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Finite Crack in a Thermoelastic Transversely Isotropic Medium Under Green-Naghdi Theory

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ABSTRACT

In this paper, we have studied a model of finite linear Mode-I crack in a thermoelastic transversely isotropic medium under Green Naghdi theory. The crack is subjected to a prescribed temperature and a known tensile stress. The plane boundary surface is considered as isothermal and all the field variables are sufficiently smooth. The heat conduction equation is written under two temperature theory (2TT) for Green Naghdi model which contains absolute temperature as well as conductive temperature. The analytical expressions of displacement components, stress components and temperature variables are obtained by normal mode analysis and matrix inversion method. Comparisons have been made within Green Naghdi (G-N) theory of type I, type II and type III for displacement, stress and absolute temperature variables against the crack width for a transversely isotropic material (Cobalt) by virtues of graphs. Also, Comparison have been made among displacement, thermal stress and absolute temperature for different depths.

Keywords: Finite crack; G-N theory; Transversely isotropic medium; Wave propagation; Thermoelastic.

1 INTRODUCTION

FRACTURE mechanics is an important branch in modern material science used to refine the performance of mechanical components and covers the study of different laws to control crack growth. Mode-I crack is created by the stress normal to the plane of the crack. In the field of Thermoelasticity dynamical crack problems are investigated by many authors [1-5]. In the theory of elasticity, non-isothermal problems have great importance due to their applications in Aerodynamics. Modern aircraft produces intense thermal stresses due to high velocity and reduces the strength of aircraft structure. The study of Thermoelasticity is worthwhile in composite engineering, geothermal engineering, and many more. Nowacki [6] and Nowinski [7] have studied Thermoelasticity with various applications in real phenomena. Biot [8] has introduced the theory of coupled Thermoelasticity after eliminating the first paradox of the classical theory to conquer the first infirmity. The first generalizations of the coupled theory have been introduced by Lord and Shulman [9]. In this theory a wave type heat equation was obtained by assuming a new law of heat conduction to replace the classical Fourier's law which contains time derivative and new constant

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acts as relaxation time. Dhaliwal and Sherief [10] extended this theory for isotropic medium in the presence of heat sources. The second generalizations of the coupled theory of Thermoelasticity with relaxation times is known as the theory of temperature-rate-dependent. Green and Laws [11] have studied a problem under this theory. Green and Naghdi [12-13] have discussed the linear and non-linear theories of the thermoelastic body with and without energy dissipation. These theories of Thermoelasticity based on entropy equality are known as G-N (type I, type II and type III) theory. Linearized G-N theory of type I describes the classical heat equation, which is parabolic in nature. G-N theory of type II and type III estimates the finite speed of a thermal wave. Othman et al. [14] studied the effect of diffusion on the two-dimensional problem of generalized Thermoelasticity with G-N theory. The linear theory of elasticity has a major role to analyze the stress for engineering structure materials. In modern time man-made materials have demand so much in engineering, geological and biological fields due to their excellent strength capacity, lesser weight and low price. A transversely isotropic material such as Epoxy, Copper, Cobalt, Bakelite, etc. having physical properties that are symmetric about an axis that is normal to the plane of isotropy. Geological layers of rocks are considered as transversely isotropic material for its effective properties of such layers. In the last few decades, considerable attempts have been made to study the failure of cracks in solids. Our geological earth structure builds with different layers of rocks, some of which are transversely isotropic in nature. Due to the collision of two plates and earthquakes, many finite cracks may be formed in that layers. We have considered this configuration in Thermoelasticity. The phenomena with finite cracks in the transversely isotropic medium have come across in the technologies of space vehicles, missile industry, ship factories and in the industry of small electronic components. There are some numerical techniques such as FEM, FDM and BEM for finding thermal stress in this type of problem. Normal mode analysis is an analytical method that determines exact expressions of stress, temperature and displacement components. We formulated this problem keeping all these things in mind. Sarkar et al. [15] have studied a two-temperature problem under G-N theory with a mode-I crack in fiber-reinforced thermoelastic medium. Thermoelastic problem has been discussed in transversely isotropic medium by Kaur [16-17] and Sur [18]. Reflection phenomena of plane waves have been discussed by Singh [19-20]. Sharma et al. [21-23] have discussed the thermomechanical interactions in transversely isotropic medium due to rotation, hall current and inclined load for two temperatures Thermoelasticity theory. Two temperature Green-Naghdi theory of type –III in a transversely isotropic thick plate is studied by Lata and Kaur [24].

