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Three Dimensional Thermal Shock Problem in Magneto-Thermoelastic Orthotropic Medium

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ABSTRACT

The paper is concerned with the study of magneto-thermoelastic interactions in three dimensional thermoelastic medium under the purview of three-phase-lag model of generalized thermoelasticity. The medium under consideration is assumed to be homogeneous orthotropic medium. The fundamental equations of the three-dimensional problem of generalized thermoelasticity are obtained as a vector-matrix differential equation form by employing normal mode analysis which is then solved by eigenvalue approach. Stresses and displacements are presented graphically for different thermoelastic models.

Keywords: Eigenvalue approach; Orthotropic medium; Three-phase-lag model; Magnetic effect.

1 INTRODUCTION

GENERALIZED thermoelasticity theory is developed to overcome the paradox of infinite speed of thermal wave inherent in the classical coupled thermoelasticity theory. Lord and Shulman [1] formulated the generalized thermoelasticity theory by introducing one relaxation time which is known as LS model. Green and Lindsay [2] introduced GL theory by incorporating two relaxation times. Later Green and Naghdi [3, 4, 5] developed three models for generalized thermoelasticity of homogeneous isotropic materials, which are labeled as G-N models I, II, III. Detailed information regarding these theories is available in [6, 7, 8]. Tzou [9] introduced two-phase lags to both the heat flux vector and the temperature gradient and considered as constitutive equation to describe the lagging behavior in the heat conduction in solids. Roy Choudhuri [10] has established a generalized mathematical model of a coupled thermoelasticity theory that includes three-phase-lags in the heat flux vector, the temperature gradient and in the thermal displacement gradient. The interplay of the Maxwell electromagnetic filed with the motion of deformable solids is largely being undertaken by many investigators [11-14] owing to the possibility of its application to geophysical problems and certain topics in optics and acoustics. El-Karamany and Ezzat [15] considered thermal shock problem in generalized thermoelasticity under four theories. Sherief et al. [16] discussed stochastic thermal shock in generalized thermoelasticity and Ezzat and Youssef [17] investigated three

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dimensional thermal shock problem of generalized thermoelastic half-space. Kalkal and Deswal [18] considered the effects of phase lags on three dimensional wave propagation with temperature dependent properties. El-Karamany and Ezzat [19] discussed three-phase-lag linear micro polar thermoelasticity theory. Ezzat et al. [20] proposed fractional order theory in thermoelastic solid with three-phase-lag heat transfer. Said and Othman [21] discussed the Effects of gravitational and hydrostatic initial stress on a two-temperature fiber-reinforced thermoelastic medium for three-phase-lag model. Lofty [22] studied two temperature generalized magneto-thermoelastic interactions in an elastic medium under three theories. Sarkar and Lahiri [23] considered electro magneto-thermoelastic interactions in an orthotropic slab with two thermal relaxation times. Das and Bhakta [24] proposed eigen function expansion method to the solution of simultaneous equations and its application in mechanics. Ezzat [25] considered the relaxation effects of the volume properties of electrically conducting viscoelastic material. Ezzat [26] discussed fundamental solution in generalized magneto-thermoelasticity with two relaxation times for perfect conductor cylindrical region. Ezzat et al. [27] studied electro-thermoelasticity theory with memory-dependent heat transfer.

The present article deals with a three dimensional electro-magneto-thermoelastic coupled problem for homogeneous orthotropic thermally and electrically conducting solid whose surface is subjected to time dependent thermal shock. The normal mode analysis and eigenvalue approach is used to solve the problem. Numerical results for the displacements and thermal stress distribution are presented graphically for dual-phase lag (DPL) model, Green-Naghdi type-III model (GN-III) and three-phase-lag model (TPL).

2 FORMULATION OF THE PROBLEM

The homogeneous orthotropic medium is supposed to be initially unstrained and unstressed. The basic equations of linear magneto-thermoelasticity with three-phase-lag model are as follows:

The equations of motion

$$\sigma_{ij,j} + \left(\vec{J} \times \vec{B}\right)_i = \rho \ddot{u}_i \tag{1}$$

Maxwell's equations (in absence of the displacement current and charge density)

$$curl\ \vec{H} = \vec{J},\ curl\ \vec{E} = -\frac{\partial \vec{B}}{\partial t},\ div\ \vec{B} = 0,\ \vec{B} = \mu_e \vec{H} \tag{2}$$

The modified Ohm's law is

$$\vec{J} = \sigma\left[\vec{E} + \left(\frac{\partial \vec{u}}{\partial t} \times \vec{B}\right)\right] \tag{3}$$

where $\vec{H}$ = the total magnetic field vector $= \left(H_x, H_y, H_z\right)$, $\vec{B}$ = magnetic inductance vector $= \left(B_x, B_y, B_z\right)$, $\vec{E}$ = electric field vector $= \left(E_x, E_y, E_z\right)$, μ_e = magnetic permeability of the medium, σ = electric conductivity of the medium, ρ = constant mass density, σ_{ij} = component of stress tensor, $i, j = x, y, z$, $\vec{u}$ = displacement vector $= \left(u, v, w\right)$. If we take $\vec{H} = \left(H_x, H_y, 0\right)$, we get from Eqs. (2) and (3), after neglecting second order differentiation of H_x and H_y,

$$-\mu_e \frac{\partial H_x}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z} \tag{4}$$

$$-\mu_e \frac{\partial H_y}{\partial t} = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \tag{5}$$

Eqs. (4) and (5) can be linearized by setting $H_x = H_0 + h_x$ and $H_y = H_0 + h_y$ where h_x and h_y denote change in the basic magnetic field H_0 (called the perturbed field) and then neglecting product terms with h_x and h_y .

After linearization, Eqs. (4) and (5) with the help of (2) become

$$h_x = -H_0\left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} + \frac{\partial w}{\partial z}\right) \tag{6}$$

$$h_y = -H_0\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} + \frac{\partial w}{\partial z}\right) \tag{7}$$

The thermal stresses in an orthotropic infinite elastic solid subject to plane strain in three dimensions are

$$\begin{aligned}
\sigma_{xx} &= c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} - \beta_1 T \\
\sigma_{yy} &= c_{12}\frac{\partial u}{\partial x} + c_{22}\frac{\partial v}{\partial y} + c_{23}\frac{\partial w}{\partial z} - \beta_2 T \\
\sigma_{zz} &= c_{13}\frac{\partial u}{\partial x} + c_{23}\frac{\partial v}{\partial y} + c_{33}\frac{\partial w}{\partial z} - \beta_3 T \\
\sigma_{yz} &= c_{44}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\
\sigma_{xz} &= c_{55}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \\
\sigma_{xy} &= c_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)
\end{aligned} \tag{8}$$

where c_{ij} are elastic constants of the orthotropic material and $\beta_1, \beta_2, \beta_3$ are thermal moduli along x , y and z axis respectively.

From Eqs. (1), (6), (7) and (8), after neglecting higher order of small quantities, we get

$$\begin{aligned}
&c_{11}\frac{\partial^2 u}{\partial x^2} + c_{66}\frac{\partial^2 u}{\partial y^2} + c_{55}\frac{\partial^2 u}{\partial z^2} + \left(c_{12} + c_{66}\right)\frac{\partial^2 v}{\partial x\,\partial y} + \left(c_{13} + c_{55}\right)\frac{\partial^2 w}{\partial x\,\partial z} - \mu_e H_0^2\left(\frac{\partial^2 w}{\partial x\,\partial z} - \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 w}{\partial y\,\partial z}\right) \\
&-\beta_1\frac{\partial T}{\partial x} = \rho\ddot{u}
\end{aligned} \tag{9}$$

$$\begin{aligned}
&c_{66}\frac{\partial^2 v}{\partial x^2} + c_{22}\frac{\partial^2 v}{\partial y^2} + c_{44}\frac{\partial^2 v}{\partial z^2} + \left(c_{12} + c_{66}\right)\frac{\partial^2 u}{\partial x\,\partial y} + \left(c_{23} + c_{44}\right)\frac{\partial^2 w}{\partial y\,\partial z} - \mu_e H_0^2\left(\frac{\partial^2 w}{\partial x\,\partial z} - \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 w}{\partial y\,\partial z}\right) \\
&-\beta_2\frac{\partial T}{\partial y} = \rho\ddot{v}
\end{aligned} \tag{10}$$

$$\begin{aligned}
&c_{55}\frac{\partial^2 w}{\partial x^2} + c_{44}\frac{\partial^2 w}{\partial y^2} + c_{33}\frac{\partial^2 w}{\partial z^2} + \left(c_{13} + c_{55}\right)\frac{\partial^2 u}{\partial x\,\partial z} + \left(c_{23} + c_{44}\right)\frac{\partial^2 v}{\partial y\,\partial z} + \mu_e H_0^2\left(\frac{\partial^2 w}{\partial z^2} - \frac{\partial^2 v}{\partial x\,\partial z} + \frac{\partial^2 u}{\partial x\,\partial z} + \frac{\partial^2 v}{\partial y\,\partial z} - \frac{\partial^2 u}{\partial y\,\partial z} + \frac{\partial^2 w}{\partial z^2}\right) \\
&-\beta_3\frac{\partial T}{\partial z} = \rho\ddot{w}
\end{aligned} \tag{11}$$

where T is the temperature above reference temperature.

