Comparison between Recursive Least Squares and Optimal Design Methods for Audio Enhancement

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ABSTRACT:
This study examines and compares the application of the optimal design method and the recursive least squares (RLS) method to improve the quality of the audio signal in noisy environments. Noise can be incorporated into audio signals through many sources, including amplification systems and electronic switches, which cause loss of signal information or affect the quality of the audio signal. RLS is an adaptive filtering procedure used to design a system that recursively minimizes the noise amplitude of a contaminated signal by comparing the filter output with a desired signal using new incoming signal samples. The optimal design is an FIR filter design technique that has been used to cut parts of the corrupted signal to improve the signal-to-noise ratio. In this study, samples of audio signals contaminated by white noise were used. The noise reduction results show that the RLS approach has vastly improved the quality of the signals. FIR filters, by contrast, can partially improve signal quality. The functionality of the RLS method depended highly on the precision of the measured noise signal. The FIR filter has shown much less signal improvement than the RLS method, but FIR filters are very practical when noise cannot be measured.

KEYWORDS: Adaptive Filtering, Error, FIR Filter, Noise, Optimal Design, RLS Filter.

1. INTRODUCTION

The recursive least squares algorithm (RLS) and FIR filters that use the optimal design are useful tools to improve the quality of a corrupted signal. Using the RLS method, an adaptive system can be established that recursively minimizes the second order error between the filter output signal and the desired signal when a new portion of the signal enters. The RLS algorithm requires a noise measurement or an estimated version of the noise as J. Gnitecki et al. in 2003 worked on canceling noise from the lung sound signal using the heart rate signal as the noise source [1]. Similar work had been done by Guohua Lu et al. in 2009 to mitigate the effect of heart noise on signals picked up by surface electromyograms [2].

The RLS method has modified or combined with other filtering algorithms for adaptation to specific applications. Cristian S. et al. in 2013, used the dichotomous coordinate descent RLS algorithm (RLS-DCD) to attenuate the echo effect on multi-microphone audio systems [3]. Alina M. et al. in 2015, used the state space RLS (SSRLS) algorithm to improve the ECG signal affected by impulsive noise [4]. Radek M. et al in 2019 carried out a project in which the active shunt power filter (SAPF) control system uses RLS and least squares algorithms to attenuate the total harmonic distortion (THD) [5].

The optimal design is an algorithm that builds an FIR filter which acts as a cutting tool and blocks out unwanted signal spectral parts. The most common techniques for implementing an FIR filter are the frequency sampling method, the window method and the optimal design method. In the Optimal or Parks-McClellan design method, the goal is to reduce the difference between the ideal filter and the designed filter. This is done using the Remez Exchange algorithm [6, 7]. The resulting filter is optimal because it minimizes the maximum error between the frequency response of the designed filter and the ideal filter. Optimal design method is popular because of its flexibility and efficiency.

Optimal design has also been modified and combined with other algorithms to be more efficient. Rana et al. in 2016, used the genetic algorithm optimization method to develop and improve the optimal design, which resulted in fewer fluctuations in the frequency response bandwidth [8]. Kwan H. K. et al. implemented the optimal design method using the cuckoo search algorithm, the results of which are similar to Parks-McClellan but with fewer errors [9]. Alwahab D. A. et al. In 2018, used an optimal design method to implement filters using neural network algorithms. The results showed the flexibility to reduce fluctuations in the stop band of the frequency spectrum [10].

Despite all the modifications and extensive research, the RLS algorithm and optimal design were basically not modified in terms of functionality. In this study, a problem was addressed by the RLS algorithm and the optimal design separately to determine to what extent they can be functional. Next, the research methodology is explained in detail and the results and conclusions are discussed.

2. METHODOLOGY

This section explains the methods and algorithms, which are the RLS algorithm and the optimal design, to implement adaptive filtering and FIR filtering for this study.

