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Effect of Thermal Environment on Vibration Analysis of Partially Cracked Thin Isotropic Plate Submerged in Fluid

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ABSTRACT

Based on a non-classical plate theory, an analytical model is proposed for the first time to analyze free vibration problem of partially cracked thin isotropic submerged plate in the presence of thermal environment. The governing equation for the cracked plate is derived using the Kirchhoff's thin plate theory and the modified couple stress theory. The crack terms are formulated using simplified line spring model whereas the effect of thermal environment is introduced using thermal moments and in-plane forces. The influence of fluidic medium is incorporated in governing equation in form fluids forces associated with inertial effects of its surrounding fluids. Applying the Galerkin method, the derived governing equation of motion is reformulated into well-known Duffing equation. The governing equation for cracked isotropic plate has also been solved to get central deflection which shows an important phenomenon of shift in primary resonance due to crack, temperature rise and internal material length scale parameter. To demonstrate the accuracy of the present model, few comparison studies are carried out with the published literature. The variation in natural frequency of the cracked plate with uniform rise in temperature is studied considering various parameters such as crack length, fluid level and internal material length scale parameter. Furthermore, the variation of the natural frequency with plate thickness is also established.

Keywords : Temperature; Crack; Vibration; Fluid-plate interaction.

1 INTRODUCTION

In recent decades thin plates or shells have been developed as important structural components in marine engineering applications which expose them to work under fluidic medium of varying temperature. Thus, the knowledge of dynamic characteristics of such thin structures under fluidic medium over a range of temperatures is essential for their designing purpose. It becomes more profound to understand the effect of temperature under fluidic medium when these structures contain various flaws in the form of cracks or holes. In literature a lot efforts have

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been devoted on vibration analysis of plates in presence of thermal environment and fluidic medium individually. However, the study on vibration analysis of cracked plates in presence of both thermal environment and surrounding fluidic medium is insignificant. Concerning the effect of thermal environment on free vibration analysis of plates, it is well known that the presence of thermal stress decreases the stiffness of plate which results in decrease of natural frequency. Murphy et al. [1] studied the effect of uniform heating of the plate on its natural frequency. They considered fully clamped rectangular plates to perform a combined theoretical and experimental study. The results for free and forced vibration characteristics of initially stressed FGM plates in presence of thermal environment are given by Yang and Shen [2]. Using higher order shear deformation theory, they studied the effect of rise in temperature, initial stresses, volume fraction index and boundary conditions on vibration characteristics of plate. Jeyaraj et al. [3,4] presented the vibration characteristics and acoustic response of an isotropic plate using ANSYS and SYSNOISE [3] and also presented the both response for a composite plate using classical laminated plate theory [4]. Li et al. [5] used three dimensional theory of elasticity and studied the free vibration problems of functionally graded plates in presence of thermal environment. Kim [6] has formulated the governing equation for initially stressed FGM rectangular plates subjected to thermal environment using third order shear deformation theory. Natarajan et al. [7] studied the linear free flexural vibrations of a functionally graded plate containing a through crack using FEM and first-order shear deformation theory. Their obtained results showed that the natural frequency decreases with increase in temperature gradient and crack length. Viola et al. [8], applied finite element method for the vibration analysis of thick composite plate with crack and extended it to plates of arbitrary shapes. The static solutions for cracked plate under influence of thermal environment are mostly done by different numerical techniques in literature, but an approximate analytical solution is only possible by means of the Line Spring Model (LSM). This concept was first proposed by Rice and Levy[9], wherein the surface crack is represented as continuous line springs with stretching and bending compliances. Later on Delalae and Erdogan [10] introduced the transverse shear deformation in the line spring model to improve the effectiveness of model. Using the line spring model, Israr et al. [11] developed the first approximate analytical model for vibration analysis of thin isotropic plate with a part-through surface crack located at its centre based on classical plate theory. Three different boundary conditions are considered for the study and the relationship between tensile and bending stress at far sides of the plate and at crack location is well expressed in their work. Ismail and Cartmell [12] extended the work of Israr et al. [11] and developed an analytical model for cracked plate considering various angular orientation of crack by establishing relations for moment and in-plane force due to orientation of the crack. It is concluded from their work that the natural frequencies of plate decreases with increase in length and angular orientation of crack. Joshi et al. [13] worked on vibration analysis of thin isotropic plate containing two perpendicular surface cracks located at its centre. Extending their work, they also studied the influence of thermal environment on vibration and buckling analysis of cracked isotropic [14] and orthotropic [15] plate. Recently, Soni et al. [16,17] performed an analytical study on non-linear vibration problem of cracked isotropic [16] and magneto-electro-elastic (MEE) plate [17] submerged in fluid. They modified the previously developed models to accommodate the effect of fluidic medium.

In recent literature on study of microstructures it has been found that it affects the vibration characteristics of plate structures [18]. Different theories which captures the size effect of the plate are developed in recent works [19–24], among them, the modified couple stress theory (MCST) which was proposed by Yang et al. [24] is found to be efficient one. They (Ref. [24]) considered a single internal material length scale parameter to capture size effect of plate in their developed theory. Tsiatas [18] proposed a new analytical model for an isotropic micro plates using the modified couple stress theory. In their study they analyzed several plates having various shapes, dimensions and Poisson's ratios to see applicability of the developed model and also to analyze the difference between the proposed model (MCST) and the Kirchhoff's plate model. Yin et al. [22] studied the vibration characteristics of micro-plate based on the modified couple stress theory. They find the variation response of the micro-plate between two different theories (MCST and CPT) and concluded that the results obtained from Classical plate model for natural frequency are always lower when compared to modified couple stress theory. Chen et al. [25] proposed an analytical model for a cross-ply laminated composite plate based a simplified couple stress theory. In their theory they used only one material scale constant to demonstrate the size effect. Gao and Zhang [26] employed Hamilton's principle to derive the governing equation of motion in which they used a material length scale parameter to capture the microstructure effect. They found the natural frequency predicted by the non-classical plate model (MCST) is higher than that of the classical plate model for very thin plates. Most recently Gupta et al. [27] used the LSM and developed an analytical model for vibration problem of partially cracked isotropic and FGM micro plate. They used classical plate theory in conjunction with modified couple stress theory and concluded that results are higher for fundamental frequencies are always higher for modified couple stress theory. Further, they showed the effect of fibre orientation on vibration characteristics of cracked orthotropic micro-plate [28]. The effect of fluid-structure interaction on vibration characteristics has received much attention due to their importance in various engineering applications. It is well known that the presence of fluidic medium significantly decreases the natural frequencies of plate in comparison with those calculated in vacuum. This decrease in natural frequency is due to the existence of the fluid around the plate which causes increase in the kinetic energy of whole system without a corresponding increase in strain energy. Using the Rayleigh's method Lamb [29] determined the natural frequencies of a thin clamped circular plate in contact with water. The developed method was theoretical and based on calculation of increase in kinetic energy of fluid. Lindholm et al. [30] and Muthuveerappan et al. [31] reported the natural frequencies of free vibrating cantilever plate in air and water. The natural frequencies of plate as affected by plate aspect ratio and thickness ratio are obtained using experimental approach. Kwak [32] studied the effect of virtual added mass on natural frequencies and mode shapes of rectangular plate coupled with water. They employed Rayleigh-Ritz method to calculate added virtual mass incremental (AVMI) factor for rectangular plate. The dry and wet dynamic characteristics of cantilever plates partially or totally immerged in water are studied by Fu and Price [33]. They employed finite element method and singularity distribution function approach to analyze the vibration response of cantilever plate in air and water. Kwak and Kim [34] determined the added virtual mass incremental (AVMI) factor which shows the increase in inertia due to presence of fluid. They studied the effect of fluidic medium on axisymmetric vibration of floating circular plate on liquid. The natural frequencies of annular plate coupled with fluid are determined using added mass approach by Amabali [35]. Haddara and Cao [36] studied the dynamic behavior of rectangular plates vibrating under water. The effect of boundary conditions and depth of submergence has been investigated experimentally and analytically in their study. Soedel and Soedel [37] found the coupled equations of motion of plates carrying fluids. They developed a closed form solution for natural frequencies of fluid-plate coupled system. Kerboua et al. [38] developed a mathematical model for free vibrating plate in contact with water using the combination of the finite element method and Sander's shell theory. Recently, Hosseini Hashemi et al. [39] worked on free vibration analysis of horizontal rectangular plates partially and totally submerged in fluid. They developed a mathematical model for moderately thick rectangular plate based on the Mindlin plate theory for six different boundary conditions. Vibration analysis of plates considering the influence of both crack and fluidic medium are found in few investigations. Liu et al. [40] employed a finite element method (FEM) for hydro elastic natural vibration of perforated plates. They investigated the influence of crack on the vibration analysis of a circular plate submerged in fluid. Recently, Si et al. [41,42] proposed a computational method for dynamic analysis of free vibrating cracked circular plate in contact with water on one side. The influences of water and crack on different modes of vibration are investigated using Rayleigh–Ritz method and finite element method.

The literature lacks in the results for free vibration analysis of cracked plates considering the effect of thermal environment and fluidic medium. Thus in order to develop theoretical understanding of influence of crack on vibration problem of submerged plate subjected to thermal environment become significant. The present work fills this gap by proposing an analytical model which addresses the following:

1. Modeling of free vibrations of partially cracked-submerged isotropic plate considering the effect of thermal environment.
2. The classical relation for central deflection of cracked isotropic plate which shows an important phenomenon of shift in primary resonance due to crack, internal material scale parameter and temperature rise.
3. New results are presented for fundamental frequencies of cracked isotropic plate as affected by crack length, internal material scale parameter, plate thickness and rise in temperature.
4. The analytical model has the obvious advantage of having efficient computation time, ease of parametric study and improved physical understanding of the problem when compared to Finite Element Models.

The present work references the analytical model proposed by Israr et al. [11], extended in the recent work of joshi et al. [14,15] and extends it to the case of partially cracked isotropic plate in the presence of thermal environment and fluidic medium. The governing equation is derived using equilibrium principle of non-classical plate theory and potential flow theory. The effect of crack is considered in the form of additional bending moment and membrane force using line spring model. Galerkin method is employed for the solution of derived governing equation. The plate configuration is shown in Fig. 1 in which the dimensions of the plate taken along x and y directions are L_1 and L_2 respectively. The thickness of the plate is denoted by h. $2a$ is the length of crack at plate centre which is parallel to the x axis and the depth of the crack is assumed to be constant.