In this paper, we used the normal mode analysis to solve the problem of linearized Mode-I finite crack in a thermoelastic transversely isotropic medium under G-N theory of type III. The analytical expression for the temperature, displacement components and thermal stresses are derived, and represented by graphs for different depths. A comparison has been shown within G-N theory of type III, type II and type I.

2 FORMULATIONS OF THE PROBLEM

Let us assume a two dimensional problem in a linear, infinite homogeneous thermoelastic transversely isotropic medium $-\infty < x < \infty$ and $-\infty < z < \infty$ with a finite crack located at $|x| \leq a, y = 0$ (Fig. 1) in Cartesian co-ordinate frame (x, y, z). The linearized mode-I crack is subjected to a known temperature and normal stress distributions. Following Sarkar et al. [15] and Youssef [25], in the absence of body forces, the governing equations are:

(i) Equation of motion

$$\rho \ddot{u}_i = \tau_{ij,j} , \tag{1}$$

(ii) The heat conductive equation of two temperature model under G-N theory of type III

$$K^*_{ij} \xi_{,ij} + K_{ij} \dot{\xi}_{,ij} = \beta_{ij} T_0 \ddot{e}_{ij} + \rho C_E \ddot{T} , \tag{2}$$

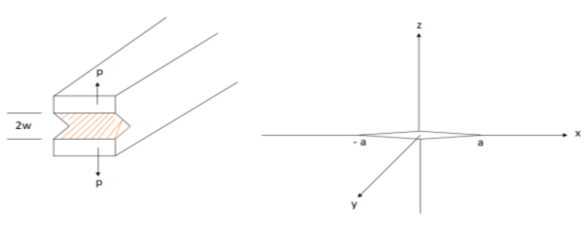

Fig.1
Geometry of the problem.

where

$$T = \xi - a_0 \xi_{,ij} , \tag{3}$$

$$\beta_{ij} = \beta_i \delta_{ij} , K_{ij} = K_i \delta_{ij} , K_{ij}^* = K_i^* \delta_{ij} , \tag{4}$$

δ_{ij} is the Kronecker delta and

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad (i, j = 1,2,3). \tag{5}$$

Here β_{ij} is the thermal elastic coupling tensor, T_0 is the reference temperature, T is the absolute temperature, ξ is the conductive temperature, C_E is the specific heat at constant strain, u_i are the displacement components, ρ is the density of the medium, K_{ij}^* is the materialistic constant, K_{ij} is the thermal conductivity and τ_{ij} are the stress tensor.

We considered this problem in xz -plane and all the functions will be dependent on time *t*. So the displacement components u_i will have the form $u_1 = u(x, z, t)$, $u_2 = v(x, z, \mathrm{t}) = 0$ and $u_3 = w(x, z, t)$. The thermal stress components in transversely isotropic medium are:

$$\tau_{xx} = C_{11}\partial_x u + C_{13}\partial_z w - \beta_1 T , \tag{6}$$

$$\tau_{zz} = C_{13}\partial_x u + C_{33}\partial_z w - \beta_3 T , \tag{7}$$

and

$$\tau_{xz} = C_{44}\left(\partial_z u + \partial_x w\right). \tag{8}$$

∂_x and ∂_z denote the partial derivative with respect to x and z variables respectively. Moreover partial derivative with respect to time t is presented by ∂_t . With the help of Eqs. (3), (6), (7) and (8) equation of motion (1) become:

$$C_{11}\partial_{xx}u + \left(C_{13} + C_{44}\right)\partial_{xz}w + C_{44}\partial_{zz}u - \beta_1\left(1 - a_0\nabla^2\right)\partial_x \xi = \rho\partial_{tt}u \tag{9}$$

and

$$C_{44}\partial_{xx}w + \left(C_{13} + C_{44}\right)\partial_{xz}u + C_{33}\partial_{zz}w - \beta_3\left(1 - a_0\nabla^2\right)\partial_z \xi = \rho\partial_{tt}w . \tag{10}$$

The heat conduction equation can be written as:

$$K_1^*\partial_{xx}\xi + K_3^*\partial_{zz}\xi + K_1\partial_{xxt}\xi + K_3\partial_{zzt}\xi = T_0\left(\beta_1\partial_{xtt}u + \beta_3\partial_{ztt}w\right) + \rho C_E\left(1 - a_0\nabla^2\right)\partial_{tt}\xi , \tag{11}$$

where $\nabla^2 \equiv \partial_{xx} + \partial_{zz}$. Considering the scalar potential functions $\phi(x, z, t)$ and $\psi(x, z, t)$ in the non-dimensional form:

$$u = \partial_x\phi + \partial_z\psi \text{ and } w = \partial_z\phi - \partial_x\psi . \tag{12}$$