Equation of three-phase-lag model in orthotropic medium is

$$K_1\left(1+\tau_T\frac{\partial}{\partial t}\right)\frac{\partial^2\dot{T}}{\partial x^2}+K_2\left(1+\tau_T\frac{\partial}{\partial t}\right)\frac{\partial^2\dot{T}}{\partial y^2}+K_3\left(1+\tau_T\frac{\partial}{\partial t}\right)\frac{\partial^2\dot{T}}{\partial z^2}+K_1^*\left(1+\tau_v\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x^2}+K_2^*\left(1+\tau_v\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial y^2}$$
$$+K_3^*\left(1+\tau_v\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial z^2}=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[\rho C_e\ddot{T}+T_0\left(\beta_1\frac{\partial\ddot{u}}{\partial x}+\beta_2\frac{\partial\ddot{v}}{\partial y}+\beta_3\frac{\partial\ddot{w}}{\partial z}\right)\right] \tag{12}$$

where T_0 is the reference temperature, $K_i\ (i=1,2,3)$ is the thermal conductivity tensor, $K_i^*\ (i=1,2,3)$ are the material constant characteristic of the theory, τ_q, τ_T and τ_v are the phase lags for heat flux vector, temperature gradient and thermal displacement gradient respectively.

Maxwell's electromagnetic stress tensor $\bar{\sigma}_{ij}$ is given by

$$\bar{\sigma}_{ij}=\left[H_i h_j+H_j h_i-\left(\vec{H}.\vec{h}\right)\delta_{ij}\right]$$

3 BOUNDARY CONDITIONS

We consider the case where the surface of the three dimensional orthotropic medium is subjected to a time dependent thermal shock and the surface is traction free. Then we consider the case where the surface of medium is rigidly fixed and a thermal shock is applied on it.

In order to determine the parameters, we need to consider the following boundary conditions at $z=0$:

3.1 Case 1

Thermal boundary condition that the surface of the medium is subjected to a time dependent thermal shock

$$T(x,y,0,t)=F(t)H(a-|x|)H(b-|y|)$$

where H denotes Heaviside function.

Mechanical boundary condition that the surface to the medium is traction free

$$\sigma_{zz}(x,y,0,t)+\bar{\sigma}_{zz}(x,y,0,t)=\sigma_{yz}(x,y,0,t)+\bar{\sigma}_{yz}(x,y,0,t)=\sigma_{xz}(x,y,0,t)+\bar{\sigma}_{xz}(x,y,0,t)=0$$

3.2 Case 2

Thermal boundary condition that the surface of the medium subjected to a time dependent thermal shock

$$T(x,y,0,t)=F(t)H(a-|x|)H(b-|y|)$$

Mechanical boundary condition that the surface of the medium is rigidly fixed

$$u(x,y,0,t)=v(x,y,0,t)=w(x,y,0,t)=0$$

4 SOLUTION OF THE PROBLEM

We take the solutions of the Eqs. (9)-(12) in the following form:

$$(u,v,w,T)(x,y,z,t)=(\bar{u},\bar{v},\bar{w},\bar{T})(z)\exp\left[i(kx+ly-\omega t)\right] \tag{13}$$

where k and l are wave number along x and y axis respectively and ω is angular frequency.

$$c_{55}\frac{d^2\bar{u}}{dz^2}+\left[ik\left(c_{13}+c_{55}\right)-i\,\mu_e H_0^2\left(k-l\right)\right]\frac{d\bar{w}}{dz}+\left[\mu_e H_0^2\left(k^2+l^2\right)+\rho\omega^2-c_{11}k^2-c_{66}l^2\right]\bar{u}$$
$$-\left[\mu_e H_0^2\left(k^2+l^2\right)+\left(c_{12}+c_{66}\right)kl\right]\bar{v}-ik\,\beta_1\bar{T}=0 \tag{14}$$

$$c_{44}\frac{d^2\bar{v}}{dz^2}+\left[il\left(c_{23}+c_{44}\right)-i\,\mu_e H_0^2\left(k-l\right)\right]\frac{d\bar{w}}{dz}+\left[\mu_e H_0^2\left(k^2+l^2\right)-\left(c_{12}+c_{66}\right)kl\right]\bar{u}$$
$$-\left[\mu_e H_0^2\left(k^2+l^2\right)+\left(c_{22}l^2+c_{66}k^2\right)-\rho\omega^2\right]\bar{v}-il\,\beta_2\bar{T}=0 \tag{15}$$

$$\left(c_{33}+2\mu_e H_0^2\right)\frac{d^2\bar{w}}{dz^2}+\left[\left(c_{13}+c_{55}\right)ik+i\,\mu_e H_0^2\left(k-l\right)\right]\frac{d\bar{u}}{dz}+\left[\left(c_{23}+c_{44}\right)il-i\,\mu_e H_0^2\left(k-l\right)\right]\frac{d\bar{v}}{dz}$$
$$+\left[\rho\omega^2-\left(c_{55}k^2+c_{44}l^2\right)\right]\bar{w}-\beta_3\frac{d\bar{T}}{dz}=0 \tag{16}$$

$$\left(K_3^*\tau_2-K_3 i\,\tau_1\omega\right)\frac{d^2\bar{T}}{dz^2}+T_0\beta_3\omega^2\frac{d\bar{w}}{dz}+ikT_0\beta_1\omega^2\bar{u}+ilT_0\beta_2\omega^2\bar{v}+\left[K_1\tau_1 i\,\omega k^2+K_2\tau_1 il^2\omega-\tau_2\left(K_1^*k^2+K_2^*l^2\right)+\rho C_e\omega^2\right]\bar{T}=0 \tag{17}$$

where $\tau_1=\dfrac{1-i\,\omega\tau_T}{1-i\,\omega\tau_q-\dfrac{\omega^2\tau_q^2}{2}}$, $\tau_2=\dfrac{1-i\,\omega\tau_v}{1-i\,\omega\tau_q-\dfrac{\omega^2\tau_q^2}{2}}$.

T, u, v and w must be bounded at infinity so as to satisfy the regularity condition at infinity. So we assume that T, u, v and w as well as their derivatives vanish at infinity.

Eqs. (14)-(17) can now be written in the form of a vector matrix differential equation as follows:

$$\frac{d\vec{V}}{dz}=A\vec{V} \tag{18}$$

where $\vec{V}=\left[\bar{u},\bar{v},\bar{w},\bar{T},\dfrac{d\bar{u}}{dz},\dfrac{d\bar{v}}{dz},\dfrac{d\bar{w}}{dz},\dfrac{d\bar{T}}{dz}\right]^T$

The matrix $\tilde{A}$ is $\tilde{A}=\begin{bmatrix}\tilde{0} & \tilde{I}\\ \tilde{P} & \tilde{Q}\end{bmatrix}$

where

$$\tilde{P}=\begin{bmatrix}A_{51} & A_{52} & 0 & A_{54}\\ A_{61} & A_{62} & 0 & A_{64}\\ 0 & 0 & A_{73} & 0\\ A_{81} & A_{82} & 0 & A_{84}\end{bmatrix},\ \tilde{Q}=\begin{bmatrix}0 & 0 & A_{57} & 0\\ 0 & 0 & A_{67} & 0\\ A_{75} & A_{76} & 0 & A_{78}\\ 0 & 0 & A_{87} & 0\end{bmatrix},\ \tilde{0}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\ \tilde{I}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$A_{51}=\frac{\left[c_{11}k^2+c_{66}l^2-\rho\omega^2-\mu_e H_0^2\left(k^2+l^2\right)\right]}{c_{55}},\ A_{52}=\frac{\left(c_{12}+c_{66}\right)kl+\mu_e H_0^2\left(k^2+l^2\right)}{c_{55}},$$

$$A_{54}=\frac{ik\,\beta_1}{c_{55}},\ A_{57}=\frac{i\,\mu_e H_0^2\left(k-l\right)-ik\left(c_{13}+c_{55}\right)}{c_{55}},\ A_{61}=\frac{\left(c_{12}+c_{66}\right)kl-\mu_e H_0^2\left(k^2+l^2\right)}{c_{44}},$$

$$A_{62}=\frac{\left(c_{66}k^2+c_{22}l^2\right)+\mu_e H_0^2\left(k^2+l^2\right)-\rho\omega^2}{c_{44}},\ A_{64}=\frac{il\,\beta_2}{c_{44}},\ A_{67}=\frac{i\,\mu_e H_0^2\left(k-l\right)-\left(c_{23}+c_{44}\right)il}{c_{44}},$$

$$A_{73}=\frac{\left(c_{55}k^{2}+c_{44}l^{2}\right)-\rho\omega^{2}}{c_{33}+2\mu_e H_0^2},\ A_{75}=\frac{-\left[i\,\mu_e H_0^2\left(k-l\right)+ik\left(c_{13}+c_{55}\right)\right]}{c_{33}+2\mu_e H_0^2},\ A_{76}=\frac{i\,\mu_e H_0^2\left(k-l\right)-\left(c_{23}+c_{44}\right)il}{c_{33}+2\mu_e H_0^2},$$

$$A_{78}=\frac{\beta_3}{\left(c_{33}+2\mu_e H_0^2\right)},\ A_{81}=\frac{-ik\,\omega^2 T_0\beta_1}{\left(K_3^{*}\tau_2-i\,\omega\tau_1 K_3\right)},\ A_{82}=\frac{-il\,\omega^2 T_0\beta_2}{\left(K_3^{*}\tau_2-i\,\omega\tau_1 K_3\right)},$$

$$A_{84}=\frac{\left[K_1\tau_1 i\,\omega k^2+K_2\tau_1 il^2\omega-\tau_2\left(K_1^{*}k^2+K_2^{*}l^2\right)+\rho C_e\omega^2\right]}{\left(K_3^{*}\tau_2-i\,\tau_1\omega K_3\right)},\ A_{87}=\frac{\omega^2 T_0\beta_3}{\left(K_3^{*}\tau_2-i\,\tau_1\omega K_3\right)}$$

