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Rigidity and Irregularity Effect on Surface Wave Propagation in a Fluid Saturated Porous Layer

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ABSTRACT

The propagation of surface waves in a fluid- saturated porous isotropic layer over a semi-infinite homogeneous elastic medium with an irregularity for free and rigid interfaces have been studied. The rectangular irregularity has been taken in the half-space. The dispersion equation for Love waves is derived by simple mathematical techniques followed by Fourier transformations. It can be seen that the phase velocity is strongly influenced by the wave number, the depth of the irregularity, homogeneity parameter and the rigid boundary. The dimensionless phase velocity is plotted against dimensionless wave number graphically for different size of rectangular irregularities and homogeneity parameter with the help of MATLAB graphical routines for both free and rigid boundaries for several cases. The numerical analysis of dispersion equation indicates that the phase velocity of surface waves decreases with the increase in dimensionless wave number. The obtained results can be useful to the study of geophysical prospecting and understanding the cause and estimating of damage due to earthquakes.

Keywords: Surface waves; Rectangular irregularity; Phase velocity; Dispersion equation; Semi-infinite medium.

1 INTRODUCTION

SURFACE waves propagation with and without the presence of irregularities and rigidity has been studied by many researchers at the interface (e.g., Love [1], Ewing et al [2],Chatopadhyay [3], Gupta et al [4] and others) . However, most of the research done on this subject does not concern porous media filled with fluid with irregular interface. Many researchers have studied the propagation of Love waves by taking various irregularities, in-homogeneities and boundaries of the Earth. Kundu et al. [5] studied the effect rigidity of the propagation of Love waves in porous layer lying over pre-stressed half space. The dispersion equation of Love waves propagating in an irregular pre-stressed anisotropic porous stratum under initial stress had been studied by Chattaraj et al.[6]. Madan et al. [7] and Kumar et al [8] studied the Love wave propagation in an irregular fluid saturated porous anisotropic layer and shear waves propagation in multilayered medium including an irregular fluid saturated porous stratum with rigid

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boundary respectively. The propagation of Love waves in a heterogeneous layer lying between homogeneous and inhomogeneous isotropic elastic half-spaces had been discussed by Kakar and Gupta [9]. Kumari [10] studied about reflection and transmission of longitudinal wave at micropolar viscoelastic solid/fluid saturated incompressible porous solid interface. Kumar et al. [11] derived the dispersion equation for Love waves in a model consisting of porous isotropic layer over a non-homogeneous elastic medium with half-space rectangular irregularity and observed that the phase velocity is significantly influenced by wave number, irregularity and in-homogeneity parameter. Kakar [12] discussed the propagation of Love waves in an isotropic layer lying between orthotropic and inhomogeneous half spaces by considering five different cases and also derived the dispersion equations for each case. Barak and Kaliraman [13, 14] investigate reflection and refraction phenomena of elastic waves propagating through imperfect interface of solids and propagation of elastic waves at micropolar viscoelastic solid/fluid saturated incompressible porous solid interface. Kaliraman and Poonia [15] discussed about elastic wave propagation at imperfect boundary of micropolar elastic solid and fluid saturated porous solid half space. Kumar et al.[16] studied the effect of in-homogeneity and rigid interfaced on propagation of Love waves in a porous isotropic layer over an elastic medium with a rectangular irregularity and obtained dispersion equation as a function of phase velocity and wave number.

In this paper the effect of rectangular irregularity and rigidity on Love wave propagation in a fluid- saturated porous layer over a homogeneous elastic half space has been studied. By using Fourier transformations the dispersion equation have been derived. It has been observed that the phase velocity is significantly influenced by the wave number, the depth of the irregularity, homogeneity parameter and the rigid boundary. The dimensionless phase velocity is plotted against dimensionless wave number and is shown graphically for different size of rectangular irregularities and homogeneity parameter for both free and rigid boundaries for several cases. The numerical analysis of dispersion equation indicates that the dimensionless wave number increases with the decrease in the phase velocity for Love waves. The obtained results are very useful to study the cause and estimating of damage due to earthquakes.

2 FREE SURFACE PROBLEM

2.1 Formulation of the problem

In this paper, a fluid saturated porous isotropic layer of thickness T, resting on a homogeneous elastic half space with a rectangular irregularity at the interface with length s and depth T' has been assumed. The Cartesian coordinate system (x_1, x_2, x_3) is chosen with x_3-axes taken vertically downward in the half space and x_1-axes is chosen parallel to the layer in the direction of propagation of the disturbance. The origin is placed at the middle point of the interface irregularity and the source of the disturbance is placed on positive x_3 axes at a distance d ($d > T'$) from the origin. Therefore, the fluid saturated porous layer describes the medium M_I: $-T \le x_3 \le 0$, and the lower half space describes the medium M_{II}: $0 \le x_3 < \infty$. The geometry of the considered problem is shown in Fig.1.

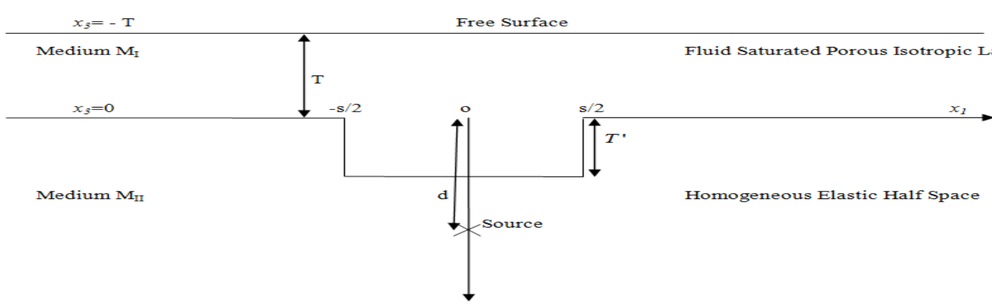

Fig.1
Geometry of the model considered for fluid saturated porous layer over homogeneous half space with free surface.