1.1 Modified couple stress theory

The behavior of micro-plates has been proven to be size dependent and hence implementation of strain gradient theories (containing internal material length scale parameters) is essential due to inadequacy of the classical plate theory. From literature review it is seen that many of researchers used strain gradient (higher order) theories [19–24] to capture the size effect of microstructures in form of internal material length scale parameter. It is seen that in their work, the scale factor used in form of internal material length scale parameter is accompanied by a second order derivative operator. However, the main drawback of their models are that the presence of the micro structural effect raises the order of the resulting partial differential equation from four (classical case) to six (gradient case) [19]-[23]. As well as the classical boundary conditions are supplemented by additional (non-classical) ones containing higher order traction and higher order moments. Hence, the employed analytical solutions are restricted only to simple geometric shapes. To resolve this problem of handling plates with complex geometries and boundary conditions Tsiatas [18] and Yin et al. [22] proposed a new non classical Kirchhoff's plate model for the static and dynamic analysis of isotropic micro-plates with arbitrary shape based on the simplified couple stress theory of Yang et al. [24].

In the simplified couple stress theory, the strain energy density (*U*) in three-dimensional body occupying a volume *V* bounded by the surface *G* is given by Yang et al. [24] as:

$$U = \frac{1}{2}\int\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}N_{ij}\right)dV \tag{1}$$

where

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial j} + \frac{\partial u_j}{\partial i}\right) \tag{2}$$

$$N_{ij} = \frac{1}{2}\left(\frac{\partial \theta_i}{\partial j} + \frac{\partial \theta_j}{\partial i}\right) \tag{3}$$

The strain tensor (ε_{ij}) and the symmetric part of the curvature tensor (N_{ij}), respectively, u_{ij} is the displacement vector and θ_{ij} is the rotation vector which can defined as:

$$\theta_i = \frac{1}{2}\theta_{ijk}\frac{\partial uk}{\partial j} \tag{4}$$

where θ_{ijk} is the permutation symbol. As per modified couple stress theory, the stress tensor (σ_{ij}) and the deviatoric part of the couple stress tensor (m_{ij}) can be expressed as (Ref.[18]),

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{5}$$

$$m_{ij} = 2\mu_0 l^2 N_{ij} \tag{6}$$

where, λ and μ_0 are the Lamé constants, δ_{ij} is the Kronecker delta and *l* is a material length scale parameter. This Eqs. (5) and (6) described the two dimensional state of stress. From Eq. (6) it is observed that the couple stress tensor m_{ij} is symmetric and from Eq. (3) the curvature tensor N_{ij} is also symmetric. That is, only the symmetric part of the rotation gradient and the symmetric part of displacement gradient contribute to the deformation energy (Ref.[24]) which is different from that in the classical couple stress theory.

In the work of Tsiatas [18], after the appropriate replacement of the Lamé constants by the modulus of elasticity *E* and the Poisson's ratio v , the stress tensor (σ_{ij}) and the couple stress tensor (m_{ij}) can be expressed as:

$$\sigma_{\alpha\beta} = \frac{E}{1-v^2}\left[v\,\varepsilon_{kk}\delta_{\alpha\beta} + (1-v)\,\varepsilon_{\alpha\beta}\right] \tag{7}$$

$$m_{\alpha\beta} = 2Gl^2 N_{\alpha\beta} \tag{8}$$

where $G = E/2(1+v)$ are the shear modulus, *l* is a material length scale parameter and N_{ij} is the curvature tensor. From Eq. (7) and (8) the expression for the bending moment and couple moment tensors can be written as [18],

$$M_{\alpha\beta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha\beta} z dz \tag{9}$$

$$Y_{\alpha\beta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} m_{\alpha\beta} dz \tag{10}$$

Expressing the strain and curvature tensors in form of lateral deflection of plate we have (Ref. [18]),

$$M_x = M_{11} = -D\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) \qquad Y_x = Y_{11} = 2D^l \frac{\partial^2 w}{\partial x \, \partial y}$$

$$M_y = M_{22} = -D\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right) \qquad Y_y = Y_{22} = -2D^l \frac{\partial^2 w}{\partial x \, \partial y}$$

$$M_{xy} = M_{11} = M_{yx} = M_{22} = D(1-v)\frac{\partial^2 w}{\partial xy} \qquad Y_{xy} = Y_{yx} = Y_{12} = Y_{21} = D^l\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2}\right)$$

where, $D = \dfrac{Eh^3}{12(1-v^2)}$ is the flexural rigidity of plate and $D^l = \dfrac{El^2 h}{2(1+v)}$ shows the bending rigidity due to couple stress of micro plate and *l* is a material length scale parameter. This D^l also shows the contribution of rotation gradients to the bending rigidity.

Tsiatas [18] employed the Gauss divergence theorem to the total potential energy of a deformable body and arrived at the expression of bending moment which shows two components of bending; (i) the bending due to microstructure and (ii) pure plate bending. This expression for moment can be written as:

$$M_{ii}^* = M_{ii} = M_{ii}^l = -\left(D + D^l\right)\left(\frac{\partial^2 w}{\partial i^2} + v\frac{\partial^2 w}{\partial j^2}\right) \qquad M_{ij}^* = M_{ij} = M_{ij}^l = \left(D + D^l\right)(1-v)\frac{\partial^2 w}{\partial i \, \partial j}$$

From the above expression, it is seen that the effect of microstructure in the form of a single material length scale parameter "*l*", contributing to the bending moment and increasing the flexural rigidity by $D^l = \dfrac{El^2 h}{2(1+v)}$. The advantage of the modified couple stress theory developed by Tsiatas [18] is that a single parameter can capture the microstructure effect and its contribution to the flexural rigidity can be easily coupled with the rigidity used in classical plate theory. It is important here to note that Yin et al. [22] employed the additional rigidity (D^l) due to microstructure in their analysis of dynamics of micro-plate. The present work employs the additional flexural rigidity established by Tsiatas [18] and applies it to the case of cracked-submerged plate in the presence of thermal environment.

2 GOVERNING EQUATION

Based on the equilibrium principle of Kirchhoff's thin plate theory, Joshi et al. [14] showed the governing equation of motion for a partially cracked isotropic plate in presence of thermal environment as:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=-\rho h\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 M_T}{\partial x^2}-\frac{\partial^2 M_T}{\partial y^2}+\frac{\partial^2 m_y}{\partial y^2}-N_{Tx}\frac{\partial^2 w}{\partial x^2}-N_{Ty}\frac{\partial^2 w}{\partial y^2}-n_y\frac{\partial^2 w}{\partial y^2}+P_z \tag{11}$$

where, $D=\dfrac{Eh^3}{12(1-\nu^2)}$ is the flexural rigidity of plate and ρ is the effective density of the plate. w is the transverse deflection of the plate. m_y and n_y are the additional bending moment and in-plane force due to effect of crack. M_T denotes the thermal bending moment. N_{Tx} and N_{Ty} represents the in-plane forces per unit length due to the thermal environment.

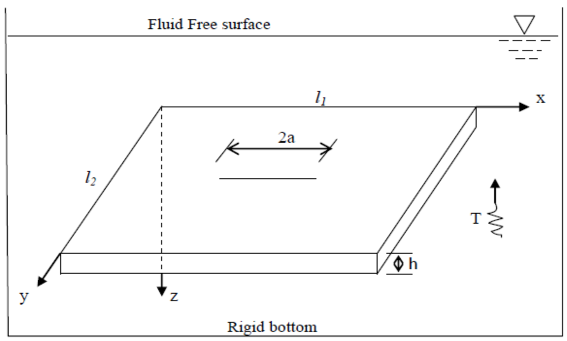


Fig.1
Plate configuration showing a surface crack at the centre of plate.

In this section the new governing equation for a partially cracked isotropic plate (as shown in Fig. 1) considering the effects of thermal environment and surrounding fluid medium is derived based on the Kirchhoff's thin plate theory and modified couple stress theory (MCST). The assumptions involved in the modeling are: (1) The plate is assumed as thin, impeccably elastic and homogenous composed of isotropic material and has a uniform thickness '*h*' which is minutely diminutive as compared to its other dimensions. (2) The mid-plane stays unstrained consequent to bending, therefore the normal strain ε_z, resulting from transverse loading, may be omitted. (3) The normal stress σ_z acting in the transverse direction of plate is considered to be diminutive compared to the other stress components and therefore, it can be neglected from stress-strain relationship in the modelling. (4) Effects of shear deformation and rotary inertia are neglected. (5) The fluid flow is potential (i.e., homogeneous, incompressible, inviscid and its motion is irrotational). (6) The temperature variation is thought to be linear all through the thickness of the plate; $T(z)=T_{avg}+\left((\Delta T)z\right)/2$, where $\Delta T=T_t-T_b$ is the temperature difference between the top and the bottom surface of the plate and $T_{avg}=(T_t-T_b)/2$ is average temperature.

Consider a plate element containing a part through crack of length $2a$ at its centre as shown in Fig. 2. The bending moments and internal forces acting on mid plane of the plate are considered as per classical thin plate theory. On resolving the forces along z direction and taking moment equilibrium about x and y axis we get the following equilibrium equations.

$$\sum F_z=0; \qquad \frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}=\rho h\frac{\partial^2 w}{\partial t^2}+\Delta P-P_z \tag{12}$$

where, Q_x and Q_y are the forces per unit length in the transverse direction, ΔP is the fluid dynamic pressure difference between the top and bottom surface of the plate. $\rho h\dfrac{\partial^2 w}{\partial t^2}$ represents the inertia force in which ρ is the density of plate and h is the thickness of the plate and P_z is the lateral load per unit area.