2.1. Recursive Least Squares Algorithm

The recursive least-squares algorithm is an improved form of Wiener's theory [11] where the basic configuration of this theory requires finding optimal weights of w for the filter $y(n)$ so that the filter gets as close as possible to the noise signal $n(n)$. The functional diagram of adaptive filtering using the RLS algorithm is illustrated in Fig. 1. By subtracting $y(n)$ from the corrupted signal $v(n)$, the rest will be the clean signal $s(n)$ where $v(n) = s(n) + n(n)$. An N sampled filter $y(n)$ in FIR format can be given as follows[12].

$$y(n) = W^T X(n) \quad (1)$$

In which, $X(n) = [x(n)\ x(n-1)\ ...\ x(n-N+1)]^T$ is an isolated and achievable observation of the noise present. Wiener's theory suggests that the weights $W = [w_0\ w_1\ ...\ w_{N-1}]^T$ of the filter should be calculated so that the mean squared error (MSE) of $v(n) - y(n)$ becomes as small as possible. The MSE allows a formal solution to be found as:

$$J = \sigma^2 - 2WP + W^T RW \quad (2)$$

Where J represents MSE, σ^2 is the power of $v(n)$, P represents the cross-correlation between $v(n)$ and $X(n)$, and R is the autocorrelation of $X(n)$. The best approximation for W can be obtained by solving $dJ/dW = 0$ which gives $W^* = R^{-1}P$. However, the calculation of R^{-1} for a large number of weights will not be possible in real applications.

In the RLS algorithm, the matrix inversion lemma was used to deal with the R^{-1} computational aspects. In this method, R and P are computed in a recursive form as follows[12, 13].

$$R(n) = \lambda R(n-1) + X(n)X^T(n) \quad (3)$$

$$P(n) = \lambda P(n-1) + v(n)X(n) \quad (4)$$

Where λ is obtained empirically and $0 < \lambda < 1$. Solving $dJ/dW = 0$ for W yields:

$$W(n) = W(n-1) + k(n)\alpha(n) \quad (5)$$

In which, $\alpha(n)$ and $k(n)$ are calculated as follows.

$$\alpha(n) = v(n) - X^T(n)W(n-1) \quad (6)$$

$$k(n) = \frac{\lambda^{-1}Q(n-1)X(n)}{1 + \lambda^{-1}X^T(n)Q(n-1)X(n)} \quad (7)$$

$$Q(n) = \lambda^{-1}Q(n-1) - \lambda^{-1}k(n)X^T(n)Q(n-1) \quad (8)$$

For the initial condition $n = 0$, $Q(n-1) = \delta I$ where I is the identity matrix and δ is inverse of the power of $X(n)$. In this way, the weights are calculated so that the filtering output $y(n)$ comes as close as possible to the noise signal $n(n)$.

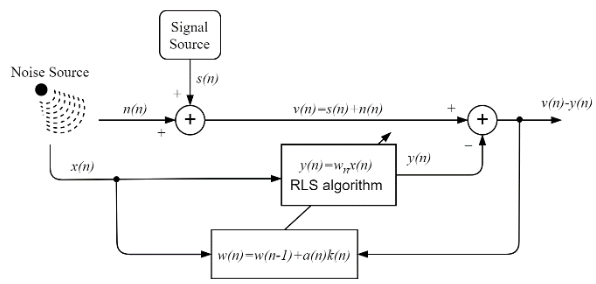

Fig. 1. Adaptive filtering block diagram using recursive least squares algorithm.