The irregularity interface is defined as:

$$x_3 = \varepsilon h(x_1) \tag{1}$$

where

$$h(x_1) = \begin{cases} 0; x_1 \le -\dfrac{s}{2}, x_1 \ge \dfrac{s}{2} \\ f(x_1); -\dfrac{s}{2} \le x_1 \le \dfrac{s}{2} \end{cases} \quad \text{where } \varepsilon = \frac{T'}{s} \text{ and } \varepsilon \ll 1. \tag{2}$$

2.2 Displacement equations

For Love waves propagating in x_1-direction in Medium M_I, the displacements in x_3-direction are expressed as:

$$\begin{aligned} &u^{(I)} \equiv w^{(I)} \equiv 0, \qquad v^{(I)} \equiv v^{(I)}(x_1, x_3, t), \\ &U^{(I)} \equiv W^{(I)} \equiv 0, \qquad V^{(I)} \equiv V^{(I)}(x_1, x_3, t), \end{aligned} \tag{3}$$

and the equation of motion for the fluid saturated porous isotropic layer in the absence of body forces are given by [10]:

$$\left\{ \frac{\mu^{(I)}}{2}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \right) - \left[\rho_{11}\partial_t^2 + b_{11}\partial_t - \frac{(\rho_{12}\partial_t^2 - b_{11}\partial_t)^2}{\rho_{22}\partial_t^2 + b_{11}\partial_t} \right] \right\} (v^{(I)}, V^{(I)}) = 0. \tag{4}$$

where $\lambda^{(I)}$ and $\mu^{(I)}$ are Lame's elastic constants. The displacement equations of homogeneous half space are

$$u^{(II)} \equiv w^{(II)} \equiv 0, \qquad v^{(II)} \equiv v^{(II)}(x_1, x_3, t),$$

and corresponding equation of motion for Love waves, are [12]

$$\left\{ \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \right\} v^{(II)} = \frac{1}{\beta_2^2} \frac{\partial^2 v^{(II)}}{\partial t^2}. \tag{5}$$

2.3 Boundary conditions

The boundary conditions for the considered Love wave propagation problem are:

At the free surface $x_3 = -T$, the shear stress component vanishes, i.e.,

$$\sigma_{32}^{(I)}(x_1, x_3 = -T, t) = 0. \tag{6}$$

The stresses are continuous at the interface $x_3 = \varepsilon h(x_1)$:

$$\frac{\mu^{(I)}}{2}\left(\frac{\partial v^{(I)}}{\partial x_3} - \varepsilon h'(x_1) \frac{\partial v^{(I)}}{\partial x_1} \right) = \mu^{(II)}\left(\frac{\partial v^{(II)}}{\partial x_3} - \varepsilon h'(x_1) \frac{\partial v^{(II)}}{\partial x_1} \right) \tag{7}$$

where $h'(x_1) = \dfrac{dh(x_1)}{dx_1}$.

At the interface $x_3 = \varepsilon h(x_1)$, the displacements are continuous:

$$v^{(I)}(x_1, x_3 = \varepsilon h(x_1), t) = v^{(II)}(x_1, x_3 = \varepsilon h(x_1), t). \tag{8}$$

2.4 Solution of the problem
2.4.1 Mathematical analysis

For Love wave of angular frequency ω changing harmonically with time and propagating in x_1-direction in Medium M_I and M_{II}, we consider

$$\begin{aligned}
v^{(I)}(x_3,x_1,t) &= v_0^{(I)}(x_3,x_1)\exp(i\omega t),\\
V^{(II)}(x_3,x_1,t) &= V_0^{(II)}(x_3,x_1)\exp(i\omega t),\\
v^{(II)}(x_3,x_1,t) &= v_0^{(II)}(x_3,x_1)\exp(i\omega t),
\end{aligned} \tag{9}$$

On substituting from Eq. (9) into Eqs. (4) and (5), we obtain

$$\left(\frac{\mu^{(I)}}{2}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}\right)+\xi_1^2\right)(v_0^{(I)},V_0^{(I)})=0, \tag{10}$$

$$\frac{\partial^2 v_0^{(II)}}{\partial x_1^2}+\frac{\partial^2 v_0^{(II)}}{\partial x_3^2}+\frac{\omega^2}{\beta^2}v_0^{(II)}=0, \tag{11}$$

where

$$\begin{aligned}
&\xi_1^2=\alpha_1+i\alpha_2,\alpha_1=F\omega^2/c_G^2,\alpha_2=R\omega^2/c_G^2,F=F(\omega)=\frac{1+\Omega^2\gamma_{22}C'}{1+(\Omega\gamma_{22})^2}.\frac{\gamma_{22}}{C'},\\
&R=R(\omega)=\frac{(C'-\gamma_{22})\Omega}{1+(\Omega\gamma_{22})^2}.\frac{\gamma_{22}}{C'},C'=\gamma_{11}\gamma_{22}-\gamma_{12}^2,\gamma_{kl}=\frac{\rho_{kl}}{\rho}(k,l=1,2),\\
&c_G^2=(\rho_{11}-\rho_{12}^2/\rho_{22})^{-1},\Omega=\frac{\rho\omega}{b_{11}}.
\end{aligned}$$

where Ω and c_G are the dimensionless frequency and velocity of shear wave in the porous layer respectively.

Fourier transformations of Eqs. (10) and (11) are, therefore

$$\begin{aligned}
&\frac{d^2\bar{v}_0^{(I)}}{dx_3^2}+\chi_1^2\bar{v}_0^{(I)}=0,\\
&\frac{d^2\bar{V}_0^{(I)}}{dx_3^2}+\chi_1^2\bar{V}_0^{(I)}=0,\\
&\frac{d^2\bar{v}_0^{(II)}}{dx_3^2}-\chi_2^2\bar{v}_0^{(II)}=0.
\end{aligned} \tag{12}$$

where $\chi_1^2=\left(\frac{2\xi_1^2}{\mu^{(1)}}-\eta^2\right),\chi_2^2=\left(\eta^2-\frac{\omega^2}{\beta_2^2}\right).$

2.4.2 Solution analysis

The solutions of Eqs. (12) are:

$$\bar{v}_0^{(I)} = A\cos\chi_1 x_3 + B\sin\chi_1 x_3,$$
$$\bar{V}_0^{(I)} = \bar{A}\cos\chi_1 x_3 + \bar{B}\sin\chi_1 x_3, \tag{13}$$
$$\bar{v}_0^{(II)} = D\exp(-\chi_2 x_3),$$

where $A, B, \bar{A}, \bar{B}, D$ are functions of η.