5 SOLUTION OF THE VECTOR MATRIX DIFFERENTIAL EQUATION

As for the solution of the Eq. (18), we follow the method of eigenvalue approach as in Das and Bhakta [24],The characteristic equation of the matrix $\tilde{A}$ takes the form

$$\lambda^8-B_1\lambda^6+B_2\lambda^4-B_3\lambda^2-B_4=0 \tag{19}$$

where

$$\begin{aligned}
&B_1=A_{78}A_{87}+A_{67}A_{76}+A_{57}A_{75}+A_{81}+A_{62}+A_{73}+A_{84},\\
&B_2=-A_{81}A_{54}-A_{81}A_{57}A_{78}-A_{82}A_{64}-A_{82}A_{67}A_{78}+A_{51}A_{78}A_{87}+A_{62}A_{78}A_{87}-A_{75}A_{54}A_{87}-A_{87}A_{64}A_{76}+A_{62}A_{73}\\
&+A_{51}A_{62}+A_{51}A_{73}+A_{51}A_{67}A_{76}-A_{52}A_{61}-A_{52}A_{67}A_{75}-A_{57}A_{61}A_{76}+A_{75}A_{62}+A_{84}A_{51}+A_{84}A_{73}+A_{84}A_{62}\\
&+A_{84}A_{67}A_{76}+A_{84}A_{57}A_{75},\\
&B_3=-A_{81}A_{54}A_{62}-A_{81}A_{54}A_{73}-A_{81}A_{54}A_{67}A_{76}-A_{81}A_{57}A_{62}A_{78}+A_{81}A_{57}A_{64}A_{76}+A_{52}A_{64}A_{81}+A_{81}A_{52}A_{67}A_{78}\\
&-A_{82}A_{64}A_{73}-A_{82}A_{51}A_{64}+A_{82}A_{54}A_{61}-A_{82}A_{51}A_{67}A_{78}+A_{82}A_{57}A_{61}A_{78}-A_{82}A_{57}A_{64}A_{75}+A_{82}A_{54}A_{75}A_{67}\\
&+A_{75}A_{52}A_{64}A_{87}-A_{75}A_{62}A_{54}A_{87}+A_{78}A_{87}A_{51}A_{62}-A_{78}A_{52}A_{61}A_{67}+A_{76}A_{61}A_{54}A_{87}-A_{76}A_{87}A_{51}A_{64}\\
&+A_{51}A_{62}A_{73}-A_{52}A_{61}A_{73}+A_{84}A_{62}A_{73}+A_{84}A_{51}A_{62}+A_{51}A_{84}A_{73}+A_{84}A_{51}A_{67}A_{76}-A_{84}A_{52}A_{61}-A_{52}A_{67}A_{75}A_{84}\\
&-A_{84}A_{57}A_{61}A_{76}+A_{84}A_{57}A_{75}A_{62},\\
&B_4=A_{73}\left\{A_{51}\left(A_{62}A_{84}-A_{64}A_{82}\right)-A_{52}\left(A_{61}A_{84}-A_{64}A_{81}\right)+A_{54}\left(A_{61}A_{82}-A_{62}A_{81}\right)\right\}
\end{aligned}$$

The roots of the characteristic Eq. (19) which are also the eigenvalues of the matrix $\tilde{A}$ are of the form

$$\lambda=\pm\lambda_1,\ \lambda=\pm\lambda_2,\ \lambda=\pm\lambda_3,\ \lambda=\pm\lambda_4 \tag{20}$$

The right and left eigenvectors $\vec{X}$ and $\vec{Y}$ of the matrix $\tilde{A}$ corresponding to the eigenvalue λ can be taken as follows:

$$\vec{X}=\left[x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\right]^T \tag{21}$$

$$\vec{Y}=\left[y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8\right] \tag{22}$$

where

$$\begin{aligned}
&x_1=-\lambda^4\left(A_{57}A_{78}+A_{54}\right)+\lambda^2\left(A_{54}A_{62}+A_{54}A_{73}-A_{52}A_{64}+A_{54}A_{67}A_{76}-A_{52}A_{67}A_{78}-A_{57}A_{64}A_{76}+A_{57}A_{62}A_{78}\right)\\
&+A_{52}A_{64}A_{73}-A_{54}A_{62}A_{73},\\
&x_2=-\lambda^4\left(A_{67}A_{78}+A_{64}\right)+\lambda^2\left(A_{64}A_{73}+A_{51}A_{67}A_{78}-A_{54}A_{67}A_{75}-A_{57}A_{61}A_{78}+A_{57}A_{75}A_{64}+A_{64}A_{51}-A_{61}A_{54}\right)\\
&-A_{73}A_{51}A_{64}+A_{73}A_{61}A_{54},\\
&x_3=-\lambda\left[\lambda^4A_{78}-\lambda^2\left(A_{51}A_{78}+A_{62}A_{78}-A_{64}A_{76}-A_{54}A_{75}\right)+\left(A_{51}A_{62}A_{78}-A_{51}A_{64}A_{76}+A_{61}A_{54}A_{76}-A_{52}A_{61}A_{78}+A_{52}A_{64}A_{75}-A_{54}A_{62}A_{75}\right)\right],
\end{aligned}$$

$$x_4=-\lambda^6+\lambda^4\left(A_{51}+A_{62}+A_{73}+A_{57}A_{75}+A_{67}A_{76}\right)-\lambda^2(A_{51}A_{62}+A_{51}A_{73}-A_{52}A_{61}+A_{62}A_{73}+A_{51}A_{67}A_{76}$$
$$-A_{52}A_{75}A_{67}+A_{57}A_{62}A_{75}-A_{57}A_{61}A_{76})+\left(A_{51}A_{62}A_{73}-A_{61}A_{52}A_{73}\right),$$
$$x_5=\lambda x_1,\ \ x_6=\lambda x_2,\ \ x_7=\lambda x_3,\ \ x_8=\lambda x_4$$

and

$$y_1=\lambda^7-\lambda^5\left(A_{62}+A_{73}+A_{84}+A_{57}A_{75}+A_{76}A_{67}+A_{87}A_{78}\right)+\lambda^3(-A_{62}A_{78}A_{67}-A_{82}A_{64}+A_{62}A_{87}A_{78}+A_{62}A_{73}$$
$$+A_{62}A_{84}+A_{84}A_{73}-A_{76}A_{87}A_{64}+A_{67}A_{76}A_{84}-A_{75}A_{52}A_{67}+A_{75}A_{62}A_{57}+A_{75}A_{57}A_{84}-A_{75}A_{54}A_{87})+\lambda A_{55}$$
$$\left(A_{52}A_{67}A_{84}-A_{52}A_{64}A_{87}-A_{62}A_{57}A_{84}+A_{62}A_{54}A_{87}+A_{82}A_{57}A_{64}-A_{82}A_{67}A_{54}\right),$$
$$y_2=\lambda^5A_{52}-\lambda^3(A_{52}A_{87}A_{78}+A_{52}A_{84}+A_{52}A_{73}-A_{82}A_{57}A_{78}-A_{82}A_{54}+A_{76}A_{57}A_{84}-A_{76}A_{54}A_{87}+A_{52}A_{67}A_{76}$$
$$-A_{62}A_{57}A_{76}-A_{76}A_{57}A_{84}+A_{76}A_{54}A_{87})+\lambda(A_{52}A_{84}A_{73}-A_{82}A_{54}A_{73}-A_{52}A_{67}A_{84}+A_{52}A_{64}A_{87}+A_{62}A_{57}A_{84}$$
$$-A_{62}A_{54}A_{87}-A_{82}A_{57}A_{64}+A_{82}A_{67}A_{54}),$$
$$y_3=A_{73}[\lambda^4A_{57}+\lambda^2\left(A_{52}A_{67}-A_{62}A_{57}-A_{57}A_{84}+A_{54}A_{87}\right)-(A_{52}A_{67}A_{84}-A_{52}A_{64}A_{87}-A_{62}A_{57}A_{84}+A_{62}A_{54}A_{87}$$
$$+A_{82}A_{57}A_{64}-A_{82}A_{67}A_{54})],$$
$$y_4=\lambda^5A_{54}-\lambda^3\left(-A_{78}A_{57}A_{84}+A_{78}A_{54}A_{87}+A_{62}A_{54}+A_{54}A_{73}-A_{52}A_{64}-A_{76}A_{57}A_{64}+A_{76}A_{67}A_{54}\right)+\lambda$$
$$\left(A_{78}A_{52}A_{67}A_{84}-A_{78}A_{52}A_{64}A_{87}-A_{78}A_{62}A_{57}A_{84}+A_{78}A_{62}A_{54}A_{87}+A_{78}A_{82}A_{57}A_{64}-A_{78}A_{82}A_{67}A_{54}-A_{52}A_{64}A_{73}+A_{62}A_{54}A_{73}\right),$$
$$y_5=\lambda^6-\lambda^4\left(A_{62}+A_{73}+A_{84}+A_{67}A_{76}+A_{78}A_{87}\right)+\lambda^2(A_{62}A_{73}+A_{84}A_{62}+A_{84}A_{73}+A_{84}A_{67}A_{76}+A_{87}A_{62}A_{78}$$
$$-A_{87}A_{64}A_{76}-A_{82}A_{64}-A_{82}A_{67}A_{78})-A_{84}A_{62}A_{73}+A_{82}A_{64}A_{73},$$
$$y_6=\lambda^4\left(A_{52}+A_{57}A_{76}\right)-\lambda^2\left(A_{52}A_{73}+A_{52}A_{84}+A_{84}A_{57}A_{76}-A_{54}A_{82}-A_{82}A_{57}A_{78}+A_{87}A_{52}A_{78}-A_{87}A_{54}A_{76}\right)$$
$$+A_{52}A_{73}A_{84}-A_{82}A_{54}A_{73},$$
$$y_7=\lambda^5A_{57}+\lambda^3\left(A_{67}A_{52}-A_{57}A_{62}-A_{84}A_{57}+A_{54}A_{87}\right)-\lambda(A_{52}A_{67}A_{64}-A_{52}A_{64}A_{87}-A_{62}A_{57}A_{84}+A_{62}A_{54}A_{87}$$
$$+A_{82}A_{57}A_{64}-A_{82}A_{67}A_{54}),$$
$$y_8=\lambda^4\left(A_{54}+A_{57}A_{78}\right)-\lambda^2\left(A_{54}A_{73}+A_{54}A_{62}-A_{64}A_{52}+A_{57}A_{62}A_{78}-A_{52}A_{67}A_{78}+A_{54}A_{76}A_{67}-A_{57}A_{64}A_{76}\right)$$
$$+A_{54}A_{62}A_{73}-A_{52}A_{64}A_{73}$$