Eqs. (9-10) became

$$C_{11}\partial_{xx}\phi+\left(C_{13}+2C_{44}\right)\partial_{zz}\phi-\beta_1\left(1-a_0\nabla^2\right)\xi-\rho\partial_{tt}\phi=0 \tag{13}$$

and

$$\left(C_{11}-C_{13}-C_{44}\right)\partial_{xx}\psi+C_{44}\partial_{zz}\psi-\rho\partial_{tt}\psi=0\,. \tag{14}$$

The solution of the Eqs. (11), (13) and (14) can be broken up in terms of normal mode (Othman [5] and Atwa [26]) and written in the following form:

$$\left[\phi,\psi,\xi,T,\tau_{ij}\right](x,z,t)=\left[\phi',\psi',\xi',T',\tau'_{ij}\right](x)e^{\omega t+ikz}\,, \tag{15}$$

where $\left[\phi',\psi',\xi',T',\tau'_{ij}\right](x)$ are the amplitudes of the functions $\left[\phi,\psi,\xi,T,\tau_{ij}\right](x,z,t)$, ω is the angular frequency and k is the wave number in the z -direction.

Substituting Eq. (15) in Eqs. (13), (14) and (11) we obtain,

$$\left(\delta_1 D^2-\delta_2\right)\phi'+\left(\delta_3 D^2-1\right)\xi'=0\,, \tag{16}$$

$$\left(D^2-\delta_4\right)\psi'=0 \tag{17}$$

and

$$\left(\varepsilon_1 D^2-\varepsilon_3\right)\xi'=\left(\varepsilon_1^* D^2-\varepsilon_3^*\right)\phi'+\varepsilon_2 D\psi'\,, \tag{18}$$

where $\delta_1=\dfrac{C_{11}}{\beta_1\left(1+a_0k^2\right)}$, $\delta_2=\dfrac{\left(C_{13}+2C_{44}\right)k^2+\rho\omega^2}{\beta_1\left(1+a_0k^2\right)}$, $\delta_3=\dfrac{a_0}{1+a_0k^2}$, $\delta_4=\dfrac{C_{44}k^2+\rho\omega^2}{C_{11}-C_{13}-C_{44}}$, $\varepsilon_1=K_1^*+\omega K_1+\rho C_E a_0\omega^2$, $\varepsilon_3=K_3^*k^2+\omega k^2K_3+\rho C_E\omega^2+\rho C_E a_0k^2\omega^2$, $\varepsilon_1^*=T_0\beta_1\omega^2$, $\varepsilon_3^*=T_0\beta_3\omega^2k^2$, $\varepsilon_2=ikT_0\omega^2\left(\beta_1-\beta_3\right)$, $D\equiv\dfrac{d}{dx}$ and $D^2\equiv\dfrac{d^2}{dx^2}$.

The solution of Eq. (17), bounded as $x\to\infty$, has the form

$$\psi'=A_3e^{-m_3x}\,, \tag{19}$$

where A_3 is arbitrary constant to be determined by the boundary conditions and m_3 is the root of the characteristic equation of Eq. (17).

Eliminating ϕ' and ξ' from the Eqs. (16) and (18), we obtain the fourth order ODE for ϕ' and ξ' in the following form:

$$\left(D^4-PD^2+Q\right)\left(\phi',\xi'\right)=A_3\varepsilon e^{-m_3x}\,, \tag{20}$$

where $P=\dfrac{\varepsilon_1\delta_2+\varepsilon_3\delta_1+\varepsilon_1^*+\delta_3\varepsilon_3^*}{\varepsilon_1\delta_1+\varepsilon_1^*\delta_3}$, $Q=\dfrac{\varepsilon_3\delta_2+\varepsilon_3^*}{\varepsilon_1\delta_1+\varepsilon_1^*\delta_3}$ and $\varepsilon=\dfrac{m_3^3\delta_3\varepsilon_2-m_3\varepsilon_2}{\varepsilon_1\delta_1+\varepsilon_1^*\delta_3}$.

We write the solution (bounded for $x\to\infty$) of Eq. (20) in the following form:

$$\phi'=A_1e^{-m_1x}+A_2e^{-m_2x}+\frac{A_3\varepsilon e^{-m_3x}}{m_3^4-Pm_3^2+Q}\,, \tag{21}$$

and

$$\xi' = A_1' e^{-m_1 x} + A_2' e^{-m_2 x} + \frac{A_3 \varepsilon e^{-m_3 x}}{m_3^4 - P m_3^2 + Q}, \tag{22}$$

where A_1, A_2, A_1' and A_2' are some parameters and $m_i^2\left(i = 1,2\right)$ are the roots of the characteristic Eq. (20).

