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Vibration Analysis of Size-Dependent Piezoelectric Nanobeam Under Magneto-Electrical Field

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ABSTRACT

The damping vibration characteristics of magneto-electro-viscoelastic (MEV) Nano beam resting on viscoelastic foundation based on nonlocal strain gradient elasticity theory (NSGT) is studied in this article. For this purpose, by considering the effects of Winkler-Pasternak, the viscoelastic medium consists of linear and viscous layers. With respect to the displacement field in accordance with the refined shear deformable beam theory (RSDT) and the Kelvin-Voigt viscoelastic-damping model, the governing equations of motion are obtained using Hamilton's principle based on nonlocal strain gradient theory (NSGT). Using Fourier Series Expansion, The Galerkin's method adopted to solving differential equations of Nano beam with both of simply supported and clamped boundary conditions. Numerical results are obtained to show the influences of nonlocal parameter, the length scale parameter, slenderness ratio and magneto-electro-mechanical loadings on the vibration behavior of Nano beam for both types of boundary conditions. It is found that by increasing the magnetic potential, the dimensionless frequency can be increased for any value of the damping coefficient and vice versa. Moreover, negative/positive magnetic potential decreases/increases the vibration frequencies of thinner Nano beam. In addition, the vibrating frequency decreases and increases with increasing nonlocal parameter and length scale parameter respectively.

Keywords: Piezoelectric Nano beam; Vibration analysis; Viscoelastic damping; Nonlocal strain gradient; Magneto-electro-viscoelastic.

1 INTRODUCTION

THE magneto-electro-elastic (MEE) material was first used in the 1970s, also MEE composite consisting of the Piezomagnetic and piezoelectric phase was discovered in this year by Boomgard *et al* [4]. The MEE nanomaterial, (including BiFeO3, NiFe2O4-PZT, BiTiO3-CoFe2O4) and their nanostructures became significant role in researches of Zheng *et al* [10], Martin *et al* [13], Wang *et al* [17] and Prashanthi *et al* [20]. For this reasons to

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ascent the major potential of nanostructure for amplification many usages, their mechanical characteristics must be studied and well known before the introduction of the new designs. According to these considerations, the classical mechanic continuum theories are no longer suitable to predict the behavior of structures with a minimum specified size, why so they cannot provide accurate forecasts. So to fix this problem, Eringen [1-3-5-6-7-12] presented the nonlocal theories as an effective tool for considering size effects in continuum modeling. Besides various researchers like Lam *et al* [8] and Li *et al* [26] shown that the increase in stiffness not considered in nonlinear elasticity. Therefore, the nonlocal strain gradient theory is presented, upon which the stress field accounts for not only the nonlocal stress field but also the strain gradients stress field. In this paper, the models that have been developed extensively based on nonlocal elasticity theory of Eringen have been studied. Hence, Peddieson *et al* [9] have developed a nonlocal Euler-Bernoulli beam model by presenting a version of the nonlocal elasticity theory. They solved some representative problems especially for cantilever beams to illustrate the magnitude of predicted nonlocal effects. Also, several researchers like Wang [11], Wang *et al* [14], Civalek and Demir [18] have investigated the wave propagation and bending in carbon nanotubes (CNTs) and microtubules for nonlocal Euler-Bernoulli and Timoshenko beam models. Murmu and Pradhan [15] analysed the small-size effects on single-walled carbon nanotubes (SWCNTs) in recent years. They described the stability response of SWCNT based on the nonlocal Timoshenko beam theory and considering elastic medium. The nonlocal parameter, aspect ratio of the SWCNT, Winkler and Pasternak parameters was studied. On the other hand, Yang *et al* [16] studied nonlinear free vibration of SWCNTs based on Eringen's nonlocal elasticity theory and von Kármán geometric nonlinearity and solved the obtained equations by using the differential quadrature (DQ) method. The free vibration, buckling and bending of Timoshenko Nano beams based on a meshless method investigated by Roque *et al* [19]. Zenkour and Sobhy [21] studied the thermal buckling of single-layered graphene sheets on an elastic medium by using the sinusoidal shear deformation plate theory. Simsek and Yurtcu [22] analysed static bending and buckling of a functionally graded (FG) Nano beam under transvers distributed loads based on the nonlocal Timoshenko and Euler–Bernoulli beam theory. They extracted the governing equations by applying the principal of the minimum total potential energy and solved analytically the resulting equations. Ke *et al* [23] investigated free vibration of magneto-electro-elastic (MEE) nanoplates based on Kirchhoff plate and theory Eringen's nonlocal theory. In this analysis, the governing equations and boundary conditions of a MEE Nanoplate that is under external magnetic potential, external electric potential, the biaxial force and temperature rise are extracted using the Hamilton's principle and then solved analytically to obtain the natural frequencies of MEE nanoplates. They also studied the free vibration of the MEE Nano beam based on Timoshenko beam theory and solved numerically the resulting equations. Moreover, bending analysis of a thermo-magneto-electro-elastic Nano beam integrated with functionally graded Piezomagnetic layers studied by Arefi and Zenkour [28]. Buckling and vibration of Piezomagnetic and piezoelectric Nano beams based on third order beam model are verified by Ebrahimi and Barati [29-30-31-32-33]. However, they did not investigated magnet-electro-viscoelastic Nano beam via nonlocal strain gradient theory in their research's.

This paper investigates the free vibration of MEV Nano beam via nonlocal strain gradient theory. The Governing equations of Nano beam resting on viscoelastic layer are extracted based on Hamilton's principle. Galerkin method is employed to analytically solve of the governing equations. Effects of various factors such as applied magnetic potential, nonlocal parameter, length scale parameter, the internal damping parameters and slenderness ratio on vibration characteristics of a Nano beam are studied.

2 THEORY AND FORMULATION

In Fig.1, a piezoelectric Nano beam with length L, width b, thickness h is illustrated. Based on refined shear deformable beam theory, the arbitrary point displacement of Nano beam can be expressed by:

$$\bar{u}\left(x,z,t\right)=u\left(x,t\right)-z\,\frac{\partial w_b}{\partial x}-f\,\left(z\right)\frac{\partial w_s}{\partial x} \tag{1}$$

$$\bar{w}\left(x,z,t\right)=w_b\left(x,t\right)+w_s\left(x,t\right) \tag{2}$$

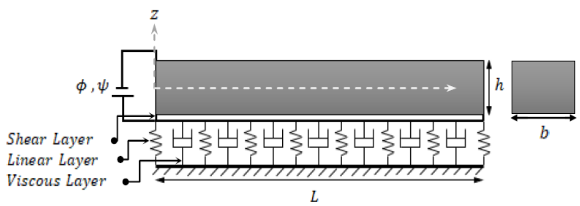


Fig.1
Geometry of MEV nanobeam with Winkler-Pasternak elastic medium.

