Dimensional and Geometrical Tolerance Analysis of Two Flexible Curved Sheet Metal Parts Assembly

Parisa Omidvar
Department of Mechanical Engineering, AmirKabir University of Technology, Iran
E-mail: parisaomidvar@aut.ac.ir

Maryam Ghassabzadeh Saryazdi*
Technology Institute of Mechanical Engineering (TIME), AmirKabir University of Technology, Iran
E-mail: mghsaryazdi@aut.ac.ir
*Corresponding author

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Abstract: Sheet metal assemblies are widely used in the automobile, aerospace, and shipbuilding industries. Sheet metals deform during the manufacturing and assembly process due to their high flexibility. Traditional tolerance analysis approaches were developed for rigid assemblies; however, new approaches of tolerance analysis and variation simulations have been proposed for flexible (compliant) assemblies using FEM. In this paper, a new method called Interactive Worst Case (IWC) is introduced for tolerance analysis of flexible assemblies, which demands a few FEM simulations and is based on traditional Worst Case (WC) method. IWC method guarantees that all the parts will assemble accurately and have proper function. The case study of this paper is two flexible sheets in the form of quarter cylinders, joined together by six spot welding to form a half-cylinder assembly. The accuracy of IWC is verified by comparing the results to uniform MIC. The results of MIC are also compared to the results of the Monte-Carlo simulation (MCS).

Keywords: Assembly Spring-Back, Compliant Assembly, Interactive Worst Case Method, Method Of Influence Coefficients, Sheet Metal Assembly, Tolerance Analysis

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Biographical notes: Maryam Ghassabzadeh Saryazdi received her BSc in Mechanical Engineering at the Sharif University of Technology, Iran, in 1998. She studied and received her MSc at Amirkabir University of Technology, Iran in 1999. She received her PhD from Sharif University of Technology in 2009. In 2016, she was appointed to Chair at the Amirkabir University of Technology. She is currently Assistant Professor at the Technology Institute of Mechanical Engineering (TIME), AmirKabir University of Technology, Tehran, Iran. Her current research interest includes optimization design and tolerance analysis. **Parisa Omidvar** received her MSc in Mechanical Engineering (Applied Mechanics) from AmirKabir University of Technology in 2020 and her BSc in Mechanical Engineering (Solid Mechanics) from Bu-Ali Sina University in 2017. Her current research interests are tolerance analysis, finite element analysis, mechanical metamaterials, compliant mechanisms, and sheet metals.

1 INTRODUCTION

A mechanical assembly consists of various parts produced with different methods, materials, and geometric properties. Sheet metals, widely used in the automobile and aerospace industries, are highly flexible and deform under their own weight, the interaction between sheets and external forces such as clamping forces, welding forces, as well as internal forces. In addition, after releasing the clamps, the springs back will occur in the assembly because of the former deformations. Therefore, the designer should choose the appropriate tolerances for each part based on these factors. Using the tolerance analysis could help determine which parameters have the largest impact on the assembly specifications. On the other hand, the assembly performance is controlled by these parameters [1-2].

The flexibility of the parts is ignored in the classic tolerance analysis, such as the worst-case method and statistical method, and it is assumed that components are rigid. However, new approaches gradually started to consider the flexibility of parts, which created a new branch in tolerance analysis called compliant tolerance analysis [3-4]. Liu and Hu [5-7] are the pioneers in the compliant tolerance analysis study. They presented Monte-Carlo Simulation (MCS) and Invented Method of Influence Coefficients (MIC) for variation analysis of sheet metal assembly. In these methods, the deviations and deformations of the parts are calculated by the finite element method (FEM).

In 1996, Merkley and chase [3] presented a model based on the stiffness matrix of parts inspired by equations of spring deformation, which could predict tolerances of 1-D blocks. The assembly was simulated with MSC/NASTRAN software. In 2003, Camelio and Hu [8] extended the MIC method for multi-station compliant sheet metal assemblies using homogeneous transformation. In 2007, Dahlstrom and Lindkvist [9] improved MIC by adding elastic contact algorithm to the classic MIC. This algorithm could also be used in 3-D contact surfaces without needing to match meshes. Non-linear variation analysis of sheet metal assembly with elastic contact was presented by Liao and Wang [10] using Ansys software. Xie et al. [11], instead of the MCS, used eDRM to estimate the statistical response of the system. The total number of the FEM simulation for eDRM in their problem was 17, which significantly reduces the computing time compared to the MCS method. However, the accuracy and number of required simulations of this approach are unclear for more complicated models. In 2015, Cho and Chung [12] simulated a sheet metal assembly with MIC, assuming distortion during the welding process in shipbuilding.