Applying Eqs. (21) and (22) in Eq. (16), we acquired the relation

$$\xi' = J_{11} A_1 e^{-m_1 x} + J_{12} A_2 e^{-m_2 x} + J_{13} A_3 e^{-m_3 x}, \tag{23}$$

where $J_{11} = \dfrac{\delta_1 m_1^2 - \delta_2}{1 - \delta_3 m_1^2}$, $J_{12} = \dfrac{\delta_1 m_2^2 - \delta_2}{1 - \delta_3 m_2^2}$, $J_{13} = \dfrac{\left(\delta_1 m_3^2 - \delta_2\right)\varepsilon}{\left(1 - \delta_3 m_3^2\right)\left(m_3^4 - P m_3^2 + Q\right)}$.

Using Eq. (23) in the Eq. (3), we obtain the expression of T as:

$$T = \left(H_{11} A_1 e^{-m_1 x} + H_{12} A_2 e^{-m_2 x} + H_{13} A_3 e^{-m_3 x}\right) e^{\omega t + ikz}, \tag{24}$$

where $H_{11} = \dfrac{\delta_1 m_1^2 - \delta_2}{1 - \delta_3 m_1^2}\left(1 - a_0 m_1^2 + a_0 k^2\right)$, $H_{12} = \dfrac{\delta_1 m_2^2 - \delta_2}{1 - \delta_3 m_2^2}\left(1 - a_0 m_2^2 + a_0 k^2\right)$ and $H_{13} = \dfrac{\left(\delta_1 m_3^2 - \delta_2\right)\varepsilon}{\left(1 - \delta_3 m_3^2\right)\left(m_3^4 - P m_3^2 + Q\right)}\left(1 - a_0 m_3^2 + a_0 k^2\right)$.

The displacement and stress components in generalized thermoelastic transversely isotropic medium are obtained with the help of Eqs. (19), (21), and (24)

$$u'(x) = -m_1 A_1 e^{-m_1 x} - m_2 A_2 e^{-m_2 x} - \frac{m_3 \varepsilon}{m_3^4 - P m_3^2 + Q} A_3 e^{-m_3 x} + ik A_3 e^{-m_3 x}, \tag{25}$$

$$w'(x) = ik A_1 e^{-m_1 x} + ik A_2 e^{-m_2 x} + \frac{ik\,\varepsilon}{m_3^4 - P m_3^2 + Q} A_3 e^{-m_3 x} + m_3 A_3 e^{-m_3 x}, \tag{26}$$

$$\tau'_{zz}(x) = a_1 A_1 e^{-m_1 x} + a_2 A_2 e^{-m_2 x} + a_3 A_3 e^{-m_3 x}, \tag{27}$$

$$\tau'_{xx}(x) = a_4 A_1 e^{-m_1 x} + a_5 A_2 e^{-m_2 x} + a_6 A_3 e^{-m_3 x}, \tag{28}$$

and

$$\tau'_{xz}(x) = b_1 A_1 e^{-m_1 x} + b_2 A_2 e^{-m_2 x} + b_3 A_3 e^{-m_3 x}, \tag{29}$$

where, $a_1 = C_{13} m_1^2 - k^2 C_{33} - \beta_3 H_{11}$, $a_2 = C_{13} m_2^2 - k^2 C_{33} - \beta_3 H_{12}$, $a_3 = \dfrac{m_3^2 \varepsilon C_{13}}{m_3^4 - P m_3^2 + Q} - \dfrac{k^2 \varepsilon C_{33}}{m_3^4 - P m_3^2 + Q} - ikC_{13} m_3 + ikC_{33} m_3 - \beta_3 H_{13}$,

$a_4 = C_{11} m_1^2 - k^2 C_{13} - \beta_1 H_{11}$, $a_5 = C_{11} m_2^2 - k^2 C_{13} - \beta_1 H_{12}$, $a_6 = \dfrac{m_3^2 \varepsilon C_{11}}{m_3^4 - P m_3^2 + Q} - \dfrac{k^2 \varepsilon C_{13}}{m_3^4 - P m_3^2 + Q} - ikC_{11} m_3 + ikC_{13} m_3 - \beta_1 H_{13}$,

$b_1 = -2iC_{44} k m_1$, $b_2 = -2iC_{44} k m_2$ and $b_3 = -\dfrac{2iC_{44} k m_3 \varepsilon}{m_3^4 - P m_3^2 + Q} - \left(m_3^2 + k^2\right) C_{44}$.