where u, w_s and w_b denote axial displacement, shear component and bending component of transverse displacement of mid-plane, respectively. While, $f(z)$ is a function that shows the shape of the shear strain/stress distribution along the beam thickness so that shear correction factor is not necessary. In the present study, a trigonometric nature is considered for shape function as described by Mantari *et al* [25]:

$$f(z) = z - \tan(mz), \quad m = 0.03 \tag{3}$$

According to the proposed beam model, non-zero strains can be written as follows:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2} \tag{4}$$

$$\gamma_{xz} = g(z)\frac{\partial w_s}{\partial x} \tag{5}$$

where $g(z) = 1 - df(z)/dz$. According to Maxwell's equation, the electric potential Φ and magnetic potential Ψ distributions across the thickness are approximated as follows (Ke et al [24]):

$$\Phi(x,z,t) = -\cos(\beta z)\phi(x,t) + \frac{2z}{h}V \tag{6}$$

$$\Psi(x,z,t) = -\cos(\beta z)\psi(x,t) + \frac{2z}{h}\Omega \tag{7}$$

In which $\beta = \pi/h$. Also, Ω and V are the external magnetic potential and electric voltage applied to the Nano beam. The relation between electric field (E_x, E_z) and electric potential (ϕ) and magnet field (H_x, H_z) and magnet potential (ψ), can be obtained as:

$$E_x = -\Phi_{,x} = \cos(\beta z)\frac{\partial \phi}{\partial x} \tag{8}$$

$$E_z = -\Phi_{,z} = -\beta\sin(\beta z)\phi(x,t) - \frac{2V}{h} \tag{9}$$

$$H_x = -\Psi_{,x} = \cos(\beta z)\frac{\partial \psi}{\partial x} \tag{10}$$

$$H_z = -\Psi_{,z} = -\beta\sin(\beta z)\psi(x,t) - \frac{2\Omega}{h} \tag{11}$$

The governing equations can be extracted by applying Hamilton's principle as follows:

$$\int_0^t \delta\left(\Pi_S - \Pi_K + \Pi_W\right)dt \tag{12}$$

where Π_S , Π_K and Π_W are the total strain energy, the kinetic energy and the work done by external applied forces, respectively. The relation of strain energy Π_S for MEV Nano beam can be derived as:

$$\Pi_S = \frac{1}{2}\int_v \left(\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz} - D_x E_x - D_z E_z - B_x H_x - B_z H_z\right)dv \tag{13}$$

By calculating the first variation of strain energy Π_S and then substituting Eqs. (4)-(5) into resultant equation, we obtain:

$$\delta\Pi_S = \int_0^L \left(N\frac{\partial \delta u}{\partial x} - M_b\frac{\partial^2 \delta w_b}{\partial x^2} - M_s\frac{\partial^2 \delta w_s}{\partial x^2} + Q\frac{\partial \delta w_s}{\partial x}\right)dx + \int_0^L \left(-\bar{D}\frac{\partial \delta\phi}{\partial x} + \bar{D}_z\delta\phi - \bar{B}_x\frac{\partial \delta\psi}{\partial x} + \bar{B}_z\delta\psi\right)dx \tag{14}$$

In which the defined variables in Eq. (14) are obtained as follows:

$$\left(N, M_b, M_s\right) = \int_{-h/2}^{+h/2}\sigma_{xx}\left(1, z, f\left(z\right)\right)dz \tag{15}$$

$$Q = \int_{-h/2}^{+h/2}\sigma_{xz}\, g\left(z\right)dz \tag{16}$$

$$\left(\bar{D}_x, \bar{B}_x\right) = \int_{-h/2}^{+h/2}\left(D_x, B_x\right)\cos\left(\beta z\right)dz \tag{17}$$

$$\left(\bar{D}_z, \bar{B}_z\right) = \int_{-h/2}^{+h/2}\left(D_z, B_z\right)\beta\sin\left(\beta z\right)dz \tag{18}$$

where $\sigma_{ij}, \varepsilon_{ij}, D_i, B_i, E_i, H_i, M_i, N$ and Q are the stress, strain, electric displacement, magnetic induction electric field, magnetic field, bending moment, the axial force and shear force resultants, respectively. The relation of kinetic energy can be expressed as follows:

$$\Pi_K = \frac{1}{2}\int_v \rho\left[\left(\frac{\partial \bar{u}}{\partial t}\right)^2 + \left(\frac{\partial \bar{w}}{\partial t}\right)^2\right]dv \tag{19}$$

In addition, the first variation of kinetic energy of present theory can be obtained in the form:

$$\begin{aligned}\delta\Pi_K = \int_0^L \Bigg[& I_0\left(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial\left(w_b + w_s\right)}{\partial t}\frac{\partial \delta\left(w_b + w_s\right)}{\partial t}\right) - I_1\left(\frac{\partial u}{\partial t}\frac{\partial^2 \delta w_b}{\partial x\,\partial t} - \frac{\partial^2 w_b}{\partial x\,\partial t}\frac{\partial \delta u}{\partial t}\right) \\ & - I_2\left(\frac{\partial u}{\partial t}\frac{\partial^2 \delta w_s}{\partial x\,\partial t} - \frac{\partial^2 w_s}{\partial x\,\partial t}\frac{\partial \delta u}{\partial t}\right) + I_3\left(\frac{\partial^2 w_b}{\partial x\,\partial t}\frac{\partial^2 \delta w_s}{\partial x\,\partial t} - \frac{\partial^2 w_s}{\partial x\,\partial t}\frac{\partial^2 \delta w_b}{\partial x\,\partial t}\right) + I_4\frac{\partial^2 w_b}{\partial x\,\partial t}\frac{\partial^2 \delta w_b}{\partial x\,\partial t} + I_5\frac{\partial^2 w_s}{\partial x\,\partial t}\frac{\partial^2 \delta w_s}{\partial x\,\partial t}\Bigg]dx\end{aligned} \tag{20}$$

In which the mass inertia are extracted as:

$$\left(I_0, I_1, I_2, I_3, I_4, I_5\right) = \int_{-h/2}^{+h/2}\rho\left(1, z, f, zf, z^2, f^2\right)dz \tag{21}$$

In addition, we obtain the work done by applied forces and the first variation of it using the following relationships:

$$\Pi_W = \int_0^L \left[F\left(w_b + w_s\right) - \frac{1}{2}\left(N^E + N^H\right)\left(\frac{\partial\left(w_b + w_s\right)}{\partial x}\right)^2 \right] dx \tag{22}$$

$$\delta\Pi_W = \int_0^L \left[F\delta\left(w_b + w_s\right) - \left(N^E + N^H\right)\frac{\partial\left(w_b + w_s\right)}{\partial x}\frac{\partial\delta\left(w_b + w_s\right)}{\partial x} \right] dx \tag{23}$$

In Eq. (22), F , denote external transverse load from viscoelastic medium which is obtained in Eq. (24):

$$F = k_w\left(w_b + w_s\right) - k_p\frac{\partial^2\left(w_b + w_s\right)}{\partial x^2} + c_d\frac{\partial\left(w_b + w_s\right)}{\partial t} \tag{24}$$

where k_p , k_w and c_d are shear, linear and damping coefficients of foundation. Also, N^E and N^H denote electric and magnet loading, respectively. By substituting Eqs. (14), (20) and (23) into Eq. (12) and then integrating by parts, the following Euler–Lagrange equations are obtained when the coefficients of resultant equation, $\delta u, \delta w_b, \delta w_s, \delta\phi, \delta\psi$, are equal to zero as:

$$\frac{\partial N}{\partial x} = I_0\frac{\partial^2 u}{\partial t^2} - I_1\frac{\partial^3 w_b}{\partial x\,\partial t^2} - I_2\frac{\partial^3 w_s}{\partial x\,\partial t^2} \tag{25}$$

$$\begin{aligned}\frac{\partial^2 M_b}{\partial x^2} &= I_0\frac{\partial^2\left(w_b + w_s\right)}{\partial t^2} + I_1\frac{\partial^3 u}{\partial x\,\partial t^2} - I_3\frac{\partial^4 w_s}{\partial x^2\partial t^2} - I_4\frac{\partial^4 w_b}{\partial x^2\partial t^2}\\ &+\left(N^E + N^H\right)\frac{\partial^2\left(w_b + w_s\right)}{\partial x^2} + k_w\left(w_b + w_s\right) - k_p\frac{\partial^2\left(w_b + w_s\right)}{\partial x^2} + c_d\frac{\partial\left(w_b + w_s\right)}{\partial t}\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{\partial^2 M_s}{\partial x^2} + \frac{\partial Q}{\partial x} &= I_0\frac{\partial^2\left(w_b + w_s\right)}{\partial t^2} + I_2\frac{\partial^3 u}{\partial x\,\partial t^2} - I_3\frac{\partial^4 w_b}{\partial x^2\partial t^2} - I_5\frac{\partial^4 w_s}{\partial x^2\partial t^2}\\ &+\left(N^E + N^H\right)\frac{\partial^2\left(w_b + w_s\right)}{\partial x^2} + k_w\left(w_b + w_s\right) - k_p\frac{\partial^2\left(w_b + w_s\right)}{\partial x^2} + c_d\frac{\partial\left(w_b + w_s\right)}{\partial t}\end{aligned} \tag{27}$$

$$\frac{\partial\bar{D}_x}{\partial x} + \bar{D}_z = 0 \tag{28}$$

$$\frac{\partial\bar{B}_x}{\partial x} + \bar{B}_z = 0 \tag{29}$$

3 THE NONLOCAL STRAIN GRADIENT THEORY FOR MEV MATERIALS

According to Eringen's nonlocal theory of elasticity, the stress state at a reference point x in an elastic continuum is regarded to be dependent not only depends on the strain state at x but also on the strains state at all other points x' of the body. In addition, based on nonlocal strain gradient theory developed in Lam *et al* [8], the stress components are calculated in a displacement field from the combination of nonlocal elastic stress field and the strain gradient stress field. Therefore, the stress components can be obtained by:

