

# Influence of Rigidity, Irregularity and Initial Stress on Shear Waves Propagation in Multilayered Media

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## ABSTRACT

The propagation of shear waves in an anisotropic fluid saturated porous layer over a pre-stressed semi-infinite homogeneous elastic half-space lying under an elastic homogeneous layer with irregularity present at the interface with rigid boundary has been studied. The rectangular irregularity has been taken in the half-space. The dispersion equation for shear waves is derived by using the perturbation technique followed by Fourier transformations. The dimensionless phase velocity is plotted against dimensionless wave number for the different size of ratios of depth of rectangular irregularity with the height of the layer and anisotropy parameters with the help of MATLAB graphical routines in presence and absence of initial stress. From the graphical results, it has been seen that the phase velocity is significantly influenced by the wave number, the depth of the irregularity, rigid boundary and initial stress. The acquired outcomes can be valuable for the investigation of geophysical prospecting and understanding the cause and evaluating of damage due to earthquakes.

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## 1 INTRODUCTION

THE study of seismic waves generated from earthquake origin gives significant information about the complex internal and layered structure of the Earth. Due to the presence of overburden layer, variety in temperature, slow movement of particles due to presence of weathered materials, and gravitational field, etc., a lot of stresses (called initial stresses) are generated in the layers. These initial stresses have a significant effect on elastic waves produced by earthquakes. These stresses have effect on the propagation of waves as discussed by Biot [1]. The Earth is a highly initially stressed medium and assumed to be under initial stress. So it became core of interest to study the influence of these stresses on the propagation of shear waves. Alam and Kundu [2] studied the effect of initial stress on propagation of Love wave in a transversely isotropic layer over an inhomogeneous half-space. Love wave propagation in non-homogeneous substratum with initially stressed heterogeneous half-space with rigidity was

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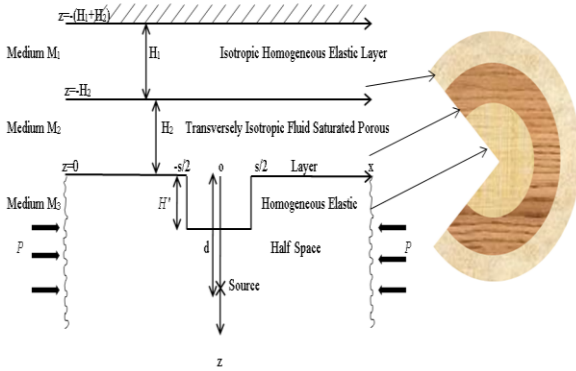
studied by Kundu et al. [3]. Gupta et al. [4] obtained the frequency equation under the influence of rigid boundary in the Love wave propagation in an inhomogeneous substratum with initial stress. Kundu et al. [5] derived the dispersion equation for Love wave propagation with initial stress over the porous half space with rigid boundary. Chattopadhyay et al. [6] obtained the dispersion equation for shear wave propagation in a monoclinic layer resting between two isotropic half-spaces. Chattopadhyay et al. [7] studied the propagation of shear waves in an inhomogeneous irregular monoclinic layer over semi-infinite monoclinic medium. Chattopadhyay et al. [8] studied the shear wave propagation in monoclinic layer lying between two isotropic layers. Rouze et al. [9] investigated the shear wave propagation in the nearly incompressible state and effects were observed on different elastic parameters. Vaishnav et al. [10] derived the dispersion equation for the torsional surface waves lying between two heterogeneous half-spaces. The propagation of shear waves in multilayered media with rigid boundary under the influence of initial stress plays an important role for understanding and prediction of seismic wave behavior at continental margins, mountain roots, mountain basins, salt, ore bodies etc. Moreover, these studies are useful in theoretical seismology as well as have practical importance in field of earthquake engineering, non-destructive evaluation, civil engineering, rock mechanics and geophysical prospecting, geophysics, signal processing, sound system and wireless communication. Inside the Earth's crust there are many permeable rocks that absorb liquid present in form of limestone, sandstone and groundwater. In general the pores contain hydrocarbon deposition such as fuel and oil. Most oil and fuel deposits are present in sandstone or limestone is very much like a hard sponge, full of holes which are not compressible. These holes or pores can be filled with water, oil or gas and rock will mainly be in saturated form made from any of these three. The holes are much smaller than sponge holes but they are still holes and referred as porosity and the layer is known as porous layer. In like manner, geological structures in the internal part of Earth may be assumed to be fluid filled porous layer at which thickness and elastic moduli change irregularly. There are many theories which describe physical properties of porous materials, and one of them is a Biot consolidation theory of fluid-saturated porous solids [11,12]. One of the model related to shear wave propagation in layered stratum consisting a transversely isotropic fluid saturated porous layer lying between an inhomogeneous elastic half space and an elastic isotropic homogeneous layer with free surface had been developed by Konczak [13] and derived the corresponding dispersion relation. The effect of irregularity, inhomogeneity and rigidity in fluid saturated porous layer over non-homogeneous elastic half space had been studied by Kumar et al. [14, 15]. Kaliraman and Poonia [16] investigated the factors on which the amplitude ratios depends when the transmission and reflection of elastic waves take place between micro polar elastic and fluid saturated porous solid half-space. Poonia et al. [17] obtained the dispersion equation with and without rigidity for Love wave propagating in the fluid saturated porous isotropic layer and homogenous elastic half space with rectangular irregularity. Gupta et al. [18] studied the effect of initial stress when Love wave propagate between anisotropic porous layer and non-homogenous elastic half-space and observed the effects of velocity on the porous layer. Kumar et al. [19] obtained the frequency equation for the shear wave with rigid boundary in multilayer medium. Kumar et al. [20] derived the dispersion relation for Love waves for the fluid saturated porous isotropic layer lying over non-homogeneous half-space and observed the effects of phase velocity on different parameters. Some researchers like Kundu and Prasad [21] and Poonia et al. [22, 23] studied the effects of gravity, irregularity, anisotropic and exponentially parameters on the wave propagation and presented the results graphically with the help of MATLAB. Mathematically modeling of shear wave propagation along with the rigid boundary of an elastic half space has been the subject of continued interest for many years.

The present paper endeavors to study the shear wave propagation in an intermediate transversely isotropic fluid saturated porous layer, which is sandwiched between isotropic homogenous elastic layer with rigid boundary and homogeneous elastic half-space under initial stress. The rectangular irregularity is taken in account for this model. The dispersion curves are depicted by means of graphs by using MATLAB for different size of ratios of depth of irregularity with the height of the layer and different values of anisotropic parameter with and without presence of initial stress.