For our further reference we shall use the following notations:

$$X_1=[X]_{\lambda=\lambda_1},\ X_2=[X]_{\lambda=-\lambda_1},\ X_3=[X]_{\lambda=\lambda_2},\ X_4=[X]_{\lambda=-\lambda_2},\ X_5=[X]_{\lambda=\lambda_3},\ X_6=[X]_{\lambda=-\lambda_3},$$
$$X_7=[X]_{\lambda=\lambda_4},\ X_8=[X]_{\lambda=-\lambda_4} \tag{23}$$

and

$$Y_1=[Y]_{\lambda=\lambda_1},\ Y_2=[Y]_{\lambda=-\lambda_1},\ Y_3=[Y]_{\lambda=\lambda_2},Y_4=[Y]_{\lambda=-\lambda_2},\ Y_5=[Y]_{\lambda=\lambda_3},\ Y_6=[Y]_{\lambda=-\lambda_3},$$
$$Y_7=[Y]_{\lambda=\lambda_4},\ Y_8=[Y]_{\lambda=-\lambda_4} \tag{24}$$

Assuming the regularity condition at infinity, the solution of the Eq. (18) can be written as:

$$\vec{V}=A_1X_2\,exp\left(-\lambda_1z\right)+A_2X_4\,exp\left(-\lambda_2z\right)+A_3X_6\,exp\left(-\lambda_3z\right)+A_4X_8\,exp\left(-\lambda_4z\right)$$

Now we get

$$\bar{u}(x,y,z,t)=\sum_{n=1}^{4}A_n[-\lambda_n^4\left(A_{57}A_{78}+A_{54}\right)+\lambda_n^2\left(A_{54}A_{62}+A_{54}A_{73}-A_{52}A_{64}+A_{54}A_{67}A_{76}-A_{52}A_{67}A_{78}-A_{57}A_{64}A_{76}+A_{57}A_{62}A_{78}\right)$$
$$+A_{52}A_{64}A_{73}-A_{54}A_{62}A_{73}]\,exp\left(-\lambda_nz\right) \tag{25}$$

$$\begin{aligned}\bar{v}\left(x, y, z, t\right) = \sum_{n=1}^{4} A_n [-\lambda_n^4 \left(A_{67}A_{78} + A_{64}\right) + \lambda_n^2 (A_{64}A_{73} + A_{51}A_{67}A_{78} - A_{54}A_{67}A_{75} - A_{57}A_{61}A_{78} + A_{57}A_{75}A_{64} \\ + A_{64}A_{51} - A_{61}A_{54}) - A_{73}A_{51}A_{64} + A_{73}A_{61}A_{54}] \exp\left(-\lambda_n z\right)\end{aligned} \tag{26}$$

$$\begin{aligned}\bar{w}\left(x, y, z, t\right) = \sum_{n=1}^{4} A_n [-\lambda_n^5 A_{78} + \lambda_n^3 \left(A_{51}A_{78} + A_{62}A_{78} - A_{64}A_{76} - A_{54}A_{75}\right) - \lambda_n (A_{51}A_{62}A_{78} - A_{51}A_{64}A_{76} + A_{61}A_{54}A_{76} \\ - A_{52}A_{61}A_{78} + A_{52}A_{64}A_{75} - A_{54}A_{62}A_{75})] \exp\left(-\lambda_n z\right)\end{aligned} \tag{27}$$

$$\begin{aligned}\bar{T}\left(x, y, z, t\right) = \sum_{n=1}^{4} A_n [-\lambda_n^6 + \lambda_n^4 \left(A_{51} + A_{62} + A_{73} + A_{57}A_{75} + A_{67}A_{76}\right) - \lambda_n^2 (A_{51}A_{62} + A_{51}A_{73} - A_{52}A_{61} + A_{62}A_{73} \\ + A_{51}A_{67}A_{76} - A_{52}A_{75}A_{67} + A_{57}A_{62}A_{75} - A_{57}A_{61}A_{76}) + A_{51}A_{62}A_{73} - A_{61}A_{52}A_{73}] \exp\left(-\lambda_n z\right)\end{aligned} \tag{28}$$

The stress components are obtained as:

$$\begin{aligned}\sigma_{xx} = \{ & c_{11} \sum_{n=1}^{4} ikA_n [-\lambda_n^4 \left(A_{57}A_{78} + A_{54}\right) + \lambda_n^2 \left(A_{54}A_{62} + A_{54}A_{73} - A_{52}A_{64} + A_{54}A_{67}A_{76} - A_{52}A_{67}A_{78} - A_{57}A_{64}A_{76} + A_{57}A_{62}A_{78}\right) + A_{52}A_{64}A_{73} - A_{54}A_{62}A_{73}] \\ & + c_{12} \sum_{n=1}^{4} ilA_n [-\lambda_n^4 \left(A_{67}A_{78} + A_{64}\right) + \lambda_n^2 (A_{64}A_{73} + A_{51}A_{67}A_{78} - A_{54}A_{67}A_{75} - A_{57}A_{61}A_{78} + A_{57}A_{75}A_{64} + A_{64}A_{51} - A_{61}A_{54}) - A_{73}A_{51}A_{64} + A_{73}A_{61}A_{54}] \\ & - c_{13} \sum_{n=1}^{4} \lambda_n A_n [-\lambda_n^5 A_{78} + \lambda_n^3 \left(A_{51}A_{78} + A_{62}A_{78} - A_{64}A_{76} - A_{54}A_{75}\right) - \lambda_n (A_{51}A_{62}A_{78} - A_{51}A_{64}A_{76} + A_{61}A_{54}A_{76} - A_{52}A_{61}A_{78} + A_{52}A_{64}A_{75} - A_{54}A_{62}A_{75})] \\ & - \beta_1 \sum_{n=1}^{4} A_n [-\lambda_n^6 + \lambda_n^4 \left(A_{51} + A_{62} + A_{73} + A_{57}A_{75} + A_{67}A_{76}\right) - \lambda_n^2 (A_{51}A_{62} + A_{51}A_{73} - A_{52}A_{61} + A_{62}A_{73} + A_{51}A_{67}A_{76} - A_{52}A_{75}A_{67} + A_{57}A_{62}A_{75} - A_{57}A_{61}A_{76}) \\ & + A_{51}A_{62}A_{73} - A_{61}A_{52}A_{73}] \} \exp\left[-\lambda_n z + i\left(kx + ly - \omega t\right)\right]\end{aligned}$$

$$\begin{aligned}\sigma_{zz} = \{ & c_{13} \sum_{n=1}^{4} ikA_n [-\lambda_n^4 \left(A_{57}A_{78} + A_{54}\right) + \lambda_n^2 \left(A_{54}A_{62} + A_{54}A_{73} - A_{52}A_{64} + A_{54}A_{67}A_{76} - A_{52}A_{67}A_{78} - A_{57}A_{64}A_{76} + A_{57}A_{62}A_{78}\right) + A_{52}A_{64}A_{73} - A_{54}A_{62}A_{73}] \\ & + c_{23} \sum_{n=1}^{4} A_n il [-\lambda_n^4 \left(A_{67}A_{78} + A_{64}\right) + \lambda_n^2 (A_{64}A_{73} + A_{51}A_{67}A_{78} - A_{54}A_{67}A_{75} - A_{57}A_{61}A_{78} + A_{57}A_{75}A_{64} + A_{64}A_{51} - A_{61}A_{54}) - A_{73}A_{51}A_{64} + A_{73}A_{61}A_{54}] \\ & - c_{33} \sum_{n=1}^{4} A_n \lambda_n [-\lambda_n^5 A_{78} + \lambda_n^3 \left(A_{51}A_{78} + A_{62}A_{78} - A_{64}A_{76} - A_{54}A_{75}\right) - \lambda_n \left(A_{51}A_{62}A_{78}A_{51}A_{64}A_{76} + A_{61}A_{54}A_{76} - A_{52}A_{62}A_{78} + A_{52}A_{64}A_{75} - A_{54}A_{62}A_{75}\right) \\ & - \beta_3 \sum_{n=1}^{4} A_n [-\lambda_n^6 + \lambda_n^4 \left(A_{51} + A_{62} + A_{73} + A_{57}A_{75} + A_{67}A_{76}\right) - \lambda_n^2 (A_{51}A_{62} + A_{51}A_{73} - A_{52}A_{61} + A_{62}A_{73} + A_{51}A_{67}A_{76} - A_{52}A_{75}A_{67} + A_{57}A_{62}A_{75} - A_{57}A_{61}A_{76}) \\ & + A_{51}A_{62}A_{73} - A_{61}A_{52}A_{73}] \} \exp\left[-\lambda_n z + i\left(kx + ly - \omega t\right)\right]\end{aligned}$$