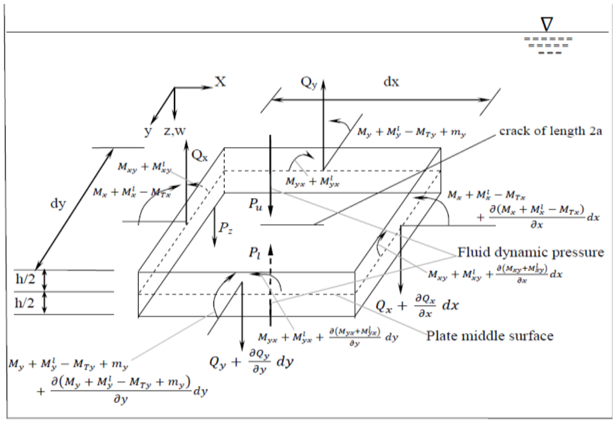


Fig.2
Plate element showing bending moments and transverse forces on middle surface.

Taking moment equilibrium about the x and y axis, we get

$$\sum M_x = 0; \qquad \frac{\partial M_y}{\partial y}+\frac{\partial M_y^l}{\partial y}-\frac{\partial M_{Ty}}{\partial y}+\frac{\partial m_y}{\partial y}+\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_{xy}^l}{\partial x}=Q_y \tag{13}$$

$$\sum M_y = 0; \qquad \frac{\partial M_x}{\partial x}+\frac{\partial M_x^l}{\partial x}-\frac{\partial M_{Tx}}{\partial x}+\frac{\partial M_{yx}}{\partial y}+\frac{\partial M_{yx}^l}{\partial y}=Q_x \tag{14}$$

On substituting Eq.(13) and (14) into Eq.(12) one obtains

$$\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_x^l}{\partial x^2}+\frac{\partial^2 M_y}{\partial y^2}+\frac{\partial^2 M_y^l}{\partial y^2}+\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_{xy}^l}{\partial x\,\partial y}+\frac{\partial^2 M_{yx}}{\partial x\,\partial y}+\frac{\partial^2 M_{yx}^l}{\partial x\,\partial y}+\frac{\partial^2 m_y}{\partial y^2}-\frac{\partial^2 M_{Tx}}{\partial x^2}-\frac{\partial^2 M_{Ty}}{\partial y^2}=\rho h\frac{\partial^2 w}{\partial t^2}+\Delta P-P_z \tag{15}$$

where, $M_{Tx}=M_{Ty}=\dfrac{\alpha E}{1-v}\int_{-h/2}^{h/2}\left(T(z)\right)z\,dz$ is the moment due to thermal environment. M_x, M_y and $M_{xy}=M_{yx}$ are the internal bending and twisting moments respectively. Similarly M_x^l, M_y^l and $M_{xy}^l=M_{yx}^l$ are bending and twisting moments due to the microstructure of the plate. m_y is additional bending moment applied by the net ligament (intact portion) which represents the effect of line crack as deduced in line spring model. The bending moments can be expressed in terms of transverse deflection as:

$$\left.\begin{aligned}
M_x &= -D\left(\frac{\partial^2 w}{\partial x^2}+v\frac{\partial^2 w}{\partial y^2}\right) & M_y &= -D\left(\frac{\partial^2 w}{\partial y^2}+v\frac{\partial^2 w}{\partial x^2}\right)\\
M_x^l &= -D^l\left(\frac{\partial^2 w}{\partial x^2}+v\frac{\partial^2 w}{\partial y^2}\right) & M_y^l &= -D^l\left(\frac{\partial^2 w}{\partial y^2}+v\frac{\partial^2 w}{\partial x^2}\right)\\
M_{xy} = M_{xy} &= -D\left(1-v\right)\frac{\partial^2 w}{\partial x\,\partial y} & M_{xy}^l = M_{yx}^l &= -D^l\left(1-v\right)\frac{\partial^2 w}{\partial x\,\partial y}
\end{aligned}\right\} \tag{16}$$

On expressing the moments in terms of transverse deflection from Eq. (16) in Eq.(15) we get

$$\left(D+D^l\right)\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=-\rho h\frac{\partial^2 w}{\partial t^2}-\Delta P-\frac{\partial^2 M_{Tx}}{\partial x^2}-\frac{\partial^2 M_{Ty}}{\partial y^2}+\frac{\partial^2 m_y}{\partial y^2}+P_z \tag{17}$$

where $D^l = \dfrac{El^2h}{2(1+v)}$ is the additional bending rigidity due to couple stress for micro plates (Refs.[22][27][28]). l represents the internal material scale parameter. With the in-plane displacements in both x and y direction being restricted, the uniform or non-uniform heating of the cracked plate causes membrane or in-plane forces[27]. The membrane forces considered for the cracked plate due to thermal environment are shown in Fig. 3. According to Berger's formulation the stretching effect produced by the lateral deflection introduces nonlinearity to the governing equation but does not affect the stiffness of plate Ref. [11,13–15]. In this work these stretching effects are neglected and only the membrane forces induced due to temperature variation are considered. The shear force is neglected in Fig. 3 as the temperature does not affect the shear components [43]. The equilibrium in the z direction is considered by assuming the two adjacent edges as fixed as shown in Fig. 3. Other boundary conditions are equally possible. The projection of the in-plane forces along z axis leads to:

$$\sum F_z = -\left(N_{Tx} + \frac{\partial N_{Tx}}{\partial x}dx\right)dy\,\frac{\partial^2 w}{\partial x^2}dx - \left(N_{Ty} + \frac{\partial N_{Ty}}{\partial y}dy + n_y + \frac{\partial n_y}{\partial y}dy\right)dx\,\frac{\partial^2 w}{\partial y^2}dy \tag{18}$$

After neglecting the higher order terms (containing dx^2 and dy^2), the transverse in-plane forces due to thermal stress can be given by

$$\sum F_z = -N_{Tx}\frac{\partial^2 w}{\partial x^2} - N_{Ty}\frac{\partial^2 w}{\partial y^2} - n_y\frac{\partial^2 w}{\partial y^2} \tag{19}$$

where N_{Tx} and N_{Ty} are the in-plane forces per unit length due to the thermal environment and n_y is the additional membrane force per unit length due to the crack [14]. On adding of the lateral force given by Eq. (19) to the moment equilibrium (Eq. (17)) yields the governing equation of cracked isotropic submerged plate subjected to thermal environment and can be stated as:

$$\left(D + D^l\right)\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho h\frac{\partial^2 w}{\partial t^2} - \Delta P - \frac{\partial^2 M_{Tx}}{\partial x^2} - \frac{\partial^2 M_{Ty}}{\partial y^2} + \frac{\partial^2 m_y}{\partial y^2} - N_{Tx}\frac{\partial^2 w}{\partial x^2} - N_{Ty}\frac{\partial^2 w}{\partial y^2} - n_y\frac{\partial^2 w}{\partial y^2} + P_z \tag{20}$$

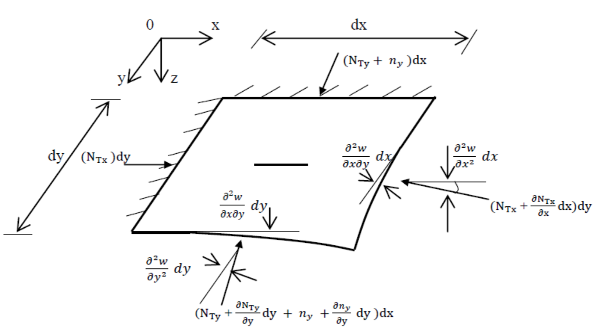


Fig.3
In-plane forces on a plate element due to thermal environment and crack of length $2a$.

The crack terms (m_y and n_y) are obtained using simplified line spring model (LSM) given by Rice and Levy [9]. According to the LSM, the three-dimensional problem can be reduced to quasi-two-dimensional by reducing the net ligament stresses to the neutral plane of the plate as an unknown membrane and bending load. It gives the relationship between the constraining effect produced by the net ligament stresses and the tensile and bending moments at far sides of the plate. Israr et al. [11] used the relationship of bending and tensile stresses at crack tips and far edges of plate given by Rice and Levy [9] and transformed it for force effects. They give a new relationship between tensile and bending loads at far sides of the plate with those at crack surface. Recently, Joshi et al. [14,15] derived the above relationship of the tensile and bending loads at the far sides of the plate and at the crack location in presence of thermal environment for the isotropic and orthotropic plates. Thus the relations for resultant in-plane forces n_y, N_{Ty} and bending moments m_y, M_y can be written as [14],

$$n_y = \frac{2a}{\left(6\alpha_{bt} + \alpha_{tt}\right)\left(1-\nu^2\right)h + 2a} N_{Ty} \tag{21}$$

$$m_y = -\frac{2a}{3\left(\frac{\alpha_{bt}}{6} + \alpha_{bb}\right)(3+\nu)(1-\nu)h + 2a}\left(M_y + M_y^l - M_{Ty}\right) \tag{22}$$

where, the terms $\alpha_{tt}, \alpha_{bb}, \alpha_{bt} = \alpha_{tb}$ are crack compliance coefficients for stretching, bending and tensile-bending respectively. These crack compliance coefficients depend on crack depth (*d*) to thickness (*h*) ratio and vanish when $d = 0$. The required expressions for compliance coefficients as a function of the ratio of crack depth to plate thickness $(\delta = d/h)$ can be written as (Ref. [11]),

$$\alpha_{tt} = 1.154\delta^2\left[1.98 - 0.54\delta^1 + 18.65\delta^2 - 33.70\delta^3 + 99.26\delta^4 - 211.90\delta^5 + 436.84\delta^6 - 460.48\delta^7 + 289.98\delta^8\right]$$
$$\alpha_{bb} = 1.154\delta^2\left[1.98 - 3.28\delta^1 + 14.43\delta^2 - 31.26\delta^3 + 63.56\delta^4 - 103.36\delta^5 + 147.52\delta^6 - 127.69\delta^7 + 61.50\delta^8\right]$$
$$\alpha_{bt} = \alpha_{tb} = 1.154\delta^2\left[1.98 - 1.91\delta^1 + 16.015\delta^2 - 34.84\delta^3 + 83.93\delta^4 - 153.65\delta^5 + 256.72\delta^6 - 244.67\delta^7 + 133.55\delta^8\right]$$

It is noted that these above expressions for compliance coefficients are valid only for $(\delta = d/h)$ values within the range 0.1– 0.7 and in the present model these δ is taken 0.6.