## 2 FORMULATION OF THE PROBLEM

A transversely isotropic fluid saturated porous layer of thickness  $H_2$  resting on a homogeneous elastic half space, lying between elastic isotropic half-space and homogeneous layer of thickness  $H_1$  has been considered. The lower half-space ( $M_3$ ) under initial stress  $P$  is considered to be homogenous in nature. The Cartesian coordinate system ( $x, y, z$ ) is chosen with  $z$ -axes taken vertically downward in the half space and  $x$ -axes is chosen parallel to the layer in the direction of propagation of the disturbance. We assume the irregularity in the form of a rectangle with length  $s$

and depth  $H'$ . The origin is placed at the middle point of the interface irregularity. The source of the disturbance is placed on positive  $z$ - axes at a distance  $d$  ( $d > H'$ ) from the origin. Therefore, the upper layer describes the medium  $M_1$ :  $-(H_1 + H_2) \leq z \leq H_2$ , the intermediate layer describes the medium  $M_2$ :  $-H_2 \leq z \leq 0$  and the homogeneous elastic half space describes the medium  $M_3$ :  $0 \leq z < \infty$ . The geometry of the problem is shown in Fig. 1.



**Fig.1**  
Geometry of the Model.

The interface between the layer and half space is defined as:

$$z = \varepsilon h(x) \tag{1}$$

where

$$h(x) = \begin{cases} 0; & x \leq -\frac{s}{2}, x \geq \frac{s}{2} \\ f(x); & -\frac{s}{2} \leq x \leq \frac{s}{2} \end{cases} \tag{2}$$

where  $\varepsilon = \frac{H'}{s}$  and  $\varepsilon \ll 1$ .

### 3 BASIC EQUATIONS AND BOUNDARY CONDITIONS

The basic equations for the medium considered are as follows:

For Medium  $M_1$ : The equations of motion, without body force (Ewing [24]) are given by

$$\sigma_{ij,j}^{(1)} = \rho^{(1)} \ddot{u}_i^{(1)}, \tag{3}$$

where  $\sigma_{ij}^{(1)}$  are the components of stress tensor,  $u_i^{(1)}$  are the components of displacement vector, and  $\rho^{(1)}$  is the density. The comma denotes differentiation with respect to position and dot denotes differentiation with respect to time. The constitutive relations are given by

$$\sigma_{ij}^{(1)} = \lambda^{(1)} e_{kk}^{(1)} \delta_{ij} + 2\mu^{(1)} e_{ij}^{(1)}, \tag{4}$$

where  $\lambda^{(1)}$  and  $\mu^{(1)}$  are Lamé's elastic coefficients and  $\delta_{ij}$  is the Kronecker delta and

$$2e_{ij}^{(1)} = (u_{i,j}^{(1)} + u_{j,i}^{(1)}), \quad e_{kk}^{(1)} = u_{k,k}^{(1)} = e^{(1)}. \tag{5}$$

For Medium  $M_2$ : The equations of motion for the intermediate fluid saturated porous layer in the absence of body forces are (Biot [22])

$$\begin{aligned}\sigma_{ij,j}^{(2)} &= \rho_1 \ddot{u}_i^{(2)} + \rho_2 \ddot{U}_i^{(2)} - b_{ij} (\dot{U}_j^{(2)} - \dot{u}_j^{(2)}), \\ \sigma_i^{(2)} &= \rho_1 \ddot{u}_i^{(2)} + \rho_2 \ddot{U}_i^{(2)} + b_{ij} (\dot{U}_j^{(2)} - \dot{u}_j^{(2)})\end{aligned}\quad (6)$$

where  $\sigma_{ij}^{(2)}$  are the components of stress tensor in the solid skeleton,  $\sigma^{(2)} = -fp$  is the reduced pressure of the fluid ( $p$  is the pressure in the fluid, and  $f$  is the porosity of the medium),  $u_i^{(2)}$  are the components of the displacement vector of the solid skeleton and  $U_i^{(2)}$  are those of fluid.

The stress-strain relations for the transverse-isotropic fluid saturated porous layer are (Biot [25])

$$\begin{aligned}\sigma_{11}^{(2)} &= (2C_1 + C_2) e_{11}^{(2)} + C_2 e_{22}^{(2)} + C_3 e_{33}^{(2)} + C_6 e^{(2)} \\ \sigma_{22}^{(2)} &= C_2 e_{11}^{(2)} + (2C_1 + C_2) e_{22}^{(2)} + C_3 e_{33}^{(2)} + C_6 e^{(2)} \\ \sigma_{33}^{(2)} &= C_3 e_{11}^{(2)} + C_3 e_{22}^{(2)} + 2C_4 e_{33}^{(2)} + C_7 e^{(2)} \\ \sigma_{23}^{(2)} &= 2C_5 e_{23}^{(2)} \\ \sigma_{31}^{(2)} &= 2C_5 e_{31}^{(2)} \\ \sigma_{12}^{(2)} &= 2C_1 e_{12}^{(2)} \\ \sigma^{(2)} &= C_6 e_{11}^{(2)} + C_6 e_{22}^{(2)} + C_7 e_{33}^{(2)} + C_8 e^{(2)}\end{aligned}\quad (7)$$

where

$$2e_{ij}^{(2)} = (u_{i,j}^{(2)} + u_{j,i}^{(2)}), e^{(2)} = \text{div} U^{(2)} \equiv U_{j,j}^{(2)}, e_{kk}^{(2)} = \text{div} u^{(2)} \equiv u_{k,k}^{(2)} \quad (8)$$

and  $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8$  are the material constants.