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla\sigma_{ij}^{(1)} \tag{30}$$

where the stress $\sigma_{ij}^{(0)}$ and higher-order stress $\sigma_{ij}^{(1)}$ correspond to strain ε_{ij} and strain gradient $\nabla\varepsilon_{ij}$, respectively which are defined by:

$$\sigma_{ij}^{(0)} = \int_0^L \alpha_0\left(x, x', e_0 a/l\right)\sigma_{ij}^c\left(x'\right)dx' \tag{31}$$

$$\sigma_{ij}^{(1)} = l^2\int_0^L \alpha_1\left(x, x', e_1 a/l\right)\nabla\sigma_{ij}^c\left(x'\right)dx' \tag{32}$$

In which L is the length of the Nano beam and σ_{ij}^c is the classical stress components at any point x' in the body, and $\alpha_i\left(x, x', e_i a/l\right)$ is the kernel of the integral equation, in which a and l parameters correspond to the non-localness and denote internal characteristic length (size of grain, granular distance, or lattice parameter) and external characteristic length of the system (crack length, wavelength, size, or dimensions of sample) respectively and e_i is a constant appropriate to the material and has to be determined for each material independently. When the non-local functions $\alpha_i\left(x, x', e_i a/l\right)$ satisfy the developed conditions by Eringen [6], the linear nonlocal differential operator can be assumed as the following:

$$L_i = 1-\left(e_i a\right)^2\nabla^2 \quad \text{for} \quad i = 0,1 \tag{33}$$

By exerting Eq. (33) into Eq. (30), a general constitutive relation in a differential form for a Nano beam can be stated as:

$$\left[1-\left(e_1 a\right)^2\nabla^2\right]\left[1-\left(e_0 a\right)^2\nabla^2\right]\sigma_{ij} = \left[1-\left(e_1 a\right)^2\nabla^2\right]\sigma_{ij}^c - \left[1-\left(e_0 a\right)^2\nabla^2\right]l^2\nabla^2\sigma_{ij}^c \tag{34}$$

where $\nabla^2 = \partial^2/\partial^2 x$ denotes the Laplacian operator. By retaining terms of order $o\left(\nabla^2\right)$ and assuming $e_0 = e_1 = e$, we can be written the general constitutive relation in simplified form as follows:

$$\left[1-\left(ea\right)^2\nabla^2\right]\sigma_{ij} = \left(1-l^2\nabla^2\right)\sigma_{ij}^c \tag{35}$$

Similarly, the following equations are obtained for a MEV Nano beam:

$$\left[1-\left(ea\right)^2\nabla^2\right]D_j = \left(1-l^2\nabla^2\right)D_j^c \tag{36}$$

$$\left[1-\left(ea\right)^2\nabla^2\right]B_j = \left(1-l^2\nabla^2\right)B_j^c \tag{37}$$

In above equations, σ_{ij}^c , D_j^c and B_j^c are given by [29] as follows:

$$\sigma_{ij}^c = C_{ijkl}\varepsilon_{kl} - e_{mij}E_m - q_{nij}H_n \tag{38}$$

$$D_j^c = e_{jkl}\varepsilon_{kl} + \chi_{jm}E_m + d_{jn}H_n \tag{39}$$

$$B_j^c = q_{jkl}\varepsilon_{kl} + d_{jm}E_m + \lambda_{jn}H_n \tag{40}$$

where ε_{kl} is the strain and $C_{ijkl}, e_{mij}, \chi_{jm}, q_{nij}, d_{jn}$ and λ_{jn} denote the elastic, piezoelectric, dielectric, Piezomagnetic, magnetoelectric and magnetic constants, respectively. Finally, the stress-strain relations of a MEV solid can be expressed as:

$$\left(1-\mu\nabla^2\right)\sigma_{xx} = \left(1-\lambda\nabla^2\right)\left(C_{11}\varepsilon_{xx} - e_{31}E_z - q_{31}H_z\right) \tag{41}$$

$$\left(1-\mu\nabla^2\right)\sigma_{xz} = \left(1-\lambda\nabla^2\right)\left(C_{55}\gamma_{xz} - e_{15}E_x - q_{15}H_x\right) \tag{42}$$

$$\left(1-\mu\nabla^2\right)D_x = \left(1-\lambda\nabla^2\right)\left(e_{15}\gamma_{xz} + \chi_{11}E_x + d_{11}H_x\right) \tag{43}$$

$$\left(1-\mu\nabla^2\right)D_z = \left(1-\lambda\nabla^2\right)\left(e_{31}\varepsilon_{xx} + \chi_{33}E_z + d_{33}H_z\right) \tag{44}$$

$$\left(1-\mu\nabla^2\right)B_x = \left(1-\lambda\nabla^2\right)\left(q_{15}\gamma_{xz} + d_{11}E_x + \lambda_{11}H_x\right) \tag{45}$$

$$\left(1-\mu\nabla^2\right)B_z = \left(1-\lambda\nabla^2\right)\left(q_{31}\varepsilon_{xx} + d_{33}E_z + \lambda_{33}H_z\right) \tag{46}$$

where $\mu=\left(ea\right)^2$ and $\lambda=l^2$. Applying the Kelvin-Voigt viscoelastic-damping model with internal damping parameter$\left(g_0\right)$ and integrating Eqs. (41)- (46) over the cross-section area of Nano beam, the nonlocal Eqs. (47)-(54) are obtained for magneto-electro-viscoelastic Nano beam as:

$$N-\mu\frac{\partial^2 N}{\partial x^2} = \left(1-\lambda\nabla^2\right)\left(1+g_0\frac{\partial}{\partial t}\right)\left(J_{11}\frac{\partial u}{\partial x} - J_{11}^{z}\frac{\partial^2 w_b}{\partial x^2} - J_{11}^{f}\frac{\partial^2 w_s}{\partial x^2}\right) + \left(1-\lambda\nabla^2\right)\left(K_{31}^{e}\phi + K_{31}^{m}\psi - N^E - N^H\right) \tag{47}$$

$$M_b-\mu\frac{\partial^2 M_b}{\partial x^2} = \left(1-\lambda\nabla^2\right)\left(1+g_0\frac{\partial}{\partial t}\right)\left(J_{11}^{z}\frac{\partial u}{\partial x} - J_{11}^{zz}\frac{\partial^2 w_b}{\partial x^2} - J_{11}^{zf}\frac{\partial^2 w_s}{\partial x^2}\right) + \left(1-\lambda\nabla^2\right)\left(X_{31}^{e}\phi + X_{31}^{m}\psi - M_b^E - M_b^H\right) \tag{48}$$

