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Free Vibration Analysis of Multi-Layer Rectangular Plate with Two Magneto-Rheological Fluid Layers and a Flexible Core

M. Shekarzadeh [1], M.M. Najafizadeh [1,*], P. Yousefi [1], A.R. Nezamabadi [1], K. Khorshidi [2]

[1]*Department of Mechanical Engineering, Arak Branch, Islamic Azad University, Arak, Iran*
[2]*Mechanical Engineering Department, Faculty of Engineering, Arak University, Arak, Iran*

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ABSTRACT

In the present article, the free vibration analysis of a multi-layer rectangular plate with two magneto-rheological (MR) fluid layers and a flexible core is investigated based on exponential shear deformation theory for the first time. In exponential shear deformation theory, exponential functions are used in terms of thickness coordinate to include the effect of transverse shear deformation and rotary inertia. The displacement of the flexible core is modeled using Frostig's second order model which contains a polynomial with unknown coefficients. MR fluids viscosity can be varied by changing the magnetic field intensity. Therefore, they have the capability to change the stiffness and damping of a structure. The governing equations of motion have been derived using Hamilton`s principle. The Navier technique is employed to solve derived equations. To validate the accuracy of the derived equations, the results in a specific case are compared with available results in the literature, and a good agreement will be observed. Then, the effect of variation of some parameters such as magnetic field intensity, core thickness to panel thickness ratio $(\frac{h_c}{h})$ and MR layer thickness to panel thickness ratio $(\frac{h_{MR}}{h})$ on natural frequency of the sandwich panel is investigated.

Keywords: Vibration; Plate; Magneto-rheological; Flexible core; Exponential shear deformation theory.

1 INTRODUCTION

THE use of sandwich structures in marine, transport, civil construction and aerospace applications is growing rapidly because of advantageous features such as high strength to weight ratio and low maintenance cost. With the increased interest and the use of sandwich structures in many challenging situations over the years, it is therefore

*Corresponding author.Tel.:+98 863 4134025.
E-mail address: m-najafizadeh@iau-arak.ac.ir (M.M. Najafizadeh).

so important to study its behavior in various circumstances [1]. In addition, they play a key role to control the vibration of structures. Since it is important to achieve a way to control the vibration with higher efficiency and short time response, many researches notice the use of sandwich structures with smart fluids layer or flexible core or a combination of them. Smart fluids such as electrorheological (ER) and magnetorheological (MR) fluids has controllable rheology. However, the ER fluids exhibit a number of shortcomings compared to the MR fluids including low yield strength, requirement of high voltage and greater sensitivity to common impurities. On the other hand, the MR fluids are known to exhibit considerably higher dynamic yield strength and greater insensitivity to temperature variation and contaminants compared to ER fluids [2, 3]. Briefly, the excellence of MR fluids on ER fluids is that they have greater changes in their characteristics. MR fluids usually contain soluble micron sized magnetic particles in a carrier fluid. When they involve with a magnetic field, their yield stress changes with the intensity changes of the magnetic field in a few milliseconds [4, 5]. So, they alter rapidly from a liquid to nearly solid state [6]. Sandwich structures coupled with controllable magnetic field yield continually vary stiffness and damping properties, and thereby could provide enhanced vibration isolation in a wide frequency range. Based on its outstanding properties, various high performance MR fluid devices have been designed and tested [7-13]. From the review of reported researches, it is observed that the sandwich structures like sandwich beams and sandwich plates have been studied from different aspects that some of them illustrate the following. Yeh and Shih [14], analyzed the dynamic characteristics and instability of MR adaptive structures under buckling loads. Rajamohan et al. [15], derived finite-element and Ritz formulations for a sandwich beam with uniform and partial MR-fluid treatment, and demonstrated their validity through experiments conducted on a cantilever sandwich beam. It was demonstrated that the natural frequencies increase with increasing in magnetic field. Mohammadi and Sedaghati [16], investigated the nonlinear vibration behavior of sandwich shell structures with a constrained ER fluid. Yeh [17] presented the vibration of the sandwich plate with MR elastomer damping treatment. The finite element method is used to model the sandwich plate with MR elastomer core. Rajamohan et al [5], studied the dynamic properties of a sandwich beam with magnetorheological (MR) fluid as a core material between the two layers of the continuous elastic structure. Yeh [18], investigated vibration characteristics of orthotropic rectangular sandwich plate with magnetorheological (MR) elastomer core and constraining layer. Hoseinzadeh and Rezaeepazhand [19], studied the influence of external electrorheological (ER) dampers on the dynamic behavior of composite laminated plates. Eshaghi et al. [20], studied analytical and experimental free vibration characteristics of sandwich annular circular plates comprising magnetorheological (MR) fluid as the core layer. Payganeh et al. [21] studied the free vibrational behavior of sandwich panels with flexible core in the presence of magnetorheological layers. Eshaghi et al. [22], investigated free vibration of a cantilevered sandwich plate with a magnetorheological (MR) fluid layer considering different boundary conditions. Ghorbanpour and Soleymani [23], studied a size-dependent vibration analysis of a rotating MR doubly-tapered sandwich beam in supersonic airflow.

In this study, the free vibration analysis of a sandwich panel with a flexible core and MR layers is investigated. The displacement of the core is modeled using Frostig's second order model. In this model, the displacement is assumed in the form of a polynomial with unknown coefficients. The displacement of the sheets is modeled using exponential shear deformation theory. Hamilton principle is used to derive motion equations. Simple support on upper and lower sheets is considered as boundary conditions. Derived equations are solved using Navier technique. Finally, the influence of magnetic field intensity, core thickness to panel thickness ratio, and MR layer thickness to panel thickness ratio, on natural frequency of the sandwich panel are presented.

2 THEORETICAL FORMULATION

The studied sandwich panel is shown in Fig.1. It is a rectangular panel that it's length, width and thickness are named with *a, b* and *h*, respectively. It has a foam flexible core that is denoted by '*c*' index in this text. The flexible core is enclosed by two composite sheets with a MR layer between them at both top and bottom. '*t*' and '*b*' indices are used for upper and lower layers, respectively. '*1*' index is used for face composite sheets, similarly '*2*' for MR layers and '3' for the nearest layers to flexible core.

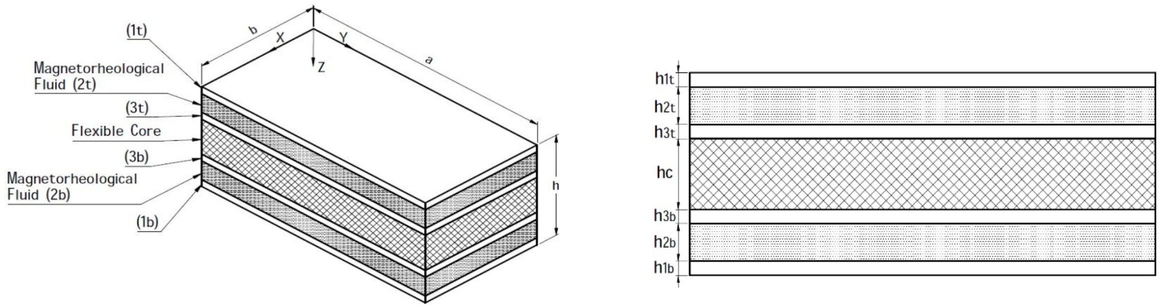

Fig.1
Schematics of the Sandwich panel with two MR fluid layers and a flexible core.

In this study, the displacement field of sheets is modeled by exponential shear deformation theory as below [24]:

$$
\begin{aligned}
u_i\left(x,y,z,t\right) &= u_0^i\left(x,y,t\right) - z\,\frac{\partial w_0^i\left(x,y,t\right)}{\partial x} + f\left(z_i\right)\varphi^i\left(x,y,t\right) \\
v_i\left(x,y,z,t\right) &= v_0^i\left(x,y,t\right) - z\,\frac{\partial w_0^i\left(x,y,t\right)}{\partial y} + f\left(z_i\right)\psi^i\left(x,y,t\right) \\
w_i\left(x,y,z,t\right) &= w_0^i\left(x,y,t\right)
\end{aligned}
\tag{1}
$$

where $i=1t,3t,1b,3b$ and $f\left(z_i\right)=z_i\left(e^{-2\left(\frac{z_i}{h_i}\right)^2}\right)$. Also, u_i, v_i and w_i are displacements in the *x, y* and *z* directions, respectively. u_0^i, v_0^i and w_0^i are the mid-plane displacements. φ^i and ψ^i are the rotation functions. According to the Frostig's second model, displacement field for the flexible core is as below [25]:

$$
\begin{aligned}
u_c(x,y,z,t) &= u_0^c(x,y,t) + z_c u_1^c(x,y,t) + z_c^2 u_2^c(x,y,t) + z_c^3 u_3^c(x,y,t) \\
v_c(x,y,z,t) &= v_0^c(x,y,t) + z_c v_1^c(x,y,t) + z_c^2 v_2^c(x,y,t) + z_c^3 v_3^c(x,y,t) \\
w_c(x,y,z,t) &= w_0^c(x,y,t) + z_c w_1^c(x,y,t) + z_c^2 w_2^c(x,y,t)
\end{aligned}
\tag{2}
$$