$$\begin{aligned}\sigma_{yz} = c_{44} \{ & -\sum_{n=1}^{4} A_n \lambda_n [-\lambda_n^4 \left(A_{67}A_{78} + A_{64}\right) + \lambda_n^2 (A_{64}A_{73} + A_{51}A_{67}A_{78} - A_{54}A_{67}A_{75} - A_{57}A_{61}A_{78} + A_{57}A_{75}A_{64} \\ & + A_{64}A_{51} - A_{61}A_{54}) - A_{73}A_{51}A_{64} + A_{73}A_{61}A_{54}] + \sum_{n=1}^{4} A_n il [-\lambda_n^5 A_{78} + \lambda_n^3 \left(A_{51}A_{78} + A_{62}A_{78} - A_{64}A_{76} - A_{54}A_{75}\right) \\ & - \lambda_n (A_{51}A_{62}A_{78} - A_{51}A_{64}A_{76} + A_{61}A_{54}A_{76} - A_{52}A_{61}A_{78} + A_{52}A_{64}A_{75} - A_{54}A_{62}A_{75})] \} \exp\left[-\lambda_n z + i\left(kx + ly - \omega t\right)\right]\end{aligned}$$

$$\begin{aligned}\sigma_{xz} = c_{55} \{ & \sum_{n=1}^{4} ik A_n [-\lambda_n^5 A_{78} + \lambda_n^3 \left(A_{51}A_{78} + A_{62}A_{78} - A_{64}A_{76} - A_{54}A_{75}\right) - \lambda_n (A_{51}A_{62}A_{78} - A_{51}A_{64}A_{76} + A_{61}A_{54}A_{76} - A_{52}A_{61}A_{78} \\ & + A_{52}A_{64}A_{75} - A_{54}A_{62}A_{75})] - \sum_{n=1}^{4} \lambda_n A_n [-\lambda_n^4 \left(A_{57}A_{78} + A_{54}\right) + \lambda_n^2 (A_{54}A_{62} + A_{54}A_{73} - A_{52}A_{64} + A_{54}A_{67}A_{76} - A_{52}A_{67}A_{78} \\ & - A_{57}A_{64}A_{76} + A_{57}A_{62}A_{78}) + A_{52}A_{64}A_{73} - A_{54}A_{62}A_{73}] \} \exp\left[-\lambda_n z + i\left(kx + ly - \omega t\right)\right]\end{aligned}$$

$$\begin{aligned}\sigma_{xy} = c_{66} [& il \{ -\lambda_n^4 \left(A_{57}A_{78} + A_{54}\right) + \lambda_n^2 \left(A_{54}A_{62} + A_{54}A_{73} - A_{52}A_{64} + A_{54}A_{67}A_{76} - A_{52}A_{67}A_{78} - A_{57}A_{64}A_{76} + A_{57}A_{62}A_{78}\right) \\ & + A_{52}A_{64}A_{73} - A_{54}A_{62}A_{73} \} + ik \{ -\lambda_n^4 \left(A_{67}A_{78} + A_{64}\right) + \lambda_n^2 (A_{64}A_{73} + A_{51}A_{67}A_{78} - A_{54}A_{67}A_{75} - A_{57}A_{61}A_{78} + A_{57}A_{75}A_{64} \\ & + A_{64}A_{51} - A_{61}A_{54}) - A_{73}A_{51}A_{64} + A_{73}A_{61}A_{54} \}] \exp\left[-\lambda_n z + i\left(kx + ly - \omega t\right)\right]\end{aligned}$$

The constants $A_n \left(n = 1, 2, 3, 4\right)$ can be obtained by using boundary conditions.

For case 1 we obtain the constants as follows:

$$A_1 = \frac{D_1}{D},\ \ A_2 = \frac{D_2}{D},\ \ A_3 = \frac{D_3}{D},\ \ A_4 = \frac{D_4}{D}$$

In which

$$\begin{aligned}
&D = M_1\left[N_2\left(P_3Q_4 - P_4Q_3\right) - N_3\left(P_2Q_4 - P_4Q_2\right) + N_4\left(P_2Q_3 - P_3Q_2\right)\right]\\
&-M_2\left[N_1\left(P_3Q_4 - P_4Q_3\right) - N_3\left(P_1Q_4 - P_4Q_1\right) + N_4\left(P_1Q_3 - P_3Q_1\right)\right]\\
&+M_3\left[N_1\left(P_2Q_4 - P_4Q_2\right) - N_2\left(P_1Q_4 - P_4Q_1\right) + N_4\left(P_1Q_2 - P_2Q_1\right)\right]\\
&-M_4\left[N_1\left(P_2Q_3 - P_3Q_2\right) - N_2\left(P_1Q_3 - P_3Q_1\right) + N_3\left(P_1Q_2 - P_2Q_1\right)\right]\\
&D_1 = F_1\left[N_2\left(P_3Q_4 - P_4Q_3\right) - N_3\left(P_2Q_4 - P_4Q_2\right) + N_4\left(P_2Q_3 - P_3Q_2\right)\right]\\
&D_2 = -F_1\left[N_1\left(P_3Q_4 - P_4Q_3\right) - N_3\left(P_1Q_4 - P_4Q_1\right) + N_4\left(P_1Q_3 - P_3Q_1\right)\right]\\
&D_3 = F_1\left[N_1\left(P_2Q_4 - P_4Q_2\right) - N_2\left(P_1Q_4 - P_4Q_1\right) + N_4\left(P_1Q_2 - P_2Q_1\right)\right]\\
&D_4 = -F_1\left[N_1\left(P_2Q_3 - P_3Q_2\right) - N_2\left(P_1Q_3 - P_3Q_1\right) + N_3\left(P_1Q_2 - P_2Q_1\right)\right]\\
&M_n = -\lambda_n^6 + \lambda_n^4\left(A_{51} + A_{62} + A_{73} + A_{57}A_{75} + A_{67}A_{76}\right) - \lambda_n^2\left(A_{51}A_{62} + A_{51}A_{73} - A_{52}A_{61} + A_{62}A_{73}\right.\\
&\left.+A_{51}A_{67}A_{76} - A_{52}A_{75}A_{67} + A_{57}A_{62}A_{75} - A_{57}A_{61}A_{76}\right) + A_{51}A_{62}A_{73} - A_{61}A_{52}A_{73}\\
&N_n = ik\left(c_{13} + \mu_e H_0^2\right)\left[-\lambda_n^4\left(A_{57}A_{78} + A_{54}\right) + \lambda_n^2\left(A_{54}A_{62} + A_{54}A_{73} - A_{52}A_{64} + A_{54}A_{67}A_{76} - A_{52}A_{67}A_{78} - A_{57}A_{64}A_{76} + A_{57}A_{62}A_{78}\right) + A_{52}A_{64}A_{73} - A_{54}A_{62}A_{73}\right]\\
&+il\left(c_{23} + \mu_e H_0^2\right)\left[-\lambda_n^4\left(A_{67}A_{78} + A_{64}\right) + \lambda_n^2\left(A_{64}A_{73} + A_{51}A_{67}A_{78} - A_{54}A_{67}A_{75} - A_{57}A_{61}A_{78} + A_{57}A_{75}A_{64} + A_{64}A_{51} - A_{61}A_{54}\right) - A_{73}A_{51}A_{64} + A_{73}A_{61}A_{54}\right]\\
&-\left(c_{33} + \mu_e H_0^2\right)\lambda_n\left[-\lambda_n^5 A_{78} + \lambda_n^3\left(A_{51}A_{78} + A_{62}A_{78} - A_{64}A_{76} - A_{54}A_{75}\right) - \lambda_n\left(A_{51}A_{62}A_{78}A_{51}A_{64}A_{76} + A_{61}A_{54}A_{76} - A_{52}A_{62}A_{78} + A_{52}A_{64}A_{75} - A_{54}A_{62}A_{75}\right)\right]\\
&-\beta_3 M_n\\
&P_n = c_{44}\left\{ik\left[-\lambda_n^5 A_{78} + \lambda_n^3\left(A_{51}A_{78} + A_{62}A_{78} - A_{64}A_{76} - A_{54}A_{75}\right) - \lambda_n\left(A_{51}A_{62}A_{78} - A_{51}A_{64}A_{76} + A_{61}A_{54}A_{76} - A_{52}A_{61}A_{78}\right.\right.\right.\\
&\left.\left.+A_{52}A_{64}A_{75} - A_{54}A_{62}A_{75}\right)\right] - \lambda_n\left[-\lambda_n^4\left(A_{57}A_{78} + A_{54}\right) + \lambda_n^2\left(A_{54}A_{62} + A_{54}A_{73} - A_{52}A_{64} + A_{54}A_{67}A_{76} - A_{52}A_{67}A_{78}\right.\right.\\
&\left.\left.\left.-A_{57}A_{64}A_{76} + A_{57}A_{62}A_{78}\right) + A_{52}A_{64}A_{73} - A_{54}A_{62}A_{73}\right]\right\}\\
&Q_n = c_{55}\left\{-\lambda_n\left[-\lambda_n^4\left(A_{67}A_{78} + A_{64}\right) + \lambda_n^2\left(A_{64}A_{73} + A_{51}A_{67}A_{78} - A_{54}A_{67}A_{75} - A_{57}A_{61}A_{78} + A_{57}A_{75}A_{64}\right.\right.\right.\\
&\left.\left.+A_{64}A_{51} - A_{61}A_{54}\right) - A_{73}A_{51}A_{64} + A_{73}A_{61}A_{54}\right] + il\left[-\lambda_n^5 A_{78} + \lambda_n^3\left(A_{51}A_{78} + A_{62}A_{78} - A_{64}A_{76} - A_{54}A_{75}\right)\right.\\
&\left.\left.-\lambda_n\left(A_{51}A_{62}A_{78} - A_{51}A_{64}A_{76} + A_{61}A_{54}A_{76} - A_{52}A_{61}A_{78} + A_{52}A_{64}A_{75} - A_{54}A_{62}A_{75}\right)\right]\right\}\\
&F_1\left(x, y, t\right) = F\left(t\right)H\left(a - \left|x\right|\right)H\left(b - \left|y\right|\right)exp\left[-i\left(kx + ly - \omega t\right)\right]
\end{aligned}$$

