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Research Paper

Transient Thermal Stress Intensity Factors for An Edge Crack in A Thin Elastic Plate via Fractional-Order Framework

V. Varghese [1*], A.S. Balwir [2], D. Kamdi [3]

[1] *Department of Mathematics, Sushilabai Bharti Science College, Arni, Yavatmal, India*
[2] *Gondwana University, Gadchiroli, India*
[3] *Rashtrapita Mahatma Gandi, Art & 039;s, Comm & Amp; Sci College, Saoli, Gadchiroli, India*

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ABSTRACT
In this study, the analysis focuses on a transient thermoelastic problem in an isotropic homogeneous elastic plate that is exposed to heat loading within the framework of the fractional-order theory. The sectional heat supply is applied on both the front edge and the farthest edge of the rectangular plate. The integral transformation was considered as a means to solve the main governing equations. The Mittag-Leffler function is utilized to express the analytical solution for temperature change, displacement, and stress response. The investigation also encompasses the study of thermoelastic behaviors in a plate featuring a central crack. The stress intensity factors at the fracture tip are determined numerically using the weight function method in this proposed solution. The findings are depicted using numerical computations, considering the material as a media, and visually represented in graphical form.

Keywords: Fractional calculus; Non-Fourier heat conduction; Thermal stress; Integral transform approach; Integral transform.

1 INTRODUCTION

SOLID objects are frequently employed as structural components in several engineering disciplines, including civil, mechanical, and aerospace engineering. Various structural elements can potentially be subjected to seismic, mechanical, hydrodynamic, blast, aerodynamic, and thermal loads. The utilization of fractional-order equations in scientific and technical domains has witnessed a notable rise. These equations serve as effective models for dynamical systems, enabling the representation of memory and heredity characteristics exhibited by diverse substances. Therefore, mathematically representing real-world situations results in formulating fractional differential

*Corresponding author. Tel.: +96 65040845.
E-mail address: vino7997@gmail.com (V. Varghese)
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equations and other challenges that involve special functions of mathematical physics and their expansions and generalizations in one or more variables. Several writers have researched fractional derivatives within thermoelastic analysis in recent scholarly literature. The essence of their work can be succinctly described as follows: Povstenko [1] introduced a theoretical framework for quasi-static uncoupled thermoelasticity, which is founded on the heat conduction equation incorporating a time-fractional derivative. In his study, Povstenko [2] also investigated the temperature distribution and thermal stresses within an unbounded medium, including a spherical cavity. This investigation employed a quasi-static uncoupled theory of thermoelasticity, which relied on the heat conduction equation incorporating a time-fractional derivative. Youssef and Al-Lehaibi [3] employed heat conduction principles in deformable bodies and the Riemann-Liouville fractional integral operator to establish a novel framework for fractional order generalized thermoelasticity. In their study, Sherief et al. [4] proposed a fractional calculus approach that incorporates coupled and generalized thermoelasticity theory, specifically focusing on a single relaxation period. Ezzat and El-Karamany [5] attempted to employ these findings in the context of two-temperature thermoelasticity in the presence of a magnetic field. In their study, Youssef and Al-Lehaibi [6] developed a mathematical model based on the idea of fractional order generalized thermoelasticity to represent a cylindrical hollow elastic material. Povstenko [7] conducted a study on axisymmetric thermal stresses in a cylinder by employing the heat conduction equation in conjunction with the Caputo time-fractional derivative. Sur and Kanoria [8] developed a theoretical framework for a two-temperature generalized thermoelasticity model that incorporates fractional-order heat conduction.

The study conducted by Wang et al. [9] examined the thermoelastic processes associated with thermal inertia in the context of both macro- and microscale heat conduction. In their study, Zenkour and Abouelregal [10] derived the thermoelastic displacement, stress, conductive temperature, and thermodynamic temperature within an infinite isotropic elastic body, including a spherical hollow. The study conducted by Yadav et al. [11] employed the framework of generalized thermoelasticity with fractional order strain to investigate the behavior of one-dimensional disturbances in a viscoelastic solid subjected to a moving internal heat source and mechanical stress. The topic at hand pertains to the field of Green-Naghdi thermoelasticity, specifically focusing on incorporating energy dissipation. In their study, Gupta and Das [12] employed the Laplace transform and the eigenvalue technique to address the deformation of an unbounded transversely isotropic material within the fractional order generalized thermoelasticity framework. In their study, Sheoran and Kundu [13] conducted a comprehensive evaluation of pertinent literature to elucidate the role of fractional calculus in the field of thermoelasticity. This paper provides an overview of the generalizations of the standard heat conduction equation and the concepts of fractional thermoelasticity.

In the study conducted by Abbas [14], an investigation was carried out to analyze the effects of thermal shock loading on the inner surface cavity in an infinite medium with a cylindrical hollow. The study focused on examining the temperature, displacement, and stresses induced by this phenomenon using the framework of fractional order generalized thermoelasticity theory. The magneto-thermoelastic response of a homogeneous isotropic two-dimensional rotating elastic half-space solid was investigated by Bachher and Sarkar [15]. The study employed generalized thermoelasticity, specifically utilizing the Caputo time-fractional derivative. The study conducted by Povstenko et al. [16] examined the regulation of thermal stress in an infinite cylindrical body. The researchers used the time-fractional heat conduction equation with the Caputo derivative to analyze the temperature distribution. Xiong and Niu [17] devised a fractional-order thermoelastic diffusion model in anisotropic and linear diffusive media. This analysis focused on the dynamic response of a semi-infinite medium subjected to thermal and chemical potential shocks at one of its ends, employing the Laplace transform. Chirilă and Marin [18] researched dipolar thermoelastic materials within multipolar continuum mechanics. In their study, Abbas [19] examined the impact of fractional order derivatives on a two-dimensional thermal shock problem with varying conductivity levels, namely weak, normal, and strong conductivity. The investigation involved utilizing Laplace, exponential Fourier transforms, and eigenvalues to analyze the problem. In their study, Lata [20] investigated the thermal response of a thick circular plate with uniform isotropic properties within the context of the two-temperature thermoelasticity hypothesis. In their research, Mondal et al. [21] investigated transient events in a cylindrical cavity of a fiber-reinforced medium subjected to an induced magnetic field. They employed the three-phase-lag model of generalized thermoelasticity and introduced a novel derivative of the Caputo-Fabrizio type in the heat transport equation. Mittal and Kulkarni [22] employed fractional thermoelasticity within the framework of the two-temperature theory to examine the thermal fluctuations occurring within a confined spherical region. In their seminal work, Mondal [23] proposed an innovative mathematical model to investigate transient phenomena in a rod inside the Lord-Shulman thermoelastic framework based on Eringen's nonlocal elasticity theory.