The kinematics (strain-displacement) equations based on Von-Karman strain assumptions are as below [26]:

$$
\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad i,j = x,y,z \tag{3}
$$

So, the strains in upper and lower sheets and in the core can be written as (4) and (5), respectively:

$$
\begin{aligned}
\varepsilon_{xx}^i &= u_{0,x}^i - z_i w_{0,xx}^i + f\left(z_i\right)\varphi_{,x}^i \\
\gamma_{xy}^i &= 2\varepsilon_{xy}^i = u_{0,y}^i + v_{0,x}^i - 2z_i w_{0,xy}^i + f\left(z_i\right)\varphi_{,y}^i + f\left(z_i\right)\psi_{,x}^i \\
\varepsilon_{yy}^i &= v_{0,y}^i - z_i w_{0,yy}^i + f\left(z_i\right)\psi_{,y}^i \\
\gamma_{xz}^i &= 2\varepsilon_{xz}^i = \frac{df\left(z_i\right)}{dz}\varphi^i \\
\gamma_{yz}^i &= 2\varepsilon_{yz}^i = \frac{df\left(z_i\right)}{dz}\psi^i \qquad\qquad i = 1t,3t,3b,1b
\end{aligned}
\tag{4}
$$

$$\begin{aligned}
\varepsilon_{xx}^{c} &= u_{0,x} + z_c u_{1,x} + z_c^2 u_{2,x} + z_c^3 u_{3,x} \\
\varepsilon_{yy}^{c} &= v_{0,y} + z_c v_{1,y} + z_c^2 v_{2,y} + z_c^3 v_{3,y} \\
\varepsilon_{zz}^{c} &= w_1 + 2 z_c w_2 \\
\gamma_{xy}^{c} &= u_{0,y} + z_c u_{1,y} + z_c^2 u_{2,y} + z_c^3 u_{3,y} + v_{0,x} + z_c v_{1,x} + z_c^2 v_{2,x} + z_c^3 v_{3,x} \\
\gamma_{xz}^{c} &= u_1 + 2 z_c u_2 + 3 z_c^2 u_3 + w_{0,x} + z_c w_{1,x} + z_c^2 w_{2,x} \\
\gamma_{yz}^{c} &= v_1 + 2 z_c v_2 + 3 z_c^2 v_3 + w_{0,y} + z_c w_{1,y} + z_c^2 w_{2,y}
\end{aligned} \tag{5}$$

In order to reduce number of unknown parameters in Eq. (2), it is assumed that sheets and core are attached to each other completely. By using this assumption, the displacement compatibility conditions at the joint surface of $3t$ and $3b$ layers with flexible core can be described as:

$$\begin{aligned}
u_{3t}\big|_{z_{3t}=-\frac{h_{3t}}{2}} &= u_c\big|_{z_c=\frac{h_c}{2}} \\
u_{3b}\big|_{z_{3b}=\frac{h_{3b}}{2}} &= u_c\big|_{z_c=-\frac{h_c}{2}} \\
v_{3t}\big|_{z_{3t}=-\frac{h_{3t}}{2}} &= v_c\big|_{z_c=\frac{h_c}{2}} \\
v_{3b}\big|_{z_{3b}=\frac{h_{3b}}{2}} &= v_c\big|_{z_c=-\frac{h_c}{2}} \\
w_{3t}\big|_{z_{3t}=-\frac{h_{3t}}{2}} &= w_c\big|_{z_c=\frac{h_c}{2}} \\
w_{3b}\big|_{z_{3b}=\frac{h_{3b}}{2}} &= w_c\big|_{z_c=-\frac{h_c}{2}}
\end{aligned} \tag{6}$$

By substituting relations (1) and (2) in relation (6), the compatibility equations can be written as:

$$\begin{aligned}
u_2 &= \frac{2\left(u_0^{3t} + u_0^{3b} - 2u_0^{c}\right) + h_{3t}\left(w_{,x}^{t} - e^{-\frac{1}{2}}\varphi^{t}\right) + h_{3b}\left(e^{-\frac{1}{2}}\varphi^{b} - w_{,x}^{b}\right)}{h_c^2} \\
u_3 &= \frac{4\left(u_0^{3t} - u_0^{3b} - h_c u_1^{c}\right) + 2h_{3t}\left(w_{,x}^{t} - e^{-\frac{1}{2}}\varphi^{t}\right) + 2h_{3b}\left(w_{,x}^{b} - e^{-\frac{1}{2}}\varphi^{b}\right)}{h_c^3} \\
v_2 &= \frac{2\left(v_0^{3t} + v_0^{3b} - 2v_0^{c}\right) + h_{3t}\left(w_{,y}^{t} - e^{-\frac{1}{2}}\psi^{t}\right) + h_{3b}\left(e^{-\frac{1}{2}}\psi^{b} - w_{,y}^{b}\right)}{h_c^2} \\
v_3 &= \frac{4\left(v_0^{3t} - v_0^{3b} - h_c v_1^{c}\right) + 2h_{3t}\left(w_{,y}^{t} - e^{-\frac{1}{2}}\psi^{t}\right) + 2h_{3b}\left(w_{,y}^{b} - e^{-\frac{1}{2}}\psi^{b}\right)}{h_c^3} \\
w_1 &= \frac{w_0^{t} - w_0^{b}}{h_c} \qquad w_2 = \frac{2\left(w_0^{t} + w_0^{b} - 2w_0^{c}\right)}{h_c^2}
\end{aligned} \tag{7}$$

The relationship between transverse strains and stresses in MR layers can be expressed as:

$$\tau_{xz}^{2i} = G^{*}\gamma_{xz}^{2i} \qquad \tau_{yz}^{2i} = G^{*}\gamma_{yz}^{2i} \qquad (i = t,b) \tag{8}$$

According to geometrical relationships between displacement components and rotation functions in layers 1 and 3, and also this assumption that a no slip condition is existed between sheet layers and MR layers, the components of strain in MR layers can be obtained as:

$$\begin{aligned} \gamma_{xz}^{2i} &= w_{,x}^{i} + \frac{1}{h_{2j}}\left(u_0^{1i} - u_0^{3i}\right) - \frac{\varphi^{i}}{2h_{2i}}\left(h_{1i} + h_{3i}\right) \\ \gamma_{yz}^{2i} &= w_{,y}^{i} + \frac{1}{h_{2i}}\left(v_0^{1i} - v_0^{3i}\right) - \frac{\psi^{i}}{2h_{2i}}\left(h_{1i} + h_{3i}\right) \qquad (i = t,b) \end{aligned} \tag{9}$$

In the pre-yield regime, the MR material demonstrates the viscoelastic behavior, which has been described in terms of the complex modulus G^{*}.

$$G^{*} = G' + iG'' \tag{10}$$

where G' is the storage modulus, and G'' is the loss modulus. They are described as[27]:

$$\begin{aligned} G' &= -3.3691B^{2} + 4.9975\times10^{3}B + 0.873\times10^{6} \\ G'' &= -0.9B^{2} + 0.8124\times10^{3}B + 0.1855\times10^{6} \end{aligned} \tag{11}$$

where B is the intensity of the magnetic field in Gauss.
In order to extract the motion equations, the Hamilton’s principle is employed as below:

$$\int_{t_1}^{t_2}(\delta U + \delta V - \delta T)dt = 0 \tag{12}$$

where δ is the variation operator, U is the strain energy, V is the work done by external forces, and T is the kinetic energy. The work done by external forces is zero in the present research, and:

$$\begin{aligned} \delta U = &\sum_{i=1t,3t,3b,1b}\left\{\int_{V_i}(\sigma_{xx}^{i}\,\delta\varepsilon_{xx}^{i} + \sigma_{yy}^{i}\,\delta\varepsilon_{yy}^{i} + \tau_{xy}^{i}\,\delta\gamma_{xy}^{i} + \tau_{XZ}^{i}\,\delta\gamma_{XZ}^{i} + \tau_{yz}^{i}\,\delta\gamma_{yz}^{i})dV_i\right\} \\ &+ \sum_{i=2t,2b}\left\{\int_{V_i}(\tau_{xz}^{i}\,\delta\gamma_{xz}^{i} + \tau_{yz}^{i}\,\delta\gamma_{yz}^{i})dV_i\right\} \\ &+ \int_{V_c}(\sigma_{zz}^{c}\,\delta\varepsilon_{zz}^{c} + \tau_{xz}^{c}\,\delta\gamma_{xz}^{c} + \tau_{yz}^{c}\,\delta\gamma_{yz}^{c} + \sigma_{xx}^{c}\,\delta\varepsilon_{xx}^{c} + \sigma_{yy}^{c}\,\delta\varepsilon_{yy}^{c} + \tau_{xy}^{c}\,\delta\gamma_{xy}^{c})dV_c \end{aligned} \tag{13}$$