For Medium  $M_3$ : For the lower non-homogeneous half-space the basic equations of motion, without body force are {Ewing [24]}:

$$\begin{aligned}\frac{\partial \sigma_{xx}^{(3)}}{\partial x} + \frac{\partial \sigma_{xy}^{(3)}}{\partial y} + \frac{\partial \sigma_{xz}^{(3)}}{\partial z} - P \left[ \frac{\partial \Omega_3}{\partial y} - \frac{\partial \Omega_2}{\partial z} \right] &= \rho^{(3)} \ddot{u}_1^{(3)} \\ \frac{\partial \sigma_{yx}^{(3)}}{\partial x} + \frac{\partial \sigma_{yy}^{(3)}}{\partial y} + \frac{\partial \sigma_{yz}^{(3)}}{\partial z} - P \left[ \frac{\partial \Omega_3}{\partial x} \right] &= \rho^{(3)} \ddot{u}_2^{(3)} \\ \frac{\partial \sigma_{zx}^{(3)}}{\partial x} + \frac{\partial \sigma_{zy}^{(3)}}{\partial y} + \frac{\partial \sigma_{zz}^{(3)}}{\partial z} - P \left[ \frac{\partial \Omega_2}{\partial x} \right] &= \rho^{(3)} \ddot{u}_3^{(3)}\end{aligned}\quad (9)$$

$$\begin{aligned}\Omega_1 &= \frac{1}{2} \left[ \frac{\partial u_3^{(3)}}{\partial y} - \frac{\partial u_2^{(3)}}{\partial z} \right] \\ \Omega_2 &= \frac{1}{2} \left[ \frac{\partial u_1^{(3)}}{\partial z} - \frac{\partial u_3^{(3)}}{\partial x} \right] \\ \Omega_3 &= \frac{1}{2} \left[ \frac{\partial u_2^{(3)}}{\partial x} - \frac{\partial u_1^{(3)}}{\partial y} \right]\end{aligned}$$

where  $\sigma_{ij,j}^{(3)}$  are the components of stress tensor,  $u_i^{(3)}$  are the components of displacement vector,  $P$  is the initial stress in the absence of body forces and  $\rho^{(3)}$  is the density. Here,  $\Omega_j^{(3)}$  are the rotational components in the lower half-space.

The constitutive relations are given by

$$\begin{aligned} \sigma_{ij}^{(3)} &= \lambda^{(3)} e_{kk}^{(3)} \delta_{ij} + 2\mu^{(3)} e_{ij}^{(3)}, \\ 2e_{ij}^{(3)} &= (u_{i,j}^{(3)} + u_{j,i}^{(3)}), \\ e_{kk}^{(3)} &= u_{k,k}^{(3)} = e^{(3)}. \end{aligned} \tag{10}$$

where  $\lambda^{(3)}$  and  $\mu^{(3)}$  are Lamé's elastic coefficients and are functions of  $x, y$  and  $z$ .

In this paper, attention is confined to shear waves propagating in the  $xy$ -plane. The displacements are parallel to  $y$  direction and are independent of the  $y$  coordinate. Thus,

$$\begin{aligned} u^{(1)} \equiv w^{(1)} &\equiv 0, & v^{(1)} &\equiv v^{(1)}(x, z, t), \\ u^{(2)} \equiv w^{(2)} &\equiv 0, & v^{(2)} &\equiv v^{(2)}(x, z, t), \\ u^{(3)} \equiv w^{(3)} &\equiv 0, & v^{(3)} &\equiv v^{(3)}(x, z, t), \\ U^{(2)} \equiv W^{(2)} &\equiv 0, & V^{(2)} &\equiv V^{(2)}(x, z, t), \end{aligned} \tag{11}$$

and the equations of motion (3), (6) and (9) with the help of (4), (5), (7), (8) and (10) respectively reduce to the form

$$\left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right\} v^{(1)} = \frac{1}{\beta_1^2} \frac{\partial^2 v^{(1)}}{\partial t^2}, \tag{12}$$

$$\left\{ C_1 \frac{\partial^2}{\partial x^2} + C_5 \frac{\partial^2}{\partial z^2} - \left[ \rho_{11} \partial_t^2 + b_{11} \partial_t - \frac{(\rho_{12} \partial_t^2 - b_{11} \partial_t)^2}{\rho_{22} \partial_t^2 + b_{11} \partial_t} \right] \right\} (v^{(2)}, V^{(2)}) = 0 \tag{13}$$

$$\left\{ \left[ 1 - \frac{P}{2\mu^{(3)}} \right] \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right\} v^{(3)} = \frac{1}{\beta_3^2} \frac{\partial^2 v^{(3)}}{\partial t^2} \tag{14}$$

where  $\beta_1^2 = \sqrt{\frac{u^{(1)}}{\rho^{(1)}}}$  and  $\beta_3^2 = \sqrt{\frac{u^{(3)}}{\rho^{(3)}}}$

The appropriate boundary conditions for the considered problem are as:

- i. At the rigid surface  $z = -(H_1 + H_2)$ , the displacement component vanishes, i.e.

$$v^{(1)}(x, z = -(H_1 + H_2), t) = 0. \tag{15}$$

- ii. At the interface  $z = -H_2$ , the displacements are continuous, i. e. ,

$$v^{(1)}(x, z = -H_2, t) = v^{(2)}(x, z = -H_2, t). \tag{16}$$

- iii. At the interface  $z = -H_2$ , the shear stress components are continuous, i.e.,

$$\sigma_{32}^{(1)}(x, z = -H_2, t) = \sigma_{32}^{(2)}(x, z = -H_2, t). \tag{17}$$

- iv. At the interface  $z = \varepsilon h(x)$ , the displacements are continuous, that is,

$$v^{(2)}(x, z = \varepsilon h(x), t) = v^{(3)}(x, z = \varepsilon h(x), t). \tag{18}$$

- v. The stresses are continuous at the interface  $z = \varepsilon h(x)$ , i.e.,

$$C_5 \frac{\partial v^{(2)}}{\partial z} - C_1 \varepsilon h'(x) \frac{\partial v^{(2)}}{\partial x} = \mu \left( \frac{\partial v^{(3)}}{\partial z} - \varepsilon h'(x) \frac{\partial v^{(3)}}{\partial x} \right) \quad (19)$$

where  $h'(x) = \frac{dh(x)}{dx}$ . Thus Eqs. (12)-(14) with above boundary conditions are the governing equations for the problem considered.