2.2. Optimal Design Algorithm

In the optimal design or Parks-McClellan method, the goal is to gradually get the minimum error between the frequency response of the FIR filter and the frequency response of the ideal filter. The transfer function of a FIR filter with a number of $k+1$ coefficients can be given as:

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_k z^{-k} \quad (9)$$

The purpose of the filter calculation is to obtain the coefficients of b_0 to b_k for Equation 9 to satisfy system requirements. Assuming $H_d(e^{j\omega T})$ is the ideal filter for project design, the approximate error compared to the designed filter is calculated as follows.

$$E(\omega) = W(\omega)[H(e^{j\omega T}) - H_d(e^{j\omega T})] \quad (10)$$

Where $E(\omega)$ is the approximate value of the error, $H(e^{j\omega T})$ is the frequency response of the designed filter and $W(\omega)$ is the weight function used to reduce the balancing error between the passband and the stopband. The design process involves minimizing the maximum error in the filter coefficient design or $\min(\max|E(\omega)|)$, which is done by the Remez Exchange algorithm [14].

3. SIMULATION AND RESULTS

In this section, we applied the RLS algorithm and the optimal design to a sample audio signal contaminated with white noise, using Matlab software.

3.1. RLS Algorithm Implementation

In this study, an audio sample was used as the original signal $s(n)$ and a random white noise was used as the interfered signal $n(n)$, where the corrupted signal is formed as $v(n)=s(n)+n(n)$. The signal $s(n)$ was sampled at 8000 Hz. The measurable noise signal $x(n)$ of current noise $n(n)$ was obtained by applying a moving average filter to the signal $n(n)$ in Matlab. The measured noise was assumed to be captured in three different qualities, such as $x_1(n)$, $x_2(n)$, and $x_3(n)$, where they respectively represent low precision, average precision, and good precision measurement. Fig. 2 shows the actual noise $n(n)$, which is a random signal and a medium precision noise measurement $x_2(n)$.

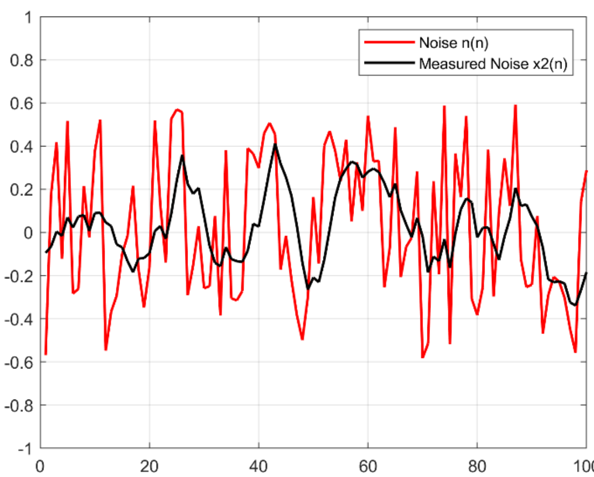

Fig. 2. The noise signal $n(n)$ in red and the measured noise $x_2(n)$ in black.

Matlab software was used to run the RLS algorithm with a filter tap 51 and $\lambda=1$. Fig. 3 shows five waveforms that include the original signal in green, the contaminated signal in red, and three cleaned signals, which used the RLS algorithm, are displayed in blue. The contaminated signal shows that the original signal has been heavily degraded by a white noise. However, the adaptive filter using the RLS algorithm partially suppressed the noise. The performance of the RLS algorithm largely depends on how close the measured noise quality is to actual noise. In Fig. 3, the cleaned signals are labeled CLN1, CLN2 and CLN3, which are the system responses to the measured noise signals $x_1(n)$, $x_2(n)$, and $x_3(n)$ respectively.

In this case study, the contaminated signal had a signal-to-noise ratio of 0.24. After cleaning this signal with the RLS algorithm the cleaned signals CLN1, CLN2, and CLN3 had a signal-to-noise ratio of 7.7, 10.1, and 12.3, respectively. We can see that the more accurate noise measurement provided a cleaner signal.