$$\begin{aligned} \delta T = &-\sum_{i=1t,3t,3b,1b}\left\{\int_0^a\int_0^b \rho_i h_i\left(\dot{u}_i\,\delta\dot{u}_i + \dot{v}_i\,\delta\dot{v}_i + \dot{w}_i\,\delta\dot{w}_i\right)dxdy\right\} \\ &- \sum_{i=2t,2b}\left\{\int_0^a\int_0^b\left(\rho_j h_j \dot{w}_j\,\delta\dot{w}_j + I_{MR}\left(\dot{\gamma}_{xz}^{j}\,\delta\dot{\gamma}_{xz}^{j} + \dot{\gamma}_{yz}^{j}\,\delta\dot{\gamma}_{yz}^{j}\right)\right)dxdy\right\} \\ &- \int_0^a\int_0^b \rho_c h_c\left(\dot{u}_c\,\delta\dot{u}_c + \dot{v}_c\,\delta\dot{v}_c + \dot{w}_c\,\delta\dot{w}_c\right)dxdy \end{aligned} \tag{14}$$

In Eq. (14), ρ is the mass density, dot-over script convention index indicates the differentiation with respect to the time variable, also it is assumed that there is no normal stress in MR layers (2*t* and 2*b*). Eqs. (15) introduce the stress resultant that are used to extract the motion equations.

$$\left\{N_{xx}^{i},N_{yy}^{i},N_{xy}^{i},Q_{xz}^{i},Q_{yz}^{i}\right\}=\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}\left\{\sigma_{xx}^{i},\sigma_{yy}^{i},\tau_{xy}^{i},\tau_{xz}^{i},\tau_{yz}^{i}\right\}dz_i \qquad (i=1t,3t,1b,3b,c) \tag{15a}$$

$$\left\{M_{xx}^{i},M_{yy}^{i},M_{xy}^{i}\right\}=\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}z_i\left\{\sigma_{xx}^{i},\sigma_{yy}^{i},\tau_{xy}^{i}\right\}dz_i \qquad (i=1t,3t,1b,3b) \tag{15b}$$

$$\left\{M_{nxx}^{c},M_{nyy}^{c},M_{nxy}^{c},M_{Qnxz}^{c},M_{Qnyz}^{c}\right\}=\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}z_c^{n}\left\{\sigma_{xx}^{c},\sigma_{yy}^{c},\tau_{xy}^{c},\tau_{xz}^{c},\tau_{yz}^{c}\right\}dz_c \tag{15c}$$

$$\left\{R_{z}^{c},M_{z}^{c}\right\}=\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}(1,z_c)\sigma_{zz}^{c}dz_c \tag{15d}$$

$$\left\{\mathrm{Q}_{xz}^{i},\mathrm{Q}_{yz}^{i}\right\}=\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}\left(\tau_{xz}^{i},\tau_{yz}^{i}\right)dz_i\ (i=2t,2b) \tag{15e}$$

$$I_{n}^{i}=\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}z_i^{n}\rho_i dz_i \qquad i=c,1t,3t,1b,3b \qquad n=0,1,2,\ldots \tag{15f}$$

$$\left\{J_{0}^{i},J_{1}^{i},J_{2}^{i}\right\}=\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}\rho_i\{(f(z_i),z_i f(z_i),(f(z_i))^2\}dz_i \qquad i=1t,3t,1b,3b \tag{15g}$$

$$\left\{R_{xx}^{i},R_{yy}^{i},R_{xy}^{i}\right\}=\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}\left(\sigma_{xx}^{i},\sigma_{yy}^{i},\tau_{xy}^{i}\right)f(z_i)dz_i \qquad i=1t,3t,1b,3b \tag{15h}$$

$$\left\{P_{x}^{i},P_{y}^{i}\right\}=\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}\left(\tau_{xz}^{i},\tau_{yz}^{i}\right)\frac{df(z_i)}{dz_i}dz_i \qquad i=1t,3t,1b,3b \tag{15i}$$

By using Eq. (12) and considering Eqs. (15a)-(15i), the motion equations are extracted as:

δu_0^{1t} :

$$N^{1t}_{xx,x} + N^{1t}_{xy,y} + \frac{1}{h_{2t}} Q^{2t}_{xz} - I_0^{1t}\ddot{u}_0^{1t} + I_1^{1t}\ddot{w}^{t}_{0,x} - J_0^{1t}\ddot{\varphi}^t + A_{16} J_0^{2t}\ddot{\varphi}^t - \frac{1}{h_{2t}{}^2} I_0^{2t}\ddot{u}_0^{1t} + \frac{1}{h_{2t}{}^2} I_0^{2t}\ddot{u}_0^{3t} - \frac{1}{h_{2t}} I_0^{2t}\ddot{w}^{t}_{0,x} = 0 \tag{16a}$$

δu_0^{1b} :

$$N^{1b}_{xx,x} + N^{1b}_{xy,y} + \frac{1}{h_{2b}} Q^{2b}_{xz} - I_0^{1b}\ddot{u}_0^{1b} + I_1^{1b}\ddot{w}^{b}_{0,x} - J_0^{1b}\ddot{\varphi}^b + A_{17} I_0^{2b}\ddot{\varphi}^b - \frac{1}{h_{2b}{}^2} I_0^{2b}\ddot{u}_0^{1b} + \frac{1}{h_{2b}{}^2} I_0^{2b}\ddot{u}_0^{3b} - \frac{1}{h_{2b}} I_0^{2b}\ddot{w}^{b}_{0,x} = 0 \tag{16b}$$

δu_0^{3t} :

$$\begin{aligned} & N^{3t}_{xx,x} + N^{3t}_{xy,y} + \frac{4}{h_c{}^3} M^{c}_{3xx,x} + \frac{2}{h_c{}^2} M^{c}_{2xx,x} + \frac{2}{h_c{}^2} M^{c}_{2xy,y} + \frac{4}{h_c{}^3} M^{c}_{3xy,y} + \frac{4}{h_c{}^2} M^{c}_{Q1xz} + \frac{12}{h_c{}^3} M^{c}_{Q2xz} - \frac{1}{h_{2t}} Q^{2t}_{xz} + A_1\ddot{u}_0^{3t} \\ & + A_2\ddot{u}_0^{3b} + A_3\ddot{w}^{t}_{0,x} + A_4\ddot{w}^{b}_{0,x} - J_0^{3t}\ddot{\varphi}^t + A_5\ddot{\varphi}^b + A_6\ddot{\varphi}^t + A_7\ddot{u}_1^c + A_8\ddot{u}_0^c - A_{16} I_0^{2t}\ddot{\varphi}^t + \frac{1}{h_{2t}{}^2} I_0^{2t}\ddot{u}_0^{1t} \\ & - \frac{1}{h_{2t}{}^2} I_0^{2t}\ddot{u}_0^{3t} + \frac{1}{h_{2t}} I_0^{2t}\ddot{w}^{t}_{0,x} = 0 \end{aligned} \tag{16c}$$

δu_0^{3b} :

$$\begin{aligned} & N^{3b}_{xx,x} + N^{3b}_{xy,y} + \frac{4}{h_c{}^3} M^{c}_{3xx,x} + \frac{2}{h_c{}^2} M^{c}_{2xx,x} + \frac{2}{h_c{}^2} M^{c}_{2xy,y} - \frac{4}{h_c{}^3} M^{c}_{3xy,y} + \frac{4}{h_c{}^2} M^{c}_{Q1xz} - \frac{12}{h_c{}^3} M^{c}_{Q2xz} - \frac{1}{h_{2b}} Q^{2b}_{xz} + A_9\ddot{u}_0^{3b} \\ & + A_2\ddot{u}_0^{3b} + A_{10}\ddot{w}^{b}_{0,x} + A_{11}\ddot{w}^{t}_{0,x} - J_0^{3b}\ddot{\varphi}^b + A_{12}\ddot{\varphi}^b + A_{13}\ddot{\varphi}^t + A_{14}\ddot{u}_1^c + A_{15}\ddot{u}_0^c - A_{17} I_0^{2b}\ddot{\varphi}^b + \frac{1}{h_{2b}{}^2} I_0^{2b}\ddot{u}_0^{1b} \\ & - \frac{1}{h_{2b}{}^2} I_0^{2b}\ddot{u}_0^{3b} + \frac{1}{h_{2b}} I_0^{2b}\ddot{w}^{b}_{0,x} = 0 \end{aligned} \tag{16d}$$