3 BOUNDARY CONDITIONS AND APPLICATIONS

The plane boundary subjects to a direct normal point force and the boundary surface is isothermal. The mechanical and thermal boundary conditions on the plane $y = 0$ and in the midpoint of the crack at $x = 0$ are shown in Fig. 1.

$$\tau_{zz}(0,z,t) = -p(z,t), \tag{30}$$

$$\tau_{xz}(0,z,t) = 0, \tag{31}$$

and

$$\xi(0,z,t) = f(z,t). \tag{32}$$

Using the above boundary conditions, we obtain the following relations

$$a_1A_1 + a_2A_2 + a_3A_3 = -p^*, \tag{33}$$

$$b_1A_1 + b_2A_2 + b_3A_3 = 0, \tag{34}$$

and

$$J_{11}A_1 + J_{12}A_2 + J_{13}A_3 = f^*. \tag{35}$$

After applying the Matrix inversion method for the system of Eqs. (33-35), we get the unknown coefficients

$$A_1 = \frac{1}{\Delta}\left[p^*\left(b_3J_{12} - b_2J_{13}\right) + f^*\left(b_3a_2 - a_3b_2\right)\right],$$
$$A_2 = \frac{1}{\Delta}\left[p^*\left(b_1J_{13} - b_3J_{11}\right) + f^*\left(b_1a_3 - a_1b_3\right)\right],$$

and

$$A_3 = \frac{1}{\Delta}\left[p^*\left(b_2J_{11} - b_1J_{12}\right) + f^*\left(b_2a_1 - a_2b_1\right)\right],$$

where

$$\Delta = a_1\left(b_2J_{13} - b_3J_{12}\right) - a_2\left(b_1J_{13} - b_3J_{11}\right) + a_3\left(b_1J_{12} - b_2J_{11}\right).$$

4 NUMERICAL RESULTS AND DISCUSSIONS

We represent numerical results graphically for displacement, stress and absolute temperature of a Mode-I finite crack in thermoelastic transversely isotropic medium. We considered the following Cobalt material data (Dhaliwal [27]) to depict the impact of GN-I, GN-II, and GN-III theories of thermoelasticity.

$$C_{11} = 3.07\times10^{11}Nm^{-2},\ C_{13} = 1.027\times10^{10}Nm^{-2},\ C_{33} = 3.581\times10^{11}Nm^{-2},\ C_{44} = 1.51\times10^{11}Nm^{-2}.$$
$$C_E = 4.27\times10^2 jkg^{-1}\deg^{-1},\ \beta_1 = 7.04\times10^6 Nm^{-2}\deg^{-1},\ \beta_3 = 6.90\times10^6 Nm^{-2}\deg^{-1},\ \rho = 8.836\times10^3 kgm^{-3},$$
$$K_1 = 0.690\times10^2 Wm^{-1}\deg^{-1},\ K_3 = 0.690\times10^2 Wm^{-1}K^{-1},\ K_1^* = 0.02\times10^2 N\sec^{-2}\deg^{-1},\ K_3^* = 0.04\times10^2 N\sec^{-2}\deg^{-1}.$$

Moreover, the following data has been taken into consideration (Othman [5] and Sarkar [15]):

$$\omega = 2+i,\ T_0 = 293K,\ k = 2,\ t = 0.1,\ p^* = 4,\ f^* = 0.5 \text{ and } a_0 = 2.7.$$

Fig. 2 consists of six figures with 3 curves in each figure shows the influence of G-N theories of type-I $\left(K_{ij}^{*}=0\right)$, type-II $\left(K_{ij}=0\right)$ and type-III $\left(K_{ij}\neq 0\right)$ for displacement thermal stress and absolute temperature variables at $z=0$ and $t=0.1$. Fig.3 consists six figures exhibiting the effect of vertical depth of crack for various components such as displacement, thermal stress and absolute temperature.

In Fig. 2(a) horizontal displacement (u) has been plotted against crack length (x). Horizontal displacement (u) has a maximum value at almost center of the crack and u falls down just near the edge of the crack. In Fig. 2(b) vertical displacement (w) has been plotted against crack's length (x). Vertical displacement (w) has a sharp increment near the edge of the crack for type I and type III theories, while w decreases near the tip of the crack for type II theory. In Fig. 2(c) absolute temperature (T) has been mapped against crack's length (x).Temperature (T) decreases with increase in crack's length for type I and type III, while type II shows the reverse effect. Temperature (T) converges to zero for each G-N theory. Fig. 2(d) is the plot of horizontal stress (τ_{xx}) versus displacement(x). It is evident from the graph that τ_{xx} decreased at the tip of the crack and start increasing in the context of type I, type II and type III at the end of the crack. Fig. 2(e) is the map of stress (τ_{xz}) versus crack's length(x). τ_{xz} decreases from $x=0$ but after some point it increases and tends to zero for type II and type III; whereas type I shows the reverse effect. In Fig. 2(f) vertical stress (τ_{zz}) has been mapped against x . τ_{zz} increases with increase in x for each instances and tends to zero with increasing value of x .