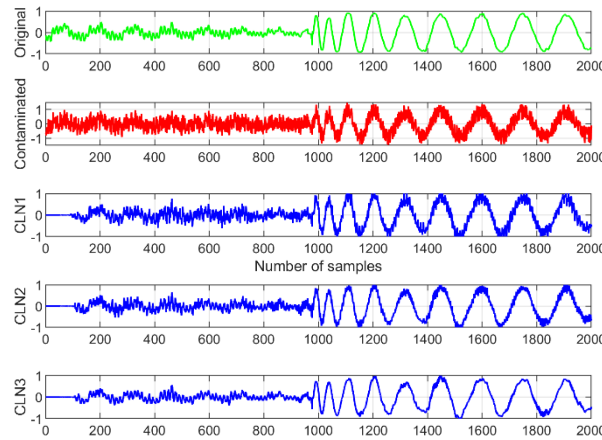

Fig. 3. Original and contaminated signals and the cleaned signals using RLS algorithm.

Fig. 4 illustrates the frequency spectrum of the original, corrupt and cleaned signals. It appears from the cleaned spectrum that the RLS algorithm has successfully eliminated noise from the contaminated spectrum.

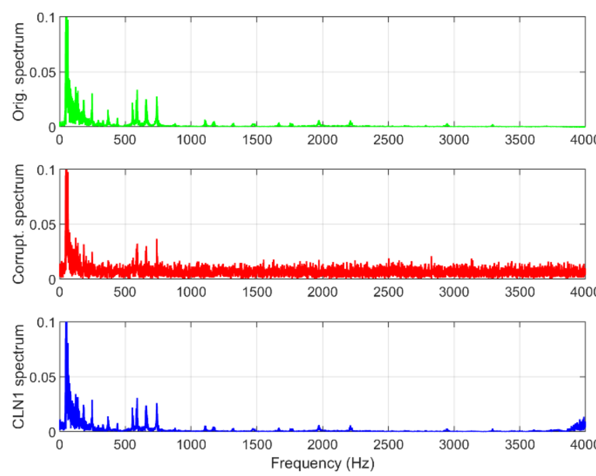

Fig. 4. Frequency spectrum of the original, corrupt and cleaned signals.

3.2. Optimal Design Method Implementation

The same signal and noise sample was used to implement the optimal design as for the RLS algorithm. As can be seen in Fig. 4, high amplitude noise occupied the entire frequency range. Therefore, it is not possible to completely separate the noise from the main signal using the FIR filter. So the goal is to reduce the noise effect and not eliminate it entirely. It can be seen that the most effective signal content in the damaged frequency spectrum is up to 1000 Hz. The contaminated spectrum was blocked above 1000 Hz by a low pass filter. Therefore, part of the original signal was also sacrificed to block most of the noise pollution.

For noise reduction, a low pass filter with a pass band of 0 to 1000 Hz and a stop band of 1500 to 4000 Hz was used. Therefore, for this design, the transition area is 500 Hz. Filter coefficients were calculated using the $firpm(n,f,m,w)$ function in MATLAB software. In this function, n is the order of the filter, f is a vector contains the normalized edge frequencies of the filter, and m the ideal edge frequency of the filter in the same order as f . The w in $firpm(n,f,m,w)$ is a weighting factor used to establish a balance between the ripple amplitude of the passband and the stopband of the filter. The vibration in the passband and the attenuation in the stopband were selected to be 1dB and 40dB, respectively. So the absolute value can be calculated as:

$$\delta_p = 10^{\left(\frac{1}{20}\right)} - 1 = 0.122$$
$$\delta_s = 10^{\left(\frac{-40}{20}\right)} = 0.01 \qquad (11)$$

From the results of Equation 11, the weighting factor $w = \delta_p / \delta_s \approx 12$. By determining the order of the filter at 25, the noise reduction results of the contaminated spectrum are presented in Fig. 5.