δv_0^{1t} :

$$N^{1t}_{yy,y} + N^{1t}_{xy,y} + \frac{1}{h_{2t}} Q^{2t}_{yz} - I_0^{1t}\ddot{v}_0^{1t} + I_1^{1t}\ddot{w}^{t}_{0,y} - J_0^{1t}\ddot{\psi}^t + A_{16} I_0^{2t}\ddot{\psi}^t - \frac{1}{h_{2t}{}^2} I_0^{2t}\ddot{v}_0^{1t} + \frac{1}{h_{2t}{}^2} I_0^{2t}\ddot{v}_0^{3t} - \frac{1}{h_{2t}} I_0^{2t}\ddot{w}^{t}_{0,y} = 0 \tag{16e}$$

δv_0^{1b} :

$$N^{1b}_{yy,y} + N^{1b}_{xy,y} + \frac{1}{h_{2b}} Q^{2b}_{yz} - I_0^{1b}\ddot{v}_0^{1b} + I_1^{1b}\ddot{w}^{b}_{0,y} - J_0^{1b}\ddot{\psi}^b + A_{17} I_0^{2b}\ddot{\psi}^b - \frac{1}{h_{2b}{}^2} I_0^{2b}\ddot{v}_0^{1b} + \frac{1}{h_{2b}{}^2} I_0^{2b}\ddot{v}_0^{3b} - \frac{1}{h_{2b}} I_0^{2b}\ddot{w}^{b}_{0,y} = 0 \tag{16f}$$

δv_0^{3t} :

$$\begin{aligned} & N^{3t}_{yy,y} + N^{3t}_{xy,x} + \frac{4}{h_c{}^3} M^{c}_{3yy,y} + \frac{2}{h_c{}^2} M^{c}_{2yy,y} + \frac{2}{h_c{}^2} M^{c}_{2xy,x} + \frac{4}{h_c{}^3} M^{c}_{3xy,x} + \frac{4}{h_c{}^2} M^{c}_{Q1yz} + \frac{12}{h_c{}^3} M^{c}_{Q2yz} - \frac{1}{h_{2t}} Q^{2t}_{yz} + A_1\ddot{v}_0^{3t} \\ & + A_2\ddot{v}_0^{3b} + A_3\ddot{w}^{t}_{0,y} + A_4\ddot{w}^{b}_{0,y} - J_0^{3t}\ddot{\psi}^t + A_5\ddot{\psi}^b + A_6\ddot{\psi}^t + A_7\ddot{v}_1^c + A_8\ddot{v}_0^c - A_{16} I_0^{2t}\ddot{\psi}^t + \frac{1}{h_{2t}{}^2} I_0^{2t}\ddot{v}_0^{1t} - \frac{1}{h_{2t}{}^2} I_0^{2t}\ddot{v}_0^{3t} + \frac{1}{h_{2t}} I_0^{2t}\ddot{w}^{t}_{0,y} = 0 \end{aligned} \tag{16g}$$

δv_0^{3t} :

$$\begin{aligned} & N^{3b}_{yy,y} + N^{3b}_{xy,x} - \frac{4}{h_c{}^3} M^{c}_{3yy,y} + \frac{2}{h_c{}^2} M^{c}_{2yy,y} + \frac{2}{h_c{}^2} M^{c}_{2xy,x} - \frac{4}{h_c{}^3} M^{c}_{3xy,x} + \frac{4}{h_c{}^2} M^{c}_{Q1yz} - \frac{12}{h_c{}^3} M^{c}_{Q2yz} - \frac{1}{h_{2b}} Q^{2b}_{yz} + A_9\ddot{v}_0^{3b} \\ & + A_2\ddot{v}_0^{3t} + A_{11}\ddot{w}^{t}_{0,y} + A_{10}\ddot{w}^{b}_{0,y} - J_0^{3b}\ddot{\psi}^b + A_{12}\ddot{\psi}^b + A_{13}\ddot{\psi}^t + A_{14}\ddot{v}_1^c + A_{15}\ddot{v}_0^c - A_{17} I_0^{2b}\ddot{\psi}^b + \frac{1}{h_{2b}{}^2} I_0^{2t}\ddot{v}_0^{1b} - \frac{1}{h_{2b}{}^2} I_0^{2b}\ddot{v}_0^{3b} + \frac{1}{h_{2b}} I_0^{2b}\ddot{w}^{b}_{0,y} = 0 \end{aligned} \tag{16h}$$

$\delta w_0^t:$

$$
\begin{aligned}
&M^{1t}_{xx,xx}+M^{3t}_{xx,xx}+M^{1t}_{xy,xy}+M^{3t}_{xy,xy}+M^{1t}_{yy,yy}+M^{3t}_{yy,yy}-\frac{h_{3t}}{h_c^2}M^c_{2xx,xx}-\frac{2h_{3t}}{h_c^3}M^c_{3xx,xx}-\frac{h_{3t}}{h_c^2}M^c_{2yy,yy}-\frac{2h_{3t}}{h_c^3}M^c_{3yy,yy}\\
&+\frac{1}{h_c}R^c_z+\frac{4}{h_c^2}M^c_z-\frac{4h_{3t}}{h_c^3}M^c_{3xy,xy}+(\frac{2h_{3t}}{h_c^2}+\frac{1}{h_c})M^c_{Q1xz,x}+(\frac{6h_{3t}}{h_c^3}+\frac{2}{h_c^2})M^c_{Q2xz,x}+(\frac{2h_{3t}}{h_c^2}+\frac{1}{h_c})M^c_{Q1yz,y}\\
&+(\frac{6h_{3t}}{h_c^3}+\frac{2}{h_c^2})M^c_{Q2yz,y}+Q^{2t}_{xz,x}+Q^{2t}_{yz,y}+I_1^{1t}\ddot{u}^{1t}_{0,x}+A_3\ddot{u}^{3t}_{0,x}+A_{11}\ddot{u}^{3b}_{0,x}+A_{33}\ddot{w}^t_{0,xx}+A_{20}\ddot{w}^b_{0,xx}+A_{34}\ddot{w}^t_{0,yy}+A_{35}\ddot{\varphi}^t_{0,x}\\
&+A_{35}\ddot{\psi}^t_{,y}+A_{23}\ddot{\varphi}^b_{,x}+A_{20}\ddot{w}^b_{0,yy}+A_{36}\ddot{w}^t_0+I_1^{1t}\ddot{v}^{1t}_{0,y}+A_{37}\ddot{v}^{3t}_{0,y}+A_{11}\ddot{v}^{3b}_{0,y}+A_{38}\ddot{u}^c_{0,x}+A_{38}\ddot{v}^c_{0,y}+A_{39}\ddot{u}^c_{1,x}\\
&+A_{23}\ddot{\psi}^b_{,y}+A_{39}\ddot{v}^c_{1,y}+A_{30}\ddot{w}^b_0+A_{29}\ddot{w}^t_0+A_{40}\ddot{w}^c_0-I_0^{2t}\ddot{w}^t_0+A_{41}I_0^{2t}\ddot{\varphi}^t_{,x}-\frac{1}{h_{2t}}I_0^{2t}\ddot{u}^{1t}_{0,x}+\frac{1}{h_{2t}}I_0^{2t}\ddot{u}^{3t}_{0,x}-I_0^{2t}\ddot{w}^t_{0,xx}\\
&-I_0^{2t}\ddot{w}^t_{0,yy}+A_{41}I_0^{2t}\ddot{\psi}^t_{,y}-\frac{1}{h_{2t}}I_0^{2t}\ddot{v}^{1t}_{0,y}+\frac{1}{h_{2t}}I_0^{2t}\ddot{v}^{3t}_{0,x}=0
\end{aligned}
\tag{16i}
$$