#### 4 SOLUTION OF THE PROBLEM

For waves changing harmonically with time  $t$  and propagating in  $x$ -direction, we obtain

$$\begin{aligned} v^{(1)}(z, x, t) &= v_0^{(1)}(z, x) \exp(i \omega t), \\ v^{(2)}(z, x, t) &= v_0^{(2)}(z, x) \exp(i \omega t), \\ V^{(2)}(z, x, t) &= V_0^{(2)}(z, x) \exp(i \omega t), \\ v^{(3)}(z, x, t) &= v_0^{(3)}(z, x) \exp(i \omega t), \end{aligned} \quad (20)$$

where  $\omega$  is the angular frequency. Thus equations of motions (12)-(14) take the form of

$$\begin{aligned} \frac{\partial^2 v_0^{(1)}}{\partial x^2} + \frac{\partial^2 v_0^{(1)}}{\partial z^2} + \frac{\omega^2}{\beta_1^2} v_0^{(1)} &= 0, \\ \left( C_1 \frac{\partial^2}{\partial x^2} + C_5 \frac{\partial^2}{\partial z^2} + \xi_1^2 \right) (v_0^{(2)}, V_0^{(2)}) &= 0, \\ \left[ 1 - \frac{P}{2\mu^{(3)}} \right] \frac{\partial^2 v_0^{(3)}}{\partial x^2} + \frac{\partial^2 v_0^{(3)}}{\partial z^2} + \frac{\omega^2}{\beta_3^2} v_0^{(3)} &= 0 \end{aligned} \quad (21)$$

where

$$\begin{aligned} \xi_1^2 &= \alpha_1 + i \alpha_2, \\ \alpha_1 &= F \omega^2 / c_G^2, \alpha_2 = R \omega^2 / c_G^2, \\ F = F(\omega) &= \frac{1 + \Omega^2 \gamma_{22} C'}{1 + (\Omega \gamma_{22})^2} \cdot \frac{\gamma_{22}}{C'}, R = R(\omega) = \frac{(C' - \gamma_{22}) \Omega}{1 + (\Omega \gamma_{22})^2} \cdot \frac{\gamma_{22}}{C'}, \\ C' &= \gamma_{11} \gamma_{22} - \gamma_{12}^2, \gamma_{kl} = \frac{\rho_{kl}}{\rho} (k, l = 1, 2), \\ c_G^2 &= (\rho_{11} - \rho_{12}^2 / \rho_{22})^{-1}, \Omega = \frac{\rho \omega}{b_{11}}. \end{aligned}$$

$\Omega$  is the dimensionless frequency and  $c_G$  is the velocity of shear wave in the porous layer. Define the Fourier transform  $\bar{v}_0^{(1)}(z, \eta)$  of  $v_0^{(1)}(z, \eta)$  as:

$$\bar{v}_0^{(1)}(z, \eta) = \int_{-\infty}^{\infty} v_0^{(1)}(z, x) e^{i \eta x} dx \quad (22)$$

and inverse Fourier transform is given by

$$v_0^{(1)}(z, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{v}_0^{(1)}(z, \eta) e^{-i\eta x} d\eta, \tag{23}$$

The Fourier transform of Eqs. (21) are then

$$\begin{aligned} \frac{\partial^2 \bar{v}_0^{(1)}}{\partial z^2} + \chi_1^2 \bar{v}_0^{(1)} &= 0, \\ \frac{\partial^2 \bar{v}_0^{(2)}}{\partial z^2} + \chi_2^2 \bar{v}_0^{(2)} &= 0, \\ \frac{\partial^2 \bar{V}_0^{(2)}}{\partial z^2} + \chi_2^2 \bar{V}_0^{(2)} &= 0, \\ \frac{\partial^2 \bar{v}_0^{(3)}}{\partial z^2} - \chi_3^2 \bar{v}_0^{(3)} &= 0. \end{aligned} \tag{24}$$

where

$$\begin{aligned} \chi_1^2 &= \left( \frac{\omega^2}{\beta_1^2} - \eta^2 \right), \\ \chi_2^2 &= \frac{C_1}{C_5} \left( \frac{\xi_1^2}{C_1} - \eta^2 \right), \\ \chi_3^2 &= \left( \eta^2 \left( 1 - \frac{P}{2\mu^{(3)}} \right) - \frac{\omega^2}{\beta_3^2} \right). \end{aligned} \tag{25}$$

The solution of Eqs. (24) are

$$\begin{aligned} \bar{v}_0^{(1)} &= A \cos \chi_1 z + B \sin \chi_1 z, \\ \bar{v}_0^{(2)} &= C \cos \chi_2 z + D \sin \chi_2 z, \\ \bar{V}_0^{(2)} &= \bar{C} \cos \chi_2 z + \bar{D} \sin \chi_2 z, \\ \bar{v}_0^{(3)} &= E \exp(-\chi_3 z), \end{aligned} \tag{26}$$

where  $A, B, \bar{A}, \bar{B}, D$  are functions of  $\eta$ .

Thus, by inverse Fourier transform, we obtain

$$v_0^{(1)}(z, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (A \cos \chi_1 z + B \sin \chi_1 z) e^{-i\eta x} d\eta, \tag{27}$$

$$v_0^{(2)}(z, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (C \cos \chi_2 z + D \sin \chi_2 z) e^{-i\eta x} d\eta,$$

$$V_0^{(2)}(z, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (\bar{C} \cos \chi_2 z + \bar{D} \sin \chi_2 z) e^{-i\eta x} d\eta, \tag{28}$$

$$v_0^{(3)}(z, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( E e^{-\chi_3 z} + \frac{2}{\chi_3} e^{\chi_3 z} e^{-\chi_3 d} \right) e^{-i\eta x} d\eta, \tag{29}$$

where the second term in the integrand of  $v_0^{(3)}(z, x)$  is introduced due to the source in the lower half space.

The relations between the constants  $\bar{C}, \bar{D}$  and  $C, D$  are provided by Eq. (13). We set the following approximations due to small value of  $\varepsilon$

$$A \cong A_0 + A_1\varepsilon, B \cong B_0 + B_1\varepsilon, C \cong C_0 + C_1\varepsilon, D \cong D_0 + D_1\varepsilon, E \cong E_0 + E_1\varepsilon. \quad (30)$$

Since the boundary is not uniform, the terms  $A, B, C, D, E$  in Eq. (30) are also functions of  $\varepsilon$ . Expanding these terms in ascending powers of  $\varepsilon$  and keeping in view that  $\varepsilon$  is small and so retaining the terms up to the first order of  $\varepsilon$ ,  $A, B, C, D, E$  can be approximated as in Eq. (30). In physical situations, when the depth  $H'$  of the irregular boundary is too small with respect to the length of the boundary  $s$ , the above assumptions are justified. Further for small  $\varepsilon$

$$e^{\pm\alpha\varepsilon h} \cong 1 \pm \alpha\varepsilon h, \cos \chi_1\varepsilon h \cong 1, \sin \chi_1\varepsilon h \cong \chi_1\varepsilon h \quad (31)$$

where  $\alpha$  is any quantity.