$$M_s-\mu\frac{\partial^2 M_s}{\partial x^2} = \left(1-\lambda\nabla^2\right)\left(1+g_0\frac{\partial}{\partial t}\right)\left(J_{11}^{f}\frac{\partial u}{\partial x} - J_{11}^{zf}\frac{\partial^2 w_b}{\partial x^2} - J_{11}^{ff}\frac{\partial^2 w_s}{\partial x^2}\right) + \left(1-\lambda\nabla^2\right)\left(Y_{31}^{e}\phi + Y_{31}^{m}\psi - M_s^E - M_s^H\right) \tag{49}$$

$$Q-\mu\frac{\partial^2 Q}{\partial x^2} = \left(1-\lambda\nabla^2\right)\left[\left(1+g_0\frac{\partial}{\partial t}\right)\left(K_{55}\frac{\partial w_s}{\partial x}\right) - X_{15}\frac{\partial\phi}{\partial x} - Y_{15}\frac{\partial\psi}{\partial x}\right] \tag{50}$$

$$\bar{D}_x-\mu\frac{\partial^2 d_x}{\partial x^2} = \left(1-\lambda\nabla^2\right)\left(X_{15}\frac{\partial w_s}{\partial x} + X_{11}\frac{\partial\phi}{\partial x} + Y_{11}\frac{\partial\psi}{\partial x}\right) \tag{51}$$

$$\bar{B}_x-\mu\frac{\partial^2 b_x}{\partial x^2} = \left(1-\lambda\nabla^2\right)\left(Y_{15}\frac{\partial w_s}{\partial x} + Y_{11}\frac{\partial\phi}{\partial x} + K_{11}\frac{\partial\psi}{\partial x}\right) \tag{52}$$

$$\bar{B}_z-\mu\frac{\partial^2 b_z}{\partial x^2} = \left(1-\lambda\nabla^2\right)\left(K_{31}^{m}\frac{\partial u}{\partial x} - X_{31}^{m}\frac{\partial^2 w_b}{\partial x^2} - Y_{31}^{m}\frac{\partial^2 w_s}{\partial x^2} - Y_{33}\phi - K_{33}\psi - F_{33}^{m}\right) \tag{53}$$

$$\bar{D}_z-\mu\frac{\partial^2 d_z}{\partial x^2} = \left(1-\lambda\nabla^2\right)\left(K_{31}^{e}\frac{\partial u}{\partial x} - X_{31}^{e}\frac{\partial^2 w_b}{\partial x^2} - Y_{31}^{e}\frac{\partial^2 w_s}{\partial x^2} - X_{33}\phi - Y_{33}\psi - F_{33}^{e}\right) \tag{54}$$

In which the cross-sectional rigidities are expressed as follows:

$$\left(J_{11},J_{11}^{z},J_{11}^{f},J_{11}^{zz},J_{11}^{zf},J_{11}^{ff}\right) = \int_{-h/2}^{+h/2} C_{11}\left(1,z,f,z^2,zf,f^2\right)dz \tag{55}$$

$$\left(K_{31}^{e},X_{31}^{e},Y_{31}^{e}\right) = \int_{-h/2}^{+h/2} e_{31}\beta\sin\left(\beta z\right)\left(1,z,f\right)dz \tag{56}$$

$$\left(K_{31}^{m},X_{31}^{m},Y_{31}^{m}\right)=\int_{-h/2}^{+h/2}q_{31}\beta\sin\left(\beta z\right)\left(1,z,f\right)dz \tag{57}$$

$$K_{55}=\int_{-h/2}^{+h/2}C_{55}g^{2}\left(z\right)dz \tag{58}$$

$$\left(X_{15},Y_{15}\right)=\int_{-h/2}^{+h/2}\left(e_{15},q_{15}\right)g\cos\left(\beta z\right)dz \tag{59}$$

$$\left(X_{11},Y_{11},K_{11}\right)=\int_{-h/2}^{+h/2}\left(\chi_{11},d_{11},\lambda_{11}\right)\cos^{2}\left(\beta z\right)dz \tag{60}$$

$$\left(X_{33},Y_{33},K_{33}\right)=\int_{-h/2}^{+h/2}\left(\chi_{33},d_{33},\lambda_{33}\right)\beta^{2}\sin^{2}\left(\beta z\right)dz \tag{61}$$

$$\left(F_{33}^{e},F_{33}^{m}\right)=\int_{-h/2}^{+h/2}\left(\chi_{33}\frac{2V}{h}+d_{33}\frac{2\Omega}{h},d_{33}\frac{2V}{h}+\lambda_{33}\frac{2\Omega}{h}\right)\beta\sin\left(\beta z\right)dz \tag{62}$$

Also, summation of normal forces and moments due to magneto-electrical field can be defined by:

$$\left(N^{E}+N^{H},M_{b}^{E}+M_{b}^{H},M_{s}^{E}+M_{s}^{H}\right)=\int_{-h/2}^{+h/2}\left(e_{31}\frac{2V}{h}+q_{31}\frac{2\Omega}{h}\right)\left(1,z,f\right)dz \tag{63}$$

The governing equations of a refined Nano beam under electrical and magnetic field based on the nonlocal strain gradient theory in terms of the displacement can be obtained by substituting Eqs. (47)- (54) into Eqs. (25)- (29) as follows:

$$\begin{aligned}&\left(1-\mu\nabla^{2}\right)\left(I_{0}\frac{\partial^{2}u}{\partial t^{2}}-I_{1}\frac{\partial^{3}w_{b}}{\partial x\,\partial t^{2}}-I_{2}\frac{\partial^{3}w_{s}}{\partial x\,\partial t^{2}}\right)\\&-\left(1-\lambda\nabla^{2}\right)\left[\left(1+g_{0}\frac{\partial}{\partial t}\right)\left(J_{11}\frac{\partial^{2}u}{\partial x^{2}}-J_{11}^{z}\frac{\partial^{3}w_{b}}{\partial x^{3}}-J_{11}^{f}\frac{\partial^{3}w_{s}}{\partial x^{3}}\right)+K_{31}^{e}\frac{\partial\phi}{\partial x}+K_{31}^{m}\frac{\partial\psi}{\partial x}\right]=0\end{aligned} \tag{64}$$

$$\begin{aligned}&\left(1-\mu\nabla^{2}\right)\left(I_{0}\frac{\partial^{2}\left(w_{b}+w_{s}\right)}{\partial t^{2}}+I_{1}\frac{\partial^{3}u}{\partial x\,\partial t^{2}}-I_{3}\frac{\partial^{4}w_{s}}{\partial x^{2}\partial t^{2}}-I_{4}\frac{\partial^{4}w_{b}}{\partial x^{2}\partial t^{2}}\right.\\&\left.+\left(N^{E}+N^{H}\right)\frac{\partial^{2}\left(w_{b}+w_{s}\right)}{\partial x^{2}}+k_{w}\left(w_{b}+w_{s}\right)-k_{p}\frac{\partial^{2}\left(w_{b}+w_{s}\right)}{\partial x^{2}}+c_{d}\frac{\partial\left(w_{b}+w_{s}\right)}{\partial t}\right)\\&-\left(1-\lambda\nabla^{2}\right)\left[\left(1+g_{0}\frac{\partial}{\partial t}\right)\left(J_{11}^{z}\frac{\partial^{3}u}{\partial x^{3}}-J_{11}^{zz}\frac{\partial^{4}w_{b}}{\partial x^{4}}-J_{11}^{zf}\frac{\partial^{4}w_{s}}{\partial x^{4}}\right)+X_{31}^{e}\frac{\partial^{2}\phi}{\partial x^{2}}+X_{31}^{m}\frac{\partial^{2}\psi}{\partial x^{2}}\right]=0\end{aligned} \tag{65}$$