$\delta w_0^b:$

$$
\begin{aligned}
&M^{1b}_{xx,xx}+M^{3b}_{xx,xx}+M^{1b}_{xy,xy}+M^{3b}_{xy,xy}+M^{1b}_{yy,yy}+M^{3b}_{yy,yy}-\frac{h_{3b}}{h_c^2}M^c_{2xx,xx}-\frac{2h_{3b}}{h_c^3}M^c_{3xx,xx}+\frac{h_{3b}}{h_c^2}M^c_{2yy,yy}-\frac{2h_{3b}}{h_c^3}M^c_{3yy,yy}\\
&-\frac{1}{h_c}R^c_z+\frac{4}{h_c^2}M^c_z-\frac{4h_{3b}}{h_c^3}M^c_{3xy,xy}-(\frac{2h_{3b}}{h_c^2}+\frac{1}{h_c})M^c_{Q1xz,x}+(\frac{6h_{3b}}{h_c^3}+\frac{2}{h_c^2})M^c_{Q2xz,x}-(\frac{2h_{3b}}{h_c^2}+\frac{1}{h_c})M^c_{Q1yz,y}\\
&+(\frac{6h_{3b}}{h_c^3}+\frac{2}{h_c^2})M^c_{Q2yz,y}+Q^{2b}_{xz,x}+Q^{2b}_{yz,y}+I_1^{1b}\ddot{u}^{1b}_{0,x}+A_{10}\ddot{u}^{3b}_{0,x}+A_4\ddot{u}^{3t}_{0,x}+A_{18}\ddot{w}^b_{0,xx}+A_{19}\ddot{w}^t_{0,xx}+A_{20}\ddot{w}^t_{0,yy}+A_{21}\ddot{\varphi}^b_{0,x}\\
&+A_{22}\ddot{\psi}^b_{,y}+A_{25}\ddot{\varphi}^t_{,x}+I_1^{1b}\ddot{v}^{1b}_{0,y}+A_{10}\ddot{v}^{3b}_{0,y}+A_{26}\ddot{w}^b_0+A_{27}\ddot{u}^c_{1,x}+A_{28}\ddot{u}^c_{0,x}+A_{28}\ddot{v}^c_{0,y}+A_{27}\ddot{v}^c_{1,y}+A_{29}\ddot{w}^b_0+A_{30}\ddot{w}^t_0+A_{31}\ddot{w}^c_0\\
&-I_0^{2b}\ddot{w}^b_0+A_{32}I_0^{2b}\ddot{\varphi}^b_{,x}-\frac{1}{h_{2b}}I_0^{2b}\ddot{u}^{1b}_{0,x}+\frac{1}{h_{2b}}I_0^{2b}\ddot{u}^{3b}_{0,x}-I_0^{2b}\ddot{w}^b_{0,xx}-I_0^{2b}\ddot{w}^b_{0,yy}+A_{32}I_0^{2b}\ddot{\psi}^b_{,y}-\frac{1}{h_{2b}}I_0^{2b}\ddot{v}^{1b}_{0,y}+\frac{1}{h_{2b}}I_0^{2b}\ddot{v}^{3b}_{0,x}+A_4\ddot{v}^{3t}_{0,y}=0
\end{aligned}
\tag{16j}
$$

$\delta\varphi^t:$

$$
\begin{aligned}
&R^{1t}_{xx,x}+R^{3t}_{xx,x}+R^{1t}_{xy,x}+R^{3t}_{xy,x}+P^{1t}_x+P^{3t}_x-\frac{2h_{3t}}{h_c^3}e^{-\frac{1}{2}}M^c_{3xx,x}-\frac{h_{3t}}{h_c^2}e^{-\frac{1}{2}}M^c_{2xx,x}-\frac{h_{3t}}{h_c^2}e^{-\frac{1}{2}}M^c_{2xy,y}-\frac{2h_{3t}}{h_c^3}e^{-\frac{1}{2}}M^c_{3xy,y}\\
&-\frac{2h_{3t}}{h_c^2}e^{-\frac{1}{2}}M^c_{Q1xz}-\frac{6h_{3t}}{h_c^3}e^{-\frac{1}{2}}M^c_{Q2xz}-A_{41}Q^{2t}_{xz}-J_0^{1t}\ddot{u}^{1t}_0+A_{35}\ddot{w}^t_{0,x}+A_{43}\ddot{\varphi}^t+A_{42}\ddot{w}^b_{0,x}-A_{39}e^{-\frac{1}{2}}\ddot{u}^c_1+A_{13}\ddot{u}^{3b}_0\\
&+A_{44}\ddot{u}^{3t}_0+A_{45}\ddot{u}^c_0+A_{46}\ddot{\varphi}^b+A_{41}I_0^{2t}\ddot{w}^t_{0,x}+A_{16}I_0^{2t}\ddot{u}^{1t}_0-A_{16}I_0^{2t}\ddot{u}^{3t}_0-A_{47}I_0^{2t}\ddot{\varphi}^t=0
\end{aligned}
\tag{16k}
$$

$\delta\varphi^b:$

$$
\begin{aligned}
&R^{1b}_{xx,x}+R^{3b}_{xx,x}+R^{1b}_{xy,y}+R^{3b}_{xy,y}+P^{1b}_x+P^{3b}_x-\frac{2h_{3b}}{h_c^3}e^{-\frac{1}{2}}M^c_{3xx,x}-\frac{h_{3b}}{h_c^2}e^{-\frac{1}{2}}M^c_{2xx,x}-\frac{h_{3b}}{h_c^2}e^{-\frac{1}{2}}M^c_{2xy,y}-\frac{2h_{3b}}{h_c^3}e^{-\frac{1}{2}}M^c_{3xy,y}\\
&+\frac{2h_{3b}}{h_c^2}e^{-\frac{1}{2}}M^c_{Q1xz}-\frac{6h_{3b}}{h_c^3}e^{-\frac{1}{2}}M^c_{Q2xz}-A_{41}Q^{2b}_{xz}-J_0^{1b}\ddot{u}^{1b}_0+A_{21}\ddot{w}^b_{0,x}+A_{48}\ddot{\varphi}^b+A_{25}\ddot{w}^t_{0,x}-A_{27}e^{-\frac{1}{2}}\ddot{u}^c_1+A_{12}\ddot{u}^{3b}_0\\
&+A_5\ddot{u}^{3t}_0+A_{46}\ddot{\varphi}^t+A_{45}\ddot{u}^c_0+A_{32}I_0^{2b}\ddot{w}^b_{0,x}+A_{17}I_0^{2b}\ddot{u}^{1b}_0-A_{17}I_0^{2b}\ddot{u}^{3b}_0-A_{49}I_0^{2b}\ddot{\varphi}^b=0
\end{aligned}
\tag{16l}
$$

$\delta\psi^t:$

$$
\begin{aligned}
&R^{1t}_{yy,y}+R^{3t}_{yy,y}+R^{1t}_{xy,x}+R^{3t}_{xy,x}+P^{1t}_y+P^{3t}_y-\frac{2h_{3t}}{h_c^3}e^{-\frac{1}{2}}M^c_{3yy,y}-\frac{h_{3t}}{h_c^2}e^{-\frac{1}{2}}M^c_{2yy,y}-\frac{h_{3t}}{h_c^2}e^{-\frac{1}{2}}M^c_{2xy,x}-\frac{2h_{3t}}{h_c^3}e^{-\frac{1}{2}}M^c_{3xy,x}\\
&-\frac{2h_{3t}}{h_c^2}e^{-\frac{1}{2}}M^c_{Q1yz}-\frac{6h_{3t}}{h_c^3}e^{-\frac{1}{2}}M^c_{Q2yz}-A_{41}Q^{2t}_{yz}-J_0^{1t}\ddot{v}^{1t}_0+A_{35}\ddot{w}^t_{0,y}+A_{43}\ddot{\psi}^t+A_{25}\ddot{w}^b_{0,y}+A_{51}e^{-\frac{1}{2}}\ddot{v}^c_1+A_{50}\ddot{v}^{3t}_0\\
&+A_{13}\ddot{v}^{3b}_0+A_{45}\ddot{v}^c_0+A_{46}\psi^b+A_{41}I_0^{2t}\ddot{w}^t_{0,y}+A_{16}I_0^{2t}\ddot{v}^{1t}_0-A_{16}I_0^{2t}\ddot{v}^{3t}_0-A_{47}I_0^{2t}\ddot{\psi}^t=0
\end{aligned}
\tag{16m}
$$

$\delta\psi^{b}$:

$$R^{1b}_{yy,y}+R^{3b}_{yy,y}+R^{1b}_{xy,x}+R^{3b}_{xy,x}+P^{1b}_{y}+P^{3b}_{y}-\frac{2h_{3b}}{h_c^{3}}e^{-\frac{1}{2}}M^{c}_{3yy,y}-\frac{h_{3b}}{h_c^{2}}e^{-\frac{1}{2}}M^{c}_{2yy,y}-\frac{h_{3t}}{h_c^{2}}e^{-\frac{1}{2}}M^{c}_{2xy,x}-\frac{2h_{3b}}{h_c^{3}}e^{-\frac{1}{2}}M^{c}_{3xy,x}$$
$$-\frac{2h_{3b}}{h_c^{2}}e^{-\frac{1}{2}}M^{c}_{Q1yz}-\frac{6h_{3b}}{h_c^{3}}e^{-\frac{1}{2}}M^{c}_{Q2yz}-A_{32}Q^{2b}_{yz}-J^{1b}_{0}\ddot{v}^{1b}_{0}+A_{21}\ddot{w}^{b}_{0,y}+A_{25}\ddot{\psi}^{b}+A_{25}\ddot{w}^{b}_{0,y}+A_{27}e^{-\frac{1}{2}}\ddot{v}^{c}_{1}+A_{3}\ddot{v}^{3t}_{0} \tag{16n}$$
$$+A_{12}\ddot{v}^{3b}_{0}+A_{45}\ddot{v}^{c}_{0}+A_{52}\ddot{\psi}^{t}+A_{41}I^{2b}_{0}\ddot{w}^{b}_{0,y}+A_{17}I^{2b}_{0}\ddot{v}^{1b}_{0}-A_{17}I^{2b}_{0}\ddot{v}^{3b}_{0}-A_{49}I^{2b}_{0}\ddot{\psi}^{b}=0$$