Defining Fourier Transform of  $h(x)$  as:

$$\bar{h}(\lambda) = \int_{-\infty}^{\infty} h(x)e^{i\lambda x} dx, \quad (32)$$

and the inverse Fourier Transform is

$$h(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{h}(\lambda)e^{-i\lambda x} d\lambda, \quad (33)$$

Therefore,

$$h'(x) = \frac{-i}{2\pi} \int_{-\infty}^{\infty} \lambda \bar{h}(\lambda)e^{-i\lambda x} d\lambda \quad (34)$$

Now, by using boundary conditions (15)-(19) along with Eqs. (27) and (29)-(30) we obtain a system of ten equations (after equating the absolute term (terms not containing  $\varepsilon$ ) and the coefficients of  $\varepsilon$ ):

$$\begin{aligned} A_0 \cos(H_1 + H_2)\chi_1 - B_0 \sin(H_1 + H_2)\chi_1 &= 0, \\ A_0 \cos H_2\chi_1 - B_0 \sin H_2\chi_1 - C_0 \cos H_2\chi_2 + D_0 \sin H_2\chi_2 &= 0, \\ \mu\chi_1 [A_0 \sin H_2\chi_1 + B_0 \cos H_2\chi_1] - C_5\chi_2 [C_0 \sin H_2\chi_2 + D_0 \cos H_2\chi_2] &= 0, \\ C_0 - E_0 &= \frac{2}{\chi_3} e^{-\chi_3 d}, \\ \mu\chi_3 E_0 + C_5\chi_2 D_0 &= 2\mu e^{-\chi_3 d}, \\ A_1 \cos(H_1 + H_2)\chi_1 - B_1 \sin(H_1 + H_2)\chi_1 &= 0, \\ A_1 \cos H_2\chi_1 - B_1 \sin H_2\chi_1 - C_1 \cos H_2\chi_2 + D_1 \sin H_2\chi_2 &= 0, \\ \mu\chi_1 [A_1 \sin H_2\chi_1 + B_1 \cos H_2\chi_1] - C_5\chi_2 [C_1 \sin H_2\chi_2 + D_1 \cos H_2\chi_2] &= 0, \\ E_1 - C_1 &= R_1(k), \\ \mu\chi_3 E_1 + C_5\chi_2 D_1 &= R_2(k) \end{aligned} \quad (35)$$

where

$$R_1(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [(D_0\chi_2 + E_0\chi_3 - 2e^{-\chi_3 d})]^{n=k-\lambda} \bar{h}(\lambda) d\lambda \quad (36)$$



and

$$R_2(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ (C_5 \chi_2^2 C_0 - \mu(\chi_3^2 E_0 + 2\chi_3 e^{-\chi_3 d})) - \lambda k \left\{ C_0 C_1 + \mu \left( E_0 + \frac{2}{\chi_3} e^{-\chi_3 d} \right) \right\} \right\}^{\eta=k-\lambda} \bar{h}(\lambda) \chi d \lambda \quad (37)$$

Solving the above system of equations for  $A_0, B_0, C_0, D_0, E_0, A_1, B_1, C_1, D_1, E_1$  and the corresponding values are given by

$$A_0 = \frac{4\mu e^{-\chi_3 d} C_5 \chi_2 [1 + \tan^2 \chi_2 H_2] \tan \chi_2 [H_1 + H_2]}{E(k)}, \quad (38)$$

$$B_0 = \frac{4\mu e^{-\chi_3 d} C_5 \chi_2 [1 + \tan^2 \chi_2 H_2]}{E(k)}, \quad (39)$$

$$C_0 = \frac{4\mu e^{-\chi_3 d} [\mu \chi_1 \tan \chi_2 H_2 (1 + \tan \chi_1 H_2 \tan \chi_1 (H_1 + H_2)) + C_5 \chi_2 (\tan \chi_1 H_2 - \tan \chi_1 (H_1 + H_2))]}{E(k)}, \quad (40)$$

$$D_0 = \frac{4\mu e^{-\chi_3 d} [\mu \chi_1 (1 + \tan \chi_1 H_2 \tan \chi_1 (H_1 + H_2)) - C_5 \chi_2 \tan \chi_2 H_2 (\tan \chi_1 (H_1 + H_2) - \tan \chi_1 H_2)]}{E(k)}, \quad (41)$$

$$E_0 = \frac{2e^{-\chi_3 d}}{\chi_3 E(k)} \left[ \mu \chi_1 (1 + \tan \chi_1 H_2 \tan \chi_1 (H_1 + H_2)) (\mu \chi_3 \tan \chi_2 H_2 - C_5 \chi_2) + C_5 \chi_2 (\tan \chi_1 (H_1 + H_2) - \tan \chi_1 H_2) (C_5 \chi_2 \tan \chi_2 H_2 + \mu \chi_3) \right], \quad (42)$$

$$A_1 = \frac{(R_2 - \mu \chi_3 R_1) C_5 \chi_2 [(1 + \tan^2 \chi_2 H_2) \tan \chi_1 (H_1 + H_2)]}{E(k)}, \quad (43)$$

$$B_1 = \frac{(R_2 - \mu \chi_3 R_1) C_5 \chi_2 (1 + \tan^2 \chi_2 H_2)}{E(k)}, \quad (44)$$

$$C_1 = \frac{(R_2 - \mu \chi_3 R_1) [\mu \chi_1 \tan \chi_2 H_2 (1 + \tan \chi_1 H_2 \tan \chi_1 (H_1 + H_2)) + C_5 \chi_2 (\tan \chi_1 H_2 - \tan \chi_1 (H_1 + H_2))]}{E(k)}, \quad (45)$$

$$D_1 = \frac{(R_2 - \mu \chi_3 R_1) [\mu \chi_1 (1 + \tan \chi_1 H_2 \tan \chi_1 (H_1 + H_2)) - C_5 \chi_2 \tan \chi_2 H_2 (\tan \chi_1 (H_1 + H_2) - \tan \chi_1 H_2)]}{E(k)}, \quad (46)$$

$$E_1 = \frac{1}{E(k)} \left[ \mu \chi_1 (R_2 \tan \chi_2 H_2 + C_5 \chi_2 R_1) (1 + \tan \chi_1 H_2 \tan \chi_1 (H_1 + H_2)) + C_5 \chi_2 (R_2 - C_5 \chi_2 R_1 \tan \chi_2 H_2) (\tan \chi_1 (H_1 + H_2) - \tan \chi_1 H_2) \right], \quad (47)$$

where

$$E(k) = \left[ \mu \chi_1 (1 + \tan \chi_1 H_2 \tan \chi_1 (H_1 + H_2)) (\mu \chi_3 \tan \chi_2 H_2 + C_5 \chi_2) - C_5 \chi_2 (\tan \chi_1 (H_1 + H_2) - \tan \chi_1 H_2) (C_5 \chi_2 \tan \chi_2 H_2 - \mu \chi_3) \right] \quad (48)$$