On employing Eq. (21) and (22) in Eq. (20) and expressing the moment M_y and M_y^l in terms of lateral deflection from Eq. (16) we get the required governing equation of cracked plate as:

$$\begin{aligned}\left(D + D^l\right)\left(\nabla^4 w\right) = &-\rho h\frac{\partial^2 w}{\partial t^2} - \Delta P + \frac{2a}{3\left(\frac{\alpha_{bt}}{6} + \alpha_{bb}\right)(3+\nu)(1-\nu)h + 2a}\left\{\left(D + D^l\right)\left(\frac{\partial^4 w}{\partial y^4} + \nu\frac{\partial^4 w}{\partial x^2\partial y^2}\right) - \frac{\partial^2 M_{Ty}}{\partial y^2}\right\} \\ &-\frac{\partial^2 M_{Tx}}{\partial x^2} - \frac{\partial^2 M_{Ty}}{\partial y^2} - N_{Tx}\frac{\partial^2 w}{\partial x^2} - N_{Ty}\frac{\partial^2 w}{\partial y^2} - \frac{2a}{\left(6\alpha_{bt} + \alpha_{tt}\right)\left(1-\nu^2\right)h + 2a}N_{Ty}\frac{\partial^2 w}{\partial y^2} + P_z\end{aligned} \tag{23}$$

where, $\nabla^4 = \left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}\right)$ is the laplacian operator.

3 FLUID MODELING

The fluid pressure acting upon the plate surface is expressed as a function acceleration which helps to form the required governing equation of a coupled fluid-plate structure system. The following assumptions are taken to express the dynamic behavior of fluid as the mathematical formulations:

1. The fluid flow is potential (i.e., homogeneous, incompressible, inviscid and its motion is irrotational).
2. Small amplitude of bending vibrations is considered (i.e., fluid motion is small).
3. The fluid pressure acting on plate is purely normal to the plate surface and there is no shear since the fluid flow is inviscid.
4. Effect of boundary conditions on the plane wave number is neglected.
5. Interaction between the crack and fluid if any is neglected.

Based on above mentioned hypothesis, the velocity potential function $\phi\,(x, y, z, t)$ satisfying the Laplace's equation can be expressed in the Cartesian coordinate system as:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{24}$$

Using Bernoulli's equation, the fluid pressure at any point of fluid-plate interface can be expressed by

$$P_u = P_{z=0} = -\rho_f \left(\frac{\partial\phi}{\partial t}\right)_{z=0} \tag{25}$$

$$P_l = P_{z=-h} = -\rho_f \left(\frac{\partial\phi}{\partial t}\right)_{z=-h} \tag{26}$$

where ρ_f is fluid density per unit volume.

Assuming the velocity potential function ϕ be the function of two separate variables.

$$\phi(x, y, z, t) = F(z) S(x, y, t) \tag{27}$$

where $F(z)$ and $S(x, y, t)$ are the two separate functions which is to be determined. For a permanent contact between the plate surface and peripheral fluid layer, the impermeability condition of the plate surface requires that the out-of-plane velocity component of the fluid on the plate surface should match the instantaneous rate of change of the plate displacement in the transversal direction. So, at the fluid-plate interfaces the kinematic boundary conditions can be expressed as:

$$\left(\frac{\partial\phi}{\partial z}\right)_{z=0} = \frac{\partial w}{\partial t} \tag{28}$$

$$\left(\frac{\partial\phi}{\partial z}\right)_{z=-h} = \frac{\partial w}{\partial t} \tag{29}$$

On introducing Eq. (27) to Eq. (28) and Eq. (29) we get

$$S(x, y, t) = \frac{1}{\left(dF(z)/dz\right)_{z=0}} \frac{\partial w}{\partial t} \tag{30}$$

$$S(x, y, t) = \frac{1}{\left(dF(z)/dz\right)_{z=-h}} \frac{\partial w}{\partial t} \tag{31}$$

By substituting Eq. (30) and (31) in Eq. (27) the velocity potential on fluid-plate interfaces (i.e., upper and lower surface of plate) can be expressed as:

$$\phi(x, y, z, t) = \frac{F(z)}{\left(dF(z)/dz\right)_{z=0}} \frac{\partial w}{\partial t} \tag{32}$$

$$\phi(x, y, z, t) = \frac{F(z)}{\left(dF(z)/dz\right)_{z=-h}} \frac{\partial w}{\partial t} \tag{33}$$

The following differential equation of second order is obtained by substituting above Eq. (32) or (33) into Eq. (24).

$$\frac{d^2F(z)}{dz^2}-\mu^2F(z)=0 \tag{34}$$

where μ is plane wave number, which can be determined by $\mu=\pi\sqrt{\frac{1}{l_1^2}+\frac{1}{l_2^2}}$ (Ref.[38]). The general solution for Eq. (34) can be defined as:

$$F(z)=Ae^{\mu z}+Be^{-\mu z} \tag{35}$$

On substituting Eq. (35) into Eq. (32) and (33) we get following expression for velocity potential function on fluid-plate interface.

$$\phi(x,y,z,t)=\frac{Ae^{\mu z}+Be^{-\mu z}}{\left(dF(z)/dz\right)_{z=0}}\frac{\partial w}{\partial t} \tag{36}$$

$$\phi(x,y,z,t)=\frac{Ae^{\mu z}+Be^{-\mu z}}{\left(dF(z)/dz\right)_{z=-h}}\frac{\partial w}{\partial t} \tag{37}$$

where A and B are the two unknown constants and can be determined using two extreme boundary conditions fluid-plate interface and at fluid extremity surfaces $z=h_1$ and $z=(h+h_2)$.

3.1 Plate-fluid model with free surface

Assuming the disturbance due to free surface motion of the fluid is insignificant, the following boundary condition may be applied for velocity potential at the free surface of fluid [38], see Fig. 4.

$$\left(\frac{\partial\phi}{\partial z}\right)_{z=h_1}=-\frac{1}{\mathrm{g}}\left(\frac{\partial^2\phi}{\partial t^2}\right)_{z=h_1} \tag{38}$$

where, 'g' is the acceleration due to gravity. Substitution of Eq. (36) into the above Eq. (38) and (28) gives following expression for velocity potential function:

$$\phi(x,y,z,t)=\frac{1}{\mu}\left[\frac{e^{\mu z}+Ce^{-\mu(z-2h_1)}}{1-Ce^{2\mu h_1}}\right]\frac{\partial w}{\partial t} \tag{39}$$

where, $c=\frac{\mathrm{g}\mu-\omega^2}{\mathrm{g}\mu-\omega^2}$ and ω is the frequency of wave motion at free surface of fluid.

The fluid dynamic pressure acting on the upper surface of plate can be obtained by substituting above Eq. (39) of velocity potential into Eq. (25) as:

$$P_u=-\frac{\rho_f}{\mu}\left[\frac{1+Ce^{2\mu h_1}}{1-Ce^{2\mu h_1}}\right]\frac{\partial^2 w}{\partial t^2} \tag{40}$$

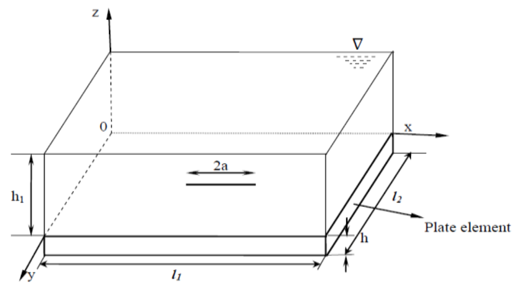


Fig.4
Plate-fluid model with free surface.

3.2 Plate-fluid model bounded by a rigid wall (Floating plate)

The boundary condition at the rigid base of tank represented in Fig. 5 is referred as null-frequency condition and can be expressed as:

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=-(h+h_2)} = 0 \tag{41}$$

On substituting Eq. (37) into Eq. (41) and (29), the following expression for velocity potential is obtained as:

$$\phi(x, y, z, t) = \frac{1}{\mu}\left[\frac{e^{\mu z} + e^{-2\mu(h+h_2)} e^{-\mu z}}{e^{-\mu h} - e^{-2\mu(h+h_2)} e^{\mu h}}\right]\frac{\partial w}{\partial t} \tag{42}$$

From Eq. (42) and Eq. (26), the fluid dynamic pressure at lower surface of plate can be expressed as:

$$P_l = -\frac{\rho_f}{\mu}\left[\frac{1+e^{-2\mu h_2}}{1-e^{-2\mu h_2}}\right]\frac{\partial^2 w}{\partial t^2} \tag{43}$$

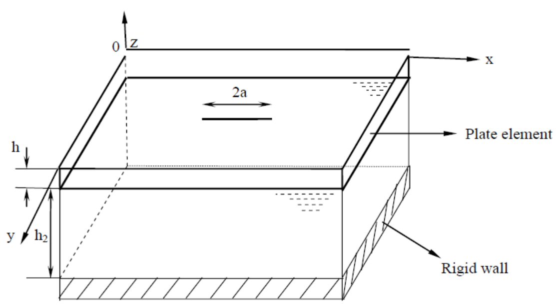


Fig.5
Plate-fluid model bounded by a rigid wall (A floating plate).

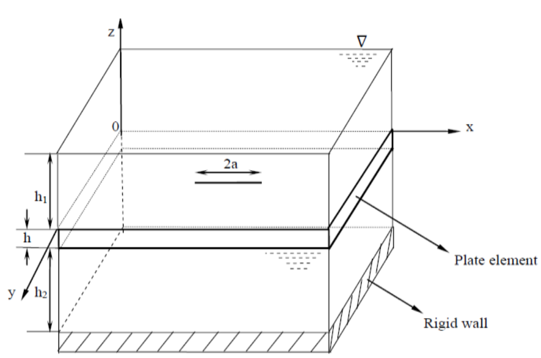


Fig.6
Plate-fluid model bounded by a rigid wall (A submerged plate).