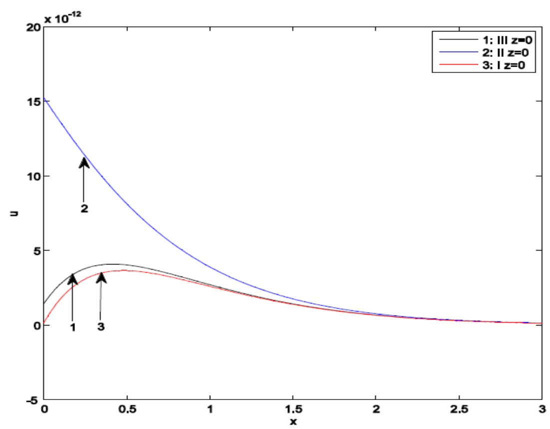

Fig.2(a)
Horizontal displacement versus crack length.

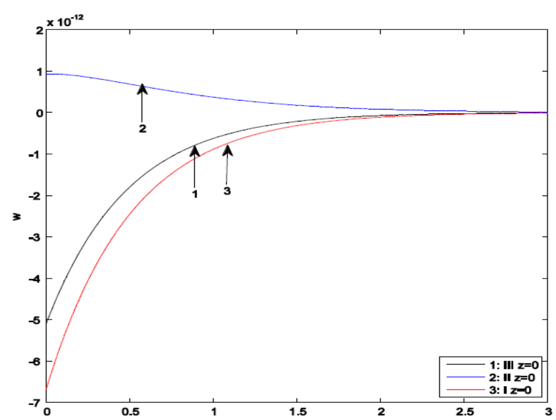

Fig.2(b)
Vertical displacement versus crack length.

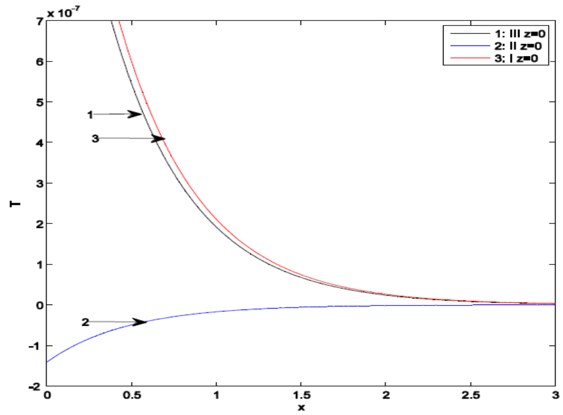

Fig.2(c)
Temperature distribution versus crack length.

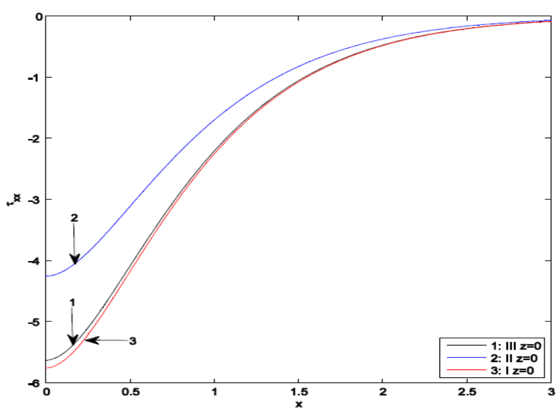

Fig.2(d)
Horizontal stress versus crack length.

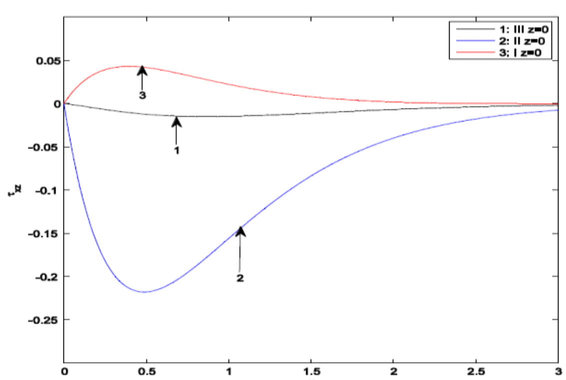

Fig.2(e)
Shear stress versus crack length.