By applying inverse Fourier transformations, we have

$$v_0^{(I)}(x_3, x_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(A\cos\chi_1 x_3 + B\sin\chi_1 x_3)e^{-i\eta x_1}d\eta,$$
$$V_0^{(I)}(x_3, x_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(\bar{A}\cos\chi_1 x_3 + \bar{B}\sin\chi_1 x_3)e^{-i\eta x_1}d\eta, \tag{14}$$
$$v_0^{(II)}(x_3, x_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(De^{-\chi_2 x_3} + \frac{2}{\chi_2}e^{\chi_2 x_3}e^{-\chi_2 d}\right)e^{-i\eta x_1}d\eta,$$

where the second term in the right side of the value of $v_0^{(II)}(x_3, x_1)$ is introduced due to the source in the lower medium.

The relations between the constants $\bar{A}, \bar{B}$ and A, B are provided by Eq. (4) and due to small value of ε, we can set A, B, D as:

$$A \cong A_0 + A_1\varepsilon, B \cong B_0 + B_1\varepsilon, D \cong D_0 + D_1\varepsilon. \tag{15}$$

Since the boundary is not uniform, the terms A, B, D in Eq. (14) are also functions of ε. Expanding these terms in ascending powers of ε and keeping in view that ε is so small that we can neglect second degree term and the terms containing higher powers of ε and A, B, D can be approximated as in Eq. (15). In physical situations, when the depth T' of the irregular boundary is too small with respect to the length of the boundary s, the above assumptions are justified. Further for small ε.

$$e^{\pm\alpha\varepsilon h} \cong 1 \pm \alpha\varepsilon h, \cos\chi_1\varepsilon h \cong 1, \sin\chi_1\varepsilon h \cong \chi_1\varepsilon h$$

where α is any quantity.

Now, by using boundary condition (6), we obtain

$$(A_0 + A_1\varepsilon)\sin\chi_1 T + (B_0 + B_1\varepsilon)\cos\chi_1 T = 0. \tag{16}$$

Now we define Fourier Transform of $h(x_1)$ as:

$$\bar{h}(\lambda) = \int_{-\infty}^{\infty} h(x_1)e^{i\lambda x_1}dx_1, \tag{17}$$

and the inverse Fourier Transform is

$$h(x_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{h}(\lambda)e^{-i\lambda x_1}d\lambda, \tag{18}$$

Therefore,

$$h'(x_1)=\frac{-i}{2\pi}\int_{-\infty}^{\infty}\lambda\,\bar{h}(\lambda)e^{-i\lambda x_1}d\lambda, \tag{19}$$

Using boundary condition (8), we have

$$\varepsilon\int_{-\infty}^{\infty}[(B_0\chi_1+\chi_2D_0-2e^{-\chi_2 d})]h(x_1)e^{-i\eta x_1}d\eta=\int_{-\infty}^{\infty}[(D_0-A_0)+\frac{2}{\chi_2}e^{-\chi_2 d})+\varepsilon(D_1-A_1)]e^{-i\eta x_1}d\eta. \tag{20}$$

Now by using Eq. (18), Eq. (20) takes the form of

$$\frac{\varepsilon}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}[(B_0\chi_1+\chi_2D_0-2e^{-\chi_2 d})]\bar{h}(\lambda)e^{-i(\eta+\lambda)x_1}d\eta\right]d\lambda=\int_{-\infty}^{\infty}[((D_0-A_0)+\frac{2}{\chi_2}e^{-\chi_2 d})+\varepsilon(D_1-A_1)]e^{-i\eta x_1}d\eta. \tag{21}$$

Putting $\eta+\lambda=k$ for the inner integral in the left hand side of Eq. (21), so that λ may be treated as a constant such that $d\eta=dk$, replacing η by k in the right hand side of Eq. (21), and finally after applying the Fourier transformations as defined in Eq. (17), we have

$$((D_0-A_0)+\frac{2}{\chi_2}e^{-\chi_2 d})+\varepsilon(D_1-A_1)=\varepsilon R_1(k) \tag{22}$$

where

$$R_1(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}[(B_0\chi_1+\chi_2D_0-2e^{-\chi_2 d})]^{\eta=k-\lambda}\,\bar{h}(\lambda)d\lambda \tag{23}$$

Similarly, by applying boundary condition (7) using Eqs. (17)-(19), we obtain

$$(B_0\chi_1\mu^{(I)}/2+\mu^{(II)}(\chi_2D_0-2e^{-\chi_2 d}))+\varepsilon(\mu^{(II)}\chi_2D_1+B_1\chi_1\mu^{(I)}/2)=\varepsilon R_2(k) \tag{24}$$

where

$$R_2(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\chi_1^2(\mu^{(I)}/2)A_0-\mu^{(II)}(q-\chi_2)(\chi_2D_0-2e^{-\chi_2 d})+\lambda k((\mu^{(I)}/2)A_0-\mu^{(II)}(D_0+\frac{2}{\chi_2}e^{-\chi_2 d}))\right]^{\eta=k-\lambda}\bar{h}(\lambda)d\lambda. \tag{25}$$

Equating the coefficients of like terms of ε from Eqs. (16), (22), and (24), we obtain

$$\begin{aligned}
&A_0\sin\chi_1T+B_0\cos\chi_1T=0,\\
&A_0-D_0=\frac{2}{\chi_2}e^{-\chi_2 d},\\
&(\mu^{(I)}/2)\chi_1B_0+\mu^{(II)}\chi_2D_0=2\mu^{(II)}e^{-\chi_2 d},\\
&A_1\sin\chi_1T+B_1\cos\chi_1T=0,\\
&D_1-A_1=R_1(k),\\
&(\mu^{(I)}/2)\chi_1B_1+\mu^{(II)}\chi_2D_1=R_2(k).
\end{aligned} \tag{26}$$