In 2018, Tao Liu et al. [13] presented a model based on the Timoshenko beam theory and virtual work to calculate the deviation of flexible beam assemblies. They extended the approach to composite curved sheet assembly in 2019 [14], by which the complexity is highly reduced.

Although curved sheet metals are usually used in the industry, they have received less attention in the research. Therefore, the case study of this paper is the geometrical tolerance analysis for two curved sheet metal parts. In order to examine the accuracy of the MIC method for the curved sheets, the MIC equations are expanded to 3-D in which the sensitivity matrix is calculated based on the sources of variation, and the variation analysis of two quarter cylinder parts is presented. The results are also compared to the results of the MSC method. In addition, the equations of MIC are directly used to find the optimum tolerances of parts based on a desired tolerance of the assembly.

A new method called Interactive Worst Case (IWC) is also introduced for variation analysis of flexible assemblies, which can also be used for curved flexible sheet metal assemblies. This method is based on the Worst Case (WC), so if all the parts are within their tolerance limit, then it could ensure that all parts will assemble and function properly. As in the MIC method, some of the parts might get rejection. [1], [7], in this paper, some changes are made in the MIC equations in a way that it can predict tolerances of the assembly like as WC.

2 FUNDAMENTALS & METHODS

In order to simulate the assembly of flexible sheet metals, the following steps are usually considered as the assembly process [7], [15]:

1- Parts are placed in their fixtures.

2- Parts are forced to move to their nominal position.

3- Parts are joined together (For instance, by spot welding).

4- Some of the fixtures are released, and the spring-back of the assembly occurs.

In this paper, it is assumed that the deformation is in the elastic range, and parts are joined by spot welding. In addition, it is assumed that fixtures and welding guns are solid compared to sheet metals. Because of the use of spot welding, distortion caused by welding is negligible. [12] After welding, the assembly is released, and the spring-back happens due to the applied forces and initial deviations [9].

IWC Method

The Interactive Worst-Case method (IWC) introduced in this paper is based on the WC method which was developed for rigid assemblies. The WC method guarantees that the deviations in all the assemblies will remain inside the desired limits. Therefore, in this method, it is assumed the deviations in parts are at the upper or lower limit of their tolerances [1]. In order to utilize the WC method for flexible assemblies, in this paper, the method is modified using finite element analysis. In the proposed method, all possible extreme cases are examined. The largest boundary calculated from these cases will be the tolerances of the assembly.

Welding gun forces [7]:

At First, the influence coefficients matrix ([C]) is determined using FEM. By simulating each part separately in a finite element software, the deformation of all nodes at spot welding is determined while a unit force is applied to the j-th weld point (j=1 to N: number of weld points). The results construct the matrix of [C] as "Eq. (1)":

$$[C]=\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1N} \\ c_{21} & c_{22} & \dots & c_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ c_{N1} & c_{N2} & \dots & c_{3N} \end{bmatrix} \tag{1}$$

Where, j-th column indicates the deformation of nodes at spot welding when the unit force is applied at j-th weld point.

Assuming the linear behavior for sheet metals, then the welding gun forces required to move the weld points to their nominal position are calculated from "Eq. (2)".

$$\{F\}=\begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{pmatrix}=[C]^{-1}\{V\}=[K]\{V\}=\sum_{j=1}^{N}\begin{pmatrix} K_{1j} \\ K_{2j} \\ \vdots \\ K_{Nj} \end{pmatrix}V_j \tag{2}$$

Where, [K] is stiffness matrix and {V} is the vector of any arbitrary deviation at weld points. Instead of deviation in weld points, the deviations at sources of variation are known in actual situations. Therefore, it is necessary to determine the relationship between deviation at weld points and sources of variation as follows in "Eq. (3)":

$$\{V\}_{N\times1}=[C]_{N\times N'}\{V'\}_{N'\times1} \tag{3}$$

Where, $\{V'\}$ is the vector of deviation at sources of variation, and N' is the number of sources of variation. Considering unit deviation only at m-th source of variation, parts are simulated in a finite element software. The parts are fixed at the ideal position of fixtures, then the deviations of nodes at spot welding are extracted. So the m-th $(m=1:N')$ column of [P] matrix is determined.

Finally, the welding gun forces based on the deviation at sources of variation are calculated as follows in "Eq. (4)":

$$\{F\}_{N\times1}=[K]_{N\times N}[P]_{N\times N'}\{V'\}_{N'\times1} \tag{4}$$

The Flow Chart of IWC:

Figure 1 represents the flow chart of the IWC method. After determining the sources of variation, boundary conditions, and their upper and lower limits, the extreme cases are determined in which all the sources of variation have their extreme values.