δu^{c}_{0} :

$$N^{c}_{xx,x}-\frac{4}{h_c^{2}}M^{c}_{2xx,x}+N^{c}_{xy,y}-\frac{4}{h_c^{2}}M^{c}_{2xy,y}-\frac{8}{h_c^{2}}M^{c}_{Q1xz}-I^{c}_{0}\ddot{u}^{c}_{0}-I^{c}_{1}\ddot{u}^{c}_{1}-A_{28}e^{-\frac{1}{2}}\ddot{\varphi}^{b}-A_{38}e^{-\frac{1}{2}}\ddot{\varphi}^{t}+A_{35}\ddot{u}^{c}_{0}+A_{54}\ddot{u}^{c}_{1} \tag{16o}$$
$$+A_{15}\ddot{u}^{3b}_{0}+A_{55}\ddot{u}^{3t}_{0}+A_{28}\ddot{w}^{b}_{0,x}+A_{38}\ddot{w}^{t}_{0,x}=0$$

δu^{c}_{1} :

$$M^{c}_{1xx,x}+M^{c}_{1xy,y}-\frac{4}{h_c^{2}}M^{c}_{3xx,x}+N^{c}_{xy,x}-\frac{4}{h_c^{2}}M^{c}_{3xy,y}+Q^{c}_{xz}-\frac{12}{h_c^{2}}M^{c}_{Q2xz}-I^{c}_{1}\ddot{u}^{c}_{0}-I^{c}_{2}\ddot{u}^{c}_{1}-A_{27}e^{-\frac{1}{2}}\ddot{\varphi}^{b}-A_{39}e^{-\frac{1}{2}}\ddot{\varphi}^{t} \tag{16p}$$
$$+A_{54}\ddot{u}^{c}_{0}+A_{56}\ddot{u}^{c}_{1}+A_{58}\ddot{u}^{3b}_{0}+A_{7}\ddot{u}^{3t}_{0}+A_{27}\ddot{w}^{b}_{,x}+A_{39}\ddot{w}^{t}_{,x}=0$$

δv^{c}_{0} :

$$N^{c}_{yy,y}-\frac{4}{h_c^{2}}M^{c}_{2yy,y}-\frac{4}{h_c^{2}}M^{c}_{2xy,y}-\frac{8}{h_c^{2}}M^{c}_{Q1yz}-I^{c}_{0}\ddot{v}^{c}_{0}-I^{c}_{1}\ddot{v}^{c}_{1}-A_{28}e^{-\frac{1}{2}}\ddot{\psi}^{b}-A_{38}e^{-\frac{1}{2}}\ddot{\psi}^{t}+A_{53}\ddot{v}^{c}_{0}+A_{54}\ddot{v}^{c}_{1} \tag{16q}$$
$$+A_{57}\ddot{v}^{3b}_{0}+A_{8}\ddot{v}^{3t}_{0}+A_{28}\ddot{w}^{b}_{0,y}+A_{38}\ddot{w}^{t}_{0,y}=0$$

δv^{c}_{1} :

$$M^{c}_{1yy,y}+M^{c}_{1xy,x}-\frac{4}{h_c^{2}}M^{c}_{3yy,y}-\frac{4}{h_c^{2}}M^{c}_{3xy,x}+Q^{c}_{yz}-\frac{12}{h_c^{2}}M^{c}_{Q2yz}-I^{c}_{1}\ddot{v}^{c}_{0}-I^{c}_{2}\ddot{v}^{c}_{1}-A_{27}e^{-\frac{1}{2}}\ddot{\psi}^{b}-A_{39}e^{-\frac{1}{2}}\ddot{\psi}^{t} \tag{16r}$$
$$+A_{54}\ddot{v}^{c}_{0}+A_{56}\ddot{v}^{c}_{1}+A_{57}\ddot{v}^{3b}_{0}+A_{8}\ddot{v}^{3t}_{0}+A_{27}\ddot{w}^{b}_{,y}+A_{39}\ddot{w}^{t}_{,y}=0$$

δw^{c}_{0} :

$$Q^{c}_{xz}-\frac{8}{h_c^{2}}M^{c}_{z}-\frac{4}{h_c^{2}}M^{c}_{Q2xz,x}+Q^{c}_{yz,y}-\frac{4}{h_c^{2}}M^{c}_{Q2yz,y}+A_{31}\ddot{w}^{b}_{0}+A_{59}\ddot{w}^{t}_{0}+A_{60}\ddot{w}^{c}_{0}=0 \tag{16s}$$

In order to compress the motion equations, the coefficients A_1 to A_{60} are considered and presented in Appendix A.

3 ANALAYSIS

Considering simply supported boundary condition for all edges, the Navier technique is employed to solve derived equations as below [24]:

$$\left\{u^{ij}_{0},u^{c}_{0},u^{c}_{1}\right\}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{u^{ij}_{mn},u^{c}_{0mn},u^{c}_{1mn}\right\}\cos\alpha x\ \sin\beta y\ e^{J\omega t}$$
$$\left\{v^{ij}_{0},v^{c}_{0},v^{c}_{1}\right\}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{v^{ij}_{mn},v^{c}_{0mn},v^{c}_{1mn}\right\}\sin\alpha x\ \cos\beta y\ e^{J\omega t} \tag{17}$$

$$\left\{w_0^j, w_0^c\right\} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{w_{mn}^j, w_{mn}^c\right\}\sin\alpha x\ \sin\beta y\ e^{J\omega t}$$
$$\left\{\varphi^j\right\} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\{\varphi_{mn}^j\}\cos\alpha x\sin\beta y e^{J\omega t} \tag{17}$$
$$\left\{\psi^j\right\} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\{\psi_{mn}^j\}\sin\alpha x\cos\beta y e^{J\omega t} \qquad (i=1,3\ j=t,b)\ \left(\alpha=\frac{n\pi}{a}\quad \beta=\frac{m\pi}{b}\right)$$

By substituting (14) in presented equations as Appendix A, simplified equation of motion can be provided as:

$$\left([K]-\omega^2[M]\right)\{\Delta\}=\{0\} \tag{18}$$

where

$$\Delta = \{u_{mn}^{1t}, u_{mn}^{1b}, u_{mn}^{3t}, u_{mn}^{3b}, v_{mn}^{1t}, v_{mn}^{1b}, v_{mn}^{3t}, v_{mn}^{3b}, w_{mn}^{t}, w_{mn}^{b}, \varphi_{mn}^{t}, \varphi_{mn}^{b}, \psi_{mn}^{t}, \psi_{mn}^{b},$$
$$u_{0mn}^{c}, u_{1mn}^{c}, v_{0mn}^{c}, v_{1mn}^{c}, w_{0mn}^{c}\}^T$$

which ω, *K, M* and Δ are stiffness matrix, mass matrix, frequency and constant vector of mode shape of the panel, respectively. To solve the Eq. (18), the determinant of $\left([K]-\omega^2[M]\right)$ must be zero. After solving the equations system, the minimum value of ω must be selected.

4 RESULTS AND DISCUSSION

In order to verify the accuracy of the present approach, the obtained natural frequency of the panel in the present study is compared with [21] . For this purpose, mechanical and geometric properties of the panel are considered as Table 1 and Table 2.

Table 1
Mechanical properties of the sandwich panel.

Property	Face Sheets	Flexible Core	MR Layers
$E_1(Gpa)$	24.51	0.10363	-
$E_2(Gpa)$	7.77	0.10363	-
$E_2(Gpa)$	7.77	0.10363	-
$G_{12}(Gpa)$	3.34	0.05	Equation (10)
$G_{12}(Gpa)$	3.34	0.05	Equation (10)
$G_{12}(Gpa)$	1.34	0.05	Equation (10)
ϑ_{12}	0.078	0.036	-
ϑ_{12}	0.078	0.036	-
ϑ_{12}	0.49	0.036	-
$\rho\left(kg/m^2\right)$	1800	130	3500

Table 2
Geometric properties of the sandwich panel.