Solving the above six equations given as in (26), we deduce that

$$
\begin{aligned}
A_0 &= \frac{2e^{-\chi_2 d}\,(\mu^{(II)}+\chi_2^2)\tan\chi_1 T}{\chi_2 E(k)},\\
B_0 &= \frac{-4\mu^{(II)}e^{-\chi_2 d}\,\tan\chi_1 T}{E(k)},\\
D_0 &= \frac{2e^{-\chi_2 d}\,(\mu^{(II)}\chi_2+(\mu^{(I)}/2)\chi_1\tan\chi_1 T)}{\chi_2 E(k)},\\
A_1 &= \frac{R_2-\mu^{(II)}\chi_2 R_1}{E(k)},\\
B_1 &= \frac{-(R_2-\mu^{(2)}\chi_2 R_1)\tan\chi_1 T}{E(k)},\\
D_1 &= \frac{R_2-R_1(\mu^{(I)}/2)\chi_1\tan\chi_1 T}{E(k)}.
\end{aligned}
\tag{27}
$$

where $E(k)=\mu^{(II)}\chi_2-(\mu^{(I)}/2)\chi_1\tan(\chi_1 T)$

2.4.3 Displacements equation for surface waves

The displacement vector in the fluid saturated isotropic layer is

$$
v_0^{(I)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu^{(II)}e^{-\chi_2 d}}{E(k)}\left[1+\frac{\varepsilon(R_2-\mu^{(II)}\chi_2 R_1)e^{\chi_2 d}}{4\mu^{(II)}}\right](\cos\chi_1 x_3-\tan(\chi_1 T)\sin\chi_1 x_3)e^{-ikx_1}dk\,, \tag{28}
$$

Therefore, by following Kumar et al. [11], the displacement in the isotropic layer is

$$
v_0^{(I)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu^{(II)}e^{-\chi_2 d}}{E(k)\left(1-\frac{T'}{2}\phi(k)e^{\chi_2 d}\right)}(\cos\chi_1 x_3-\tan(\chi_1 T)\sin\chi_1 x_3)e^{-ikx_1}dk\,. \tag{29}
$$

The value of this integral depends entirely on the contribution of the poles of the integrand and poles are given by

$$
E(k)\left[1-\frac{T'}{2}\phi(k)e^{\chi_2 d}\right]=0 \tag{30}
$$

This implies

$$
\left(\mu^{(II)}\chi_2-\frac{\mu^{(I)}\chi_1\tan(\chi_1 T)}{2}\right)-T'\left(\frac{\mu^{(I)}\chi_1\left(\chi_1-q\tan\chi_1 T\right)}{2}+\chi_1\chi_2\tan\chi_1 T\right)=0 \tag{31}
$$

If c is the common wave velocity of wave propagating along the surface, then we can set in Eq. (31) $\omega=ck$, (ω is the circular frequency and k is the wave number), $\chi_1=P_1 k$ and $\chi_2=P_2 k$ where

$$
P_1=\left(\frac{2}{\mu^{(I)}}\left(\frac{c^2}{c_G^2}.F(\omega)-\frac{\mu^{(I)}}{2}\right)+i\,\frac{2}{\mu^{(I)}}.\frac{c^2}{c_G^2}.R(\omega)\right)^{\frac{1}{2}}\ \ \&\ \ P_2=\sqrt{1-\frac{c^2}{\beta_2^2}}
$$

Solving Eq. (31), we obtain

$$\tan P_1 kT = \frac{\mu^{(II)} P_2 - T' P_1^2 k \, \mu^{(I)}/2}{P_1(\mu^{(I)}(1-QkT')/2 + P_2 T' k)} \tag{32}$$

Since the quantity P_1^2 is complex, so we have

$$P_1 = k_1 + ik_2, \tag{33}$$

where

$$k_{1,2} = \left\{ \frac{1}{2} \left(\left[\left\{ \left(\frac{2}{\mu^{(I)}} \frac{c^2}{c_G^2} F(\omega) - 1 \right) \right\}^2 + \left(\frac{2}{\mu^{(I)}} . \frac{c^2}{c_G^2} R(\omega) \right)^2 \right]^{\frac{1}{2}} \pm \left\{ \left(\frac{2}{\mu^{(I)}} \frac{c^2}{c_G^2} F(\omega) - 1 \right) \right\} \right) \right\}^{\frac{1}{2}} \tag{34}$$

Therefore with the help of Eq. (33), the dispersion Eq. (32) for Love waves, reduces to

$$\tan(k_1 + ik_2)kH = A_r + iA_i \tag{35}$$

where

$$A_r = \frac{\left(\mu^{(II)} P_2 - T' \left(\frac{c^2}{c_G^2} F(\omega) - \frac{\mu^{(I)}}{2} \right) k \right) k_1 + T' k \, \frac{c^2}{c_G^2} .R(\omega) k_2}{\left(k_1^2 + k_2^2 \right) \left(\frac{\mu^{(I)}(1-QkT')}{2} + P_2 T' k \right)},$$

$$A_i = -\frac{\left(\mu^{(II)} P_2 - T' \left(\frac{c^2}{c_G^2} F(\omega) - \frac{\mu^{(I)}}{2} \right) k \right) k_2 + T' k \, \frac{c^2}{c_G^2} .R(\omega) k_1}{\left(k_1^2 + k_2^2 \right) \left(\frac{\mu^{(I)}(1-QkT')}{2} + P_2 T' k \right)}.$$

For small values of k_2, we obtain

$$\tan(k_1 + ik_2)kT \approx \frac{\tan k_1 kT + ik_2 kh}{1 - ik_2 kT \, \tan k_1 kT}. \tag{36}$$

Using the expressions for A_r, A_i and Eq. (36) and separating real and imaginary parts of Eq. (35) to obtain two real equations

$$\tan k_1 kT = \frac{A_r}{1 - A_i k_2 kT}, \qquad k_2 kT \,(1 + A_r k_2 kT \,.\tan k_1 kT) = A_i \tag{37}$$

The dispersion equation for Love waves can be obtain from the real part of Eq. (35), i.e. ,

$$\tan k_1 kT = \frac{A_r}{1 - A_i k_2 kT} \approx A_r (1 + A_i k_2 kT). \tag{38}$$

3 RIGID BOUNDARY PROBLEM

3.1 Formulation of the problem

A fluid saturated porous isotropic layer of thickness T with rigid boundary, resting on a homogeneous elastic half space has been considered. The Cartesian coordinate system (x_1, x_2, x_3) is chosen with x_3-axes taken vertically downward in the half space and x_1-axes is chosen parallel to the layer in the direction of propagation of the disturbance by taking a rectangular irregularity with length s and depth T' with origin at the middle point of the interface irregularity and the source of the disturbance is placed on positive x_3-axes at a distance d ($d>T'$) from the origin. Therefore, the upper layer describes the medium M_I: $-T \le x_3 \le 0$, and the homogeneous elastic half space describes the medium M_{II}: $0 \le x_3 < \infty$. The geometry of the problem is shown in Fig. 2.