This research presents the development of a transient heat conduction model utilizing time-fractional equations to investigate the thermoelastic response within a fractured plate. The Laplace and finite Fourier sine transform technique is utilized in the solution of fractional equations. The analytical solution is derived with the assistance of the Mittag-Leffler function. The weight function approach is utilized to determine the stress intensity factor for an edge crack in a thin elastic plate via fractional-order framework. This study examines the impacts of the temperature, moisture, stress response, and stress intensity components.

2 BASIC ASSUMPTIONS AND GOVERNING EQUATIONS

In our study, we employed a hypothesis based on time-fractional thermoelasticity. The thermal impact on elastic stresses and deformation is considered, but conversely, elastic deformation does not exert an influence on temperature. The mathematical expression for the time-fractional heat conduction equation is given by:

i. The classical Fourier's law of heat conduction [24]

$$q(x,t) = -k\nabla T(x,t) \tag{1}$$

in which $q(x,t)$ is the heat flux vector, t is the time, x is the position of any point on solid, k is the conductivity, ∇ is the gradient operator, and T is the temperature gradient, respectively. The primary flaw of the conventional Fourier's law is that it generates a parabolic equation for temperature, which leads to infinitely fast thermal wave propagation and renders it useless in its present form.

ii. The single-phase-lag model was proposed by Maxwell-Cattaneo as a means to address the inconsistency observed between the mathematical model [25,26] and experimental results [27]. This modification transforms the original parabolic equation into a hyperbolic equation.

$$q(x,t) + \tau_0 \frac{\partial q(x,t)}{\partial t} = -k\nabla T(x,t) \tag{2}$$

One recovers the classical Fourier's law with an infinitely fast propagation as a limiting case when $\tau_0 \to 0$. In this context, the flux undergoes relaxation with a designated characteristic time constant, denoted as τ_0, representing the heat flux's phase lag or relaxation time. As a result, the velocity of propagation is limited. In current literature, a generalization of Eq. (2) involves substituting the conventional integer-order derivative with a fractional-order derivative. This concept is discussed in reference [28] and the sources cited inside.

$$q(x,t) + \tau_0 \frac{\partial^\alpha q(x,t)}{\partial t^\alpha} = -k\nabla T(x,t) \tag{3}$$

with the solution as

$$q(x,t) = -\frac{k}{\tau_0}\int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left[-\frac{(t-\tau)^\alpha}{\tau_0}\right]\nabla T(x,\tau)d\tau \tag{4}$$

in which the fractional Caputo derivative of order α with a lower limit zero

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha} = {}_0^C D_t^\alpha f(t) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_0^t (t-\gamma)^{m-\alpha-1}\frac{\partial^m f(\gamma)}{\partial \gamma^m} d\gamma, m-1<\alpha<m \\ \dfrac{\partial^m f(t)}{\partial t^m},\ \alpha = m,\ m\in N \end{cases} \tag{5}$$

whereas $f(t)$ is a Lebesgue integrable function and the Riemann-Liouville fractional derivative is taken as

$$D_{RL}^\alpha f(t) = \frac{\partial^m}{\partial t^m}\left[\frac{1}{\Gamma(m-\alpha)}\int_0^t (t-\gamma)^{m-\alpha-1} f(\gamma)\, d\gamma\right],\ m-1<\alpha<m \tag{6}$$

wherein Eq. (3), without losing the generality $\Gamma(1+\alpha)$ appearing in the Taylor series is merged in τ_0 terms, Γ is the gamma function, α is introduced to keep the dimension in order, and $\partial^\alpha / \partial t^\alpha$ is the fractional time derivative based on Caputo fractional definition [29].

By combining Eq. (3) with the continuity equation, which is given as

$$-\rho C_v \frac{\partial T(x,t)}{\partial t} = \nabla \cdot q(x,t) \tag{7}$$

leads to the hyperbolic heat conduction equation

$$\frac{\partial}{\partial t}\left(1+\tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right) T(x,t) = \kappa \, \Delta T(x,t) \tag{8}$$

in which $\kappa = k / \rho C_v$ is the thermal diffusivity coefficient, ρ is the density, C_v is the calorific value and $\Delta = \nabla^2 = \nabla \cdot \nabla$ is the gradient operator, respectively.
For the limiting case:

(i) Taking $\tau_0 = 0$ or $\alpha = 0$, Eq. (8) reduces to classical Fourier heat conduction,

(ii) Taking $\alpha \in (0,1]$, Eq. (8) is identified as a fractional generalization of the Cattaneo approach,

3 FORMULATION OF THE PROBLEM

3.1 Time fractional heat conduction equation in the single-phase-lag model

For our investigation, we consider the transient response of the fractional heat conduction in a plate of thickness h that has a crack along one of its edges. It is decided to use the Cartesian coordinate system $O-xyz$, with the plate having an infinite extent in the *y* and *z* directions but having a finite extent in the *x* direction (i.e., $0 < x \leq h$).

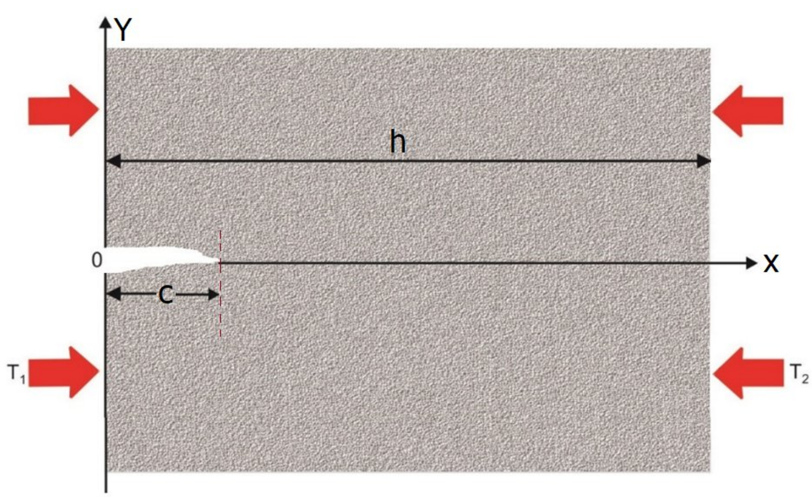

Fig. 1
Infinite plate with a crack at its edge subject to uniform thermal loading.

