluid weight (Hydrostatic effect) is neglected. Both ends of the fluid volume (in correspondence of the sell edge) are assumed to be open, so that a zero pressure is assumed there, and boundary conditions are $(\phi)_{x=0} = (\phi)_{x=L} = 0$ (velocity potential of the fluid).With these assumptions, the fluid–structure interaction can be described by potential flow theory[19, 23, 24].
q_f is the flow pressure which is considered as:

$$q_f = \rho_f(\frac{L}{m\pi})\frac{I_n(\frac{m\pi R}{L})}{I'_n(\frac{m\pi R}{L})}\frac{\partial^2 w}{\partial t^2}$$

$$\gamma = \rho_f(\frac{L}{m\pi})\frac{I_n(\frac{m\pi R}{L})}{I'_n(\frac{m\pi R}{L})} \tag{9}$$

$$q_f = \gamma\frac{\partial^2 w}{\partial t^2}$$

where ρ_f is the mass density of the internal fluid, I_n is the modified Bessel Function of order n and I'_n is the derivative of I_n with respect to argument.[19]

4   THE NONLOCAL ELASTIC SHELL THEORY

In the nonlocal elasticity (Eringen, 1976)[16], the stress at a reference point x is considered to be function of the strain field at every point in the body. Thus, the theory of nonlocal continuum mechanics contains information about

the long-range forces between atoms, and the internal length scale is introduced into the constitutive equations simply as a material parameter The basic equations for linear, homogeneous, isotropic, and nonlocal elastic solids with zero body force are given as follows (Eringen, 1983)[17]:

$$\sigma = \int_v \alpha(|x'-x|,\tau)T(x')dx'$$

$$T(x) = C(x):\varepsilon(x) \tag{10}$$

where $T(x)$ is the classic stress tensor at point x, $\varepsilon(x)$ is the strain tensor, $C(x)$ is the fourth-order elasticity tensor and denotes the 'double-dot product' $(\alpha(|x'-x|),\tau)$ is the nonlocal modulus or attenuation function incorporating into the constitutive equations the nonlocal effects at the reference point x produced by the local strain at the source $x'.|x'-x|$ is the Euclidean distance, $\tau = e_0.a/1$ is defined as small scale factor where e_0 is a constant to adjust the model to match the reliable results by experiments or other models, and a and l are the internal and the external characteristic length (e.g. the length of C–C bond, the lattice spacing and the granular distance.), respectively. In the context of two-dimensional nonlocal elasticity, the equivalent of Eq. (11) in a differential form can be expressed as:

$$(1-(e_0 a)^2 \nabla^2)\sigma = t \ , \ \ \nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2}{\partial \theta^2} \tag{11}$$

when the nonlocal parameter $e_0.a$ becomes zero, the nonlocal elasticity reduces to the classical (local) elasticity. Following Eringen's theory and using Eq. (11), the nonlocal elasticity based on the stress-strain relationships are in the following form:

$$\sigma_x - (e_0 a)^2 \nabla^2 \sigma_x = \frac{E}{(1-\upsilon^2)}(\varepsilon_x + \upsilon\varepsilon_\theta)$$

$$\sigma_\theta - (e_0 a)^2 \nabla^2 \sigma_\theta = \frac{E}{(1-\upsilon^2)}(\varepsilon_\theta + \upsilon\varepsilon_x) \tag{12}$$

$$\tau_{x\theta} - (e_0 a)^2 \nabla^2 \tau_{x\theta} = \frac{E}{2(1+\upsilon)}\gamma_{x\theta}$$