$$\begin{aligned}&\left(1-\mu\nabla^{2}\right)\left(I_{0}\frac{\partial^{2}\left(w_{b}+w_{s}\right)}{\partial t^{2}}+I_{2}\frac{\partial^{3}u}{\partial x\,\partial t^{2}}-I_{3}\frac{\partial^{4}w_{b}}{\partial x^{2}\partial t^{2}}-I_{5}\frac{\partial^{4}w_{s}}{\partial x^{2}\partial t^{2}}\right.\\&\left.+\left(N^{E}+N^{H}\right)\frac{\partial^{2}\left(w_{b}+w_{s}\right)}{\partial x^{2}}+k_{w}\left(w_{b}+w_{s}\right)-k_{p}\frac{\partial^{2}\left(w_{b}+w_{s}\right)}{\partial x^{2}}+c_{d}\frac{\partial\left(w_{b}+w_{s}\right)}{\partial t}\right)\\&-\left(1-\lambda\nabla^{2}\right)\left(1+g_{0}\frac{\partial}{\partial t}\right)\left(J_{11}^{f}\frac{\partial^{3}u}{\partial x^{3}}-J_{11}^{zf}\frac{\partial^{4}w_{b}}{\partial x^{4}}-J_{11}^{ff}\frac{\partial^{4}w_{s}}{\partial x^{4}}+K_{55}\frac{\partial^{2}w_{s}}{\partial x^{2}}\right)\\&-\left(1-\lambda\nabla^{2}\right)\left(\left(Y_{31}^{e}-X_{15}\right)\frac{\partial^{2}\phi}{\partial x^{2}}+\left(Y_{31}^{m}-Y_{15}\right)\frac{\partial^{2}\psi}{\partial x^{2}}\right)=0\end{aligned} \tag{66}$$

$$\left(1-\lambda\nabla^{2}\right)\left(K_{31}^{e}\frac{\partial u}{\partial x}-X_{31}^{e}\frac{\partial^{2}w_{b}}{\partial x^{2}}+\left(X_{15}-Y_{31}^{e}\right)\frac{\partial^{2}w_{s}}{\partial x^{2}}+X_{11}\frac{\partial^{2}\phi}{\partial x^{2}}+Y_{11}\frac{\partial^{2}\psi}{\partial x^{2}}-X_{33}\phi-Y_{33}\psi-F_{33}^{e}\right)=0 \tag{67}$$

$$\left(1-\lambda\nabla^{2}\right)\left(K_{31}^{m}\frac{\partial u}{\partial x}-X_{31}^{m}\frac{\partial^{2}w_{b}}{\partial x^{2}}+\left(Y_{15}-Y_{31}^{m}\right)\frac{\partial^{2}w_{s}}{\partial x^{2}}+Y_{11}\frac{\partial^{2}\phi}{\partial x^{2}}+K_{11}\frac{\partial^{2}\psi}{\partial x^{2}}-Y_{33}\phi-K_{33}\psi-F_{33}^{m}\right)=0 \tag{68}$$

4 SOLUTION PROCEDURE

The following boundary conditions for exact solution of the governing equations of a magneto-electro-viscoelastic Nano beam are expressed as:

Simply-supported (S):

$$w_{b}=w_{s}=M=\frac{\partial u}{\partial x}=0 \qquad at \quad x=0,L \tag{69}$$

Clamped (C):

$$u=w_{b}=w_{s}=\frac{\partial\left(w_{b}+w_{s}\right)}{\partial x}=0 \qquad at \quad x=0,L \tag{70}$$

According to the defined boundary conditions, the displacement components can be given by Fourier series expansion as:

$$u\left(x,t\right)=\sum_{n=1}^{\infty}U_{n}\frac{\partial X_{n}\left(x\right)}{\partial x}e^{i\omega_{n}t} \tag{71}$$

$$w_{b}\left(x,t\right)=\sum_{n=1}^{\infty}W_{bn}X_{n}\left(x\right)e^{i\omega_{n}t} \tag{72}$$

$$w_{s}\left(x,t\right)=\sum_{n=1}^{\infty}W_{sn}X_{n}\left(x\right)e^{i\omega_{n}t} \tag{73}$$

$$\phi\left(x,t\right)=\sum_{n=1}^{\infty}\Phi_{n}X_{n}\left(x\right)e^{i\omega_{n}t} \tag{74}$$

$$\psi\left(x,t\right)=\sum_{n=1}^{\infty}\Psi_{n}X_{n}\left(x\right)e^{i\omega_{n}t} \tag{75}$$

with substituting Eqs. (71)-(75) into Eqs. (64)-(68) and using Galerkin's method, we can be obtained the following equations:

$$\begin{aligned}
&\left[I_{0}\left(\mu\bar{X}_{13}-\bar{X}_{11}\right)\omega_{n}^{2}+g_{0}J_{11}\left(\lambda\bar{X}_{15}-\bar{X}_{13}\right)i\,\omega_{n}+J_{11}\left(\lambda\bar{X}_{15}-\bar{X}_{13}\right)\right]U_{n}\\
&\quad+\left[I_{1}\left(\bar{X}_{11}-\mu\bar{X}_{13}\right)\omega_{n}^{2}+g_{0}J_{11}^{z}\left(\bar{X}_{13}-\lambda\bar{X}_{15}\right)i\,\omega_{n}+J_{11}^{z}\left(\bar{X}_{13}-\lambda\bar{X}_{15}\right)\right]W_{bn}\\
&\quad+\left[I_{2}\left(\bar{X}_{11}-\mu\bar{X}_{13}\right)\omega_{n}^{2}+g_{0}J_{11}^{f}\left(\bar{X}_{13}-\lambda\bar{X}_{15}\right)i\,\omega_{n}+J_{11}^{f}\left(\bar{X}_{13}-\lambda\bar{X}_{15}\right)\right]W_{sn}\\
&\quad+K_{31}^{e}\left(\lambda\bar{X}_{13}-\bar{X}_{11}\right)\Phi_{n}+K_{31}^{m}\left(\lambda\bar{X}_{13}-\bar{X}_{11}\right)\Psi_{n}=0
\end{aligned} \tag{76}$$

$$\begin{aligned}
&\left[I_1\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)\omega_n^2+g_0J_{11}^z\left(\lambda\bar{X}_{60}-\bar{X}_{40}\right)i\,\omega_n+J_{11}^z\left(\lambda\bar{X}_{60}-\bar{X}_{40}\right)\right]U_n\\
&\quad+\Big[\left(I_0\left(\mu\bar{X}_{20}-\bar{X}_{00}\right)+I_4\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)\right)\omega_n^2\\
&\quad+\left(c_d\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+g_0J_{11}^{zz}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right)\right)i\,\omega_n+\left(N^E+N^H\right)\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)\\
&\quad+k_w\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+k_p\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)+J_{11}^{zz}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right)\Big]W_{bn}\\
&\quad+\Big[\left(I_0\left(\mu\bar{X}_{20}-\bar{X}_{00}\right)+I_3\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)\right)\omega_n^2\\
&\quad+\left(c_d\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+g_0J_{11}^{zf}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right)\right)i\,\omega_n+\left(N^E+N^H\right)\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)\\
&\quad+k_w\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+k_p\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)+J_{11}^{zf}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right)\Big]W_{sn}\\
&\quad+X_{31}^e\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right)\Phi_n+X_{31}^m\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right)\Psi_n=0
\end{aligned}\tag{77}$$