As seen in Figure 1, the edge fracture situated in the plane $y = 0$ may be found at the coordinates $0 < x < c, -\infty < y < +\infty$ in which c as crack length, and is perpendicular to the plate's free surface. T_0 denotes the temperature reference at the beginning of the process. Eq. (8) can be rewritten in the dimensionless form as a time-fractional conduction equation in the single-phase-lag model with its corresponding boundary conditions after dropping primes for convenience.

$$\frac{\partial}{\partial t}\left(1+\tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)(\Theta+\varepsilon e)=\frac{\partial^2\Theta}{\partial x^2} \tag{9}$$

subjected to conditions

$$\Theta(x,0)=0, \frac{\partial\Theta(x,0)}{\partial t}=0 \tag{10}$$

$$\Theta(0,t)=\Theta_1 H(t),\ \Theta(h,t)=\Theta_2 H(t) \tag{11}$$

where Θ_1 and Θ_2 are prescribed heat constants, e is the strain dilatation along the x direction, $H(t)$ is the Heaviside unit step function and the following non-dimensional variables are used

$$x'=c_1\eta x,\ y'=c_1\eta y,\ u'=c_1\eta u,\ t'=c_1^2\eta t,\ \tau_0'=c_1^2\eta\tau_0,\ \eta=1/\kappa=\rho C_v/k,$$

$$\sigma_{ij}'=\sigma_{ij}/\rho c_1^2,\ c_1^2=(\lambda+2\mu)/\rho,\ \varepsilon=T_0\gamma^2/(\lambda+2\mu)k\eta,$$

$$\Theta=\gamma(T-T_0)/\rho c_1^2,\ \gamma=\alpha_t(2\lambda+3\mu)$$

with α_t is the coefficients of linear thermal expansion of the material, λ and μ are the Lame constants, T_0 is the reference temperature, respectively. Here, to simplify, the equations in subsequent sections are converted to dimensionless form, and later on, the single quote mark is removed for simplicity's sake.

3.2 The thermal stress function

To calculate the thermoelastic response of the plate with an edge fracture, we will assume that both of the plate's surfaces $x=0$ and $x=h$, do not experience any traction as

$$\begin{aligned}&\sigma_{xx}(0,t)=0,\sigma_{xx}(h,t)=0,\sigma_{xy}(0,t)=0,\\&\sigma_{xy}(h,t)=0,\sigma_{xz}(0,t)=0,\sigma_{xz}(h,t)=0\end{aligned} \tag{12}$$

Now, suppose the temperature $\Theta=\Theta(x,t)$ is the excess of temperature over Θ_0, the absolute temperature of the plate in a state of zero stress and strain; then, the thermal stress $\sigma=\sigma_{yy}(x,t)$ is connected with u and Θ by the relation

$$\sigma=e-\omega\Theta \tag{13}$$

where quantity $\omega=T_0\gamma/(\lambda+2\mu)$, E denotes Young's modulus, α_t the linear expansion coefficient, and ε_{yy} is the strain component which can be obtained using compatibility condition $\partial^2\varepsilon_{yy}/\partial x^2=0$ that gives

$$e=\varepsilon_{yy}(x)=C_1x+C_2 \tag{14}$$

where C_1 and C_2 are coefficients to be determined from the boundary conditions of the plate structure. Thus, for the thermoelastic medium in plane strain, using generalized Hooke's law, the thermal stress in the absence of crack, as shown in equation (13), can be rewritten as

$$\sigma=C_1x+C_2-\omega\Theta \tag{15}$$

4 THE SOLUTION TO THE PROBLEM

Following Liang et al. [30], if $\alpha > 0$, $n = [\alpha] + 1$, and functions $f(t),\ f'(t),\ f''(t), ...\ f^{(n-1)}(t)$ are continuous in $[0, \infty)$ and of exponential order, while ${}^{C}D_0^{\alpha} f(t)$ with order α is piecewise continuous on $[0, \infty)$, then Laplace transform of Caputo fractional derivative of $f(t)$, is defined as follows

$$L[{}^{C}D_0^{\alpha} f(t)] = s^{\alpha} L[f(t)] - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0) \tag{16}$$

In view of the above theorem, assuming

$$f(x,0) = \frac{\partial}{\partial t} f(x,0) = \frac{\partial^2}{\partial t^2} f(x,0) = ... = 0 \tag{17}$$

Using Eqs. (13) and (14), applying the Laplace transforms to the Eqs. (9) and (11), bearing Eq. (15) in mind, one obtains

$$(s + \tau_0 s^{\alpha+1})[\overline{\Theta} + \varepsilon(C_1 x + C_2)] = \frac{\partial^2 \overline{\Theta}}{\partial x^2} \tag{18}$$

subjected to conditions

$$\Theta(0,s) = \Theta_1 / s,\ \Theta(h,s) = \Theta_2 / s \tag{19}$$

where s is the parameter and $\overline{f}$ stands for the Laplace transform of *f,* respectively.

To obtain the solution to Eq. (18), we recall the following property of the finite Fourier sine transform in the domain $0 \le x \le h$

$$F\left\{\frac{\partial^2 \overline{f}(x,s)}{\partial x^2}\right\} = -\xi_n^2 \overline{\overline{f}}(\xi_n, s) + \xi_n[\overline{f}(0,s) - (-1)^n \overline{f}(h,s)] \tag{20}$$

where $\overline{\overline{f}}$ stand for the finite Fourier transform of $\overline{f}$, and $\xi_n = n\pi / h,\ \ k = 1, 2, ...$ respectively.

Performing the finite Fourier sine transform of both sides of Eq. (18) subject to conditions (19), one obtains

$$\overline{\overline{\Theta}}(\xi_n, s) = \frac{\Theta_{12}\xi_n^2 - \Psi\varepsilon\Omega}{s\xi_n(\Psi + \xi_n^2)} \tag{21}$$

where $\Theta_{12} = \Theta_1 - (-1)^n \Theta_2$, $\Psi = s + \tau_0 s^{\alpha+1}$ and $\Omega = C_1(-1)^{n+1} h + C_2[1 - (-1)^n]$.

Then, we perform the inverse finite Sine-Fourier transform to both sides of Eq. (21)

$$\overline{\Theta}(x,s) = 2\sum_{n=1}^{\infty}\left[\frac{\Theta_{12}\xi_n^2 - \Psi\varepsilon\Omega}{s\xi_n(\Psi + \xi_n^2)}\right]\sin(\xi_n x) \tag{22}$$

Using Eqs. (16) and (17), applying the Laplace transforms to the dimensionless governing Eq. (15), the transformed equations are given as

$$\overline{\sigma} = \overline{\sigma}_{yy} = C_1 x + C_2 - \omega\overline{\Theta}(x,s) \tag{23}$$

If the plate is only subjected to thermal shock without constraint along its boundaries, then the unknown constants C_1 and C_2 can be solved from the following conditions

$$\int_0^h \overline{\sigma}_{yy}(x,s)dx = \int_0^h \overline{\sigma}_{yy}(x,s)x dx = 0 \tag{24}$$

which may be used to determine two constants C_1 and C_2. Thus, Eqs. (22)−(24) describe the analytical solutions of thermal parameters $\bar{\Theta}$ and $\bar{\sigma}$, respectively, in the Laplace domain.