In the nonlocal elastic shell theory, the stress and moment resultants are defined based on the stress components in Eq. (12), and thus can be expressed as follows:

$$N_x = C(\varepsilon_x + \upsilon\varepsilon_\theta)+(e_0 a)^2 \nabla^2 N_x \ , \ \ M_x = D(k_x + \upsilon k_\theta)+(e_0 a)^2 \nabla^2 M_x$$

$$N_\theta = C(\varepsilon_\theta + \upsilon\varepsilon_x)+(e_0 a)^2 \nabla^2 N_\theta \ , \ \ M_\theta = D(k_\theta + \upsilon k_x)+(e_0 a)^2 \nabla^2 M_\theta$$

$$\tau_{x\theta} - (e_0 a)^2 \nabla^2 \tau_{x\theta} = \frac{E}{2(1+\upsilon)}\gamma_{x\theta} \ , \ \ M_{x\theta} = D\frac{(1-\upsilon)}{2}K_{x\theta}+(e_0 a)^2 \nabla^2 M_{x\theta} \tag{13}$$

$$C = \frac{Eh}{(1-\upsilon^2)}$$

By using Eqs. (13), the nonlocal governing equations based on Donnell's shell theory can be obtained as follows:

$$D\nabla^4 w - \frac{1}{R}N_\theta - (N_x w_{,xx} - \frac{2}{R}N_{x\theta}w_{,x\vartheta}+\frac{1}{R^2}N_\theta w_{,\theta\theta}) = -\rho h\ddot{w}+\rho h(e_0 a)^2(\ddot{w}_{xx}+\frac{\ddot{w}_{\theta\theta}}{R^2}).$$

$$\nabla^4 F - (e_0 a)^2(\nabla^6 F) = Eh(-\frac{1}{R}\frac{\partial^2 w}{\partial x^2}+\left[\left(\frac{\partial^2 w}{R\partial x\partial\theta}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{R^2\partial\theta^2}\right]). \tag{14}$$

5 MODELLING AND FORMULAION

The coupled shell model with fluid-filled for the nonlocal model is stated as:

$$D\nabla^4 w - \frac{1}{R}N_\theta - (N_x w,_{xx} - \frac{2}{R}N_{x\theta}w,_{x\vartheta} + \frac{1}{R^2}N_\theta w,_{\theta\theta}) = -(\rho h + \gamma)\ddot{w} + (\rho h + \gamma)(e_0 a)^2(\ddot{w}_{xx} + \frac{\ddot{w}_{\theta\theta}}{R^2}).$$ (15)

with substituting Eqs. (7) and (9) into (15):

$$D\nabla^4 w = -(\rho_t.h + \gamma).\ddot{w} + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} - (\frac{1}{R^2}\frac{\partial^2 F}{\partial\theta^2}w,_{xx} - \frac{2}{R^2}\frac{\partial^2 F}{\partial x.\partial\theta}w,_{x\vartheta} + \frac{1}{R^2}\frac{\partial^2 F}{\partial x^2}w,_{\theta\theta}) + \rho h(e_0 a)^2(\ddot{w}_{xx} + \frac{\ddot{w}_{\theta\theta}}{R^2}).$$

$$\nabla^4 F - (e_0 a)^2(\nabla^6 F) = Eh(-\frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \left[\left(\frac{\partial^2 w}{R\partial x\partial\theta}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{R^2\partial\theta^2}\right]).$$ (16)

Simply Supported Boundary Conditions: $v = w = 0,$ $N_x = 0,$ $M_x = 0.$

The radial displacement *w* is expanded by using the linear shell Eigen modes as basis; in particular, the flexural response may be written as below[25]:

$$w(x,\theta,t) = A(t)\cos(n\theta)\sin(\frac{m\pi x}{1}) + \frac{n^2}{4R}A(t)^2\sin(\frac{m\pi x}{1})^2.$$ (17)

where *m* is the axial wave number (equal to the number of half-waves along the shell), and *n* is the circumferential wave number. The amplitude functions, *A* is an unknown generalized time function of the vibration.

Substituting Eq. (17), in the right-hand side of compatibility equation Eq. (16), and solving for the particular solution, we have

$$F_p = Eh(A_1\cos(2n\theta) + A_2\cos(n\theta)\sin(\frac{3m\pi x}{L}) + A_3\cos(n\theta)\sin(\frac{m\pi x}{L})).$$ (18)

A_1, A_2 and A_3 values are not reported here for the sake of brevity.