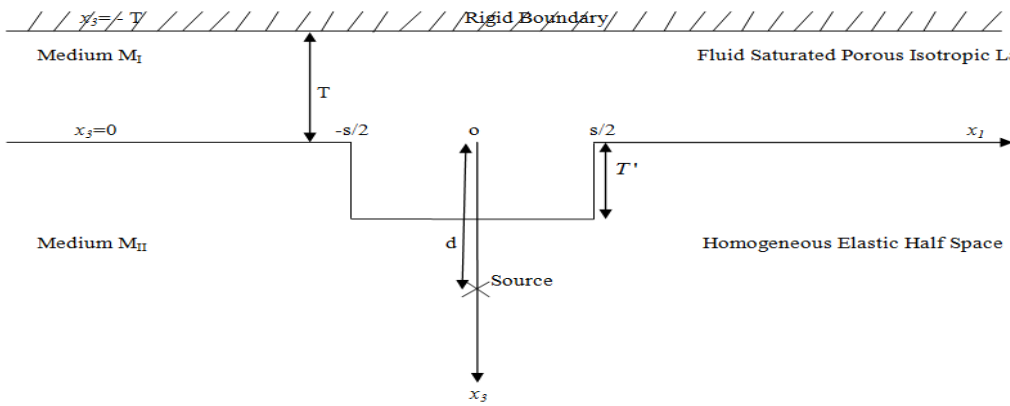

Fig.2
Geometry of the problem for fluid saturated porous layer over homogeneous half space with rigid boundary.

3.2 Equation of love waves propagation over rigid surface and boundary conditions

3.2.1 Displacement equations

The basic governing equations for this medium will be same as for the medium considered for free surface i.e., from Eqs. (1) to (5).

3.2.2 Boundary conditions

The boundary conditions for the considered model are:

The displacement component vanishes at the rigid surface $x_3 = -T$, i.e.,

$$v^{(I)}(x_1, x_3 = -T, t) = 0 \tag{39}$$

The continuity of stress component at the interface $z = \varepsilon h(x_1)$ that is

$$\frac{\mu^{(I)}}{2}\left(\frac{\partial v^{(I)}}{\partial x_3} - \varepsilon h'(x_1)\frac{\partial v^{(I)}}{\partial x_1}\right) = \mu^{(II)}\left(\frac{\partial v^{(II)}}{\partial x_3} - \varepsilon h'(x_1)\frac{\partial v^{(II)}}{\partial x_1}\right) \tag{40}$$

where $h'(x_1) = \dfrac{dh(x_1)}{dx_1}$.

The displacements are also continuous at the interface $x_3 = \varepsilon h(x_1)$:

$$v^{(I)}(x_1, x_3 = \varepsilon h(x_1), t) = v^{(II)}(x_1, x_3 = \varepsilon h(x_1), t). \tag{41}$$

Now, by using boundary condition (39), we obtain

$$(A_0 + A_1\varepsilon)\cos\chi_1 T - (B_0 + B_1\varepsilon)\sin\chi_1 T = 0. \tag{42}$$

By using boundary conditions (40) and (41), we obtain Eqs. (22) and (24).

Now, by equating the coefficients of like powers of ε from Eqs. (42), (22), and (24), we obtain a set of six equations

$$\begin{aligned}
&A_0\cos\chi_1 T - B_0\sin\chi_1 T = 0,\\
&A_0 - D_0 = \frac{2}{\chi_2}e^{-\chi_2 d},\\
&(\mu^{(I)}/2)\chi_1 B_0 + \mu^{(II)}\chi_2 D_0 = 2\mu^{(II)}e^{-\chi_2 d},\\
&A_1\cos\chi_1 T - B_1\sin\chi_1 T = 0,\\
&D_1 - A_1 = R_1(k),\\
&(\mu^{(I)}/2)\chi_1 B_1 + \mu^{(II)}\chi_2 D_1 = R_2(k).
\end{aligned} \tag{43}$$

The values of $A_0, B_0, D_0, A_1, B_1, D_1$ are obtained by solving above set of equations and the corresponding values are given by

$$\begin{aligned}
&A_0 = \frac{4\mu^{(II)}e^{-\chi_2 d}\tan\chi_1 T}{E_1(k)},\\
&B_0 = \frac{4\mu^{(II)}e^{-\chi_2 d}}{E_1(k)},\\
&D_0 = \frac{e^{-\chi_2 d}(2\mu^{(II)}\chi_2\tan\chi_1 T - \mu^{(I)}\chi_1)}{\chi_2 E_1(k)},\\
&A_1 = \frac{(R_2 - \mu^{(II)}\chi_2 R_1)\tan\chi_1 T}{E_1(k)},\\
&B_1 = \frac{R_2 - \mu^{(II)}\chi_2 R_1}{E_1(k)},\\
&D_1 = \frac{2R_2\tan\chi_1 T + R_1\mu^{(I)}\chi_1}{2E_1(k)},
\end{aligned} \tag{44}$$

where $E_1(k) = \mu^{(II)}\chi_2\tan\chi_1 T + (\mu^{(I)}/2)\chi_1$

Therefore the equation of the displacement vector in the isotropic layer is

$$v_0^{(I)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu^{(II)}e^{-\chi_2 d}}{E_1(k)}\left[1 + \frac{\varepsilon(R_2 - \mu^{(II)}\chi_2 R_1)e^{\chi_2 d}}{4\mu^{(II)}}\right](\sin\chi_1 x_3 + \tan(\chi_1 T)\cos\chi_1 x_3)e^{-ikx_1}dk, \tag{45}$$

or

$$v_0^{(I)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu^{(II)}e^{-\chi_2 d}}{E_1(k)[1 - \frac{T'}{2}\phi(k)e^{\chi_2 d}]}(\sin\chi_1 x_3 + \tan(\chi_1 T)\cos\chi_1 x_3)e^{-ikx_1}dk, \tag{46}$$