For case 2 we obtain the constants as follows:

$$A_1 = \frac{\Delta_1}{\Delta},\ \ A_2 = \frac{\Delta_2}{\Delta},\ \ A_3 = \frac{\Delta_3}{\Delta},\ \ A_4 = \frac{\Delta_4}{\Delta}$$

In which

$$\begin{aligned}
&\Delta = M_1\left[L_2\left(R_3S_4 - R_4S_3\right) - L_3\left(R_2S_4 - R_4S_2\right) + L_4\left(R_2S_3 - R_3S_2\right)\right]\\
&-M_2\left[L_1\left(R_3S_4 - R_4S_3\right) - L_3\left(R_1S_4 - R_4S_1\right) + L_4\left(R_1S_3 - R_3S_1\right)\right]\\
&+M_3\left[L_1\left(R_2S_4 - R_4S_2\right) - L_2\left(R_1S_4 - R_4S_1\right) + L_4\left(R_1S_2 - R_2S_1\right)\right]\\
&-M_4\left[L_1\left(R_2S_3 - R_3S_2\right) - L_2\left(R_1S_3 - R_3S_1\right) + L_3\left(R_1S_2 - R_2S_1\right)\right]\\
&\Delta_1 = F_1\left[L_2\left(R_3S_4 - R_4S_3\right) - L_3\left(R_2S_4 - R_4S_2\right) + L_4\left(R_2S_3 - R_3S_2\right)\right]\\
&\Delta_2 = -F_1\left[L_1\left(R_3S_4 - R_4S_3\right) - L_3\left(R_1S_4 - R_4S_1\right) + L_4\left(R_1S_3 - R_3S_1\right)\right]\\
&\Delta_3 = F_1\left[L_1\left(R_2S_4 - R_4S_2\right) - L_2\left(R_1S_4 - R_4S_1\right) + L_4\left(R_1S_2 - R_2S_1\right)\right]
\end{aligned}$$

$$\Delta_4 = -F_1\left[L_1\left(R_2S_3 - R_3S_2\right) - L_2\left(R_1S_3 - R_3S_1\right) + L_3\left(R_1S_2 - R_2S_1\right)\right]$$
$$L_n = -\lambda_n^4\left(A_{57}A_{78} + A_{54}\right) + \lambda_n^2\left(A_{54}A_{62} + A_{54}A_{73} - A_{52}A_{64} + A_{54}A_{67}A_{76} - A_{52}A_{67}A_{78} - A_{57}A_{64}A_{76} + A_{57}A_{62}A_{78}\right)$$
$$+A_{52}A_{64}A_{73} - A_{54}A_{62}A_{73}$$
$$R_n = -\lambda_n^4\left(A_{67}A_{78} + A_{64}\right) + \lambda_n^2(A_{64}A_{73} + A_{51}A_{67}A_{78} - A_{54}A_{67}A_{75} - A_{57}A_{61}A_{78} + A_{57}A_{75}A_{64} + A_{64}A_{51}$$
$$-A_{61}A_{54}) - A_{73}A_{51}A_{64} + A_{73}A_{61}A_{54}$$
$$S_n = -\lambda_n^5 A_{78} + \lambda_n^3\left(A_{51}A_{78} + A_{62}A_{78} - A_{64}A_{76} - A_{54}A_{75}\right) - \lambda_n(A_{51}A_{62}A_{78} - A_{51}A_{64}A_{76} + A_{61}A_{54}A_{76} - A_{52}A_{61}A_{78}$$
$$+A_{52}A_{64}A_{75} - A_{54}A_{62}A_{75})$$

6 SPECIAL CASES

We discuss some special cases for different values of the parameters considered in the problem.

The above discussion converts for orthotropic medium without magnetic effect when we put

$$H_0 = 0.$$

This problem reduces for isotropic medium with three-phase-lag model, if we take

$$c_{11} = c_{22} = c_{33} = \lambda + 2\mu, c_{13} = c_{23} = c_{12} = \lambda, c_{44} = \mu, \beta_1 = \beta_2 = \beta_3 = \beta, K_1 = K_2 = K_3 = K, K_1^* = K_2^* = K_3^* = K^*$$

The study reduces to the case of orthotropic elasticity if we neglect the thermal parameters i.e.

$$\beta_1 = \beta_2 = \beta_3 = 0, K_1 = K_2 = K_3 = 0, K_1^* = K_2^* = K_3^* = 0$$

7 PARTICULAR CASES

From the general Eqs. (9)-(12), we now classify the problem into three classes for our further reference and for our comparison of numerical computations of the results:

i. If $\tau_q^2 = 0,\ \tau_q \neq 0,\ \tau_T = 0,\ K_1^* = K_2^* = K_3^* = 0$, then the problem reduces to the problem of Lord-Shulman (LS) model.

ii. If $\tau_q = \tau_T = \tau_\nu = 0$ then the problem reduces to the problem of Green-Naghdi theory type-III (GN-III).

iii. If $K_1^* = K_2^* = K_3^* = 0$ then the problem reduces to the problem of dual-phase-lag model (DPL).

8 NUMERICAL RESULTS AND DISCUSSION

For numerical computations, we take the following values of the relevant parameters for cobalt material as follows:

$$c_{11} = 3.071\times10^{11} Nm^{-2},\ c_{12} = 1.650\times10^{11} Nm^{-2},\ c_{13} = 1.027\times10^{11} Nm^{-2},\ c_{23} = 1.027\times10^{11} Nm^{-2},$$
$$c_{22} = 3.071\times10^{11} Nm^{-2},\ c_{33} = 3.581\times10^{11} Nm^{-2},\ c_{44} = 1.510\times10^{11} Nm^{-2},\ \beta_1 = 7.04\times10^6 Nm^{-2}deg^{-1},$$
$$\beta_2 = 7.04\times10^6 Nm^{-2}deg^{-1},\ \beta_3 = 6.90\times10^6 Nm^{-2}deg^{-1},\ K_1 = 69Wm^{-1}deg^{-1},\ K_2 = 69Wm^{-1}deg^{-1},$$
$$K_3 = 69Wm^{-1}deg^{-1},\ K_1^* = 13.1Wm^{-1}deg^{-1}s^{-1},\ K_2^* = 15.4Wm^{-1}deg^{-1}s^{-1},\ K_3^* = 15.4Wm^{-1}deg^{-1}s^{-1},$$
$$\rho = 7.14\times10^3 Kgm^{-3},\ C_e = 381.4JKg^{-1}deg^{-1},\ T_0 = 296K,\ \tau_q = 2\times10^{-7}s,\ \tau_T = 1.5\times10^{-7}s,\ \tau_\nu = 1\times10^{-8}s.$$

We consider $F(t) = \theta_0 exp(-dt)$ where θ_0 is a constant. Further for numerical purpose we take

$$\theta_0 = 10, d = 0.1, a = b = 1m, k = 1.2, \mu_e = 1.2Hm^{-1}, \varepsilon_0 = 1.2Fm^{-1}, H_0 = 10^5 Am^{-1}.$$

In Figs. 1-12, displacements and stresses for traction free surface are presented. In Fig. 1, variation of *u* with respect to *z* for three-phase-lag model is presented. It is observed that *u* decreases with the increase of magnetic field. *u* is showing oscillatory behavior and converging towards zero with the increase of *z*.

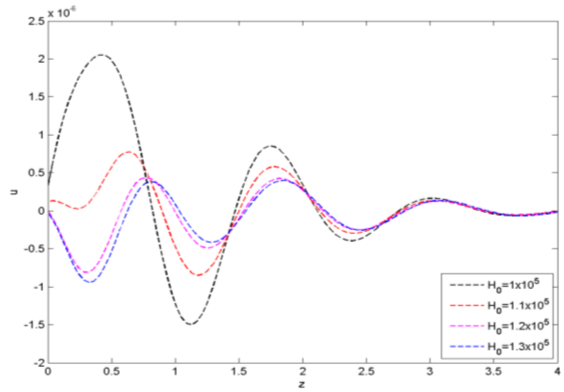

Fig.1
Variation of *u* with respect to *z*.

In Fig. 2 and 3, variation of *v* and *w* with respect to *z* for three-phase-lag model is presented. It is noticed that *v* and *w* decrease with the increase of magnetic field. Displacements are showing oscillatory behavior and converging towards zero with the increase of *z*.