$$\begin{aligned}
&\left[I_2\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)\omega_n^2+g_0J_{11}^f\left(\lambda\bar{X}_{60}-\bar{X}_{40}\right)i\,\omega_n+J_{11}^f\left(\lambda\bar{X}_{60}-\bar{X}_{40}\right)\right]U_n\\
&\quad+\Big[\left(I_0\left(\mu\bar{X}_{20}-\bar{X}_{00}\right)+I_3\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)\right)\omega_n^2\\
&\quad+\left(c_d\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+g_0J_{11}^{zf}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right)\right)i\,\omega_n+\left(N^E+N^H\right)\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)\\
&\quad+k_w\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+k_p\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)+J_{11}^{zf}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right)\Big]W_{bn}\\
&\quad+\Big[\left(I_0\left(\mu\bar{X}_{20}-\bar{X}_{00}\right)+I_5\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)\right)\omega_n^2\\
&\quad+\left(c_d\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+g_0J_{11}^{ff}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right)+g_0K_{55}\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right)\right)i\,\omega_n\\
&\quad+\left(N^E+N^H\right)\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)+k_w\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+k_p\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)\\
&\quad+K_{55}\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right)+J_{11}^{ff}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right)\Big]W_{sn}\\
&\quad+\left(Y_{31}^e-X_{15}\right)\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right)\Phi_n+\left(Y_{31}^m-Y_{15}\right)\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right)\Psi_n=0
\end{aligned}\tag{78}$$

$$\begin{aligned}
&K_{31}^e\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)U_n+X_{31}^e\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right)W_{bn}+\left(X_{15}-Y_{31}^e\right)\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)W_{sn}\\
&\quad+\left[X_{11}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)+X_{33}\left(\lambda\bar{X}_{20}-\bar{X}_{00}\right)\right]\Phi_n\\
&\quad+\left[Y_{11}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)+Y_{33}\left(\lambda\bar{X}_{20}-\bar{X}_{00}\right)\right]\Psi_n-F_{33}^e=0
\end{aligned}\tag{79}$$

$$\begin{aligned}
&K_{31}^m\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)U_n+X_{31}^m\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right)W_{bn}+\left(Y_{15}-Y_{31}^m\right)\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)W_{sn}\\
&\quad+\left[Y_{11}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)+Y_{33}\left(\lambda\bar{X}_{20}-\bar{X}_{00}\right)\right]\Phi_n\\
&\quad+\left[K_{11}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)+K_{33}\left(\lambda\bar{X}_{20}-\bar{X}_{00}\right)\right]\Psi_n-F_{33}^m=0
\end{aligned}\tag{80}$$

where:

$$\bar{X}_{11}=\int_0^L\frac{\partial X_n}{\partial x}\frac{\partial X_n}{\partial x}dx,\qquad \bar{X}_{13}=\int_0^L\frac{\partial X_n}{\partial x}\frac{\partial^3 X_n}{\partial x^3}dx,\qquad \bar{X}_{15}=\int_0^L\frac{\partial X_n}{\partial x}\frac{\partial^5 X_n}{\partial x^5}dx\tag{81}$$

$$\bar{X}_{00}=\int_0^L X_nX_n dx,\qquad \bar{X}_{20}=\int_0^L\frac{\partial^2 X_n}{\partial x^2}X_n dx\tag{82}$$

$$\bar{X}_{40}=\int_0^L\frac{\partial^4 X_n}{\partial x^4}X_n dx,\qquad \bar{X}_{60}=\int_0^L\frac{\partial^6 X_n}{\partial x^6}X_n dx\tag{83}$$

The admissible function X_n is considered as mode shape according to boundary conditions as follows (Leissa [2]):

$$S-S:\ X_n=\sin\left(\frac{n\pi}{L}x\right) \tag{84}$$

$$C-C:\ X_n=\sin^2\left(\frac{n\pi}{L}x\right) \tag{85}$$

Finally, the Eqs. (76)-(80) can be written in a matrix form as follows:

$$\{[M]\omega_n^2+[D]\omega_n+[K]\}\{\Delta\}=0 \tag{86}$$

In Eq. (86), $\{\Delta\}=\{U_n, W_{nb}, W_{sn}, \Phi_n, \Psi_n\}^T$ are the unknown coefficients and $[M]$, $[D]$ and $[K]$ are the mass, damping, and stiffness matrices for Nano beam which have been presented in Appendix. The properties materials of magneto-electro-viscoelastic Nano beam are presented in Table 1. (Ke *et al* [24]).

Table 1
Material constants for MEV $BaTiO_3$–$CoFe_2O_4$ composite.

Properties	$BiTiO_3$ - $CoFe_2O_4$
$C_{11}(GPa)$	154.81
C_{55}	44.2
$e_{31}(Cm^2)$	-7.54
e_{15}	5.8
$q_{31}(N/Am)$	89.23
q_{15}	275
$\chi_{11}(10^9 C^2 m^2 N^1)$	5.64
χ_{33}	5.95
$\lambda_{11}(10^4 Ns^2 C^2)$	-297
λ_{33}	650.3
$d_{11}(10^{12} Ns/VC)$	5.36
d_{33}	2752.56
$\rho(kgm^3)$	5550

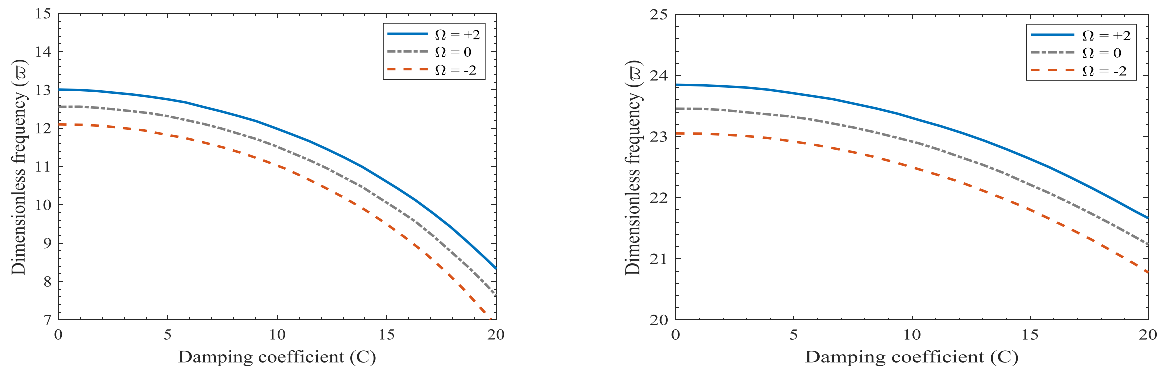
5 NUMERICAL RESULTS AND DISCUSSIONS

In this section, the vibration behavior of Nano beam made of piezoelectric material in magnetic field is investigated. In order to confirm the accuracy of the results, validation of the obtained results of the present study are carried with those provided by Ebrahimi et *al* [27]. Hence, the frequencies obtained from the two models are compared with respect to the power low index (*P*), temperature variations (ΔT) and damping coefficient of zero for different values of the nonlocal parameter and presented in Table 2. In addition, the dimensionless form of viscoelastic and dimensionless frequency parameters with $C_{11}=E$ and $I=h^3/12$ are defined by:

$$\bar{\omega}=\omega L^2\sqrt{\frac{\rho A}{EI}},\quad K_w=k_w\frac{L^4}{EI},\quad K_p=k_p\frac{L^2}{EI},\quad C=c_d\frac{L^2}{\sqrt{\rho AEI}},\quad \eta=\frac{g_0}{L^2}\sqrt{\frac{EI}{\rho A}} \tag{87}$$

Table 2
Comparison of the non-dimensional frequency obtained from the present study and Reference [32] for undamped S-S Nano beam.