For solving Eq. (16) substitute Eqs. (17) and (18) into Eq. (16), but the direct solution is impossible. Thus, the Galerkin's method was used to obtain an approximate solution.

5.1 Galerkin's method

Galerkin's method was used in most of the analyses and proved to be certainly the simplest method in the investigations of nonlinear vibrations of shells. Galerkin's procedure provides a very powerful approximate method by employing any set of basic functions φ, which reduces a system of nonlinear partial differential equations (PDEs) into a system of nonlinear ordinary differential equations (ODEs) which becomes manageable. The Galerkin's projection of the equation of motion (15), in this case, may be expressed as:

$$\left(D\nabla^4 w + \rho h \ddot{w} = ...,\varphi\right) = \int_0^{2\pi}\int_0^1 (D\nabla^4 w + \rho h \ddot{w} = ...)\times\varphi.$$ (19)

The Galerkin's weighting function is obtained from the first derivative of Eq. (17) with respect to time.

$$\varphi = \cos(n\theta)\sin(\frac{m\pi x}{1}) + \frac{n^2}{2R}A(t)\sin(\frac{m\pi x}{1})^2.$$ (20)

After the evaluation of the integrals, an ordinary non-linear differential equation is obtained.

$$\tilde{\alpha}_1 A(t) + \tilde{\alpha}_2 A(t)^3 + \tilde{\alpha}_3 A(t)^2 (\frac{d^2}{dt^2} A(t)) + \tilde{\alpha}_4 A(t)(\frac{d^2}{dt^2} A(t)) + \tilde{\alpha}_5 A(t)(\frac{d}{dt} A(t))^2 +$$

$$\tilde{\alpha}_6 (\frac{d}{dt} A(t))^2 + (e_0 a)^2 \tilde{\alpha}_7 A(t)^2 (\frac{d^2}{dt^2} A(t)) + (e_0 a)^2 \tilde{\alpha}_8 A(t)(\frac{d}{dt} A(t)^2) +$$

$$(e_0 a)^2 \tilde{\alpha}_9 (\frac{d^2}{dt^2} A(t)) = (\tilde{\alpha}_{10} A(t)^3 + \tilde{\alpha}_{11} A(t) + \tilde{\alpha}_{12} A(t)^5 + (e_0 a)^2 \tilde{\alpha}_{13} A(t)^3 + (e_0 a)^4 \tilde{\alpha}_{14} A(t)^3 +$$

$$(e_0 a)^4 \tilde{\alpha}_{15} A(t)^5 + (e_0 a)^4 \tilde{\alpha}_{16} A(t)) / (\tilde{\alpha}_{17} + (e_0 a)^2 \tilde{\alpha}_{18} + (e_0 a)^4 \tilde{\alpha}_{19} + (e_0 a)^6 \tilde{\alpha}_{20}).$$

(21)

$\tilde{\alpha}$ values are not reported here for the sake of brevity.

5.2 The method of averaging

The ordinary non-linear differential Eq. (21) cannot yet be solved exactly. But, an approximate solution can be obtained by the procedure known as the method of averaging. The method of averaging has been applied to a wide variety of problems dealing with nonlinear vibrations [26]. First of all the method is used to obtain simpler relations for the first and second order derivatives of a function $A(t)$ with slowly varying amplitude $a(t)$ and phase $\beta(t)$. [27]

$$A(t) = a(t)\cos(\omega t + \beta(t)) \tag{22}$$

Then

$$\dot{A}(t) = -a(t).\omega.\sin(\omega t + \beta(t)) + \dot{a}(t).\cos(\omega t + \beta(t)) - a.\dot{\beta}(t).\sin(\omega t + \beta(t)) \tag{23}$$

Using the assumption that $a(t)$ and $\beta(t)$ are slowly varying functions of time yields

$$\dot{a}(t).\cos(\omega t + \beta(t)) - a.\dot{\beta}(t).\sin(\omega t + \beta(t)) = 0 \tag{24}$$