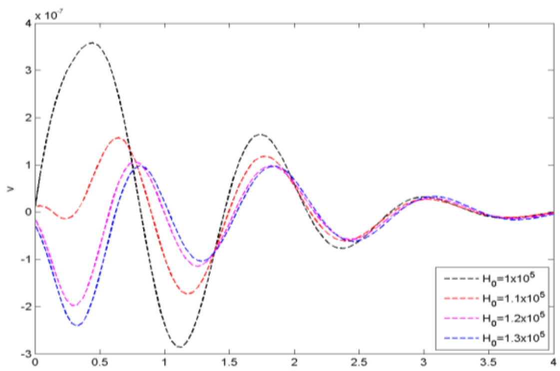

Fig.2
Variation of *v* with respect to *z*.

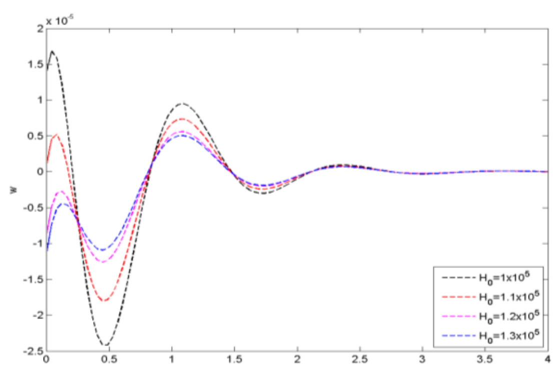

Fig.3
Variation of *w* with respect to *z*.

In Fig. 4, variation of σ_{xz} with respect to *z* for three-phase-lag model is presented. It is noticed that σ_{xz} decrease with the increase of magnetic field. Stress is showing oscillatory behavior and converging towards zero with the increase of *z*.

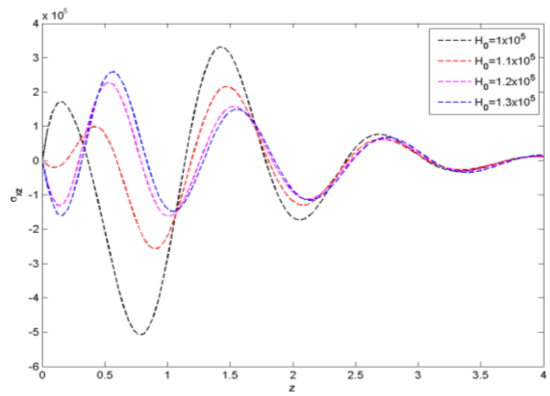

Fig.4
Variation of σ_{xz} with respect to z.

In Fig. 5, variation of σ_{yz} with respect to z for three-phase-lag model is presented. It is noticed that σ_{yz} increase with the increase of magnetic field. Stress is showing oscillatory behavior and converging towards zero with the increase of z.

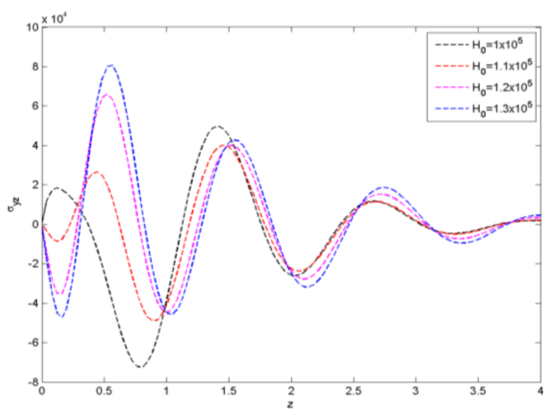

Fig.5
Variation of σ_{yz} with respect to z.

In Fig. 6, variation of σ_{zz} with respect to z for three-phase-lag model is presented. It is noticed that σ_{zz} decrease with the increase of magnetic field. Stress is showing oscillatory behavior and converging towards zero with the increase of z.

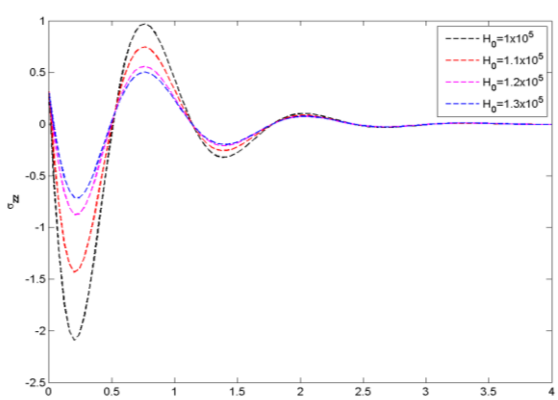

Fig.6
Variation of σ_{zz} with respect to z.

In Figs. 7 and 8, variation of u and v with respect to z for GN-III and DPL model is presented. It is noticed that u and v increase with the increase of magnetic field. Displacements for DPL model are greater than displacements for GN-III model. Displacements are showing oscillatory behavior and converging towards zero with the increase of z.

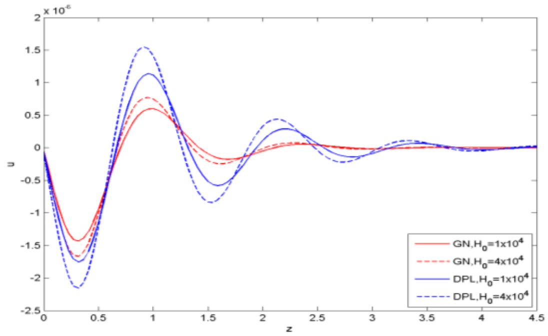

Fig.7
Comparison of u with respect to z for GN and DPL.

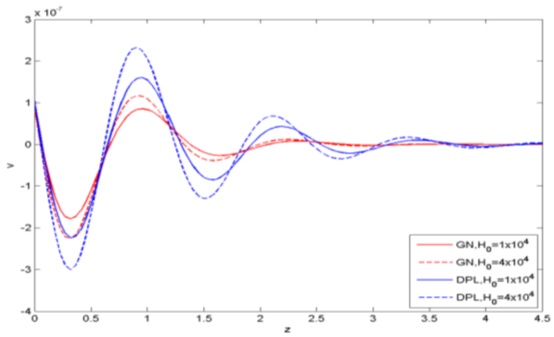

Fig.8
Comparison of v with respect to z for GN and DPL.

In Fig. 9, variation of w with respect to z for GN-III and DPL model is presented. It is noticed that w increase with the increase of magnetic field. Displacement for DPL model is greater than displacement for GN-III model. Displacement is converging towards zero with the increase of z.

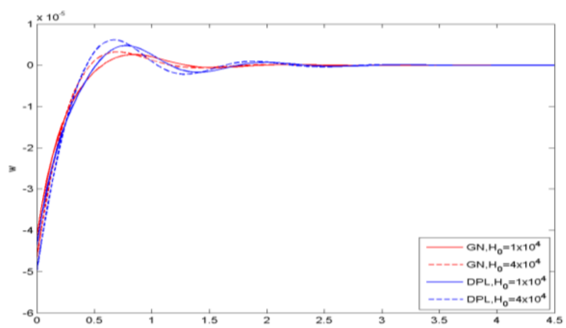

Fig.9
Comparison of w with respect to z for GN and DPL.

In Fig.10, comparison of σ_{zz} with respect to z for GN-III and DPL model is presented. It is noticed that σ_{zz} increase with the increase of magnetic field. Stress for DPL model is greater than stress for GN-III model. Stress is oscillating and converging towards zero with the increase of z.

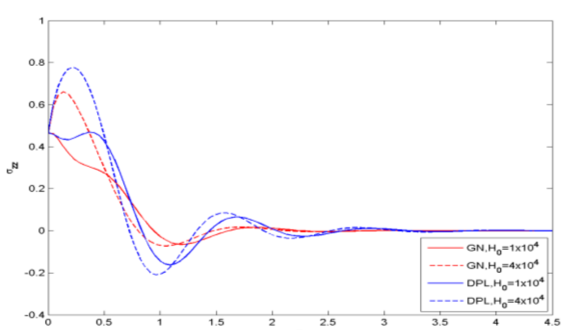

Fig.10
Comparison of σ_{zz} with respect to z for GN and DPL.

In Fig. 11, comparison of σ_{xz} with respect to z for GN-III and DPL model is presented. It is noticed that σ_{xz} increase with the increase of magnetic field. Stress for DPL model is greater than stress for GN-III model. Stress is oscillating and converging towards zero with the increase of z.

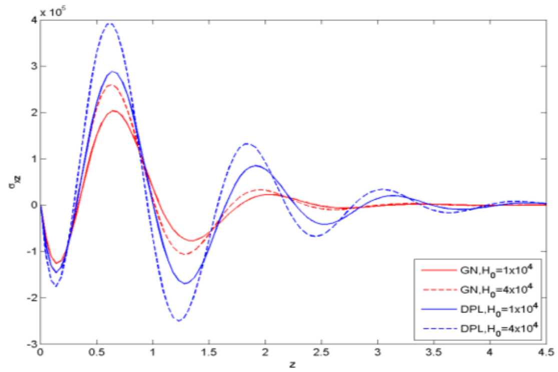

Fig.11
Comparison of σ_{xz} with respect to z for GN and DPL.

In Fig. 12, comparison of σ_{yz} with respect to z for GN-III and DPL model is presented. It is noticed that σ_{yz} decrease with the increase of magnetic field. Stress for DPL model is greater than stress for GN-III model. Stress is oscillating and converging towards zero with the increase of z.