5 THE THERMAL STRESS INTENSITY FACTOR

Following [31], the crack problem considered, we require that equal and opposite axial stress will be superposed to ensure crack faces are free. Using the weight function method, the stress intensity factor (SIF) K_I near the central crack tip can be calculated by the following integral [32]:

$$K_I = \frac{2\sqrt{h}}{\sqrt{\pi c}(1-c)^{3/2}} \int_0^c \frac{\bar{\sigma}_{yy}(x,s) F_1(x,c)}{\sqrt{1-(x/c)^2}} dx \tag{25}$$

Here, $c' = c_1 \eta c,\ F_1(x,c)$ is a non-dimensional weight function is given as

$$F_1(x,c) = f_1(c) + f_2(c)\left(\frac{x}{c}\right) + f_3(c)\left(\frac{x}{c}\right)^2 + f_4(c)\left(\frac{x}{c}\right)^3$$

where

$$f_1(c) = 0.46 + 3.06c + 0.84(1-c)^5 + 0.66c^2(1-c)^2, f_2(c) = -3.52c^2,$$
$$f_3(c) = 6.17 - 28.22c - 34.54c^2 - 14.39c^3 - (1-c)^{1/2} - 5.88(1-c)^5 - 2.64c^2(1-c)^2,$$
$$f_4(c) = -6.63 + 25.16c - 31.04c^2 + 14.41c^3 + 2(1-c)^{1/2} + 5.04(1-c)^5 + 1.98c^2(1-c)^2$$

It is noted that the above integral computation is effective for a positive (tensile) stress since a negative or compressive stress does not give rise to crack opening but closing. Of course, from another point of view, a negative stress intensity factor may be understood as a shield effect to prevent the crack from advancing. Here, for simplicity, the SIFs are normalized by $K_0 = -\sqrt{\pi h} E \gamma T_0 / (1-\upsilon)$.

6 THE NUMERICAL INVERSION OF THE LAPLACE TRANSFORMS

Consider the Gaver-Stehfest algorithm [33-35], which aims to approximate $f(t)$ by a sequence of functions, can be given as

$$f(t) \approx f_n(t) = \left[\frac{1}{t}\text{In}(2)\right] \sum_{n=1}^{L} a_n F\left[\frac{n}{t}\text{In}(2)\right],\ n \ge 1,\ t > 0 \tag{26}$$

where $F[.]$ is the Laplace transform of $f(t)$.

The coefficients a_n depend only on the number of expansion terms n, defined as

$$a_n = (-1)^{n+L/2} \sum_{k=[(n+1)/2]}^{\min(n,L/2)} \frac{k^{L/2}(2k)!}{(L/2-k)!k!(k-1)!(n-k)!(2k-n)!},\ n \ge 1,\ 1 \le L \le n \tag{27}$$

The convergence of Gaver-Stehfest algorithm for numerical inversion of the Laplace transform was developed by Kuznetsov [36]. It is well proved that the approximations $f_n(t)$ converge to $f(t)$, if *f* is continuous at *t* and of bounded variation in a neighbourhood of *t*. Mathematical evidence has been provided to illustrate that the convergence of the series can be achieved by considering a substantial number of terms from the sequence. Due to the comparative thickness of the plate, the solution proposed in this study will exhibit convergence. In summary, the convergence claim posits that by considering a significant number of distinct terms, the solutions to the series will ultimately converge toward the exact answer, and the magnitude of error will approach zero universally. In alternative terms, reducing the magnitude and size of the phase will result in a substantial enhancement in the convergence rate. In order to ensure the convergence of the infinite series inside the solution and satisfy the necessary criteria imposed by the functions at an arbitrary point, it is necessary to approximately substitute $\sum_{\infty}$ in the temperature and its stresses with $\sum_{20}$.

7 NUMERICAL RESULTS, DISCUSSION AND REMARKS

To demonstrate the accuracy of the present model, we will try to provide a realistic illustration in the next section. The results can also be organized systematically to make it easier for other researchers to compare and confirm their findings' accuracy. We adopted the copper material constants as follows to make numerical studies easier as shown in Table 1:

Table 1
Values of the constants [5]

$k=386$ N/Ks, $\alpha_t = 1.78 \times 10^{-5}$K^{-1}, $C_v = 383.1$m^2/K, $\tau_0 = 0.002$s, $\rho = 8954$kg/m^3,
$\mu = 3.86 \times 1010$ Nm^{-2}, $\lambda = 7.76 \times 10^{10}$Nm^{-2}, $T_0 = 293$ K, $\varepsilon = 0.0168$, $c_1 = 415$ m/s

For calculation simplicity, we set the sectional heat supply as a constant function as $\Theta_1 = 350K$ and $\Theta_2 = 0$. Here we perform the numerical calculation with the help of MATHEMATICA software. The thermal variations are investigated using the numerical values of the thermal properties of the heat conductor while taking into account several distinct orders (α) of fractional differential coefficients in time-varying variables. It should be noted that the fractional heat conduction model reduces to the classical Fourier heat conduction model when $\alpha = 0$ or $\tau = 0$, and to the hyperbolic heat conduction model when $\alpha = 1$ are discussed numerically. Figs 2 to 4 illustrate the transient temperature distributions shown for different values of fractional order = 0, 0.4, 0.7, 1.0. With the fractional-order increasing from 0 to 1.0, the temperature responses increase near the mid part, while the temperature responses decrease to minimum at both ends.