The displacement in the anisotropic layer is

$$v_0^{(2)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{4\mu e^{-\chi_3 d}}{E(k)} \left[ 1 + \frac{\varepsilon(R_2 - \mu\chi_3 R_1) e^{\chi_3 d}}{4\mu} \right] [B_2 + B_3] e^{-ikx} dk, \quad (49)$$

where

$$B_2 = \mu\chi_1 (1 + \tan \chi_1 H_2 \tan \chi_1 (H_1 + H_2)) \times (\sin \chi_2 z + \tan \chi_2 H_2 \cos \chi_2 z) \quad (50)$$

$$B_3 = C_5 \chi_2 (\tan \chi_1 (H_1 + H_2) - \tan \chi_1 H_2) \times (\cos \chi_2 z - \tan \chi_2 H_2 \sin \chi_2 z) \quad (51)$$

Now from Eqs. (1), (2) and (32), we have

$$\bar{h}(\lambda) = \frac{2s}{\lambda} \sin \frac{\lambda s}{2}. \quad (52)$$

Using values of  $R_1(k)$  and  $R_2(k)$  from Eqs. (36)-(37), we obtain

$$(R_2 - \mu\chi_3 R_1) = \frac{s}{\pi} \int_{-\infty}^{\infty} [\phi(k - \lambda) + \phi(k + \lambda)] \frac{1}{\lambda} \sin \frac{\lambda s}{2} d\lambda \quad (53)$$

where

$$\phi(k - \lambda) = A_2 + A_3, \quad (54)$$

$$A_2 = C_5 \chi_2^2 C_0 - \mu\chi_3 (\chi_2 D_0 - 4e^{-\chi_3 d}) \quad (55)$$

$$A_3 = -\lambda k \left[ C_1 C_0 + \mu \left( E_0 + \frac{2}{\chi_3} e^{-\chi_3 d} \right) \right] \quad (56)$$

Using asymptotic formula of Willis [26] and Tranter [27] and neglecting the terms containing  $2/s$  and highest powers of  $2/s$  for large  $s$ , we obtain

$$\int_{-\infty}^{\infty} [\phi(k - \lambda) + \phi(k + \lambda)] \frac{1}{\lambda} \sin \frac{\lambda s}{2} d\lambda \cong \frac{\pi}{2} \cdot 2\phi(k) = \pi\phi(k). \quad (57)$$

Now using Eqs. (53) and (57), we obtain

$$R_2 - \mu\chi_3 R_1 = s\phi(k) = \frac{H'}{\varepsilon} \phi(k) \quad (58)$$

Therefore the displacement in the anisotropic layer is

$$v_0^{(2)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{4\mu e^{-\chi_3 d}}{E(k)} \left[ 1 - \frac{H'}{\varepsilon} \psi(k) e^{\chi_3 d} \right] [B_2 + B_3] e^{-ikx} dk, \quad (59)$$

where

$$\psi(k) = \frac{\phi(k)}{4\mu} \quad (60)$$

The value of this integral depends entirely on the contribution of the poles of the integrand. The poles are located at the roots of the equation

$$E(k)[1 - H' \psi(k) e^{\chi_3 d}] = 0 \tag{61}$$

This equation is the dispersion equation for SH waves. If  $c$  is the common wave velocity of wave propagating along the surface, then we can set in Eq. (61)  $\omega = ck$  ( $\omega$  is the circular frequency and  $k$  is the wave number),  $\chi_1 = P_1 k$ ,  $\chi_2 = P_2 k$ ,  $\chi_3 = P_3 k$  where

$$\begin{aligned} P_1 &= \sqrt{\frac{c^2}{\beta_1^2} - 1}, \\ P_2 &= \sqrt{\left( \frac{1}{C_5} \left( \frac{c^2}{c_G^2} F(\omega) - C_1 \right) + i \frac{1}{C_5} \cdot \frac{c^2}{c_G^2} R(\omega) \right)}, \\ P_3 &= \sqrt{\left( 1 - \frac{P}{2\mu^{(3)}} \right) - \frac{c^2}{\beta_3^2}} \end{aligned} \tag{62}$$

Solving Eq. (61), we obtain

$$\begin{aligned} \frac{\mu P_1}{C_5 P_2} & \{ [1 + \tan P_1 k H_2 \tan P_1 k (H_1 + H_2)] \times [(1 - P_3 k H') [C_5 P_2 + \mu P_3 \tan P_2 k H_2] - H' P_2 k (C_5 P_2 \tan P_2 k H_2 - \mu P_3)] \} \\ &= \{ [\tan P_1 k (H_1 + H_2) - \tan P_1 k H_2] \times [(1 - P_3 k H') (C_5 P_2 \tan P_2 k H_2 - \mu P_3) + H' P_2 k (C_5 P_2 - \mu P_3 \tan P_2 k H_2)] \} \end{aligned} \tag{63}$$

Since the quantity  $P_2^2$  is complex, so we have

$$P_2 = k_1 + ik_2, \tag{64}$$

where

$$\begin{aligned} k_1 &= \left\{ \frac{1}{2} \left( \sqrt{\left\{ \frac{1}{C_5} \left( \frac{c^2}{c_G^2} F(\omega) - C_1 \right) \right\}^2 + \left( \frac{1}{C_5} \cdot \frac{c^2}{c_G^2} R(\omega) \right)^2} + \left\{ \frac{1}{C_5} \left( \frac{c^2}{c_G^2} F(\omega) - C_1 \right) \right\} \right) \right\}^{\frac{1}{2}} \\ k_2 &= \left\{ \frac{1}{2} \left( \sqrt{\left\{ \frac{1}{C_5} \left( \frac{c^2}{c_G^2} F(\omega) - C_1 \right) \right\}^2 + \left( \frac{1}{C_5} \cdot \frac{c^2}{c_G^2} R(\omega) \right)^2} - \left\{ \frac{1}{C_5} \left( \frac{c^2}{c_G^2} F(\omega) - C_1 \right) \right\} \right) \right\}^{\frac{1}{2}} \end{aligned}$$

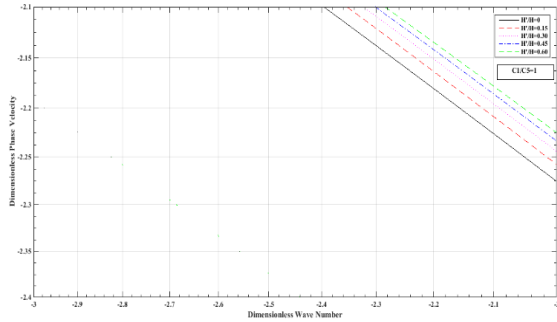
Since, the Eq. (63) is complex and hence its real part gives the dispersion equation for shear waves.