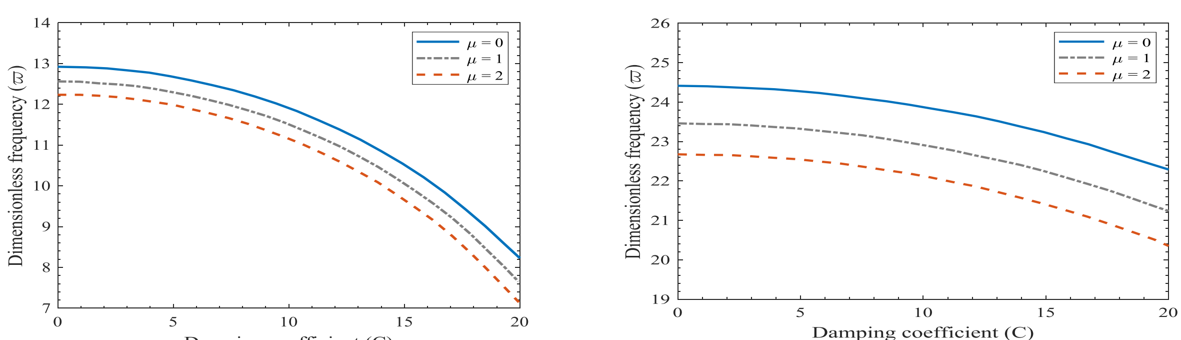
L/h	P	ΔT	μ (nm^2)	Ebrahimi and Salari [27]	Present
20	0	0	0	9.6468	9.65189
20	0	0	1	9.1859	9.19080
20	0	0	2	8.7825	8.78720
20	0	0	3	8.4254	8.42995



(a) Simply supported nanobeam (S-S) (b) Clamped nanobeam (C-C)

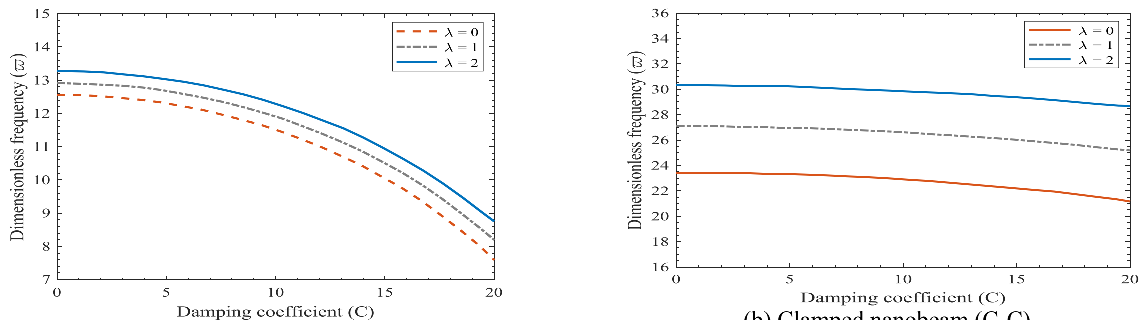
Fig.2
Effect of magnetic potential on vibration frequency of Nano beam respect to different damping coefficient ($L/h=20$, V= 0, $\mu=1$, $\lambda=0$, $K_w=15$, $K_P=5$, $\eta=0.01$).

Fig. 2 shows influence of magnetic potentials on the changes of non-dimensional frequency of magneto-electro-viscoelastic Nano beam respect to damping coefficient (C) for different boundary conditions S-S and C-C at slenderness ratio $L/h=20$, nonlocal parameter $\mu=1nm^2$, Winkler constant $K_w=15$, Pasternak constant $K_P=5$, internal damping parameter $\eta=0.01$. As can be seen, influence of damping coefficient (C) led to a decrease of non-dimensional frequencies of magneto- electro-viscoelastic Nano beams for both types of boundary conditions. This decrease in the vibrational frequency is significantly observed at larger values of the damping coefficient. Furthermore, vibrational frequency per magnitude of the damping coefficient can be affected by applying a magnetic field to the Nano beam. So can be increased the dimensionless frequency for any value of the damping coefficient by increasing the magnetic potential, and vice versa. The reason for this is the change in the stiffness of the Nano beam by exerting magnetic potentials. Hence, can be controlled the effect of the damping coefficient on the vibrational frequency according to the sign and the magnitude of the magnetic potential.



(a) Simply supported nanobeam (S-S) (b) Clamped nanobeam (C-C)

Fig.3
Effect of nonlocal parameter on vibration frequency of nanobeam respect to different damping coefficient ($L/h=20$, $V=\Omega=0$, $\lambda=0$, $K_w=15$, $K_P=5$, $\eta=0.01$).



(a) Simply supported nanobeam (S-S)

(b) Clamped nanobeam (C-C)

Fig. 4
Effect of length scale parameter on vibration frequency of nanobeam respect to different damping coefficient ($L/h=20$, $V=\Omega=0$, $\mu=1$, $K_w=15$, $K_P=5$, $\eta=0.01$).

Influence of the nonlocal and length scale parameters (μ,λ) on non-dimensional frequency of the smart magneto-electro-viscoelastic Nano beam versus damping coefficient at $L/h=20$, $V=\Omega=0$, $K_w=15$, $K_P=5$ and $\eta=0.01$ is shown in Figs. 3 and 4. It can be concluded that the increase of the nonlocal parameter leads to the reduction of the dimensionless frequency values. The cause of this decrease is the production of a stiffness-softening effect in the MEV Nano beam when the nonlocal parameter rises. On the other hand, the length scale parameter has an opposite effect on the dimensionless frequency, so that its increase leads to production a stiffness-hardening effect in the Nano beam and thus the frequency increases. It is also seen that the dimensionless frequencies of a C-C magneto-electro-viscoelastic Nano beam are larger than S-S Nano beam. In fact, Stronger support in the corners makes the beam more rigid so that the vibration frequency rises.

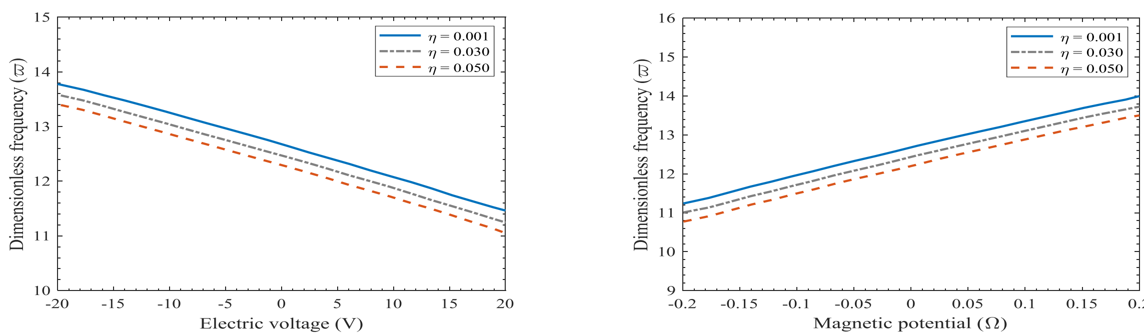


Fig. 5
Effect of internal damping parameter on vibration frequency of S-S Nano beam respect to electric voltage and magnetic potential ($L/h=20$, $\mu=1$, $\lambda=1$, $K_w=15$, $K_P=5$).

Fig. 5 presents influence of the internal damping constant (η) on vibration frequency of magneto-electro-viscoelastic Nano beam with simply- supported boundary conditions versus electric voltage (V) and magnetic potential (Ω) at $L/h=20$, $\mu=1$, $\lambda=1$, $K_w=15$, $K_P=5$ and C = 5. In this figure, with the increase of the internal damping parameter, the vibration frequency decreases for each constant value of the magnetic potential and electrical voltage. In addition, it can be seen that by increasing the applied voltage, less quantities are obtained for dimensionless frequencies. This decrease is due to the fact that the exertion of positive and negative voltages leads to the production of compressive and tensile forces in the magneto-electro-viscoelastic Nano beams, respectively. On the other hand, magnetic field has an opposite effect on vibration frequencies. Therefore, the increase of magnetic potential leads to increase of the dimensionless frequency for any constant value of internal damping parameter. It is also clear that by changing the internal damping parameter (η), the effect of electrical voltage and magnetic potential on the vibration frequencies can be controlled for each sign and value.