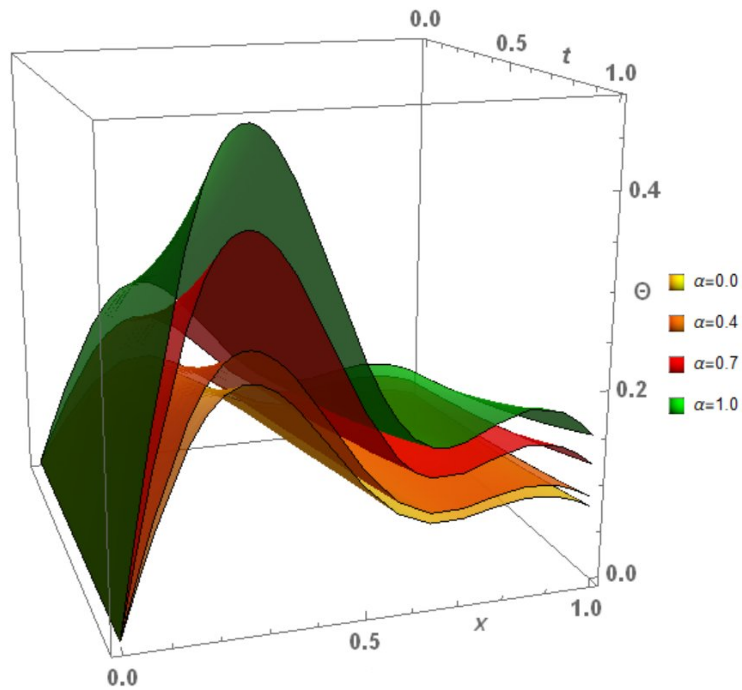

Fig. 2
3D temperature profile along and t for various α .

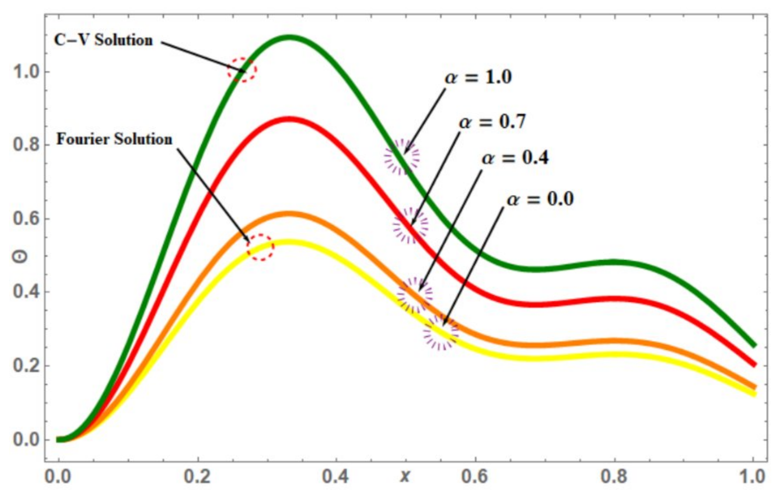

Fig. 3
The temperature distribution along the thickness when t = 0.8.

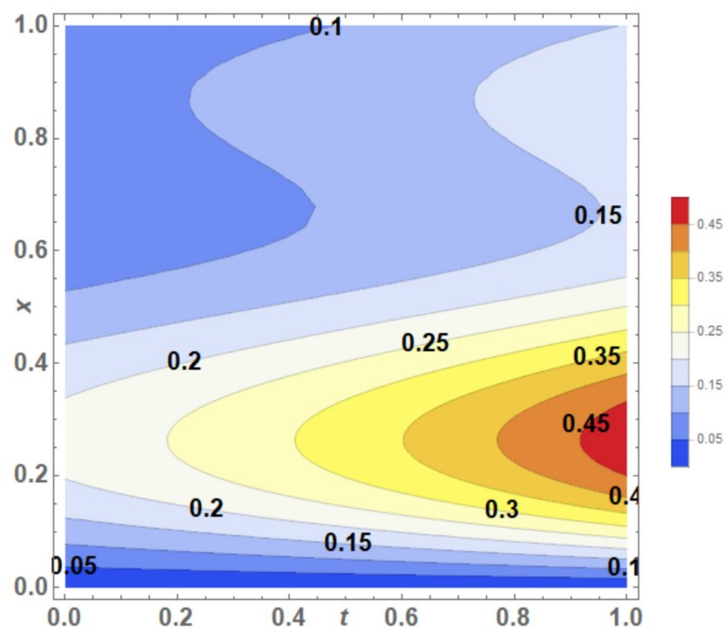

Fig. 4
The contour plot of temperature profile with fractional-order α= 0.7.

In Figure 2, the three-dimensional temperature curves are presented that illustrate the changes in temperatures in the isotropic homogeneous elastic plate when the ambient temperature is specified. With increasing dimensionless time and length *x*, the temperature increases to the peak at *x* = 0.3, and gradually declines to the end of plate. Fig. 4 shows the contour plot of the various temperature distributions presented in Figs. 2 and 3 which clearly indicates that continuous heating due to the sectional heat source on the plate surface increases the temperature gradually concerning time and location.

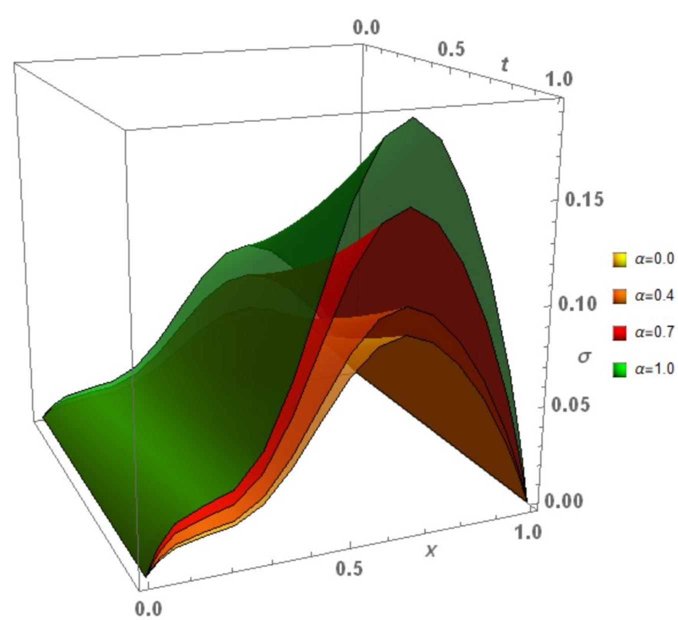

Fig. 5
3D thermal stresses distribution along and t for various α .

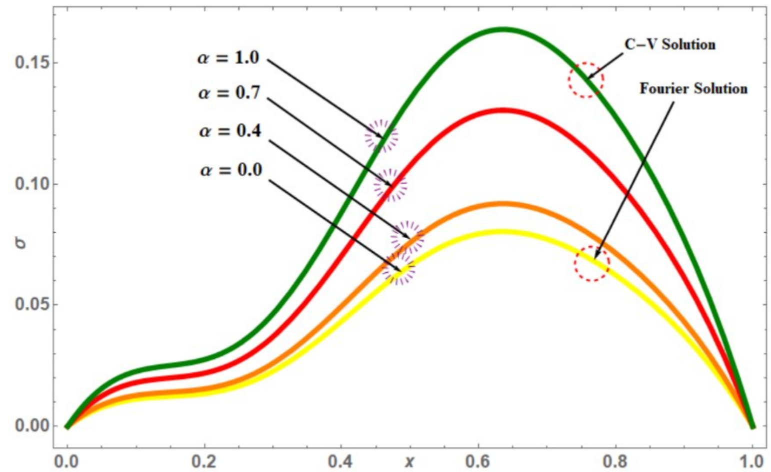

Fig. 6
The thermal stresses distribution along the thickness when t = 0.8.