Hence

$$\dot{A}(t) = -a(t)\omega\sin(\omega t + \beta(t)),$$
$$\ddot{A}(t) = -a(t)\omega^2 \cos(\omega t + \beta(t)) - \dot{a}(t)\omega\sin(\omega t + \beta(t)) - a(t)\dot{\beta}(t)\cos(\omega t + \beta(t)). \tag{25}$$

In this paper the steady-state vibrations are considered, which means the average values A and φ remain steady with time. These expressions then are substituted into the governing equation. After some regrouping, in the final state of the analysis, the equation is "averaged" by integrating over one period of the vibration. In this case, the average derivative $\dot{\beta}(t)$ is identically zero, and Eq. (21) can be reduced to:

$$\overline{\alpha}_1 A + \overline{\alpha}_2 A^3 + \overline{\alpha}_3 A^3 \omega^2 + \overline{\alpha}_4 A\omega^2 + (e_0 a)^2 \overline{\alpha}_5 A^3 \omega^2 + (e_0 a)^2 \overline{\alpha}_6 A\omega^2 = (\overline{\alpha}_7 A^3 + \overline{\alpha}_8 A +$$

$$\overline{\alpha}_9 A^5 + (e_0 a)^2 \overline{\alpha}_{10} A^2 + (e_0 a)^4 \overline{\alpha}_{11} \omega^2 A^2 + (e_0 a)^4 \overline{\alpha}_{12} A + (e_0 a)^2 \overline{\alpha}_{13} A^3 + (e_0 a)^2 \overline{\alpha}_{14} A^5 +$$

$$(e_0 a)^2 \overline{\alpha}_{15} A + (e_0 a)^4 \overline{\alpha}_{16} A^3 + (e_0 a)^4 \overline{\alpha}_{17} A^5) / (\tilde{\alpha}_{18} + (e_0 a)^2 \tilde{\alpha}_{19} + (e_0 a)^4 \tilde{\alpha}_{20} + (e_0 a)^6 \tilde{\alpha}_{21}).$$

(26)

$\overline{\tilde{\alpha}}$ values are not reported here for the sake of brevity.

The related non-linear algebraic equations in non dimensional form may conveniently be represented as follow:

$$\tilde{A} = \frac{A}{h}, \xi = \frac{m\pi R}{nl}, \varepsilon = (\frac{n^2 h}{R})^2, \Gamma = \frac{(\rho_t . h + \gamma)}{h}, \mu = \frac{e_0 a . n}{R}, \Omega^2 = \frac{2.\Gamma.\omega^2.R^2}{E} \qquad (27)$$

$$Z.\Omega^2 + Z_1.A + Z_3.A3 + Z_5.A5 = 0$$

where, $\tilde{A}$ is the non dimensional amplitude, ω is non dimensional nonlinear frequency, ε is nonlinearity parameter, ξ is an aspect ratio and Ω is the non dimensional frequency. Z values are not reported here for the sake of brevity.

6 VERIFICATION RESULTS

Most of the researches which have been done on CNTs containing fluid are those have been done on the dynamic characteristics of conveying fluid CNTs. Moreover, all of publications considered fluid-filled CNTs are based on local theory, and most of them investigated linear vibration. Yan et al. [13] have considered noncoaxial vibration of double walled CNTs which is based on local theory, and linear vibration has been considered. Also, they have considered nonlinear vibration characteristics of fluid-filled Double-Walled CNTs based on local shell theory [14], and the method of "harmonic balance" has been used for, however; we used "Averaging method", and based on nonlocal shell theory. Yan et al [28] have considered linear vibration of fluid-filled SWCNT based on local theory. Hence, In Table 1. the results of recent paper both local and nonlocal theories have been compared with the results obtained by Evensen [25].When $e_0 a = 0\,nm$ actually the results are based on local theory. As it is seen, the results which are obtained from Evensen's paper are very close to this recent paper and the difference between them is negligible. When the nonlocal parameter considered $e_0 a = 1\,nm$, with comparison the non dimensional frequencies Ω of each study, it was seen that there is a difference between these two papers. This difference is because that Evensen's equations were based on macro shell, however; this study is based on Eringen's theory in nano scale.