The value of the integral in Eq.(46) depends on the contribution of the poles of the integrand and the poles are located at the roots of the equation

$$E_1(k)[1-\frac{T'}{2}\phi(k)e^{\chi_2 d}]=0 \tag{47}$$

On simplification

$$\tan P_1 kT=\frac{T'kP_2(\mu^{(I)}Q-2\mu^{(II)}P_2)-\mu^{(I)}P_1}{2\mu^{(II)}P_2-T'k\,\mu^{(I)}P_1^2}, \tag{48}$$

where, the quantity P_1^2 is complex, so we have

$$P_1=k_1+ik_2,$$

$$k_{1,2}=\left\{\frac{1}{2}\left(\left[\left(\frac{2}{\mu^{(I)}}.\frac{c^2}{c_G^2}.F(\omega)-1\right)^2+\left(\frac{2}{\mu^{(I)}}.\frac{c^2}{c_G^2}.R(\omega)\right)^2\right]^{\frac{1}{2}}\pm\left(\frac{2}{\mu^{(I)}}.\frac{c^2}{c_G^2}.F(\omega)-1\right)\right)\right\}^{\frac{1}{2}}$$

Eq. (48) gives

$$\tan(k_1+ik_2)kT=A_{r.}+iA_i \tag{49}$$

where

$$A_r=\frac{\left[\left(T'kP_2(Q\mu^{(I)}-2\mu^{(II)}P_2)-C_5k_1\right)\left(\mu^{(II)}P_2-T'k\left(\frac{c^2}{c_G^2}.F(\omega)-\frac{\mu^{(I)}}{2}\right)\right)+T'k\,\mu^{(I)}k_2.\frac{c^2}{c_G^2}.R(\omega)\right]}{2\left[\left(\mu^{(II)}P_2-T'k\left(\frac{c^2}{c_G^2}.F(\omega)-\frac{\mu^{(I)}}{2}\right)\right)^2+\left(\frac{T'k\,\mu^{(I)}k_2}{2}.\frac{c^2}{c_G^2}.R(\omega)\right)^2\right]},$$

$$A_i=\frac{\left[\left(T'kP_2(Q\mu^{(I)}-2\mu^{(II)}P_2)-\mu^{(I)}k_1\right)T'k.\frac{c^2}{c_G^2}.R(\omega)-\mu^{(I)}k_2\left(\mu^{(II)}P_2-T'k\left(\frac{c^2}{c_G^2}.F(\omega)-\frac{\mu^{(I)}}{2}\right)\right)\right]}{2\left[\left(\mu^{(II)}P_2-T'k\left(\frac{c^2}{c_G^2}.F(\omega)-\frac{\mu^{(I)}}{2}\right)\right)^2+\left(T'kk_2.\frac{\mu^{(I)}}{2}.\frac{c^2}{c_G^2}.R(\omega)\right)^2\right]}.$$

Due to small values of k_2,we have

$$\tan(k_1+ik_2)kT\approx\frac{\tan k_1kT+ik_2kT}{1-ik_2kT\,\tan k_1kT}. \tag{50}$$

Using the expressions for A_r , A_i and Eq. (50) and separating real and imaginary parts of Eq. (49) to obtain two real equations

$$\tan k_1kT=\frac{A_r}{1-A_ik_2kT},\qquad k_2kT\,(1+A_rk_2kT\,.\tan k_1kT)=A_i. \tag{51}$$

The dispersion equation for Love waves is obtained from the real part of Eq. (49) and is of the form

$$\tan k_1 kT = \frac{A_r}{1 - A_i k_2 kT} \approx A_r (1 + A_i k_2 kT). \tag{52}$$

4 NUMERICAL ANALYSIS AND PLOTS

In this section, we wish to examine the effect of irregularity present in the layer by varying the phase velocity (c / c_G) with wave number (kT) for different values of homogeneity parameter for free as well as rigid surface. By using MATLAB graphical routines and following results:

(i) For fluid saturated isotropic layer, M_I [13]

$$\mu^{(I)} = 7.10 \times 10^{10} N / m^2$$
$$\rho^{(I)} = 3321 kg / m^3$$

(ii) For homogeneous half space, M_{II} [13]

$$\mu^{(II)} = 6.77 \times 10^{10} N / m^2$$
$$\rho^{(II)} = 3323 kg / m^3$$

We obtain following figures and graphs:

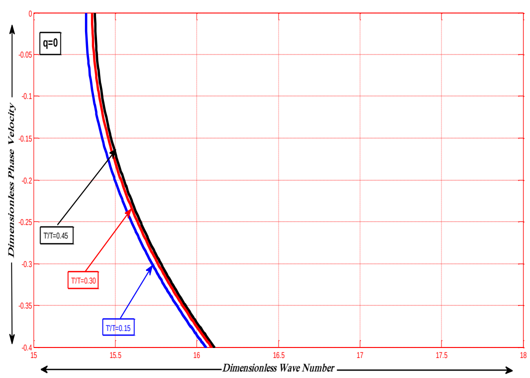

Fig.3
Variations of (c / c_G) with (kT) in a fluid saturated porous isotropic layer with free surface for different values of T'/T (=0.15, 0.30, 0.45) when q=0.

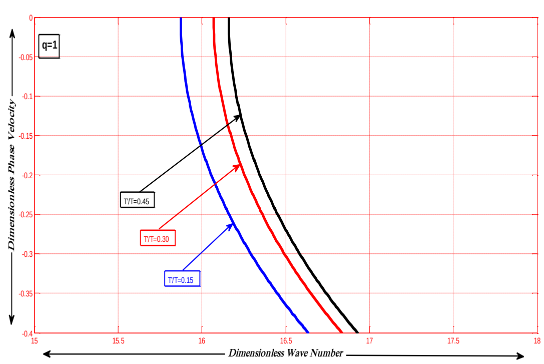

Fig.4
Variations of (c / c_G) with (kT) in a fluid saturated porous isotropic layer with free surface for different values of T'/T (=0.15, 0.30, 0.45) when q=1.