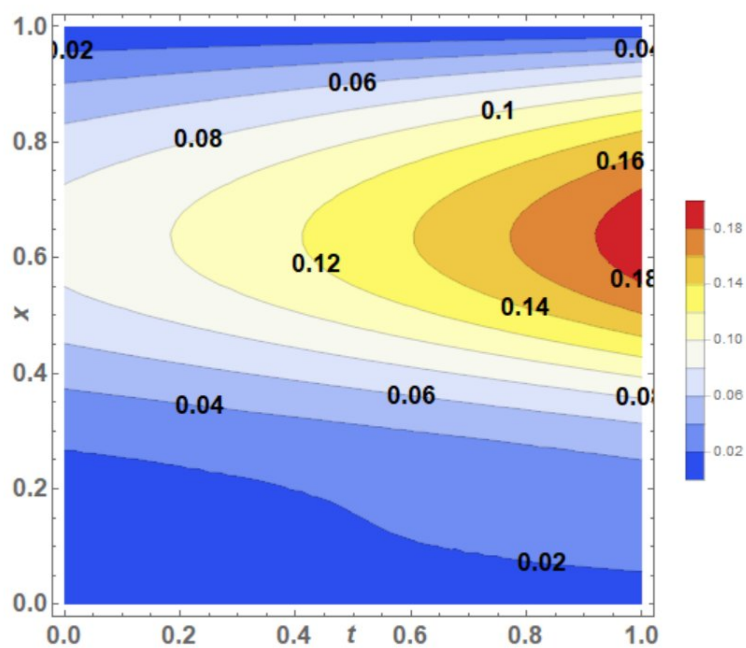

Fig. 7
The contour plot of thermal stresses with fractional-order $\alpha = 0.7$.

In Fig. 5, the dimensionless stress distribution is plotted against the dimensionless thickness x as well as time t with various values of the order of the fractional derivative α. The different values of the parameter α in the wide range ($0 < \alpha \leq 1$) cover the two cases of conductivity; ($0 < \alpha < 1$) for weak conductivity and ($\alpha = 1$) for normal conductivity. In the aforementioned figures, we noticed the difference in all value of fractional parameter α ($0 < \alpha \leq 1$). It is noted that the stress with increasing dimensionless thickness first exhibits a minimum value $x = 0$ due to compressive force and then a maximum tensile force at $x = 0.75$, before tending to minimum at the end $x = 1$. To find the effect of dimensionless time τ on the thermal stress σ_{yy}, the evolution of the thermal stress at x = 0.75 is shown in Figure 6. It is noted that it has a maximum tensile stress at the outer edge so the maximum tensile stress is occurring, and its absolute value increases with time for different values of α, which may be due to the accumulation of thermal energy dissipated by sectional heat supply. The dependence of the position of the peak value of thermal stress intensity factors on the fractional order α can be observed. Fig. 7 shows the contour plot displaying the distribution of stress due to the thermal load experienced in the plate and the same is well observed in Figs. 5 and 6.

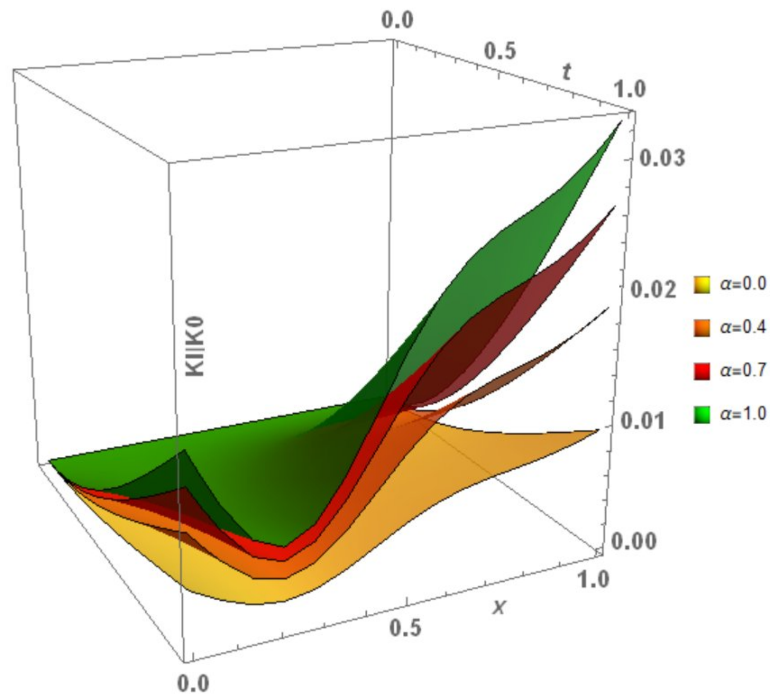

Fig. 8
The effects of crack length c on the SIFs / 0 for various α .

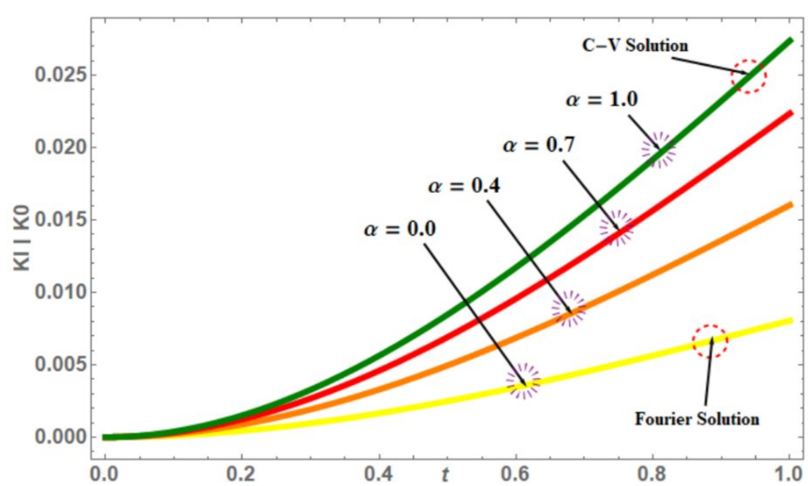

Fig. 9
The effects of crack length c on the SIFs / 0 along the τ when x = 0.2.