In case of totally submerged plate (Fig. 6.) the net fluid dynamic pressure can be determined using above two fluid boundary conditions. The resulting fluid dynamic pressure difference (ΔP) for the horizontally submerged plate can expressed as:

$$\Delta P = P_u - P_l = -\frac{\rho_f}{\mu}\left[\frac{1+Ce^{2\mu h_1}}{1-Ce^{2\mu h_1}} - \frac{1+e^{-2\mu h_2}}{1-e^{-2\mu h_2}}\right]\frac{\partial^2 w}{\partial t^2} \tag{44}$$

$$\Delta P = m_{add}\frac{\partial^2 w}{\partial t^2} \tag{45}$$

where, $m_{add} = -\frac{\rho_f}{\mu}\left[\frac{1+Ce^{2\mu h_1}}{1-Ce^{2\mu h_1}} - \frac{1+e^{-2\mu h_2}}{1-e^{-2\mu h_2}}\right]$ is the virtual added mass of submerged plate due to surrounding fluid. When a plate vibrates under a fluid medium then its mass is increased by the mass of the virtual layer of vibrating fluid and this phenomenon is called the added virtual mass effect. On substituting Eq. (45) into Eq. (23) the required governing equation for cracked-submerged plate subjected to thermal environment can be expressed as:

$$\begin{aligned}\left(D+D^l\right)\left(\nabla^4 w\right) = -\left(\rho h + m_{add}\right)\frac{\partial^2 w}{\partial t^2} + \frac{2a}{3\left(\frac{\alpha_{bt}}{6}+\alpha_{bb}\right)(3+\nu)(1-\nu)h+2a}\left\{\left(D+D^l\right)\left(\frac{\partial^4 w}{\partial y^4}+\nu\frac{\partial^4 w}{\partial x^2\partial y^2}\right)-\frac{\partial^2 M_{Ty}}{\partial y^2}\right\} \\ -\frac{\partial^2 M_{Tx}}{\partial x^2}-\frac{\partial^2 M_{Ty}}{\partial y^2}-N_{Tx}\frac{\partial^2 w}{\partial x^2}-N_{Ty}\frac{\partial^2 w}{\partial y^2}-\frac{2a}{\left(6\alpha_{bt}+\alpha_{tt}\right)\left(1-\nu^2\right)h+2a}N_{Ty}\frac{\partial^2 w}{\partial y^2}+P_z\end{aligned} \tag{46}$$

4 GENERAL SOLUTION OF GOVERNING EQUATION

The presence of external environment like rise in temperature has been included in the governing equation of cracked plate in the form of thermal bending moments and in-plane compressive forces. Three significant solution cases arise by the presence of thermal environment: (1) The plate is heated throughout its volume uniformly ($M_{Tx} = M_{Ty} = 0$) with in-plane deflections restricted. (2) Unrestricted in-plane deflections ($N_{Tx} = N_{Ty} = 0$) and only thermal moment. (3) The third solution can be found for presence of both thermal bending and in-plane compressive thermal forces. The present work restricts itself to the formulation of governing equation for all the above three cases and give results for case (1). Since, in majority of engineering applications thin plate structures having good thermal conductivity are used, there is little temperature gradient along the thickness of the plate and they can be considered as uniformly heated plates. The constant in-plane compressive forces induced due to temperature are only considered making the model geometrically linear. The modal functions, depending on the boundary conditions can be selected for general solution of governing equation as:

$$w(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} X_m Y_n \psi_{mn}(t) \tag{47}$$

where, X_m and Y_n are the characteristic or modal functions satisfying the boundary conditions of the cracked plate in the fluidic medium, A_{mn} is an arbitrary amplitude and $\psi_{mn}(t)$ is time dependent modal term. On introducing the expression for lateral deflection from Eq. (47) in the governing equation (Eq. (46)) we get

$$\begin{aligned}\left(D+D^l\right)\left(X_m^{iv}Y_n+2X_m^{ii}Y_n^{ii}+Y_n^{iv}X_m\right)A_{mn}\psi_{mn}(t) = -\left(\rho h+m_{add}\right)A_{mn}X_mY_n\frac{\partial^2\psi_{mn}(t)}{\partial t^2} \\ +\left\{\frac{2a}{3\left(\frac{\alpha_{bt}}{6}+\alpha_{bb}\right)(3+\nu)(1-\nu)h+2a}\left(D+D^l\right)\left(Y_n^{iv}X_m+\nu X_m^{ii}Y_n^{ii}\right)-N_{Tx}X_m^{ii}Y_n-N_{Ty}Y_n^{ii}X_m-\frac{2a}{\left(6\alpha_{bt}+\alpha_{tt}\right)\left(1-\nu^2\right)h+2a}N_{Ty}Y_n^{ii}X_m\right\}A_{mn}\psi_{mn}(t)+P_z\end{aligned} \tag{48}$$

In case of a uniform heated isotropic plate, the constant in-plane compressive forces can be expressed as (Refs.[14])

$$N_{Tx} = N_{Ty} = \frac{E\alpha h T_c}{1-\nu} \tag{49}$$

where, T_c is the rise in temperature above which the plate is stress free. For free vibration ($P_z = 0$), integrating over plate area and by multiplying both sides by X_m, Y_n Eq. (48) can be expressed in the form of Duffing equation.

$$M_{mn}\frac{\partial^2 \psi_{mn}(t)}{\partial t^2} + K_{mn}\psi_{mn}(t) = 0 \tag{50}$$

where,

$$M_{mn} = \frac{(\rho h + m_{add})}{D + D^l}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn}\int_0^{l_1}\int_0^{l_2} X_m^{\,2} Y_n^{\,2}\, dx\, dy \tag{51}$$

$$\begin{aligned} K_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn}\int_0^{l_1}\int_0^{l_2} \Bigg\{ &(X_m^{\,iv} Y_n + 2X_m^{\,ii} Y_n^{\,ii} + Y_n^{\,iv} X_m) - \frac{2a\left(\nu X_m^{\,ii} Y_n^{\,ii} + Y_n^{\,iv} X_m\right)}{3\left(\frac{\alpha_{bt}}{6} + \alpha_{bb}\right)(3+\nu)(1-\nu)h + 2a} + \\ &\frac{E\alpha h T_c}{\left(D + D^l\right)(1-\nu)}\left(X_m^{\,ii} Y_n + Y_n^{\,ii} X_m + \frac{2aY_n^{\,ii} X_m}{(6\alpha_{bt} + \alpha_{tt})\left(1-\nu^2\right)h + 2a}\right)\Bigg\} X_m Y_n\, dx\, dy \end{aligned} \tag{52}$$

From Eq. (50) the natural frequency for the cracked-submerged isotropic plate in presence of thermal environment can be calculated as:

$$\omega_{mn}^2 = K_{mn} / M_{mn} \tag{53}$$

5 RELATION FOR CENTRAL DEFLECTION OF PLATE

Consider an isotropic cracked plate with all edges simply supported, subjected to a lateral uniformly distributed dynamic load (P_z) harmonically varying with time. For a plate in the absence of thermal bending ($M_{Tx} = M_{Ty} = 0$) and the presence of constant in-plane forces ($N_{Tx} = N_{Ty} = \frac{E\alpha h T_c}{1-\nu}$) due to uniform rise in temperature, the governing equation (Eq. (46)) becomes:

$$\begin{aligned} &\left(D + D^l\right)\left(\nabla^4 w\right) - \frac{2a}{3\left(\frac{\alpha_{bt}}{6} + \alpha_{bb}\right)(3+\nu)(1-\nu)h + 2a}\left\{\left(D + D^l\right)\left(\frac{\partial^4 w}{\partial y^4} + \nu\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)\right\} \\ &+ \frac{E\alpha h T_c}{1-\nu}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{2a}{(6\delta_{bt} + \delta_{tt})\left(1-\nu^2\right)h + 2a}\frac{\partial^2 w}{\partial y^2}\right) + \left(\rho h + m_{add}\right)\frac{\partial^2 w}{\partial t^2} = P_z \end{aligned} \tag{54}$$

Assuming the solution for lateral deflection as:

$$w(x,y,t)=W_{mn}\sin\left(\frac{m\pi x}{l_1}\right)\sin\left(\frac{n\pi y}{l_2}\right)\sin\left(\omega t\right) \tag{55}$$

where, ω is the vibrational frequency. Now for the case of forced flexural vibrations, the applied dynamic lateral load $P_z = P_z\left(x,y,t\right)$ can be expressed as:

$$P_z(x,y,t)=P_{mn}\sin\left(\frac{m\pi x}{l_1}\right)\sin\left(\frac{n\pi y}{l_2}\right)\sin\left(\varnothing t\right) \tag{56}$$

with $\varnothing$ being the operational frequency of the load. Substituting the general solution of lateral deflection (w) from Eq. (55) with ω being replaced by $\varnothing$ and the lateral dynamic load (P_z) form Eq. (56) into the governing equation (Eq. (54)) results in an expression for W_{mn} . Thus the classical relation for central deflection of cracked isotropic submerged plate in the presence of thermal environment can be proposed as:

$$W_{mn}=\frac{P_{mn}}{\pi^4\left(D+D^l\right)\left(\left(\frac{m}{l_1}\right)^2+\left(\frac{n}{l_2}\right)^2\right)^2-\frac{2a}{H_2}\pi^4\left(D+D^l\right)\left(\left(\frac{n}{l_2}\right)^4+\nu\frac{m^2n^2}{l_1^2l_2^2}\right)-\pi^2\frac{E\alpha hT_c}{1-\nu}\left(\left(\frac{m}{l_1}\right)^2+\left(\frac{n}{l_2}\right)^2\left(1+\frac{2a}{H_1}\right)\right)-\varnothing^2\left(\rho h+m_{add}\right)} \tag{57}$$

where, $H_2=3\left(\frac{\delta_{bt}}{6}+\delta_{bb}\right)(3+\nu)(1-\nu)h+2a$ and $H_1=\left(6\delta_{bt}+\delta_{tt}\right)\left(1-\nu^2\right)h+2a$.