### 5 NUMERICAL RESULTS AND DISCUSSIONS

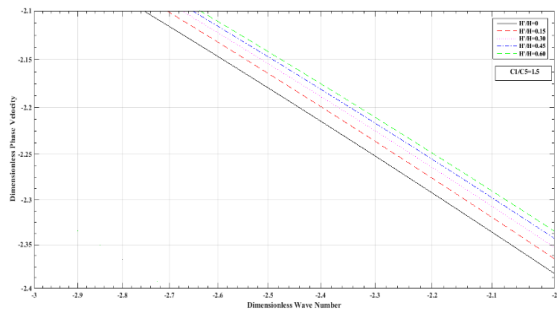
In order to examine the effect of irregularity and initial stress present in the transversely isotropic fluid saturated porous layer and homogeneous elastic half-space and to compare the results between phase velocity and wave number, we have taken the values of elastic constants given by Ding et al. [28] for medium  $M_2$  and by Konczak [13] for media  $M_1$  and  $M_3$ . And by using MATLAB, we obtain following graphs as given in Figs. 2-13 for different values of anisotropy coefficient  $C_1/C_5$  for two special cases.

Case I. When  $H_1 = 0$  and  $H_2 = H$  that is when the wave propagation in transversely isotropic fluid saturated porous layer lying over a homogeneous half-space. The variations of the dimensionless phase velocity ( $c/c_G$ ) against

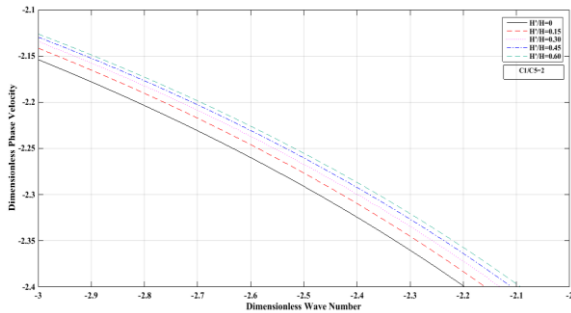
the dimensionless wave number ( $kH$ ) in an elastic isotropic homogeneous layer over a homogeneous elastic half-space for different values of the ratios of the depth of the irregularity and the height of the layer ( $H'/H=0, 0.15, 0.30, 0.45, 0.60$ ) and anisotropy parameter  $C_1/C_5 = 1, 1.5$  and  $2$  with initial stress  $P = 0$  are shown in Figs. 2, 3 and 4 and with initial stress  $P = 0.5$  are shown in Figs. 5, 6 and 7.



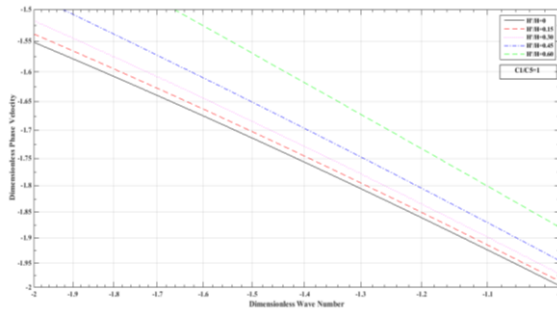
**Fig.2**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_2$  for  $H'/H=0, 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 1$  with initial stress  $P = 0$ .



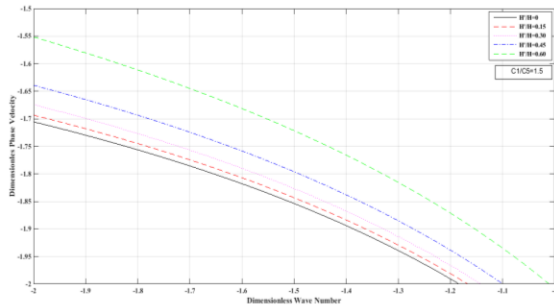
**Fig.3**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_2$  for  $H'/H=0, 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 1.5$  with initial stress  $P = 0$ .



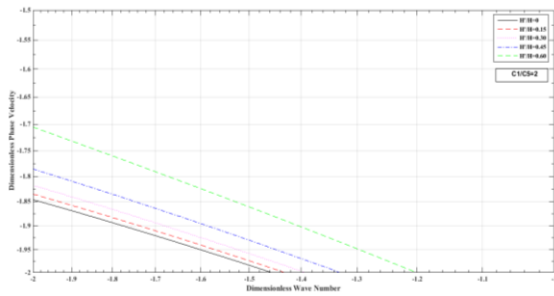
**Fig.4**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_2$  for  $H'/H=0, 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 2$  with initial stress  $P = 0$ .



**Fig.5**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_2$  for  $H'/H=0, 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 1$  with initial stress  $P = 0.5$ .



**Fig.6**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_2$  for  $H_1/H=0, 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 1.5$  with initial stress  $P = 0.5$ .

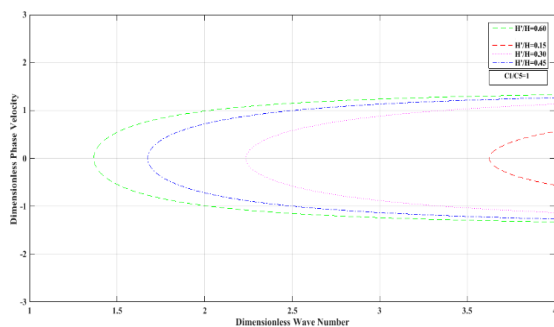


**Fig.7**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_2$  for  $H_1/H=0, 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 2$  with initial stress  $P = 0.5$ .