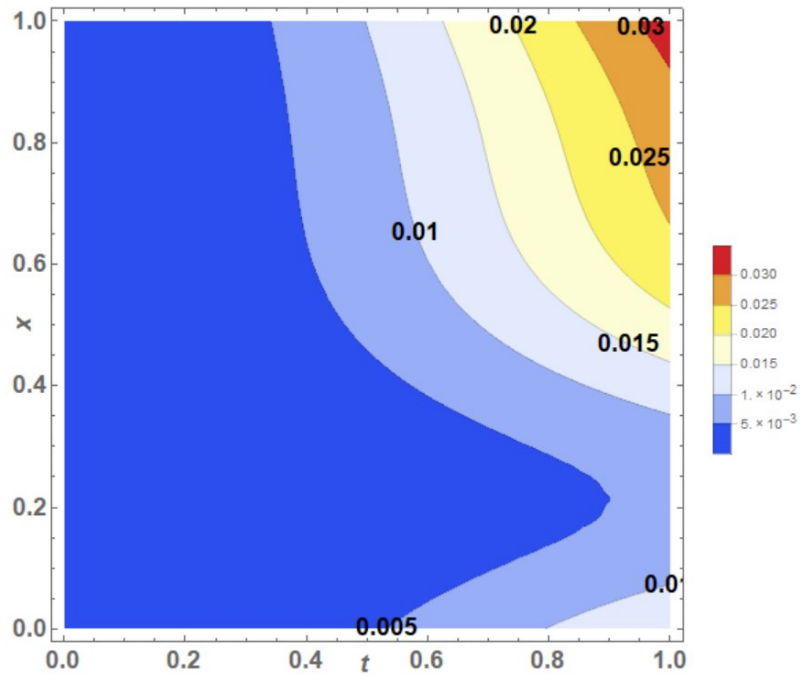

Fig. 10
The effects of crack length c on the SIFs ⁄ 0 along the time τ.

Figures 8-10 illustrates the temporal progression of the thermal SIFs at the crack tip for a plate containing an edge crack. The figures exhibit the evolution history of the SIFs for both the classical solution with $\tau_0 = 0$ or $\alpha = 0$, the CV solution with $\alpha = 1$, and between other fractional orders $\alpha = 0.4, \alpha = 0.7$. Figure 8 shows that as the length of the crack increases, there is an initial increase in the maximum magnitude of dynamic thermal stress intensity factors. Moreover, the classical and CV solutions exhibit a similar pattern in which the transient thermal stress intensity factors (SIFs) initially grow until reaching a maximum value, indicating that the thermal stresses impede crack propagation. Subsequently, the SIFs take a considerably longer time to dissipate completely. The rationale behind this phenomenon is that the fractional diffusion equation serves as an intermediary between the conventional heat conduction equation and wave equation when the value of falls within the range of 1 to 2. Furthermore, these curves demonstrate the highest magnitude compared to the response curves for classical and CV solutions scenarios. As seen in Figure 9, the normalized stress intensity factors SIFs K_I / K_0 for the situation of a central cracking embedded in a plate that is being heated sectionally are shown for different values of α with crack length $c = 0.2$ on the SIFs ⁄ 0 along the increasing period. The line curves ascend to their maximum values with a rising parameter τ_0 for $\alpha \leq 1$. It has also been learned that a central crack that is exposed to sectional heat supply is likely to progress and can go up to its length *c*, which may be at least half of the layer's thickness or even farther. The following observations have also come to our attention: (i) In the scenario involving the case ($0 < \alpha \leq 1$), the contours of the curves are smoother, as shown in Figure 10. (ii) The fractional order α substantially impacts every field. (iii) When the value of the parameter α is increased, the result is a corresponding rise in the speed of the waves that propagate the thermal stresses. (iv) As a result of the finding, there is now a reason to research conducting thermoelastic material as a new kind of thermoelastic material that may be used.

8 COCLUSION

The present work employed a fractional heat conduction model to examine the behavior of an isotropic homogeneous elastic plate subjected to a sectional heat source. This model's fundamental basis lies in utilizing the equation for heat conduction, using the Caputo fractional derivative of a certain order. The solution can be obtained by utilizing the Laplace integral transform and the finite Fourier sine transform. A numerical inversion technique used for the Laplace transform acquired the numerical outcomes in the time domain. The weight function technique was employed as the preferred methodology to obtain the numerical values for the thermal stress intensity parameters. This paper presents the graphical representation and computational analysis of numerical results pertaining to temperature, stresses, and the stress intensity factor. The subsequent content presents a succinct overview of the investigation conducted on the outcomes: (i) The influence of the fractional parameter on the outcome is significantly affected by the values of all the field variables. (ii) The temperature and stress distributions at any given point exhibit an increase when the value of α grows within the range of 0 to 1, inclusively. (iii) Furthermore, it is seen that the traditional Fourier heat conduction theories and a fractional extension of the Cattaneo approach can be derived as specific instances for further investigation. The findings of this investigation indicate that the utilization of the theory of generalized thermoelasticity with fractional order heat transfer provides a more precise depiction of the behavior shown by the constituent particles comprising an elastic body, in comparison to the theory of generalized thermoelasticity with integer order.

References

[1] Y.Z. Povstenko, Fractional heat conduction equation and associated thermal stress, *J. Therm. Stresses*, Vol. 28, No. 1, pp. 83-102, 2004.

[2] Y.Z. Povstenko, Two-dimensional axisymmetric stresses exerted by instantaneous pulses and sources of diffusion in an infinite space in a case of time-fractional diffusion equation, *Int J Solids Struct*, Vol. 44, No. 7–8, pp. 2324-2348, 2007.

[3] H.M. Youssef and E.A. Al-Lehaibi, Variational principle of fractional order generalized thermoelasticity, *Appl. Math. Lett.*, Vol. 23, No. 10, pp. 1183-1187, 2010.

[4] H.H. Sherief, A.M.A. El-Sayed, A.M. Abd El-Latief, Fractional order theory of thermoelasticity, *Int J Solids Struct*, Vol. 47, No. 2, pp. 269-275, 2010.

[5] M.A. Ezzat and A.S. El-Karamany, Two-temperature theory in generalized magneto-thermoelasticity with two relaxation times, *Meccanica*, Vol. 46, pp. 785–794, 2011.

[6] H.M. Youssef and E.A. Al-Lehaibi, Fractional order generalized thermoelastic infinite medium with cylindrical cavity subjected to harmonically varying heat, *Sci. Res.*, Vol. 3, No. 1, pp. 32-37, 2011.