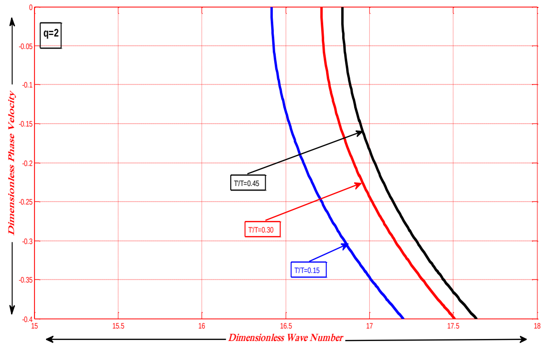

Fig.5
Variations of (c/c_G) with (kT) in a fluid saturated porous isotropic layer with free surface for different values of T'/T (=0.15, 0.30, 0.45) when q=2.

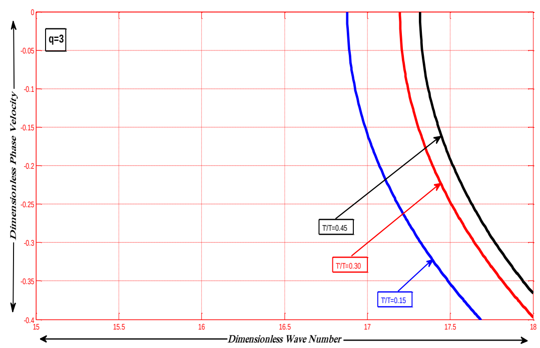

Fig.6
Variations of (c/c_G) with (kT) in a fluid saturated porous isotropic layer with free surface for different values of T'/T (=0.15, 0.30, 0.45) when q=3.

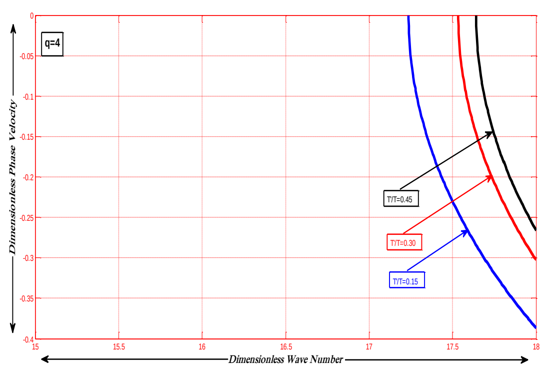

Fig.7
Variations of (c/c_G) with (kT) in a fluid saturated porous isotropic layer with free surface for different values of T'/T (=0.15, 0.30, 0.45) when q=4.

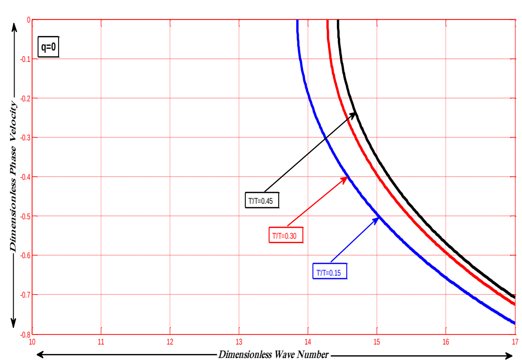

Fig.8
Variations of (c/c_G) with (kT) in a fluid saturated porous isotropic layer over rigid boundary for different values of T'/T (=0.15, 0.30, 0.45) when q=0.

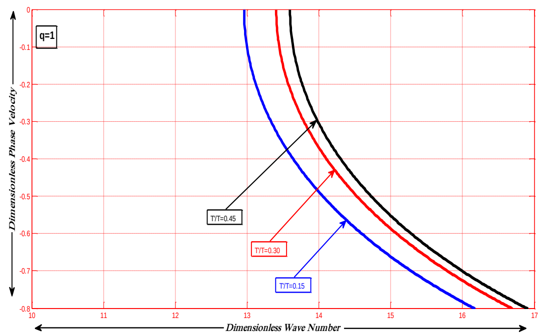

Fig.9
Variations of (c/c_G) with (kT) in a fluid saturated porous isotropic layer over rigid boundary for different values of T'/T (=0.15, 0.30, 0.45) when q=1.

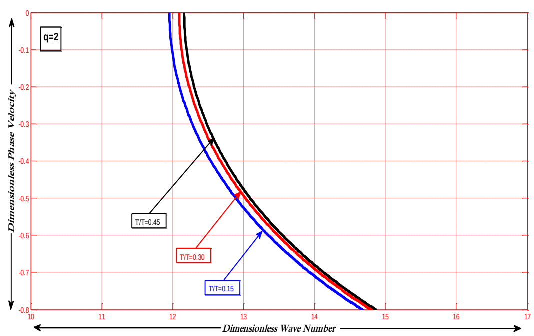

Fig.10
Variations of (c/c_G) with (kT) in a fluid saturated porous isotropic layer over rigid boundary for different values of T'/T (=0.15, 0.30, 0.45) when q=2.

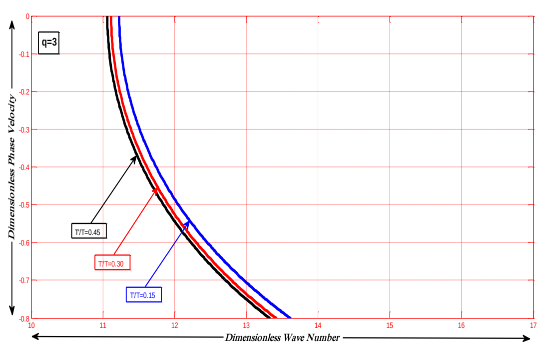

Fig.11
Variations of (c/c_G) with (kT) in a fluid saturated porous isotropic layer over rigid boundary for different values of T'/T (=0.15, 0.30, 0.45) when q=3.

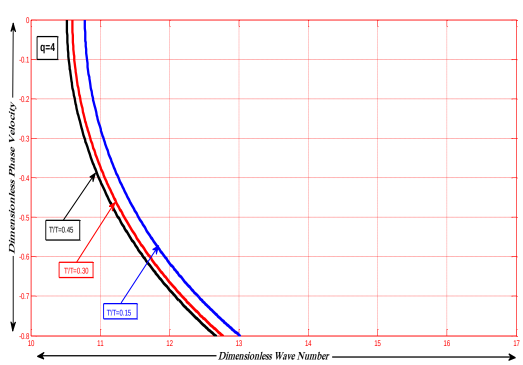

Fig.12
Variations of (c/c_G) with (kT) in a fluid saturated porous isotropic layer over rigid boundary for different values of T'/T (=0.15, 0.30, 0.45) when q=4.

