Passive Target Geo-location using Direction Finding Angles and Probability Density Matrix

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ABSTRACT:
Accurate geo-location of targets are extremely important in electronic warfare systems. In this paper we propose the use of probability density matrix (PDM), which is the sampled probability density function of observations, to evaluate the azimuth and elevation measurements of sensor arrays. The three dimensional joint probability density matrix of the observations will have the sufficient information to extract the location of the target. We also propose a localization algorithm to efficiently adapt the PDM domain with the latest estimation points and decrease the computational complexity. The simulation results indicate that with appropriate settings the localization algorithm can reduce the estimation error about 50%.

KEYWORDS: Angle of arrival, Direction Finding, Geo-location, Probability density

1. INTRODUCTION

Passive geo-location of ground targets is one of the missions of modern military surveillance aircrafts. These important tasks are carried out using information provided by sensors attached to the aircraft. Passive systems can detect and locate signal sources without the assistance of active systems. Consequently, these systems don't reveal the presence of the host platforms. However, passive systems usually provide lower localization accuracy. Direction finding (DF) angle is a passively generated sensor measurement that can be used for geo-location. A DF angle is one of several possible angles used to define the line of sight (LOS) from the aircraft to the signal source. In order to be specified, the LOS must be resolved into two angles relative to a local-level coordinate frame at the current aircraft position. These angles are typically represented as azimuth and elevation. Using DF angles for geo-location has various advantages. First of all, it is computationally less demanding compared to algorithms dealing with time difference of arrival (TDOA) and frequency difference of arrival (FDOA) measurements. More importantly, DF techniques allow a single aircraft to detect a signal and locate its source on the ground independently. Therefore, using DF angles for geo-location is very common and critically important [1], [2]. In this paper we present a novel method for geo-location of ground targets using azimuth and elevation measurements. Paper is organized as follows: section II is the background studies on the geo-location, section III formulates the problem, section IV describes the solution with the concept of discrete probability density matrix and section V has the conclusion.

2. BACKGROUND

Various papers have concentrated on the emitter geo-location using angle measurements. The so called triangulation algorithms are formulated mostly for position estimation on the plane. The well-known algorithm by Stansfield was a start; he provided a closed-form small error approximation of the maximum likelihood estimator [3]. His work was successfully developed and improved [4] [5]. These algorithms are mainly designed for two-dimensional geometry. An original algorithm for estimating target location which doesn't require flat earth approximations is presented in [6]. Another important feature of these works is that they intend to process all the angle measurements at the same time. A generalized triangulation algorithm for target geo-location on three-dimensional space is proposed in [7]. Studies that mostly focus on statistical techniques are published later. A least-squares error (LSE) algorithm is presented in [8]. This positioning method is based on the angle of arrival (AOA) measurements and provides a closed-form, non-iterative solution. [9] Proposes a grid-based probability density matrix for multi-sensor data fusion. Two kinds of measurements, AOA and range, in two-dimensional space are discussed. Using Gaussian distribution, probability values for different measurements are calculated over a local grid. Joint PDF is then determined by multiplying individual PDF matrix. Maximum values in the joint PDF matrix show the most probable locations of the targets.

3. PROBLEM FORMULATION

The problem of estimating emitter location with the use of angle measurements is formulated in this section. Let

$$p_T = [x_T \quad y_T \quad z_T]^T \tag{1}$$

Indicate the position of the target and

$$s_i = [x_i \quad y_i \quad z_i]^T \tag{2}$$

be the sensor's position, also called observation points, for $i = 1,2,\ldots,N$, where N is the number of observation points. Without loss of generality and for simplicity all the observation points are considered to be linearly distributed on a line. An observation vector is defined as a vector connecting the current measurement point to the next one. Consequently, a start point, an observation vector and number of measurements completely describe the geometry of the observation points.

In addition, let

$$\theta_i = [\phi_i \quad \varphi_i]^T \tag{3}$$

be the vector of measurements i.e., azimuth and elevation of target at the aircraft position s_i. As illustrated in Fig. 1, elevation, φ, is the angle between the target and sensor's local horizon and azimuth, ϕ, is the angle of the target around horizon. These angles are obtained from the measurements of direction of arrival (DOA) by an array of sensors (probably attached to an aircraft).

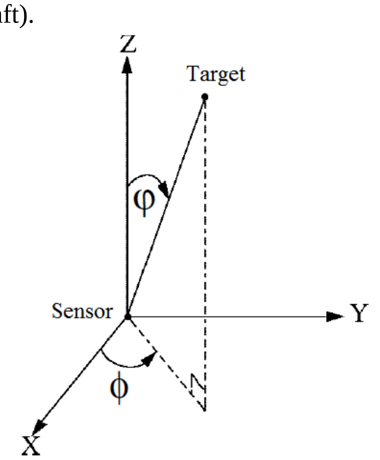

Fig. 1. Azimuth and elevation measurements

A horizontal array is required for azimuth measurement and a vertical array for elevation measurement. The target is any transmitter or emitting source which radiates intentionally or unintentionally. The measurements taken from various spatially distributed points are corrupted by noise and the problem is to estimate p_T having sensor measurements.