For a special case of a square plate with side of l_1 and $m=n=1$, the central deflection W_{11} takes the form which clearly shows the presence of crack and temperature terms.

$$W_{11}=\frac{P_{mn}}{\frac{\pi^4}{l_1^4}\left[4\left(D+D^l\right)-\frac{2a}{H_2}\left(D+D^l\right)\left(1+\nu\right)-\frac{l_1^2E\alpha hT_c}{\pi^2(1-\nu)}(2+\frac{2a}{H_1})\right]-\varnothing^2\left(\rho h+m_{add}\right)} \tag{58}$$

The result for central deflection of a cracked isotropic submerged plate without influence of thermal environment ($T_c=0$) can be expressed as:

$$W_{11}^{cracked}=\frac{P_{mn}}{\frac{\pi^4}{l_1^4}\left[4\left(D+D^l\right)-\frac{2a}{H_2}\left(D+D^l\right)\left(1+\nu\right)\right]-\varnothing^2\left(\rho h+m_{add}\right)} \tag{59}$$

Similarly, the central deflection of a uniformly heated isotropic submerged plate without any crack ($a=0$) can be expressed as:

$$W_{11}^{heated}=\frac{P_{mn}}{\frac{\pi^4}{l_1^4}\left[4\left(D+D^l\right)-\frac{2l_1^2E\alpha hT_c}{\pi^2(1-\nu)}\right]-\varnothing^2\left(\rho h+m_{add}\right)} \tag{60}$$

The central deflection of an intact isotropic submerged plate ($a=0,\ T_c=0$).

$$W_{11}^{MCST}\ or\ W_{11}^{intact}=\frac{P_{mn}}{\frac{\pi^4}{l_1^4}\left[4\left(D+D^l\right)\right]-\varnothing^2\left(\rho h+m_{add}\right)} \tag{61}$$

The corresponding expression for central deflection of an intact isotropic submerged plate ($a=0,\ T_c=0$) based on classical plate can be represented as:

$$W_{11}^{CPT}=\frac{P_{mn}}{\frac{\pi^4}{l_1^4}\left[4D\right]-\varnothing^2\left(\rho h+m_{add}\right)} \tag{62}$$

The central deflection ratio $W_{11}^{cracked}/W_{11}^{intact}$ can be obtained by dividing Eq. (59) and Eq. (61) as:

$$\frac{W_{11}^{cracked}}{W_{11}^{intact}}=\frac{\frac{\pi^4}{l_1^4}\left[4\left(D+D^l\right)\right]-\varnothing^2\left(\rho h+m_{add}\right)}{\frac{\pi^4}{l_1^4}\left[4\left(D+D^l\right)-\frac{2a}{H_2}\left(D+D^l\right)(1+\nu)\right]-\varnothing^2\left(\rho h+m_{add}\right)} \tag{63}$$

or

$$\frac{W_{11}^{cracked}}{W_{11}^{intact}}=\frac{1-\left(\varnothing/\omega_{11}^{intact}\right)^2}{1-\frac{\frac{2a}{H_2}\left(D+D^l\right)(1+\nu)}{\left[4\left(D+D^l\right)\right]}-\left(\varnothing/\omega_{11}^{intact}\right)^2} \tag{64}$$

where, $\omega_{11}^{intact}=\sqrt{\frac{\frac{\pi^4}{l_1^4}\left[4\left(D+D^l\right)\right]}{\left(\rho h+m_{add}\right)}}$ is the natural frequency of intact isotropic submerged plate based on modified couple stress theory.

Similarly, the central deflection ratio $W_{11}^{heated}/W_{11}^{intact}$ can be obtained by dividing Eq. (60) and Eq. (61) as:

$$\frac{W_{11}^{heated}}{W_{11}^{intact}}=\frac{1-\left(\varnothing/\omega_{11}^{intact}\right)^2}{1-\frac{2l_1^2E\alpha hT_c/\pi^2(1-\nu)}{\left[4\left(D+D^l\right)\right]}-\left(\varnothing/\omega_{11}^{intact}\right)^2} \tag{65}$$

The central deflection ratio $W_{11}^{MCST}/W_{11}^{CPT}$ can be obtained by dividing Eq. (61) and Eq. (62) as:

$$\frac{W_{11}^{MCST}}{W_{11}^{CPT}}=\frac{\frac{\pi^4}{l_1^4}\left[4(D)\right]-\varnothing^2\left(\rho h+m_{add}\right)}{\frac{\pi^4}{l_1^4}\left[4\left(D+D^l\right)\right]-\varnothing^2\left(\rho h+m_{add}\right)} \tag{66}$$

or

$$\frac{W_{11}^{MCST}}{W_{11}^{CPT}}=\frac{1-\left(\varnothing/\omega_{11}^{CPT}\right)^2}{1+\frac{D^l}{D}-\left(\varnothing/\omega_{11}^{CPT}\right)^2} \tag{67}$$

where, $\omega_{11}^{CPT}=\sqrt{\frac{\frac{\pi^4}{l_1^4}\left[4(D)\right]}{\left(\rho h+m_{add}\right)}}$ is the natural frequency of intact isotropic submerged plate based on classical plate theory.

6 RESULTS AND DISCUSSION

In this section, the numerical results obtained for fundamental mode of vibration ($m = n = 1$) of partially cracked plate submerged in fluid subjected to thermal environment are presented and discussed to verify the accuracy of present model. Since the literature lacks in results for partially cracked plate with influence of both the thermal environment and fluidic medium, the new results for natural frequencies of cracked plate are presented as a function half crack length (*a*), internal material scale parameter (l / l_1), fluid level (h_1 / l_1) and uniform rise in temperature (T_c). For the purpose of validation of present model, the numerical results obtained are compared with the existing results of literature. Table 1., shows the comparison of frequency parameter of an intact and cracked isotropic plate, for different crack length, material scale parameter and boundary conditions. The material properties of plate for this validation table are taken from Ref. [27]. From Table 1., it can be seen that the present results are in good agreement with the published results which verifies the validity of our proposed model. Validation of results obtained for frequency parameter of intact and cracked isotropic plate subjected to thermal environment is shown in Table 2. Again the good agreement of the results shows that the present model reduces to the model proposed in Ref [14] when the effect of fluidic medium is neglected.

As seen in literature, a few or no work has been done on vibration analysis of cracked plates under influence of fluidic medium and hence the validation of present model is carried out for intact isotropic plate coupled with fluid. Table 3., shows the comparison of results obtained for dimensionless frequency parameter of an intact plate partially/fully immersed in fluid in absence of thermal environment. The comparison is made for two different boundary conditions (SFSF and SSSS). The mechanical and geometrical properties of plate for validation Table 3., are taken from Ref. [39]. The fluid density is taken 1000 $kg\ m^{-3}$. The dimension of reservoir tank on which plate is submerged has taken as 5 m × 5 m × 5 m. In Table 3., the results of frequency parameters for different fluid levels h_1 / l_1 obtained by present theory are slightly higher than the results of existing (MPT) theory it is because of ignoring the effects of shear deformation in present model as the model is based on Classical plate theory. Considering these fact, the present results are in agreement with the available results.

Table 1
Frequency parameter $(F = \omega_{mn} l_1^2 \sqrt{\rho h / D})$ for intact plate and cracked isotropic plate in absence of thermal environment.

B.C.	Crack length	Classical plate theory		Modified couple stress theory			
		$l = 0$		$l = 0.0005$		$l = 0.001$	
	a (m)	Present	Ref. [27]	Present	Ref. [27]	Present	Ref. [27]
SSSS	0	19.739	19.736	19.838	19.835	20.132	20.129
	0.01	19.153	19.151	19.249	19.247	19.535	19.532
	0.05	17.965	17.963	18.055	18.053	18.324	18.321
CCFF	0	5.119	5.118	5.144	5.143	5.221	5.220
	0.01	4.897	4.896	4.922	4.921	4.995	4.994
	0.05	4.438	4.437	4.460	4.459	4.526	4.525
CCSS	0	28.348	28.344	28.489	28.486	28.912	28.908
	0.01	25.472	25.469	27.540	27.536	27.947	27.944
	0.05	24.485	24.481	25.602	25.597	25.980	25.976

SSSS- All edges simply supported, CCFF- Two adjacent edges clamped and other two free, CCSS- Two adjacent edges clamped and other two simply supported.

Table 2
Comparison of non dimensional frequency parameter ($\omega_{mn} l_1^2 \sqrt{\rho h / D}$) for CCSS plate, $l_1 / l_2 = 1, l = 0$.

T^*	Intact plate		Cracked plate					
			$a = 0.01\ m$		$a = 0.05\ m$		$a = 0.1 m$	
	Present	Ref. [14]	Present	Ref. [14]	Present	Ref. [14]	Present	Ref. [14]
0	28.35	28.35	27.50	27.50	25.47	25.47	24.48	24.48
0.1	26.89	26.89	26.00	26.00	24.17	24.17	23.23	23.23
0.2	25.35	25.35	24.51	24.51	22.78	22.78	21.90	21.90
0.3	23.72	23.72	22.93	22.93	21.31	21.31	20.48	20.48
0.4	21.96	21.96	21.23	21.23	19.73	19.73	18.97	18.97
0.5	20.05	20.05	19.38	19.38	18.01	18.01	17.31	17.31

Table 3
Comparison of non-dimensional frequency parameter $(\omega_{mn} l_1^2 \sqrt{\rho h / D})$ of intact plate submerged in water as a function of fluid level. $l_1 / l_2 = 1, l = 0$.

B.C.	In vacuum		In water					
			$h_2 / l_2 = 0$		$h_2 / l_2 = 0.1$		$h_2 / l_2 = 0.3$	
	Present	MPT [39]	Present	MPT [39]	Present	MPT [39]	Present	MPT [39]
SFSF	9.869	9.458	7.334	6.718	6.881	6.333	6.360	5.843
SSSS	49.348	48.301	42.274	41.429	39.229	38.463	37.685	36.958

The new results for dimensionless frequency parameter for cracked isotropic plate with combined effect of thermal environment and fluidic medium are presented in this work. The material and geometrical properties for the plate are taken as; Young's modulus $E = 207GPa$, material density $\rho = 7850$ kgm^{-3}, Poisson's ratio $v = 0.3$, coefficient of thermal expansion $\alpha = 1.2e-05/^{\circ}C$, $l_1 = 1\,m$, $l_2 = 1\,m$ and thickness $h = 0.01\,m$. The fluid density is 1000 $kg\ m^{-3}$. The reservoir tank on which plate is submerged has taken as 5 m × 5 m × 5 m in size. The depth of crack is taken 6mm and thickness of plate is taken 10 mm throughout the work. It is assumed that the material and fluid properties are independent of temperature. The influence of fluidic medium on fundamental frequencies of the plate is analyzed by placing the plate horizontally in a fluid tank with various level of submergence (h_1 / l_1).