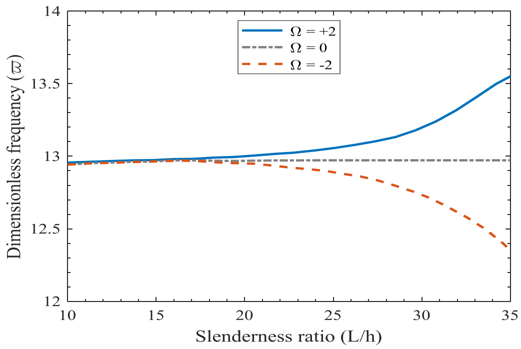


Fig. 6
Effect of slenderness ratio on vibration frequency of S-S Nano beam respect to different magnetic potential (V= 0, $\mu=1$, $\lambda=1$, $K_w=15$, $K_P=5$, $\eta=0.01$).

Fig. 6 shows the effect of slenderness ratio (L/h) on dimensionless frequency of magneto-electro-viscoelastic Nano beam with simply-supported edges for different values of magnetic potential at, $\mu=1$, $\lambda=1$, V = 0, $K_w=15$, $K_P=5$ and $\eta=0.01$. As can be seen, the effect of the magnetic field is considerable in larger values of the slenderness ratio. Thus, effect of the magnetic field on the thinner Nano beams is more than thicker Nano beams, so that the vibrational frequencies of thinner Nano beams increase for positive potential and decrease for negative potential. The cause of this fact is the production of tensile and compressive forces in the MEV Nano beam due to the exertion of positive and negative potentials, respectively. In addition, zero magnetic potential do not produce any force in the Nano beam, so it will not affect the dimensionless frequency by changing the slenderness ratio.

6 CONCLUSION

This article investigates free vibration of magneto-electro-viscoelastic Nano beam resting on viscoelastic medium based on a nonlocal refined beam theory with various boundary conditions. According to nonlocal strain gradient theory, the governing equations are obtained using Hamilton's principle and solved implementing an analytical solution. The results denote which the nonlocal and strain gradient parameters decreases the vibration frequency of magneto-electro-viscoelastic Nano beam. As shown that increase of internal damping parameter and damping coefficient leads to reduction of vibration frequencies. The most important consequence of this research is that the exertion of a positive and negative magnetic field to the MEV Nano beam produces tensile and compressive forces, respectively that have a significant effect on the vibrational frequency in larger value of slenderness ratio. Hence, negative/positive magnetic potential decreases/increases the vibration frequencies of thinner Nano beam. Therefore, the effect of these parameters should be considered in the design of devices and practical applications.

APPENDIX

$$M_{11}=I_0\left(\mu\bar{X}_{13}-\bar{X}_{11}\right)\ ,\ M_{12}=I_1\left(\bar{X}_{11}-\mu\bar{X}_{13}\right),\ M_{13}=I_2\left(\bar{X}_{11}-\mu\bar{X}_{13}\right),\ M_{21}=I_1\left(\mu\bar{X}_{40}-\bar{X}_{20}\right),$$
$$M_{22}=I_0\left(\mu\bar{X}_{20}-\bar{X}_{00}\right)+I_4\left(\bar{X}_{20}-\mu\bar{X}_{40}\right),\ M_{22}=I_0\left(\mu\bar{X}_{20}-\bar{X}_{00}\right)+I_3\left(\bar{X}_{20}-\mu\bar{X}_{40}\right),$$
$$M_{31}=I_2\left(\mu\bar{X}_{40}-\bar{X}_{20}\right),\ M_{32}=I_0\left(\mu\bar{X}_{20}-\bar{X}_{00}\right)+I_3\left(\bar{X}_{20}-\mu\bar{X}_{40}\right),$$
$$M_{33}=I_0\left(\mu\bar{X}_{20}-\bar{X}_{00}\right)+I_5\left(\bar{X}_{20}-\mu\bar{X}_{40}\right),$$
$$C_{11}=ig_0J_{11}\left(\lambda\bar{X}_{15}-\bar{X}_{13}\right),\ C_{12}=ig_0J_{11}^{z}\left(\bar{X}_{13}-\lambda\bar{X}_{15}\right),\ C_{13}=ig_0J_{11}^{f}\left(\bar{X}_{13}-\lambda\bar{X}_{15}\right),$$
$$C_{21}=ig_0J_{11}^{z}\left(\lambda\bar{X}_{60}-\bar{X}_{40}\right),\ C_{22}=i\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)c_d+ig_0J_{11}^{zz}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right),$$
$$C_{23}=i\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)c_d+ig_0J_{11}^{zf}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right),\ C_{31}=ig_0J_{11}^{f}\left(\lambda\bar{X}_{60}-\bar{X}_{40}\right),$$
$$C_{32}=i\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)c_d+ig_0J_{11}^{zf}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right),$$
$$C_{33}=i\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)c_d++ig_0J_{11}^{ff}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right)+ig_0K_{55}\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right),$$
$$K_{11}=J_{11}\left(\lambda\bar{X}_{15}-\bar{X}_{13}\right),\ K_{12}=J_{11}^{z}\left(\bar{X}_{13}-\lambda\bar{X}_{15}\right),\ K_{13}=J_{11}^{f}\left(\bar{X}_{13}-\lambda\bar{X}_{15}\right),$$

$$K_{14}=K_{31}^{e}\left(\lambda\bar{X}_{13}-\bar{X}_{11}\right),\ K_{15}=K_{31}^{m}\left(\lambda\bar{X}_{13}-\bar{X}_{11}\right),\ K_{21}=J_{11}^{z}\left(\lambda\bar{X}_{60}-\bar{X}_{40}\right),$$
$$K_{22}=\left(N^{E}+N^{H}\right)\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)+k_{w}\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+k_{p}\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)+J_{11}^{zz}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right),$$
$$K_{23}=\left(N^{E}+N^{H}\right)\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)c_{d}+k_{w}\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+k_{p}\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)+J_{11}^{zf}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right),$$
$$K_{24}=X_{31}^{e}\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right),\ K_{25}=X_{31}^{m}\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right),\ K_{31}=J_{11}^{f}\left(\lambda\bar{X}_{60}-\bar{X}_{40}\right),$$
$$K_{32}=\left(N^{E}+N^{H}\right)\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)+k_{w}\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+k_{p}\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)+J_{11}^{zf}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right),$$
$$K_{33}=\left(N^{E}+N^{H}\right)\left(\bar{X}_{20}-\mu\bar{X}_{40}\right)+k_{w}\left(\bar{X}_{00}-\mu\bar{X}_{20}\right)+k_{p}\left(\mu\bar{X}_{40}-\bar{X}_{20}\right)+K_{55}\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right)+J_{11}^{ff}\left(\bar{X}_{40}-\lambda\bar{X}_{60}\right),$$
$$K_{34}=\left(Y_{31}^{e}-X_{15}\right)\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right),\ K_{35}=\left(Y_{31}^{m}-Y_{15}\right)\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right),\ K_{41}=K_{31}^{e}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right),$$
$$K_{42}=X_{31}^{e}\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right),\ K_{43}=\left(X_{15}-Y_{31}^{e}\right)\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right),$$
$$K_{44}=X_{11}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)+X_{33}\left(\lambda\bar{X}_{20}-\bar{X}_{00}\right),\ K_{45}=Y_{11}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)+Y_{33}\left(\lambda\bar{X}_{20}-\bar{X}_{00}\right),$$
$$K_{51}=K_{31}^{m}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right),\ K_{52}=X_{31}^{m}\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right),\ K_{53}=\left(Y_{31}^{m}-Y_{15}\right)\left(\lambda\bar{X}_{40}-\bar{X}_{20}\right),$$
$$K_{54}=Y_{11}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)+Y_{33}\left(\lambda\bar{X}_{20}-\bar{X}_{00}\right),\ K_{55}=K_{11}\left(\bar{X}_{20}-\lambda\bar{X}_{40}\right)+K_{33}\left(\lambda\bar{X}_{20}-\bar{X}_{00}\right),$$
$$F_{33}^{e}=F_{33}^{m}=0$$

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