The algorithm evaluates only azimuth (ϕ) and elevation (φ) measurements. The true values of azimuth and elevation for each observation point can be calculated using the relationship between Spherical and Cartesian coordination systems. Function H describes this relation, knowing the target position.

$$H(s_i) = \begin{bmatrix} tan^{-1}\left(\dfrac{y_T - y_i}{x_T - x_i}\right) \\ tan^{-1}\left(\dfrac{z_T - z_i}{\sqrt{(x_T - x_i)^2 + (y_T - y_i)^2}}\right) \end{bmatrix}. \tag{4}$$

Additive noise of the sensor measurements are best modeled by zero mean Gaussian noise. For the vector of measurements we can write,

$$\theta_i = H(s_i) + n_i. \tag{5}$$

In which, n_i is the zero mean Gaussian noise with variance σ_i^2. The probability density function of random variable, X, with Gaussian distribution of mean, μ, and variance, σ^2, is,

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right). \tag{6}$$

4. DISCRETE PROBABILITY DENSITY MATRIX

Sensor measurements have some amount of uncertainty, which will be modeled with a probability density function (PDF). A PDF is a function describing the relative likelihood for a random variable to take a given value. In the three-dimensional (3D) case these given values can span the space. Each measurement provided by the sensors will express a specific PDF. Calculating the joint PDF in each step will give us only one functions of three variables, width, height and depth. This function can then be used to determine the most likely location of the target. In order to avoid evaluating the DPF in an infinite continuous space, we have to follow two steps. First of all, we have to limit the space as much as possible. This step depends on our prior knowledge of the target's location. An initial guess from the three-dimensional domain must contain the true location of the target. Thus a larger domain can be selected to compensate for our uncertainty about the target's location. We only consider cuboid domain as it is the most convenient structure of the Cartesian coordinate system. The second step is to discretize the cuboid, which is performed uniformly to maintain the simplicity of the problem. This process can be shown by $\{W, H, D\}$, which indicates the size of discretized matrix. We will refer to this discretized cuboid as the probability density matrix (PDM) that has $W \times H \times D$ elements. Although we have limited and discretized the space, we will keep the sum of elements for the PDM equal to unity. Each observation point comes with two measurements. The conditional probability of target being in a specific point can be calculated by adding the associated probability of each measurement to the PDM.

$$\boldsymbol{P}\,(w,h,d) = p_{\phi}(w,h,d) + p_{\varphi}(w,h,d). \qquad (7)$$

$\boldsymbol{P}$ denotes the three dimensional probability density matrix and $1 \le w \le W$, $1 \le h \le H$ and $1 \le d \le D$. $p_{\phi}(w,h,d)$ and $p_{\varphi}(w,h,d)$ denote the probability of target being at the point (w,h,d) associated with the azimuth and elevation measurements respectively.

For example if the observation point is $s_1 = [100\;100\;50]^T$ and the target's location is $p_T = [0\;0\;0]^T$ then the measurements with $\sigma = 3^{\circ}$ will give the probability density matrix as illustrated in Fig. 2.

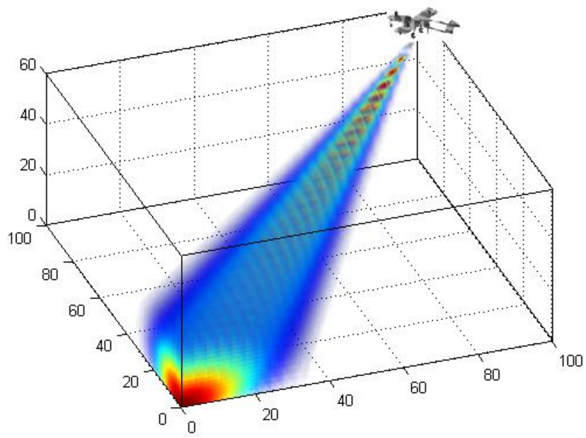

Fig. 2. A sample probability density matrix illustration

The high values of PDM mean the higher probability of target being in those positions. As mentioned before, the PDM is a function of three Cartesian coordinates, so the Fig. 2, has only the contour diagrams of the PDM.