The results obtained for natural frequency of plate as a function of crack length ($a = 0$ to 0.1m), fluid level $h_1 / l_1 = 0.1$ to 0.5) and temperature ($T_c = 0$ to 3 ^{o}C) are shown in Tables 4 and 5 for two different boundary conditions SSSS and CCSS respectively. It is seen from both the tables, for a given crack length and temperature, as the plate's submergence level goes increased form ($h_1 / l_1 = 0.1$ to 0.5) the frequency of plate is decreased. This is because of increase in virtual added mass of plate due to its surrounding fluid. Such phenomenon of variation in frequency is also found in literature Ref. [39] for intact plate. Similarly it is also observed from tables that for a given level of submergence, as the length of crack and temperature of plate is increased from $a = 0$ to 0.1m and $T_c = 0$ to 3^{o}C respectively, the frequency parameter is decreased for both the boundary conditions. This is due to reduction in stiffness of plate and this reduction is clearly shown by the second and third term on expression of resultant plate's stiffness (Eq. (52)) which is due to line crack and temperature respectively. Thus it can be concluded that the presence of thermal environment and crack in submerged plate decreases its natural frequency. Comparing the results of cracked plate for the two different boundary conditions, it is also observed that the CCSS submerged plate is more pronounced by crack length than the SSSS submerged plate.

Table 4
Frequency parameters of SSSS plate as a function fluid level, crack length and temperature $(l / l_2 = 0.001)$.

T_c	Half Crack length (a) (m)	In vacuum	In water				
			$\frac{h_2}{l_2} = 0.1$	$\frac{h_2}{l_2} = 0.2$	$\frac{h_2}{l_2} = 0.3$	$\frac{h_2}{l_2} = 0.4$	$\frac{h_2}{l_2} = 0.5$
0	$a = 0.00$	20.132	8.948	8.285	7.982	7.851	7.796
	$a = 0.01$	19.535	8.682	8.039	7.745	7.618	7.565
	$a = 0.05$	18.323	8.144	7.541	7.265	7.146	7.096
	$a = 0.10$	17.707	7.870	7.287	7.021	6.906	6.857
1	$a = 0.00$	19.170	8.520	7.889	7.601	7.476	7.424
	$a = 0.01$	18.476	8.212	7.604	7.325	7.205	7.155
	$a = 0.05$	17.026	7.568	7.007	6.751	6.640	6.594
	$a = 0.10$	16.262	7.228	6.693	6.448	6.342	6.298
2	$a = 0.00$	18.157	8.070	7.473	7.199	7.081	7.032
	$a = 0.01$	17.353	7.713	7.141	6.880	6.767	6.720
	$a = 0.05$	15.622	6.944	6.429	6.194	6.092	6.050
	$a = 0.10$	14.675	6.523	6.040	5.819	5.723	5.683
3	$a = 0.00$	17.085	7.594	7.031	6.774	6.663	6.616
	$a = 0.01$	16.151	7.179	6.647	6.404	6.299	6.255
	$a = 0.05$	14.078	6.258	5.794	5.582	5.490	5.452
	$a = 0.10$	12.894	5.732	5.307	5.113	5.029	4.994

Table 5
Frequency parameters of CCSS plate as a function fluid level, crack length and temperature (($l / l_2 = 0.001$).

T_c	Half Crack length (a) (m)	In vacuum	In water				
			$\frac{h_2}{l_2} = 0.1$	$\frac{h_2}{l_2} = 0.2$	$\frac{h_2}{l_2} = 0.3$	$\frac{h_2}{l_2} = 0.4$	$\frac{h_2}{l_2} = 0.5$
0	$a = 0.00$	28.913	12.849	11.898	11.463	11.276	11.197
	$a = 0.01$	27.948	12.421	11.501	11.081	10.899	10.823
	$a = 0.05$	25.980	11.546	10.691	10.300	10.132	10.061
	$a = 0.10$	24.971	11.098	10.277	9.901	9.739	9.671
1	$a = 0.00$	28.083	12.481	11.557	11.135	10.952	10.876
	$a = 0.01$	27.033	12.014	11.125	10.718	10.543	10.469
	$a = 0.05$	24.852	11.045	10.227	9.853	9.692	9.624
	$a = 0.10$	23.711	10.538	9.758	9.401	9.247	9.182
2	$a = 0.00$	27.229	12.101	11.206	10.796	10.619	10.545
	$a = 0.01$	26.086	11.593	10.735	10.343	10.173	10.102
	$a = 0.05$	23.671	10.520	9.741	9.385	9.231	9.167
	$a = 0.10$	22.379	9.946	9.210	8.873	8.728	8.667
3	$a = 0.00$	26.347	11.709	10.843	10.446	10.275	10.203
	$a = 0.01$	25.103	11.157	10.331	9.953	9.790	9.722
	$a = 0.05$	22.428	9.968	9.230	8.892	8.747	8.685
	$a = 0.10$	20.964	9.317	8.627	8.312	8.176	8.118

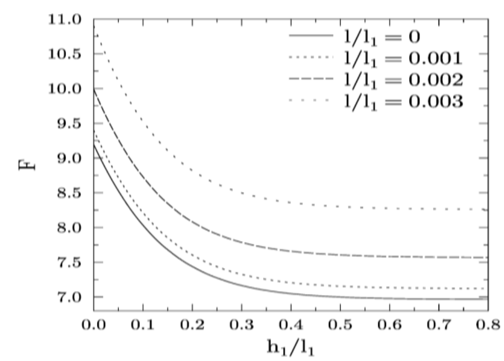
By employing the modified couple stress theory, new results for frequency parameter of submerged plate as affected by crack length, internal material scale parameter and rise in temperature are presented in Table 6 and 7 for SSSS and CCSS boundary conditions respectively. Comparing the results obtained from CPT ($l = 0$) and MCST ($l \neq 0$), it is seen that as the material scale parameter increases from $l / l_1 = 0.0005$ to 0.003, the fundamental frequency of plate is increased as a result of variation in couple stresses. This type of variation is observed in literature Ref. [27] for cracked plate in the absence of surrounding fluidic medium. Tables 6 and 7 show such variation found to be true in case of submerged plate also. Fig. 7 shows the variation of frequency parameter of plate for various fluid levels (h_1 / l_1) and various internal material scale parameters (l / l_1) for two different boundary conditions (SSSS and CCSS). Is it seen from Fig. 7(a) and (b) that for all values of h_1 / l_1, the increase in material scale parameter increases the natural frequency of plate which signifies the contribution of couple stress to the bending rigidity. Similarly it is also observed that for all values of l / l_1, the increase in level of submergence decreases the frequency as a result of increase in effective mass of plate due to inertia of fluid.

Table 6
Frequency parameters of SSSS plate as a function crack length, internal material scale parameter and temperature $(h_1 / l_1 = 0.1)$.

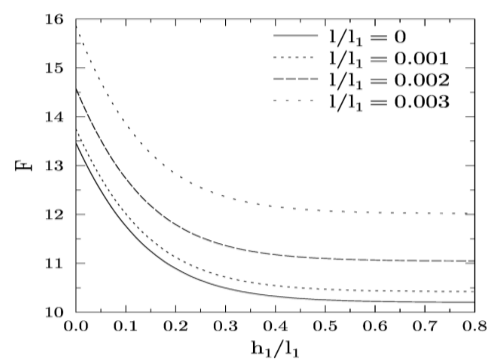
T_c	Half Crack length (a) (m)	Classical plate theory (CPT) $l / l_1 = 0$	Modified couple stress theory (MCST)			
			$l / l_1 = 0.0005$	$l / l_1 = 0.001$	$l / l_1 = 0.002$	$l / l_1 = 0.003$
0	$a = 0.00$	8.773	8.817	8.948	9.452	10.238
	$a = 0.01$	8.513	8.556	8.682	9.172	9.934
	$a = 0.05$	7.985	8.025	8.144	8.603	9.318
	$a = 0.10$	7.717	7.755	7.870	8.314	9.005
1	$a = 0.00$	8.337	8.383	8.520	9.048	9.866
	$a = 0.01$	8.032	8.078	8.212	8.728	9.525
	$a = 0.05$	7.396	7.439	7.568	8.059	8.819
	$a = 0.10$	7.060	7.103	7.228	7.709	8.449
2	$a = 0.00$	7.876	7.925	8.070	8.626	9.480
	$a = 0.01$	7.521	7.570	7.713	8.260	9.099
	$a = 0.05$	6.756	6.804	6.944	7.477	8.289
	$a = 0.10$	6.337	6.384	6.523	7.052	7.854
3	$a = 0.00$	7.387	7.439	7.594	8.182	9.078
	$a = 0.01$	6.973	7.025	7.179	7.764	8.651
	$a = 0.05$	6.049	6.102	6.258	6.844	7.724
	$a = 0.10$	5.519	5.573	5.732	6.327	7.211

Table 7
Frequency parameters of CCSS plate as a function crack length, internal material scale parameter and temperature ($h_1/l_1 = 0.1$).

T_c	Half Crack length (a) (m)	Classical plate theory (CPT) $l/l_1 = 0$	Modified couple stress theory (MCST)			
			$l/l_1 = 0.0005$	$l/l_1 = 0.001$	$l/l_1 = 0.002$	$l/l_1 = 0.003$
0	$a = 0.00$	12.599	12.662	12.849	13.574	14.702
	$a = 0.01$	12.178	12.239	12.421	13.121	14.212
	$a = 0.05$	11.321	11.378	11.546	12.197	13.211
	$a = 0.10$	10.882	10.936	11.098	11.724	12.698
1	$a = 0.00$	12.223	12.288	12.481	13.226	14.381
	$a = 0.01$	11.763	11.827	12.014	12.737	13.858
	$a = 0.05$	10.809	10.869	11.045	11.724	12.775
	$a = 0.10$	10.310	10.367	10.538	11.195	12.212
2	$a = 0.00$	11.835	11.902	12.101	12.868	14.053
	$a = 0.01$	11.333	11.399	11.593	12.341	13.494
	$a = 0.05$	10.272	10.335	10.520	11.231	12.324
	$a = 0.10$	9.704	9.765	9.946	10.640	11.705
3	$a = 0.00$	11.434	11.503	11.709	12.500	13.717
	$a = 0.01$	10.886	10.954	11.157	11.931	13.121
	$a = 0.05$	9.706	9.772	9.968	10.715	11.856
	$a = 0.10$	9.058	9.124	9.317	10.054	11.175



(a)



(b)

Fig.7
Frequency parameter ($F = \omega_{mn} l_1^2 \sqrt{\rho h / D}$) as a function of fluid level (h_1/l_1) and internal material scale parameter (l/l_1) for $T_c = 1$, $a = 0.01m$ (a) SSSS (b)CCSS.