Table 1
Results verification

ξ	ε	Ω [25]	Ω (Present study) $e_0 a = 1\,nm$	Ω (Present study) $e_0 a = 0\,nm$
$\frac{1}{4}$	0	1	0.9999999999	1
	0.01	0.9134970962	0.9712176002	0.9135113274
	0.1	0.7534025509	0.9466466704	0.7534862388
	1	0.8415883879	0.8582509652	0.8415519329
2	0	1	1	1
	0.01	1.031175412	1.045485728	1.031263692
	0.1	1.341899079	1.236556788	1.342082467
	1	2.267650503	1.506669286	2.265575360

7 RESULTS AND DISCUSSIONS

Three zigzag SWCNTs have been investigated to analyse the nonlinear vibration of the fluid filled CNTs. As a case study, mechanical and dimensional properties of the SWCNT are shown in Table 2. The related material and geometric parameters were used from Gupta et al. [18]

Table 2
The parameters used for modeling (SWCNT) as a case study.[18]

Case Study	Tube	Radius(R) (nm)	ν	h (nm)	E (TPa)
Case 1	(10, 0)	0.3713	0.265	0.0878	3.939
Case 2	(20, 0)	0.7420	0.238	0.1251	2.704
Case 3	(30, 0)	1.1129	0.227	0.1340	2.493

The related mass density of the SWCNT and the fluid are considered respectively $\rho_t = 2.3(g/cm^3)$ and $\rho_f = 1000 kg/m^3$ [13] and the related axial half wave number (*m*) and the circumferential wave number (*n*) are considered m=n=1.

7.1 The effect of the different nonlocal parameters

Fig. 2 and Fig. 3 show the effects of the different nonlocal parameters [29]on the nonlinear frequency for case 1, when the nanotube is fluid filled, and empty. It is seen that the nonlinear frequencies decreased with increment of the nonlocal parameters. The vibration behavior was softening for the low amplitudes and it showed hardening behavior for the large amplitudes in both cases. Also, it is observed that with increasing the nonlocal parameters, changing the softening behavior of the vibration into the hardening behavior occurred at the higher amplitudes for both cases. Fig.3 compared the nonlinear frequency between the fluid filled nanotube and an empty one for the local and the nonlocal theories. It is observed when nanotube is fluid filled, the frequencies decreased based on both theories.
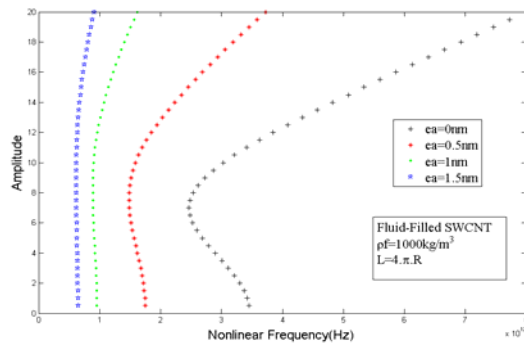

Fig. 2
The effect of the different nonlocal parameters on the non-dimensional nonlinear frequency of the fluid filled CNT.
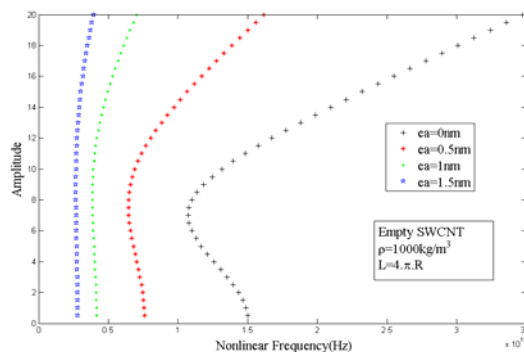

Fig. 3
The effect of the different nonlocal parameters on the non-dimensional nonlinear frequency of an empty CNT.