4.1. Data Fusion

The measurements from different observation points must be combined to form a single joint PDM. The joint PDM is,

$$\boldsymbol{P}\,(w,h,d) = \frac{1}{C}\prod_{i=1}^{N}\left[p_{i,\phi}(w,h,d) + p_{i,\varphi}(w,h,d)\right]. \qquad (8)$$

In which, w, h, d span the cuboid domain as in (7). $p_{i,\phi}$ and $p_{i,\varphi}$ are the 3D PDFs of the i^{th} observation sampled over $\{W, H, D\}$ and C is used to normalize the sum of the PDM to one.

In order to estimate the target's location, we have to find the coordinate of the maximum value of the joint PDM and translate it back into a Cartesian location. A peak in the joint probability density matrix indicates that all the measurements point into that coordinate as the possible target location.

For example imagine a target at $p_T = [0\;0\;0]^T$. Two pairs of azimuth and elevation measurements made from the observation points $s_1 = [120\;60\;50]^T$ and $s_2 = [60\;120\;50]^T$ with the standard deviation of three degrees. Each observation point corresponds to one probability density matrix as illustrated in Fig. 3. The joint PDM can then be calculated by (8), using individual PDMs. Fig. 4, shows the resulted joint PDM. The maximum value of this matrix accords with $\hat{p}_T = [0.8\;0.8\;0.1]^T$ which is an acceptable estimation of target's location.

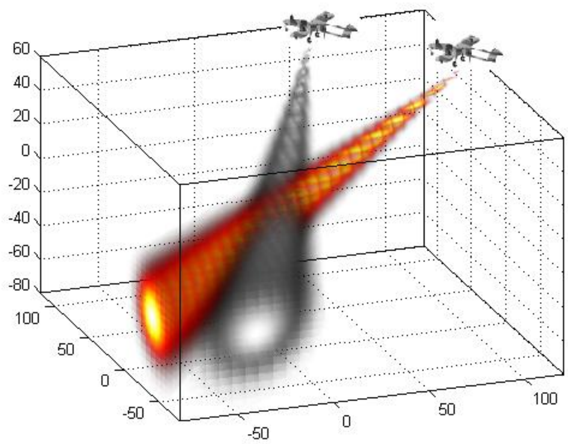

Fig. 3. PDM for two set of observations

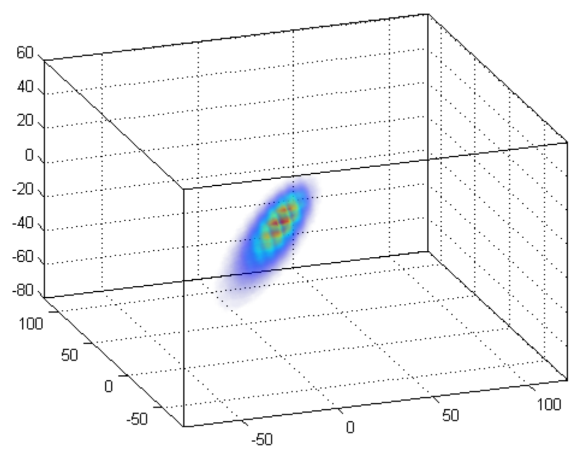

Fig. 4. The joint PDM for two set of observations

4.2. Observation Vector

In this section we provide an example to illustrate the effects that observation vectors with different lengths have on the estimation error. Imagine a target located at $\boldsymbol{s_T} = [80\ \ 50\ \ 10]^T$ meters. Azimuth and elevation measurements of this target are made by an aircraft starting from $s_1 = [3000\ 3000\ 500]^T$ meters and the observation vector is R. Thus the aircraft observation points can be described as,

$$\boldsymbol{s_i} = \begin{cases} [3000\ \ 3000\ \ 500]^T & i = 1 \\ [3000\ \ 3000\ \ 500]^T + iR & 2 \le i \le N \end{cases} \quad (9)$$

Three different scenarios are considered, $R_1 = [200, -400, 60]^T$, $R_2 = 0.5\, R_1$ and $R_3 = 0.2\, R_1$ while keeping other variables fixed. The local three dimensional grid is a rectangular cuboid with one vertex at $\{0, -5000, 0\}$, all the edges parallel to the axis and one vertex at $\{7000, 3000, 2000\}$. This cuboid is discretized with $\{100, 100, 100\}$ into one million small cuboids which make up our PDM. Fig. 5, shows the root mean square error (RMSE) of 100 simulations for each case.