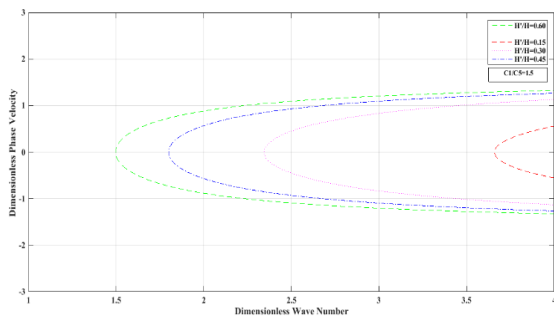
However the effect of initial stress has been seen and following observations are made:

1. The dimensionless phase velocity increases proportionally with the values of the ratio  $H_1/H$  slowly in case of there is no initial stress and increases fast in case of initial stress  $P=0.5$ .
2. The phase velocity and wave number decreases with the increment in the values of anisotropy parameter  $C_1/C_5$ .

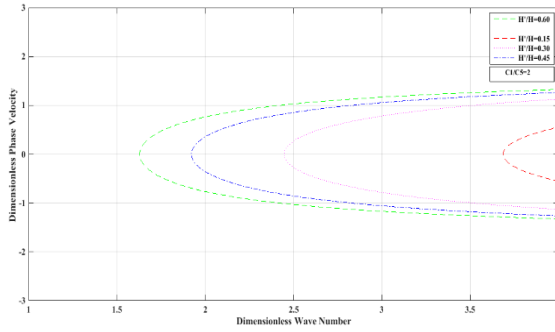
Case II. when  $H_2 = 0$  and  $H_1 = H$  that means the wave propagation in elastic homogeneous layer lying over a homogeneous half-space, the variations of the dimensionless phase velocity ( $c/c_G$ ) plotted against the dimensionless wave number ( $kH$ ) for different values of the ratios of the depth of the irregularity and the height of the layer ( $H_1/H = 0.15, 0.30, 0.45, 0.60$ ) and anisotropy ratio  $C_1/C_5 = 1, 1.5$  and  $2$  with initial stress  $P = 0$  are shown in Figs. 8, 9 and 10 and with initial stress  $P = 0.5$  are shown in Figs. 11, 12 and 13.



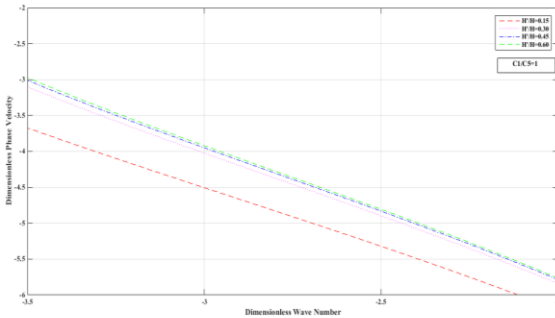
**Fig.8**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_1$  for  $H_1/H = 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 1$  with initial stress  $P = 0$ .



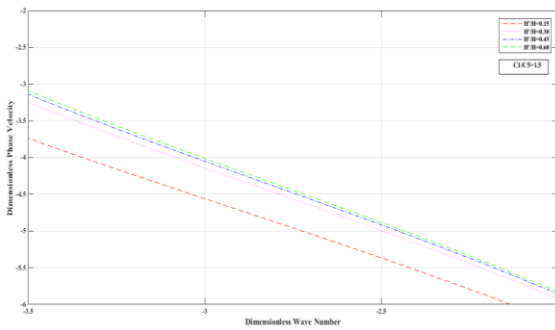
**Fig.9**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_1$  for  $H_1/H = 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 1.5$  with initial stress  $P = 0$ .



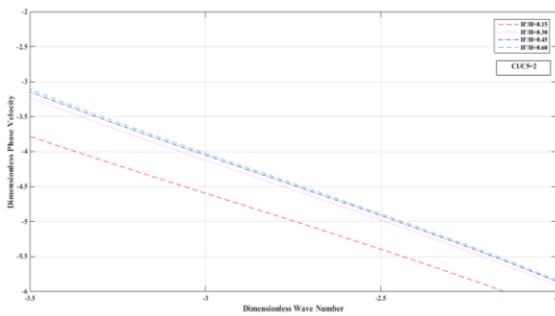
**Fig.10**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_1$  for  $H'/H = 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 2$  with initial stress  $P = 0$ .



**Fig.11**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_1$  for  $H'/H = 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 1$  with initial stress  $P = 0.5$ .



**Fig.12**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_1$  for  $H'/H = 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 1.5$  with initial stress  $P = 0.5$ .



**Fig.13**  
Variations of the dimensionless phase velocity ( $c/c_G$ ) against the dimensionless wave number ( $kH$ ) in medium  $M_1$  for  $H'/H = 0.15, 0.30, 0.45, 0.60$  and  $C_1/C_5 = 2$  with initial stress  $P = 0.5$ .

However the effect of initial stress was seen and following observations are made:

1. The dimensionless wave number decrease if there is no initial stress and increases with initial stress  $P=0.5$  with the increment in the value of the ratio  $H'/H$ .
2. The nature of the wave propagation is parabolic in the absence of initial stress and simple curve in presence of initial stress  $P=0.5$ .
3. From Figs. 11-13, it can be observed that the value of phase velocity and wave number change slightly for different values of  $H'/H$ .

## 6 CONCLUSIONS

This model discussed in this paper consisting three layers: isotropic homogenous elastic layer; transversely isotropic fluid saturated porous layer and prestressed homogeneous elastic half-space with linear variation of depth with rigidity. The displacement of shear waves in all the three mediums has been derived by using simple mathematical techniques followed by Fourier transformations, and finally, the dispersion relation has been obtained for the consider problem. From the numerical analysis and graphs, it has been observed that

- The anisotropy factor of the porous layer ( $C_1/C_5$ ) present in the medium  $M_2$  gives the direct effect on the shear wave velocity.
- The dimensionless phase velocity ( $c/c_G$ ) decrease with increase in anisotropy factor ( $C_1/C_5$ ) in the presence of initial stress
- The wave number decreases with the increment in the values of the anisotropy factor ( $C_1/C_5$ ) in all the cases.
- The phase velocity in a layer with irregularity is affected by not only the shape of irregularity but also the wave number, the ratio of the depth of the irregularity to layer width and layer structure.
- The shear wave propagation is affected significantly in the presence and absence of irregularity.
- The anisotropic factor and initial stress affects a lot the shear wave propagation in the porous medium.

The model presented in this paper is one of the more realistic forms of the Earth models. The present theoretical results may provide useful information for experimental scientists, researchers and seismologists working in the area of wave propagation in porous medium.

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