In order to compare the results obtained using modified couple stress theory and classical plate theory, the variation of natural frequency with plate thickness is shown in Fig. 8 for two different boundary conditions. For a cracked isotropic plate, the results for natural frequency as affected by plate thickness are shown in literature Ref. [27]. The present model when applied for a cracked plate in absence of fluid medium and thermal environment reduces to the model proposed by Gupta et al. [27]. Therefore, such a validation is omitted here. It is established in literature (Ref. [27]) that the effect of internal material scale parameter is more significant for very thin plates and as the thickness of plate increases this effect reduces, this variation is further augmented by presence of fluidic medium and thermal environment in Figs. 8(a) to 8(d). It is seen from figures that for all values of plate thickness and material scale parameter the increase in fluid level and temperature decreases the frequency of cracked plate. Thus it is concluded that the presence of surrounding fluid medium and thermal environment also affects the natural frequency of plate with varying thickness.

Fig. 9 shows the variation of ratio of deflection of cracked plate to intact isotropic plate. In order to investigate the primary resonance, the ratio of forcing frequency to fundamental frequency of intact plate is varied from 0.6 to 1.3. It is interesting to note that the presence of crack shifts the primary resonance and it takes place well below $\varnothing / \omega_{11}^{intact} = 1$. This is due to decrease in stiffness of plate due to centrally located crack.

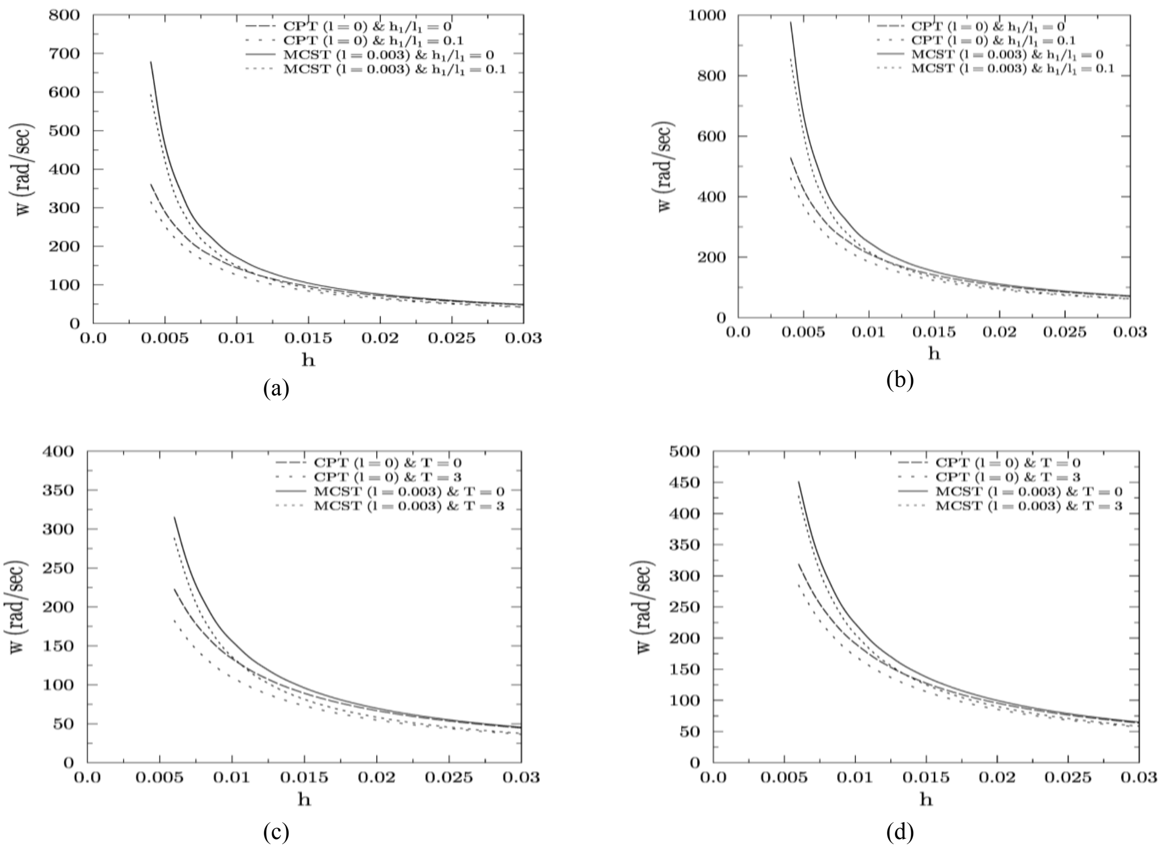


Fig.8
Natural frequency (ω) as a function of plate thickness (*h*), fluid level (h_1 / l_1), Temperature (T_c) and internal material scale parameter (*l*) for l_1 / h =100 and $a / l_1 = 0.01m$.

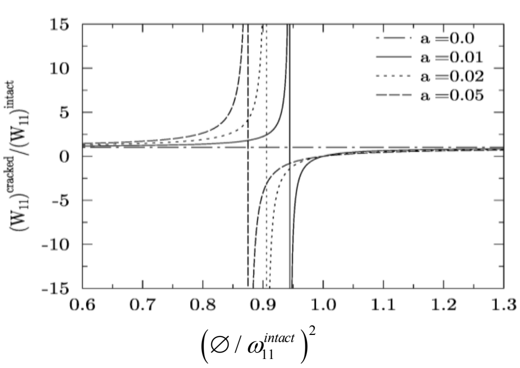


Fig.9
Central deflection ratio $W_{11}{}^{cracked} / W_{11}{}^{intact}$ versus the normalized operational frequency $\left(\varnothing / \omega_{11}^{intact}\right)^2$ for various values of crack length (*a*).

The results for variation of deflection ratio of heated intact plate to intact plate with respect to ratio of forcing frequency and fundamental frequency are shown in Fig. 10. With the rise in temperature of plate, it is known that the fundamental frequencies decreases, such a fact seen in literature is validated from the results in Fig. 10. As expected the rise of temperature decreases fundamental frequency of plate thereby increasing the deflection. The shift in primary resonance can be attributed to the decrease in stiffness due to temperature rise. Fig. 11 shows the ratio of deflections $W_{11}{}^{MCST} / W_{11}{}^{CPT}$ versus $\left(\varnothing / \omega_{11}^{CPT}\right)^2$ for various values of internal material scale parameter (l / l_1). It is seen that increasing values of l / l_1 result in shifting the primary resonance position of the classical case ($\varnothing / \omega_{11}^{\mathrm{CPT}} = 1$) to higher values of ($(\varnothing / \omega_{11}^{\mathrm{CPT}})$). Thus it can be concluded that the primary resonance occurs at higher values of operational frequency ($\varnothing$). To the best of author's knowledge Figs. 9, 10 and 11 along with Eqs.

(57) to (67) presents first time the effect of crack length, rise in temperature and internal material scale parameter on deflection and primary resonance of isotropic cracked plate.

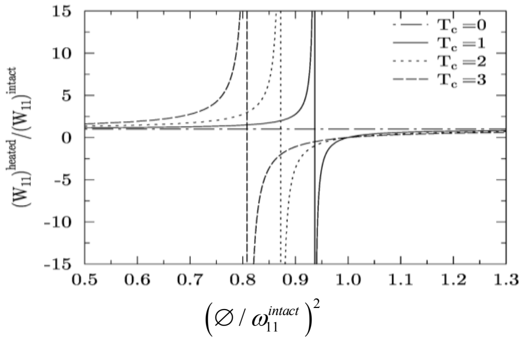


Fig.10
Central deflection ratio $W_{11}^{heated} / W_{11}^{intact}$ versus the normalized operational frequency $\left(\varnothing / \omega_{11}^{intact}\right)^2$ for various values of temperature (T_c).

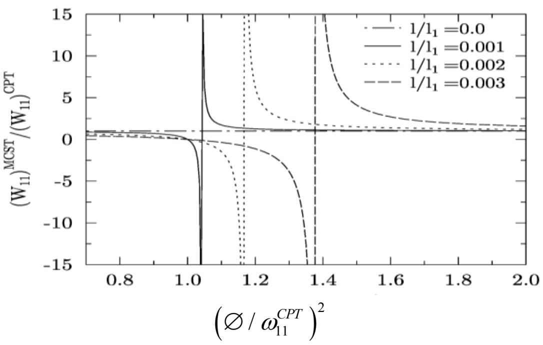


Fig.11
Central deflection ratio$W_{11}^{MCST} / W_{11}^{CPT}$ versus the normalized operational frequency $\left(\varnothing / \omega_{11}^{CPT}\right)^2$ for various values of internal material scale parameter (l / l_1).

7 CONCLUSIONS

In present work, an analytical model is presented for free vibration analysis of a uniformly heated partially cracked isotropic plate subjected to fluidic medium. It is established that the fundamental frequency is decreased by the presence of crack and uniform rise in temperature of plate and this decrease in frequency is further augmented by presence of surrounding fluid medium in present study. The influence of the fluid medium is incorporated in governing equation in the form of surrounding fluid dynamic pressure. The velocity potential and Bernoulli's equation are employed to express the fluid dynamic pressure acting on plate element. The effect of centrally located part through crack is deduced using appropriate crack compliance coefficients based on the LSM. A classical relation for central deflection of cracked isotropic plate is also proposed. The effect of varying forcing frequency, crack length, temperature rise and material scale parameter on deflection has been established. The results obtained from present study showing the influence of fluid levels, crack length, material scale parameter and uniform rise in temperature on fundamental frequencies is presented for isotropic plate with two different boundary conditions. The fact that the natural frequency for an intact plate decreases with increase in temperature and fluid level is established to be true for cracked plate also. To the best of the authors' knowledge, this is the first attempt to model vibrations of a cracked isotropic plate subjected to temperature rise in presence of fluidic medium and hence it would be instructive to formulate the analytical model for any curved structures subjected to random fluid dynamic pressures.

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