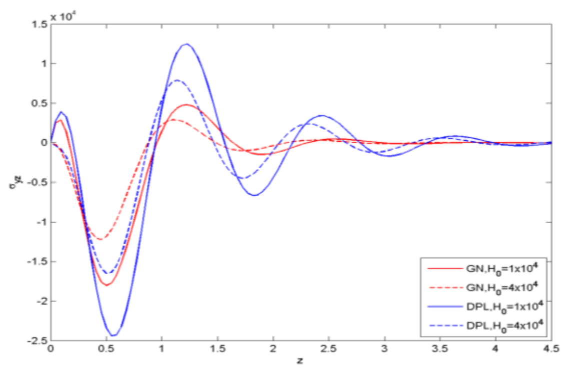

Fig.12
Comparison of σ_{yz} with respect to z for GN and DPL.

In Figs. 13-21, displacements and stresses for rigidly fixed surface are presented. In Fig. 13, variation of u with respect to z for three-phase-lag model is presented. It is observed that u increases with the increase of magnetic field. u is showing oscillatory behavior and converging towards zero with the increase of z.

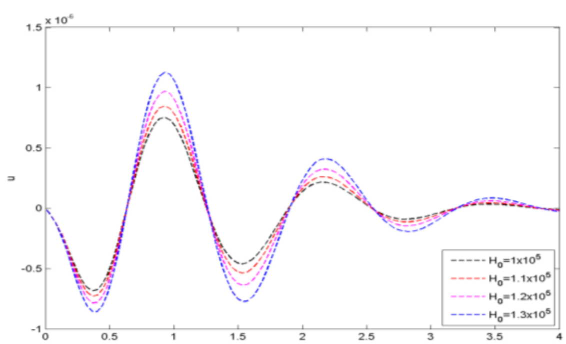

Fig.13
Comparison of u with respect to z.

In Fig.14 and 15, variation of v and w with respect to z for three-phase-lag model is presented. It is noticed that v and w increase with the increase of magnetic field. Displacements are showing oscillatory behavior and converging towards zero with the increase of z.

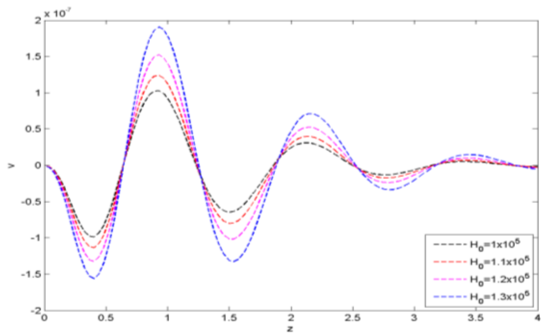

Fig.14
Comparison of *v* with respect to *z*.

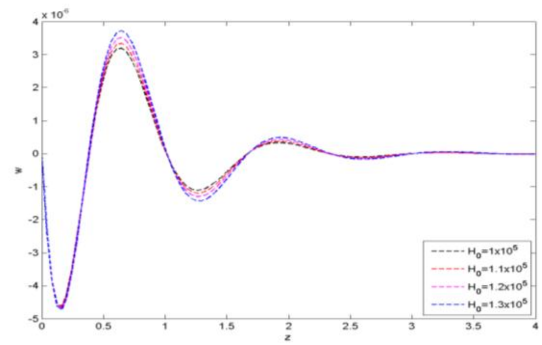

Fig.15
Comparison of *w* with respect to *z*.

In Fig. 16, variation of σ_{xz} with respect to *z* for TPL model is presented. It is noticed that σ_{xz} increase with the increase of magnetic field. Stress is oscillating and converging towards zero with the increase of *z*.

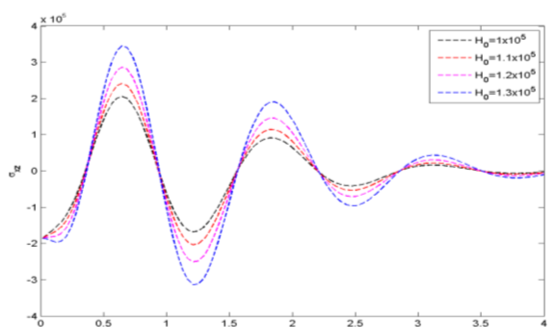

Fig.16
Comparison of σ_{xz} with respect to *z*.

In Figs. 17 and 18, variation of *u* and *v* with respect to *z* for GN-III and DPL model is presented. It is noticed that *u* and *v* increase with the increase of magnetic field. Displacements for DPL model are greater than displacements for GN-III model. Displacements are showing oscillatory behavior and converging towards zero with the increase of *z*.

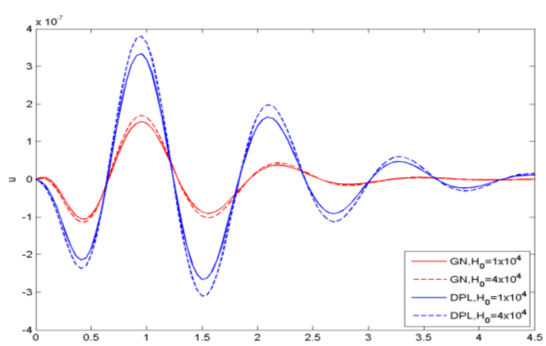

Fig.17
Comparison of *u* with respect to *z* for GN and DPL.

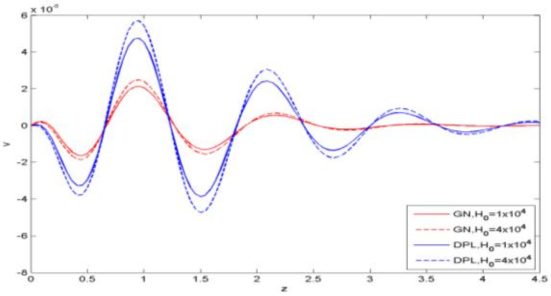

Fig.18
Comparison of v with respect to z for GN and DPL.

In Fig. 19, variation of w with respect to z for GN-III and DPL model is presented. It is noticed that w increase with the increase of magnetic field. Displacement for DPL model is greater than displacement for GN-III model. Displacement is converging towards zero with the increase of z.

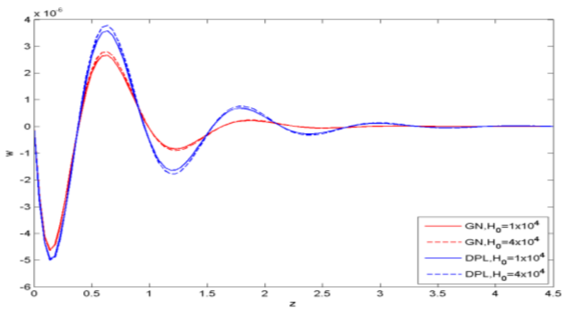

Fig.19
Comparison of w with respect to z for GN and DPL.

In Fig. 20, comparison of σ_{xz} with respect to z for GN-III and DPL model is presented. It is noticed that σ_{xz} increase with the increase of magnetic field. Stress for DPL model is greater than stress for GN-III model. Stress is oscillating and converging towards zero with the increase of z.

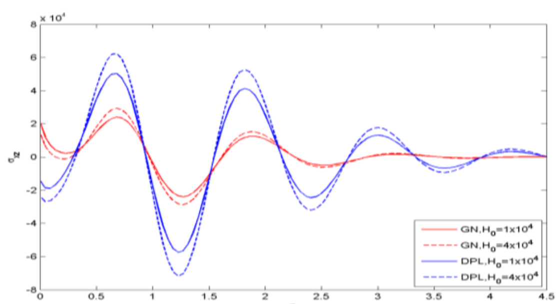

Fig.20
Comparison of σ_{xz} with respect to z for GN and DPL.

In Fig. 21, comparison of σ_{yz} with respect to z for GN-III and DPL model is presented. It is noticed that σ_{yz} decrease with the increase of magnetic field. Stress for DPL model is greater than stress for GN-III model. Stress is oscillating and converging towards zero with the increase of z.

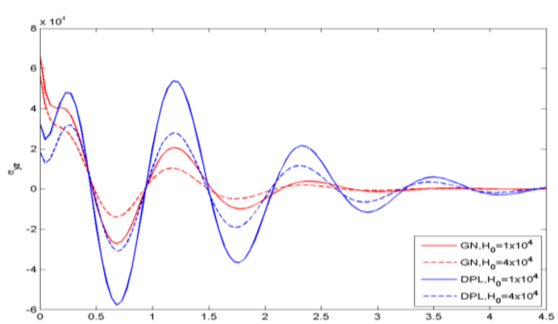

Fig.21
Comparison of σ_{yz} with respect to z for GN and DPL.

9 CONCLUSIONS

The present article provides a detail analysis of propagation of thermoelastic disturbances in an orthotropic medium in the presence of a time dependent thermal shock. With the view of theoretical analysis and numerical computation, we can conclude the following phenomena:

(i) This problem with three-phase-lag model is more general as other problem with different thermoelastic models can be derived as a special case from this.
(ii) Displacements decrease with the increase of magnetic field for TPL model but displacements increase with the increase of magnetic field for GN-III and DPL model.
(iii) Displacements and stresses are showing oscillatory behavior and converging towards zero with the increase of distance.
(iv) Displacements and stresses are showing similar nature for both traction free and rigidly fixed surface.
(v) Displacements and stresses for DPL model are greater than displacements and stresses for GN-III model.

The results presented in this article may be useful for researchers who are working on material science, mathematical physics and thermodynamics with low temperatures as well as on the development of the hyperbolic thermoelasticity theory.

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