7.2 The effect of the different nonlinear parameters

The effects of the different nonlinear parameters on the nonlinear frequency for the fluid filled SWCNTs were shown in Fig. 4 and Fig. 5, the nonlocal parameter is $e_0 a = 1 \times 10^{-9} nm$. All three cases are used for considering the different nonlinear parameters. It is seen that the nonlinear frequencies have been reduced by reduction the nonlinear parameters. Furthermore, it is shown that the softening behavior was happened at the lower amplitudes, and it changed to the hardening type at the higher amplitudes by declining the nonlinear parameters. Fig.6 compared the nonlinear frequency between the fluid filled nanotube and an empty one for nanotube (10, 0) and nanotube (20, 0). It

is observed that vibration frequencies of fluid filled CNTs were lower compared to empty one, moreover, an influence of nonlinear parameter on an empty CNT is more than fluid-filled CNT.
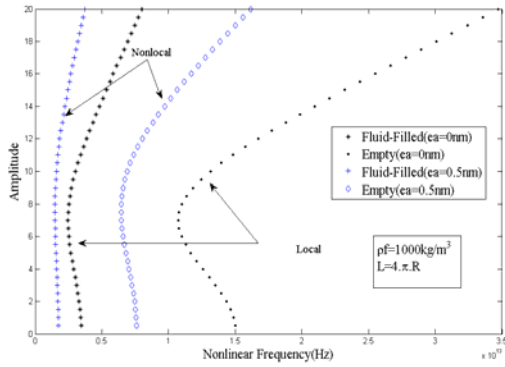

Fig. 4
Comparison of the effects of the local and nonlocal theories on the nonlinear frequency of the fluid filled CNT.
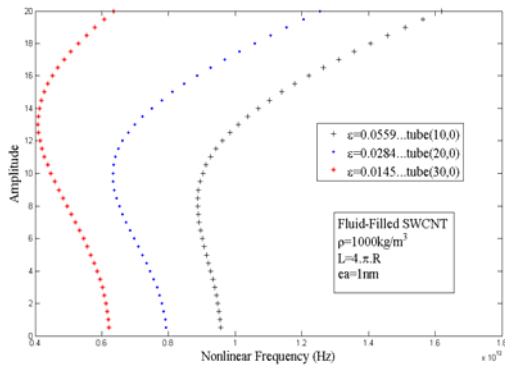

Fig. 5
The effect of the different nonlinear parameters on the non-dimensional nonlinear frequency of the fluid filled CNT.
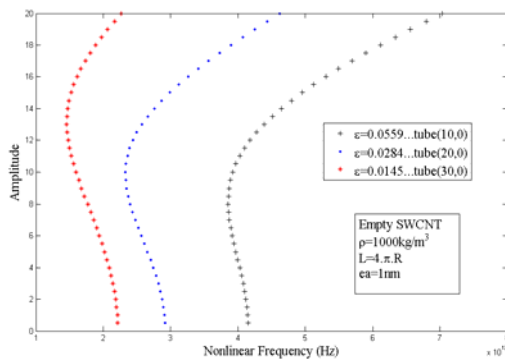

Fig. 6
The effect of the different nonlinear parameters on the non-dimensional nonlinear frequency of an empty CNT.

7.3 The effect of the different aspect ratios

Fig.7 and Fig. 8 showed an influence of the different aspect ratios on the nonlinear frequency for case 1, when the nanotube is fluid filled. The nonlocal parameter is $e_0 a = 1 \times 10^{-9}$ nm. The nonlinear frequency grew with increasing the aspect ratios (reduction of the length to radius ratio). Also, the vibration behavior is softening for the low amplitudes and turns into hardening type at the higher amplitudes in lower aspect ratios. Fig.9 compared the

nonlinear frequency amid the fluid filled CNT and an empty one for the different aspect ratios. It is observed that vibration frequency of fluid filled CNT was lower compared with empty CNT.
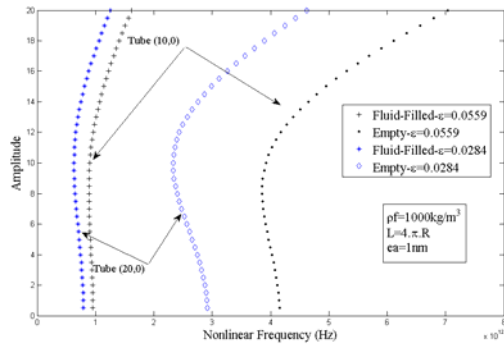