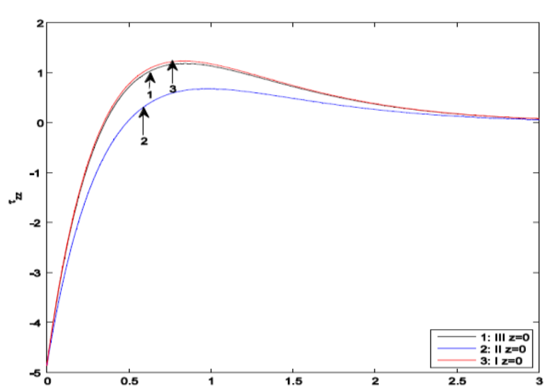

Fig.2(f)
Vertical stress versus crack length.

In Fig. 3(a) horizontal displacement (u) has been mapped against *x*. It is clear from the figure that horizontal displacement increases near the center position of the crack. Horizontal displacement converges to zero with increase in crack length. In Fig. 3(b) vertical displacement (w) has been plotted against *x*. Vertical displacement increases near the center of the crack and there is no vertical displacement near the edge of the crack. Fig. 3(c) depicts the distribution of temperature (T) against crack's length *x*. Absolute temperature decreases with increase in distance *x*. In Fig. 3(d) horizontal stress (τ_{xx}) has been plotted against crack length (x). Horizontal stress increases with increase of crack length (x) and goes to zero at the end of the crack. The impact of absolute horizontal stress is higher for lesser values of *z*. Fig. 3(e) reveals the effect of stress component (τ_{xz}) for different value of *z*. At the center of the crack there is no significant effect of τ_{xz} but with increase in *z*, τ_{xz} shows more influences on the crack. τ_{xz} is decreasing in nature near the midpoint of the crack but for $x > 0.5$, stress component τ_{xz} increases and tends to zero with increase in *x*. In Fig. 3(f) vertical stress (τ_{zz}) has been plotted against *x*. It is notable that τ_{zz} increased in the range $x < 0.5$ and after that it approaches to zero in a constant mode. The effect of τ_{zz} is more prominent for higher values of *z*.

All the graphical results are similar in nature with graphs exhibited by Othman and Atwa [5].

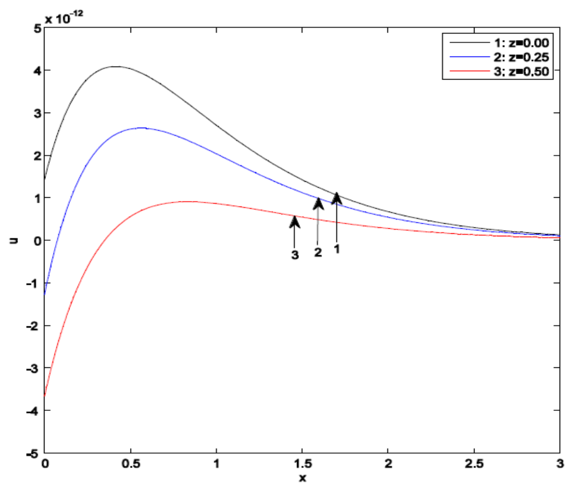

Fig.3(a)
Horizontal displacement versus crack length.

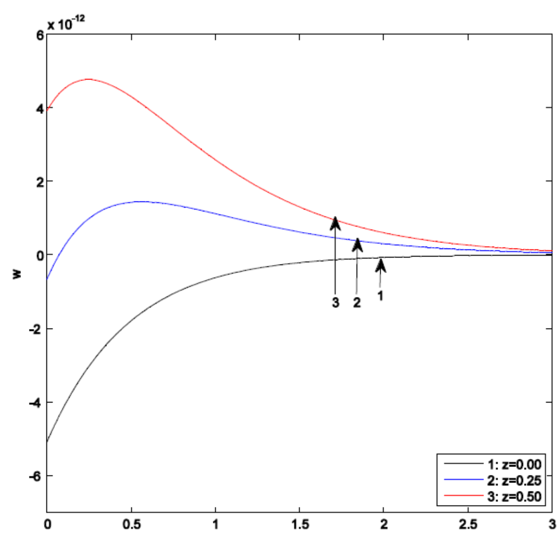

Fig.3(b)
Vertical displacement versus crack length.