$h_{2t}(mm)$	$h_{2b}(mm)$	$h_{2t}(mm)$	$h_{2b}(mm)$	$h_{2t}(mm)$	$h_{2b}(mm)$	h_c/h	h/a
1	1	1	1	2	2	0.88	0.1

The magnetic field intensity and the arrangement of layers are considered as B=150 Gauss and (0/90/MR/0/core/0/MR/90/0) respectively. In order to compare results of this method with [21], four dimensionless frequencies are derived based on:

$$\bar{\omega} = \frac{\omega a^2}{h}\sqrt{\frac{\rho_C}{E_C}} \tag{19}$$

The results are shown in Table 3 and they are close to the results of [21]. Difference between the results of present study and [21] is because the transverse shear stresses at top and bottom surfaces of the plate are zero in exponential shear deformation theory while in first order shear deformation theory, the correction factor is required due to the lack of control of shear stresses at top and bottom surfaces of the plate. Therefore, it can be concluded that, the exponential shear deformation theory can simulate the vibrational behavior of the plate better than the first-order shear deformation theory.

Table 3
Comparison of dimensionless frequencies $\bar{\omega}$ between present study and [21].

Mode Number	Present	[21]
(1,1)	18.75	20.54
(1,2)	12.69	13.72
(2,1)	32.55	33.22
(2,2)	20.51	23.31

To investigate the effect of magnetic field intensity on natural frequency, a plate with presented properties in Tables 1 and 2, is considered. The result of this investigation in four modes of vibration is shown in Fig.2. It is observed by applying the magnetic field, the natural frequency increases in each mode. Increasing the magnetic field intensity increases the shear modulus of the MR fluid, which leads to an increase in the stiffness of the structure. So, the natural frequency of the system increases.

Fig.3 depicts the effect of core thickness to overall thickness ratio h_C / h , on natural frequency in four modes. According Fig.3, the natural frequency decreases with a gentle slope when the core thickness to overall thickness ratio increases. Increasing h_C / h , increases the mass of the structure, which leads to a decrease in the natural frequency of the structure.

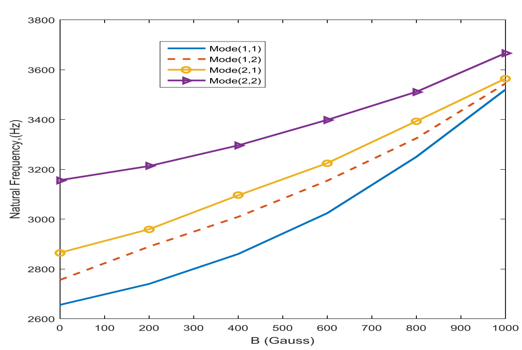

Fig.2
Effect of magnetic field intensity on natural frequency.

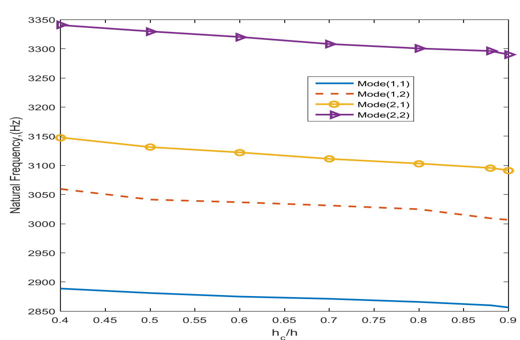

Fig.3
Effect of core thickness to overall thickness ratio on natural frequency.

The effect of MR layer thickness to overall thickness ratio h_{MR}/h , on natural frequency in four modes is depicted in Fig.4. Based on Fig.4, the natural frequency decreases by increasing of MR layer thickness to overall thickness ratio, because increasing h_{MR}/h , increases the mass of the structure, which leads to a decrease in the natural frequency of the structure.

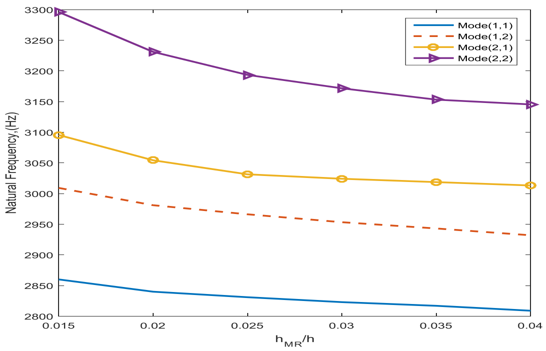

Fig.4
Effect of MR layer thickness to overall thickness ratio on natural frequency.

5 CONCLUSION

The free vibration analysis of a multi-layer rectangular plate with both magnetorheological (MR) fluid layers and a flexible core was presented. The applied theories for displacement fields in sheet layers and flexible core layer were exponential shear deformation theory and Frostig's second model, respectively. Hamilton`s principle was employed to reach the equations of motion, and the Navier technique was applied to solve them. The presented graphs had described the variation of the natural frequency by the variation of the magnetic field intensity, core thickness to panel thickness ratio, and MR layer thickness to panel thickness ratio.

APPENDIX A

The used coefficients in motion equations:

$$A_1=-I_o^{3t}-\frac{16I_6^c}{h_c^6}-\frac{16I_5^c}{h_c^5}-\frac{4I_4^c}{h_c^4}\ ,\ A_2=\frac{16I_6^c}{h_c^6}-\frac{4I_4^c}{h_c^4}\ ,\ A_3=I_1^{3t}-\frac{8h_{3t}I_6^c}{h_c^6}-\frac{8h_{3t}I_5^c}{h_c^5}-\frac{2h_{3t}I_4^c}{h_c^4}\ ,$$

$$A_4=-\frac{8h_{3b}I_6^c}{h_c^6}+\frac{2h_{3b}I_4^c}{h_c^4}\ ,\ A_5=\frac{8h_{3b}I_6^c}{h_c^6}e^{-\frac{1}{2}}-\frac{2h_{3b}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,\ A_6=\frac{8h_{3t}I_6^c}{h_c^6}e^{-\frac{1}{2}}+\frac{8h_{3t}I_5^c}{h_c^5}e^{-\frac{1}{2}}+\frac{2h_{3t}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,$$

$$A_7=\frac{16I_6^c}{h_c^5}+\frac{8I_5^c}{h_c^4}-\frac{4I_4^c}{h_c^3}-\frac{2I_3^c}{h_c^2}\ ,\ A_8=\frac{16I_5^c}{h_c^5}+\frac{8I_4^c}{h_c^4}-\frac{4I_3^c}{h_c^3}-\frac{2I_2^c}{h_c^2}\ ,\ A_9=-I_o^{3b}-\frac{16I_6^c}{h_c^6}+\frac{16I_5^c}{h_c^5}-\frac{4I_4^c}{h_c^4}\ ,$$

$$A_{10}=I_1^{3b}+\frac{8h_{3b}I_6^c}{h_c^6}-\frac{8h_{3b}I_5^c}{h_c^5}+\frac{2h_{3b}I_4^c}{h_c^4}\ ,\ A_{11}=\frac{8h_{3t}I_6^c}{h_c^6}-\frac{2h_{3t}I_4^c}{h_c^4}\ ,$$

$$A_{12}=-\frac{8h_{3b}I_6^c}{h_c^6}e^{-\frac{1}{2}}+\frac{8h_{3b}I_5^c}{h_c^5}e^{-\frac{1}{2}}-\frac{2h_{3b}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,\ A_{13}=-\frac{8h_{3t}I_6^c}{h_c^6}e^{-\frac{1}{2}}+\frac{2h_{3t}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,$$

$$A_{14}=-\frac{16I_6^c}{h_c^5}+\frac{8I_5^c}{h_c^4}+\frac{4I_4^c}{h_c^3}-\frac{2I_3^c}{h_c^2}\ ,\ A_{15}=-\frac{16I_5^c}{h_c^5}+\frac{8I_4^c}{h_c^4}+\frac{4I_3^c}{h_c^3}-\frac{2I_2^c}{h_c^2}\ ,\ A_{16}=\frac{h_{1t}+h_{3t}}{2h_{2t}^2}\ ,\ A_{17}=\frac{h_{1b}+h_{3b}}{2h_{2b}^2}\ ,$$

$$A_{18}=-I_2^{1b}-I_2^{3b}-\frac{4h_{3b}^2 I_6^c}{h_c^6}+\frac{4h_{3b}^2 I_5^c}{h_c^5}-\frac{h_{3b}^2 I_4^c}{h_c^4}\ ,\ A_{19}=\frac{4h_{3t}h_{3b}I_6^c}{h_c^6}e^{-\frac{1}{2}}+\frac{h_{3t}h_{3b}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,$$