Fig. 7
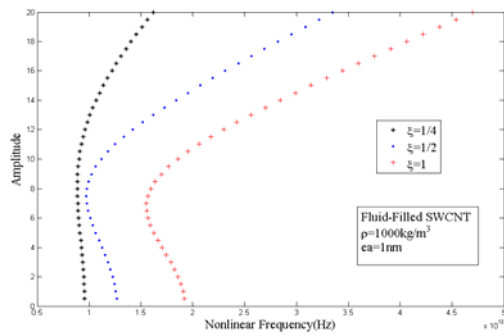Comparison of the effects of the different nonlinear parameters on the nonlinear frequency of the fluid filled CNT.

Fig. 8
The effect of the different aspect ratios on the non-dimensional nonlinear frequency of the fluid filled CNT.
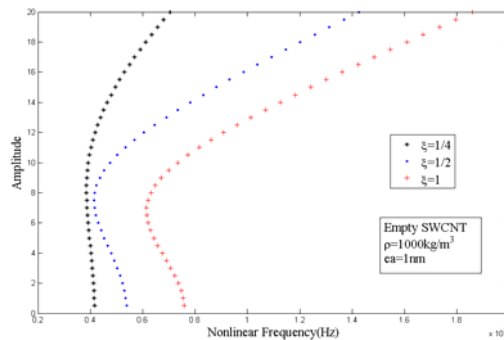

Fig. 9
The effect of the different aspect ratios on the non-dimensional nonlinear frequency of an empty CNT.

8   CONCLUSIONS

In this paper, the nonlinear vibration of an empty, and the fluid-filled SWCNTs with simply supported ends is investigated based on Von-Karman's geometric nonlinearity and the simplified Donnell's shell with the nonlocal model and the effects of an internal fluid on the coupling vibration of the SWCNT-fluid system with the different aspect ratios, the different nonlinear parameters and the different nonlocal parameters have been discussed in details. As a case study, the mechanical and dimensional properties of the SWCNT were used from Gupta et al[18]. It is seen although the nonlinear frequencies decreased with increment of the nonlocal parameters in both fluid-filled and empty CNTs, the nonlinear frequency increased with growing the aspect ratios and the nonlinear parameters for both

cases. Moreover, fluid-filled SWCNT compared with the empty one with the different aspect ratios, the different nonlinear parameters, and the different nonlocal parameters, and it was seen that for all these mentioned parameters, vibration frequencies of fluid-filled CNTs were lower than empty SWCNT. It means that fluid in SWCNT caused vibration frequencies reduced. Also, it is observed that with increasing the nonlocal parameters, the softening behavior of the vibration is occurred at the higher amplitudes for both fluid-filled and empty SWCNT, however; it is observed that with increasing the nonlinear parameter and the aspect ratios the softening behavior of the vibration is occurred at the lower amplitudes for both types of CNTs.

APPENDIX

A_1, A_2 and A_3 values mentioned in Eq (19):

$$A_1 = \frac{-1}{32} \frac{A(t)^2 R^2 m^2 \pi^2}{n^2 l^2}$$

$$A_2 = \frac{1}{4} \frac{A(t)^3 R \, m^2 \pi^2 n^4 l^2}{(n^4 l^4 + 81 R^4 m^4 \pi^4 + 18 R^2 m^2 \pi^2 n^2 l^2)}$$

$$A_3 = \frac{1}{2} \frac{A(t) \, R \, m^2 \pi^2 (2R^2 - \frac{1}{2}.n^4.A(t)^2)}{(n^4 l^4 + R^4 m^4 \pi^4 + 2 R^2 m^2 \pi^2 n^2 l^2)}$$

$$(A.1)$$

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