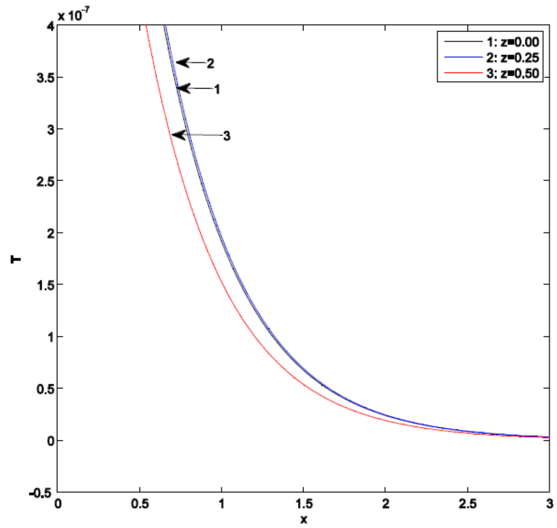

Fig.3(c)
Temperature distribution versus crack length.

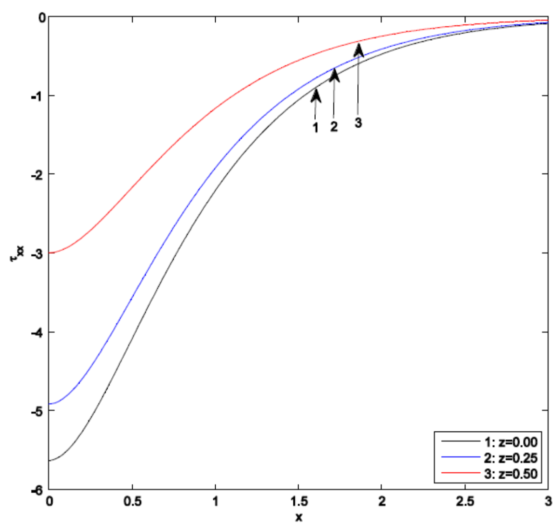

Fig.3(d)
Horizontal stress versus crack length.

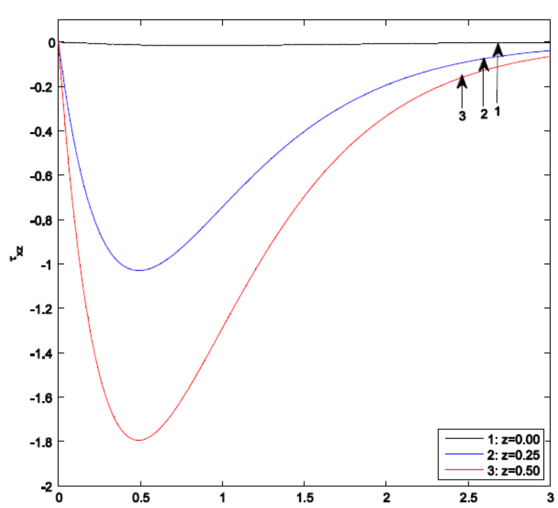

Fig.3(e)
Shear stress versus crack length.

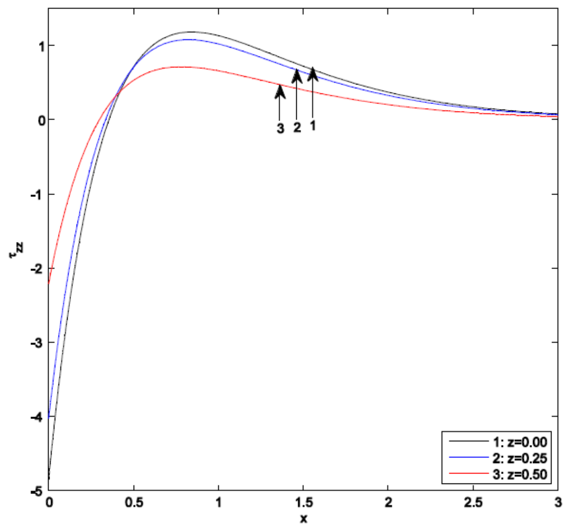

Fig.3(f)
Vertical stress versus crack length.

5 CONCLUSIONS

The objective of this current study is to unfold the nature of displacement, temperature, and stress components of a Mode-I crack in a thermoelastic transversely isotropic medium with the help of Green-Naghdi theory. Analytical solution to this problem has been obtained by normal mode analysis. The key points from this paper are noted as follows:

- There is no horizontal and vertical displacement near the edge of the crack for each type of G-N theory.
- Impact of temperature (T) is not significant for type II G-N theory.
- All the thermal stress components converge to zero near the edge of the crack for each G-N type theory.
- Impact of temperature distribution is zero at the tip position of the crack.
- The crack dimensions are crucial to explain the mechanical shape of solid.
- The cracks are static in nature but thermal stress is forced to transmit such cracks.
- This analytical solution demands its accuracy and stability to a wide range of problems in thermoelasticity.

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