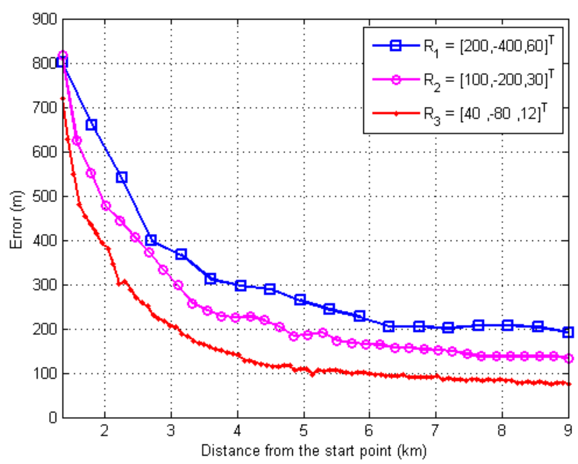

Fig. 5. RMSE for different measurement vectors

The error estimations are shown only after one kilometer of flight. As it is apparent from Fig. 5, decreasing the length of observation vector enhances our estimation. It should be mentioned that 80 meters of error, which is reached by R_3, is the least possible error based on the grid design for this example.

4.3. Localization

As the above example points out, a grid of probabilities on the large cuboid will lead to computational overloads. Furthermore, combining azimuth and elevation measurements each with independent noise can affect the estimation error significantly. However, these problems might not be conspicuous in the two dimensional cases discussed in [9], [10], [11]. In order to reduce the error of estimation while dealing with acceptable computational costs, we propose a localization method. The initial cuboid domain can cover a large area in the three dimensional space but it should have the real location of the target inside. The localization method helps to maintain this criteria while focusing on the target location along the estimation process Fig. 6, shows the flowchart of the proposed method. The estimation process continues with a fixed cuboid until a sort of steadiness is detected in the series of estimated locations.

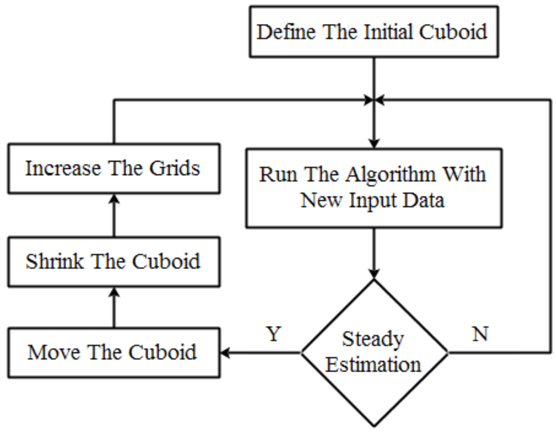

Fig. 6. Flowchart of the localization algorithm

When convergence is detected, the algorithm changes the cuboid domain before evaluating the new data. This change has three bases:

- Move the center of the cuboid to the recently estimated point
- Shrink the cuboid by a certain factor to focus further on the interested area
- Increase the number of grids to enhance the estimation

Here is an example simulated with the localization technique. Target is located at $\boldsymbol{s_T} = [80\ \ 50\ \ 10]^T$, as the previous example, and the measurement locations are described by (9), where $R_1 = [200, -400, 60]^T$. Fig. 7, illustrates the RMSE of 100 runs for localization method and fixed-cuboid method. While the fixed-cuboid method settles with 200 meters of average estimation error, the localization method reaches 100 meters of error with lower computational complexity. The initial PDM settings are same for both cases.

Steady results are detected when the distance between currently estimated point and the previous estimation is less than twice of space diagonal of PDM units. For the localized case, the domain is initially discretized with $\{10, 10, 10\}$ and when the steadiness condition meet, the number of grids along each dimension are increased by the factor of 1.5. Of course this increase is block with the limit of $\{100, 100, 100\}$. The fixed-cuboid method, however, has the PDM of one million elements with construction of $\{100, 100, 100\}$. The localization method is about five times faster than the fixed-cuboid method in the simulations and as mentions, it has lower estimation error.

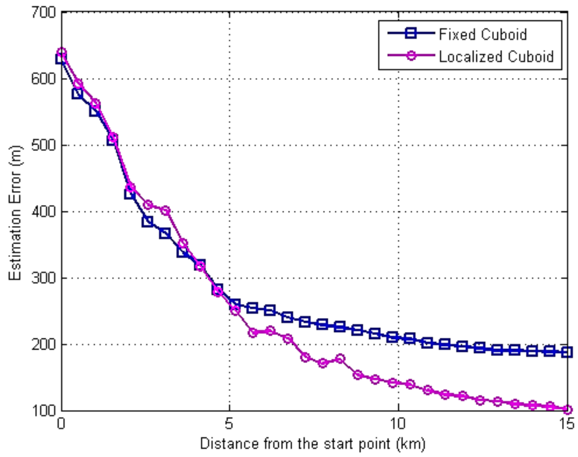

Fig. 7. RMSE of localized method and fixed-cuboid method

5. CONCLUSION

In this paper we proposed a grid based probability density matrix to evaluate the DF angles measred by sensor arrays. Although two dimensional cases have already been described and well-studied in different papers, we focued on the three dimensional kind. The main challenge faced in the 3D case is the computational complexity of dealing with an oversized PDM. This problem is overcome by the proposed localization method.

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