In above figures we have plotted the dimensionless phase velocity (c/c_G) against the dimensionless wave number (kT). It is interesting to note that $(kT)_{\frac{T'}{T}=0.15} \le (kT)_{\frac{T'}{T}=0.30} \le (kT)_{\frac{T'}{T}=0.45}$; $c/c_G < 0$ at the free surface for different values of q (i.e., q=0, 1, 2, 3, 4), but $(kT)_{\frac{T'}{T}=0.15} \leq (kT)_{\frac{T'}{T}=0.30} \leq (kT)_{\frac{T'}{T}=0.45}; c/c_G < 0$ for q (q= 0, 1, 2) and $(kT)_{\frac{T'}{T}=0.15} \geq (kT)_{\frac{T'}{T}=0.30} \geq (kT)_{\frac{T'}{T}=0.45}; c/c_G < 0$ for q (q=3, 4) over rigid boundary in all the cases. And the wave number decreases with the increase in the value of the homogeneity parameter q over rigid boundary and increases over free surface.

5 CONCLUSIONS

The propagation of Love waves in an irregular fluid saturated porous isotropic layer with free and rigid boundary over a homogeneous isotropic half space has been discussed in the present study. Simple mathematical techniques followed by Fourier transformation are applied to find the displacement vector in the porous layer. The effect of rigidity and dimensionless wave number on dispersion curve is shown graphically by using MATLAB graphical routines for different values of q and the results are compared with free surface boundaries. Variation of phase velocity for different ratio of irregularity depth to the layer width and for different values of homogeneity parameter has been studied and shown graphically. From above numerical analysis, it may be conclude that:

(i) It is observed from graphs that with the increase in the value of the homogeneity parameter q, the dimensionless wave number also increases.

(ii) The numerical analysis of dispersion equation indicates that the dimensionless wave number increases with the decrease in the phase velocity of Love waves.

(iii) It is noticed that the phase velocity c/c_G of Love waves is affected by the ratio of height of irregularity with the height of the layer i. e., T'/T and the phase velocity c/c_G increases with the increase in the value of T'/T .

(iv) It is observed from all graphs and figures that the dimensionless phase velocity c/c_G increases over free surface, but decreases over rigid boundary with the increase in the value of the parameter q.

Hence, it is concluded that the irregular fluid saturated porous isotropic layer with rigid boundary as well as free surface has a significant effect on the propagation of Love waves, and the phase velocity in a layer with irregularity is affected by not only the shape of irregularity, but also by wave number, rigid boundary, homogeneity parameter, the ratio of the depth of the irregularity to layer width and layer structure. From above discussion, it can be said that the obtained results in this research paper are useful for the study of various fields of geophysics and to understand the cause and estimating of damage due to earthquakes.

REFERENCES

[1] Love A.E.H., 1944, *A Treatise on the Mathematical Theory of Elasticity*, Dover Publications, New York.

[2] Ewing M., Jardetzky W.S., Press F., 1957, *Elastic Waves in Layered Media*, McGraw-Hill, New York.

[3] Chatopadhyay A.,1975, On the dispersion equation for Love wave due to irregularity in the thickness of non-homogeneous crustal layer, *Acta Geophysica* **23**: 307-317.

[4] Gupta S., Majhi D.K., Kundu S., Vishwakarma S.K., 2013, Propagation of Love waves in non-homogeneous substratum over initially stressed heterogeneous half-space, *Applied Mathematics and Mechanics* **34**: 249-258.

[5] Kundu S., Gupta S., Majhi D.K., 2013, Love wave propagation in porous rigid layer lying over an initially stressed half space, *Applied Physics and Mathematics* **3**(2): 140-142.

[6] Chattaraj R., Samal S.K., Mahanti N.C., 2013, Dispersion of Love wave propagating in irregular anisotropic porous stratum under initial stress, *International Journal of Geomechanics* **13**(4): 402-408.

[7] Madan D.K., Kumar R., Sikka J.S., 2014, Love wave propagation in an irregular fluid saturated porous anisotropic layer with rigid boundary, *Applied Scientific Research* **10**: 281-287.

[8] Kumar R., Madan D.K., Sikka J.S., 2014, Shear wave propagation in multilayered medium including an irregular fluid saturated porous stratum with rigid boundary, *Advances in Mathematical Physics* **2014**: 163505.

[9] Kakar R., Gupta M., 2014, Love waves in an intermediate heterogeneous layer lying in between homogeneous and inhomogeneous isotropic elastic half-spaces, *EJGE* **19**: 7165-7185.

[10] Kumari N., 2014, Reflection and transmission of longitudinal wave at micropolar viscoelastic solid/fluid saturated incompressible porous solid interface, *Journal of Solid Mechanics* **6**(3): 240-254.

[11] Kumar R., Madan D.K., Sikka J.S., 2015, Effect of irregularity and inhomogenity on the propagation of Love waves in fluid saturated porous isotropic layer, *Journal of Applied Science and Technology* **20**: 16-21.

[12] Kakar R., 2015, Dispersion of love wave in an isotropic layer sandwiched between orthotropic and prestressed inhomogeneous half-spaces, *Latin American Journal of Solids and Structures* **12**: 1934-1949.

[13] Barak M.S., Kaliraman V., 2018, Propagation of elastic waves at micropolar viscoelastic solid/fluid saturated incompressible porous solid interface, *International Journal of Computational Methods* **15**(1): 1850076(1-19).

[14] Barak M.S., Kaliraman V., 2019, Reflection and transmission of elastic waves from an imperfect boundary between micropolar elastic solid half space and fluid saturated porous solid half space, *Mechanics of Advanced Materials and Structures* **26**: 1226-1233.

[15] Kaliraman V., Poonia R.K., 2018, Elastic wave propagation at imperfect boundary of micropolar elastic solid and fluid saturated porous solid half space, *Journal of Solid Mechanics* **10**(3): 655-671.

[16] Kumar R., Madan D.K., Sikka J.S., 2016, Effect of rigidity and inhomogenity on the propagation of love waves in an irregular fluid saturated porous isotropic layer, *International Journal of Mathematics and Computation* **27**: 55-70.

[17] Gubbins D., 1990, *Seismology and Plate Tectonics,* Cambridge University Press, Cambridge.