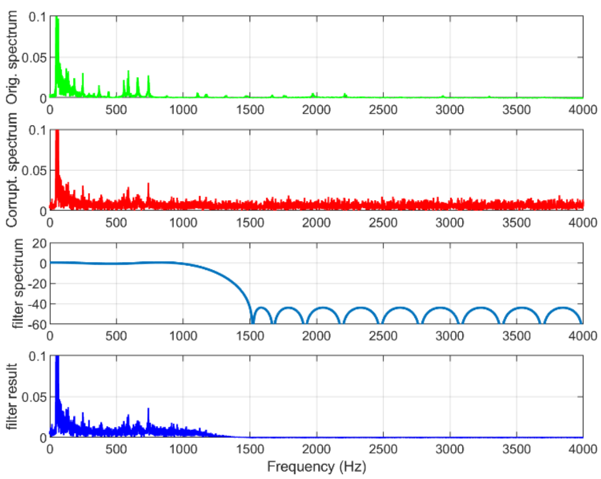

Fig. 5. Original and corrupted spectrum, filter spectrum and filtering result spectrum.

In Fig. 5, the original and corrupt spectra are displayed in green and red respectively and the FIR filter and its filtering results are displayed in blue. As discussed, to minimize the noise effect, the FIR filter was used to cut parts of the corrupted signal with the least amount of energy. As a result, parts of the original signal and a large part of the noise spectrum have been suppressed. The results of implementing the FIR filter on the corrupted signal showed a large reduction in the signal-to-noise ratio from 0.24 to 6.4, showing a great improvement in noise reduction but in the cost of losing a part of the main signal. Fig.6 shows the results in the time domain.

In Figure 6, the final result of noise reduction using the RLS algorithm for the adaptive filter and the optimal design for the FIR filter are shown in the time domain. The original and damaged signals are shown in green and red, and the filtering results are shown in blue. The result of the RLS algorithm is obtained with good noise measurement precision, which improved the signal-to-noise ratio from 0.24 to 12.3. On the other hand, optimal design filtering provided a signal-to-noise ratio enhancement from 0.24 to 6.4, which is half the improvement achieved by the RLS algorithm. The comparison of the two methods discussed shows that RLS works in a much more effective way than the optimal filter. However, as a disadvantage of the RLS method, it is necessary to provide an estimate or a separate measurement of noise to which RLS cannot be applied if no noise data is provided.

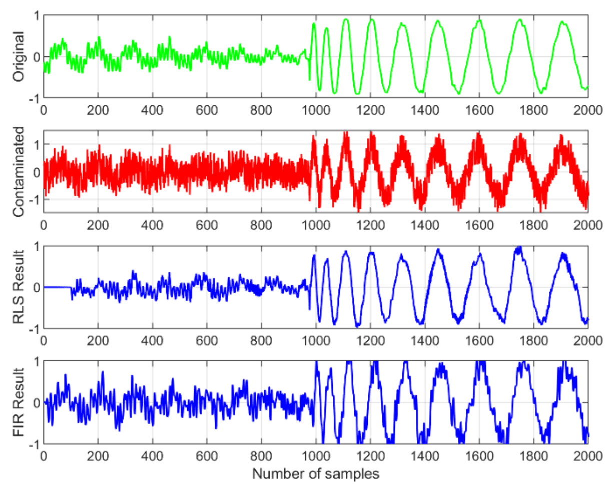

Fig. 6. The noise reduction results from RLS algorithm and optimal design.

4. CONCLUSION

In this study, the effectiveness of the RLS algorithm in adaptive filtering and the optimal design in the FIR filter were tested and compared in a problem of noise reduction in corrupted signals. From a design and application point of view, FIR filters are much easier to design and apply than adaptive filters. Both techniques were applied to clean the same damaged signal. Obviously, when the noise was white noise, the RLS method was able to reduce the noise much more effectively than the FIR filter. However, as a disadvantage, running the RLS algorithm requires measurements of noise sources that are not always available. On the other hand, FIR filters are not very suitable for removing white noise, however, when there is no noise measurement available, they can be used for noise reduction at the expense of loss of parts of the original signal information. In conclusion, the efficiency of the filters is greatly affected by the type of noise and the availability of noise measurement.

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