$$A_{20}=-\frac{4h_{3t}h_{3b}I_6^c}{h_c^6}+\frac{h_{3t}h_{3b}I_4^c}{h_c^4}\ ,\ A_{21}=\left(J_1^{1b}+J_1^{3b}+\frac{4h_{3b}^2 I_6^c}{h_c^6}e^{-\frac{1}{2}}-\frac{4h_{3b}^2 I_5^c}{h_c^5}e^{-\frac{1}{2}}+\frac{h_{3b}^2 I_4^c}{h_c^4}e^{-\frac{1}{2}}\right),$$

$$A_{22}=J_1^{1b}+J_1^{3b}+\frac{4h_{3b}^2 I_6^c}{h_c^6}e^{-\frac{1}{2}}\ ,\ A_{23}=\frac{4h_{3t}h_{3b}I_6^c}{h_c^6}e^{-\frac{1}{2}}-\frac{h_{3t}h_{3b}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,$$

$$A_{24}=-\frac{4h_{3b}^2 I_5^c}{h_c^5}e^{-\frac{1}{2}}+\frac{h_{3b}^2 I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,\ A_{25}=\frac{4h_{3t}h_{3b}I_6^c}{h_c^6}e^{-\frac{1}{2}}-\frac{h_{3t}h_{3b}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,\ A_{26}=-I_0^{1b}-I_0^{3b}\ ,$$

$$A_{27}=\frac{8h_{3b}I_6^c}{h_c^5}-\frac{4h_{3b}I_5^c}{h_c^4}-\frac{2h_{3b}I_4^c}{h_c^3}+\frac{h_{3b}I_3^c}{h_c^2}\ ,\ A_{28}=\left(\frac{8h_{3b}I_5^c}{h_c^5}-\frac{4h_{3b}I_4^c}{h_c^4}-\frac{2h_{3b}I_3^c}{h_c^3}+\frac{h_{3b}I_2^c}{h_c^2}\right),$$

$$A_{29}=-\frac{4I_4^c}{h_c^4}+\frac{4I_3^c}{h_c^3}-\frac{I_2^c}{h_c^2}\ ,\ A_{30}=-\frac{4I_4^c}{h_c^4}+\frac{I_2^c}{h_c^2}\ ,\ A_{31}=\frac{8I_4^c}{h_c^4}-\frac{4I_3^c}{h_c^3}-\frac{2I_2^c}{h_c^2}+\frac{I_1^c}{h_c}\ ,\ A_{32}=\frac{h_{1b}+h_{3b}}{2h_{2b}}\ ,$$

$$A_{33}=-I_2^{1t}-I_2^{3t}-\frac{4h_{3t}^2 I_6^c}{h_c^6}-\frac{4h_{3t}^2 I_5^c}{h_c^5}+\frac{h_{3t}h_{3b}I_4^c}{h_c^4}-\frac{h_{3t}^2 I_4^c}{h_c^4}\ ,\ A_{34}=-I_2^{1t}-I_2^{3t}-\frac{4h_{3t}^2 I_6^c}{h_c^6}-\frac{4h_{3t}^2 I_5^c}{h_c^5}-\frac{h_{3t}^2 I_4^c}{h_c^4},$$

$$A_{35}=J_1^{1t}+J_1^{3t}+\frac{4h_{3t}^2 I_6^c}{h_c^6}e^{-\frac{1}{2}}+\frac{4h_{3t}^2 I_5^c}{h_c^5}e^{-\frac{1}{2}}+\frac{h_{3t}^2 I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,\ A_{36}=-I_0^{1t}-I_0^{3t}\ ,$$

$$A_{37}=\left(I_1^{3t}-\frac{8h_{3t}I_6^c}{h_c^6}-\frac{8h_{3t}I_5^c}{h_c^5}-\frac{2h_{3t}I_4^c}{h_c^4}\right),\ A_{38}=\frac{8h_{3t}I_5^c}{h_c^5}+\frac{4h_{3t}I_4^c}{h_c^4}-\frac{2h_{3t}I_3^c}{h_c^3}-\frac{h_{3t}I_2^c}{h_c^2}\ ,$$

$$A_{39}=\frac{8h_{3t}I_6^c}{h_c^5}+\frac{4h_{3t}I_5^c}{h_c^4}-\frac{2h_{3t}I_4^c}{h_c^3}-\frac{h_{3t}I_3^c}{h_c^2}\ ,\ A_{40}=\frac{8I_4^c}{h_c^4}+\frac{4I_3^c}{h_c^3}-\frac{2I_2^c}{h_c^2}-\frac{I_1^c}{h_c}\ ,\ A_{41}=\frac{h_{1t}+h_{3t}}{2h_{2t}}\ ,$$

$$A_{42}=-\frac{h_{3t}h_{3b}I_4^c}{h_c^4}e^{-\frac{1}{2}}+\frac{4h_{3b}h_{3t}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,\ A_{43}=-J_2^{1t}-J_2^{3t}-\frac{4h_{3t}^2 I_6^c}{h_c^6}e^{-1}-\frac{4h_{3t}^2 I_5^c}{h_c^5}e^{-1}-\frac{h_{3t}^2 I_4^c}{h_c^4}e^{-1}\ ,$$

$$A_{44}=-J_0^{3t}+\frac{8h_{3t}I_6^c}{h_c^6}e^{-\frac{1}{2}}+\frac{8h_{3t}I_5^c}{h_c^5}e^{-\frac{1}{2}}+\frac{2h_{3t}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,\ A_{45}=-\frac{4h_{3t}I_4^c}{h_c^4}e^{-\frac{1}{2}}-\frac{8h_{3t}I_5^c}{h_c^5}e^{-\frac{1}{2}}+\frac{2h_{3t}I_3^c}{h_c^3}e^{-\frac{1}{2}}\ ,$$

$$A_{46}=-\frac{4h_{3t}h_{3b}I_6^c}{h_c^6}e^{-1}+\frac{4h_{3b}h_{3t}I_4^c}{h_c^4}e^{-1}\ ,\ A_{47}=\frac{(h_{1t}+h_{3t})^2}{4h_{2t}^2}\ ,$$

$$A_{48}=-J_2^{1b}-J_2^{3b}-\frac{4h_{3b}^2 I_6^c}{h_c^6}e^{-1}+\frac{4h_{3b}^2 I_5^c}{h_c^5}e^{-1}+\frac{h_{3b}^2 I_4^c}{h_c^4}e^{-1}\ ,\ A_{49}=\frac{(h_{1b}+h_{3b})^2}{4h_{2b}^2}\ ,$$

$$A_{50}=-J_0^{3t}+\frac{8h_{3t}I_6^c}{h_c^6}e^{-\frac{1}{2}}+\frac{8h_{3t}I_5^c}{h_c^5}e^{-\frac{1}{2}}+\frac{2h_{3t}I_4^c}{h_c^4}e^{-\frac{1}{2}}\ ,$$

$$A_{51}=-\frac{8h_{3t}I_6^c}{h_c^5}e^{-\frac{1}{2}}-\frac{4h_{3t}I_5^c}{h_c^4}e^{-\frac{1}{2}}+\frac{2h_{3t}I_4^c}{h_c^3}e^{-\frac{1}{2}}+\frac{h_{3t}I_3^c}{h_c^2}e^{-\frac{1}{2}}\ ,\ A_{52}=-\frac{4h_{3t}h_{3b}I_6^c}{h_c^6}e^{-1}+\frac{h_{3t}h_{3b}I_4^c}{h_c^4}e^{-1}\ ,$$

$$A_{53}=-\frac{16I_4^c}{h_c^4}+\frac{8I_2^c}{h_c^2}\ ,\ A_{54}=-\frac{16I_5^c}{h_c^4}+\frac{8I_3^c}{h_c^2}\ ,\ A_{55}=\frac{16I_5^c}{h_c^5}+\frac{8I_4^c}{h_c^4}-\frac{4I_3^c}{h_c^3}-\frac{2I_2^c}{h_c^2}\ ,\ A_{56}=-\frac{16I_6^c}{h_c^4}+\frac{8I_4^c}{h_c^2}\ ,$$

$$A_{57}=-\frac{16I_6^c}{h_c^5}+\frac{8I_5^c}{h_c^4}+\frac{4I_4^c}{h_c^3}-\frac{2I_3^c}{h_c^2}\ ,\ A_{58}=\frac{16I_6^c}{h_c^5}+\frac{8I_5^c}{h_c^4}+\frac{4I_4^c}{h_c^3}-\frac{2I_3^c}{h_c^2}\ ,\ A_{59}=\frac{8I_4^c}{h_c^4}+\frac{4I_3^c}{h_c^3}-\frac{2I_2^c}{h_c^2}-\frac{I_1^c}{h_c}\ ,$$

$$A_{60}=-I_0^c-\frac{16I_4^c}{h_c^4}+\frac{8I_2^c}{h_c^2}$$

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