[7] Y. Povstenko, Time-fractional radial heat conduction in a cylinder and associated thermal stresses, *Arch. Appl. Mech.*, Vol. 82, pp. 345–362, 2012.

[8] A. Sur and M. Kanoria, Fractional order two-temperature thermoelasticity with finite wave speed, *Acta Mech.*, Vol. 223, pp. 2685-2701, 2012.

[9] Y. Wang, X. Zhang and X. Song, A generalized theory of thermoelasticity based on thermomass and its uniqueness theorem, *Acta Mech.*, Vol. 225, pp. 797–808, 2014.

[10] A.M. Zenkour and A.E. Abouelregal, State-space approach for an infinite medium with a spherical cavity based upon two-temperature generalized thermoelasticity theory and fractional heat conduction, *Z. Angew. Math. Phys.*, Vol. 65, pp. 149–164, 2014.

[11] R. Yadav, K.K. Kalkal and S. Deswal, Two-temperature generalized thermoviscoelasticity with fractional order strain subjected to moving heat source: State space approach, *J. Math.*, 2015, Article ID 487513, 13 pages, 2015.

[12] N.D. Gupta and N.C. Das, Eigenvalue approach to fractional order generalized thermoelasticity with line heat source in an infinite medium, *J. Therm. Stresses*, Vol. 39, pp. 977-990, 2016.

[13] S.S. Sheoran and P. Kundu, Fractional order generalized thermoelasticity theories: A review, *Int. J. Adv. Appl. Math. and Mech.*, Vol. 3, No. 4, 76–81, 2016.

[14] I.A. Abbas, Fractional order generalized thermoelasticity in an unbounded medium with cylindrical cavity, *J. Eng. Mech.*, Vol. 142, pp. 04016033-1-5, 2016.

[15] M. Bachher and N. Sarkar, Fractional order magneto-thermoelasticity in a rotating media with one relaxation time, *Mathematical Models in Engineering*, Vol. 2, No. 1, pp. 56-68, 2016.

[16] Y. Povstenko, D. Avci, E. İskender and Ö. Necati, Control of thermal stresses in axissymmetric problems of fractional thermoelasticity for an infinite cylindrical domain, *Therm. Sci.*, Vol. 21, No. 1A, pp. 19-28, 2017.

[17] C. Xiong and Y. Niu, Fractional-order generalized thermoelastic diffusion theory, *Appl. Math. Mech. -Engl. Ed.*, Vol. 38, pp. 1091–1108, 2017.

[18] A. Chirilă and M. Marin, The theory of generalized thermoelasticity with fractional order strain for dipolar materials with double porosity, *J Mater. Sci.*, Vol. 53, pp. 3470–3482. 2018.

[19] I.A. Abbas, A Study on Fractional Order Theory in Thermoelastic Half-Space under Thermal Loading, *Phys Mesomech*, Vol. 21, pp. 150–156, 2018.

[20] P. Lata, Fractional order thermoelastic thick circular plate with two temperatures in frequency domain, *Appl. Appl. Math.*, Vol. 13, pp. 1216 – 1229, 2018.

[21] S. Mondal, A. Sur, M. Kanoria, Magneto-thermoelastic interaction in a reinforced medium with cylindrical cavity in the context of Caputo–Fabrizio heat transport law, *Acta Mech.*, pp. 1-18, 2019.

[22] G. Mittal and V.S. Kulkarni, Two temperature fractional order thermoelasticity theory in a spherical domain, *J. Therm. Stresses*, Vol. 42, No. 9, pp. 1136-1152, 2019.

[23] A. Sur and M. Kanoria, Fractional order two-temperature thermoelasticity with finite wave speed, *Acta Mech.*, Vol. 223, pp. 2685-2701, 2012. DOI: 10.1007/s00707-012-0736-7

[24] H. R. Ghazizadeh, M. Maerefat, and A. Azimi, Explicit and implicit finite difference schemes for fractional Cattaneo equation, *J. Comput. Phys.*, Vol. 229, No. 19, pp. 7042–7057, 2010. DOI: 10.1016/j.jcp.2010.05.039.

[25] T. T. Lam, A unified solution of several heat conduction models, *Int. J. Heat Mass Transf.*, Vol. 56, No. 1–2, pp. 653–666, 2013. DOI: 10.1016/j.ijheatmasstransfer.2012.08.055.

[26] C. Cattaneo, Sur uneForme de I'equation de la Chaleur Eliminant le Paradoxed'une Propagation Instantanee', *ComptesRendus de l'Académie des Sciences*, Vol. 247, pp. 431-433, 1958.

[27] M. N. Özisik, *Heat Conduction*, John Wiley & Sons, New York, 1993.

[28] H.H. Sherief, A. El-Sayed and A. El-Latief, Fractional order theory of thermoelasticity, *Int J Solids Struct*, Vol. 47, No. 2, pp. 269-275, 2010.

[29] A.A. Kilbas, H.M. Srivastava, and J.J. Trujillo, *Theory and applications of fractional differential equations*, Elsevier, Amsterdam, 2006.

[30] S. Liang, W. Ranchao, and L. Chen, Laplace transform of fractional order differential equation, *Electron. J. Differ. Equ.*, Vol. 139, pp. 1–15, 2015. URL: http://ejde.math.txstate.edu

[31] X. Y. Zhang, and X. F. Li, Thermal shock fracture of a cracked thermoelastic plate based on time–fractional heat conduction, *Engng Fract Mech*, Vol. 171, pp. 22-34, 2017. DOI: 10.1016/j.engfracmech.2016.11.033

[32] H. Tada, P. C. Paris, G. R. Irwin, *The stress analysis of cracks handbook*, 3rd edn, ASME, New York, 2000.

[33] D. P. Gaver, Observing stochastic processes and approximate transform inversion, *Oper. Res.*, Vol. 14, No. 3, pp. 444–459, 1966. DOI:10.1287/opre.14.3.444.

[34] H. Stehfest, Algorithm 368, Numerical inversion of Laplace transforms, *Comm. Assn. Comp. Mach.*, Vol. 13, No. 1, pp. 47–49, 1970. DOI:10.1145/361953.361969.

[35] H. Stehfest, Remark on algorithm 368: Numerical inversion of Laplace transforms, *Commun. Assn. Comput. Mach.*, Vol. 13, No. 10, pp. 624, 1970. DOI:10.1145/355598.362787.

[36] A. Kuznetsov, On the convergence of the Gaver–Stehfest algorithm, *SIAM J. Num. Anal.*, Vol. 51, No. 6, pp. 2984–2998, 2013. DOI:10.1137/13091974X.