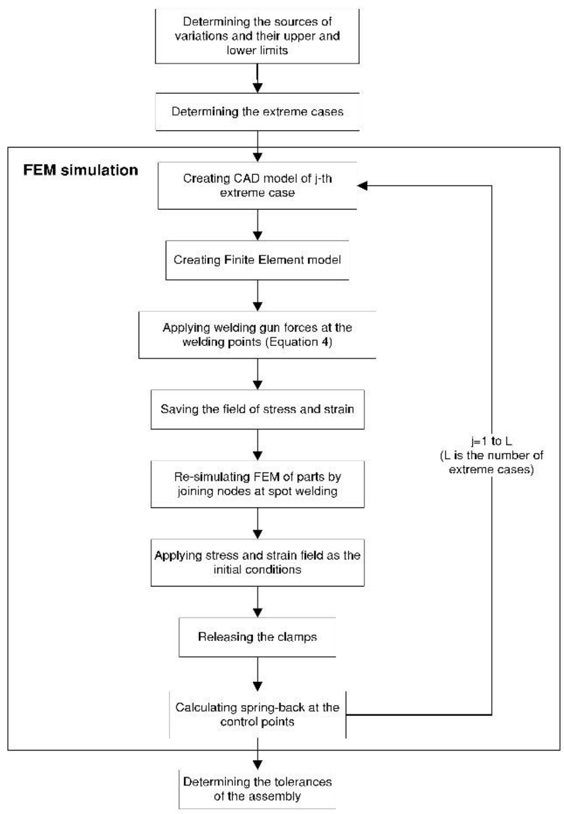

Fig. 1 The flowchart of the IWC method.

The number of cases will be equal to $(L=2^{N'})$. The assembly process is then simulated for each extreme case using the FEM. By applying the welding gun forces, nodes at spot welding are forced to move to their nominal positions, and the stress and strain fields are saved for the next step. To consider the interaction between parts, they are re-simulated in FEM, node pairs at spot welding are joined together, and the stress and strain field are applied as the initial conditions. The spring-back of the assembly is calculated after releasing some fixtures. Finally, the assembly deviation in the desired points, namely control points, is extracted. By comparing the deviation in the assembly specification in all cases, the maximum and minimum deviation is extracted, and the dimensional tolerances of the assembly specification could be determined. This method has zero rejection rate because it is developed based on WC method.

In order to provide a general overview of the effects of deviations of parts on the deviations of the assembly, especially in complicated cases, the results of all extreme cases at various nodes could be plotted in one diagram. In this way, a better understanding of the position and deviation of the assembly nodes could be obtained. The diagram also helps to compare the results of extreme cases together in a simple and quick way.

MIC

MIC is one of the most important methods developed in the field of flexible assembly tolerance analysis. This method assumes that there is a linear relationship between deviations of parts and assembly deviations [7]. This assumption is valid as long as the deviations are small and the material behavior is linear. The sensitivity matrix, in this paper, is calculated based on the deviation in sources of variation $\{V'\}$, rather than using the deviations in the weld points $\{V\}$. In this way, the assembly specifications are directly related to the sources of variation, and from the viewpoint of application, the tolerance analysis would be more practical than the original MIC.

The finite element simulation is done according to the FEM simulation box in Fig. 1, except that the geometry model is created with unit deviation in i-th source of variation. Then, the $\{S_i\}$ vector, which includes the deviations of the control points, is recorded as “Eq. (5)”, in which M represents the number of control points.

$$\{S_i\} = \begin{pmatrix} s_{x_{1i}} \\ s_{y_{1i}} \\ s_{z_{1i}} \\ \vdots \\ s_{x_{M\,i}} \\ s_{y_{M\,i}} \\ s_{z_{M\,i}} \end{pmatrix} \tag{5}$$

Where, x, y, and z represent the axis of the coordinate system. Combining vectors of $\{S_i\}$ would easily construct the sensitivity matrix [S]. The assembly deviations at the control points {U} for arbitrary deviation in sources of variation are calculated using “Eq. (6)”:

$$\{U\} = \begin{pmatrix} u_{x_1} \\ u_{y_1} \\ u_{z_1} \\ \vdots \\ u_{x_M} \\ u_{y_M} \\ u_{z_M} \end{pmatrix} = \sum_{i=1}^{N} \begin{pmatrix} s_{x_{1i}} \\ s_{y_{1i}} \\ s_{z_{1i}} \\ \vdots \\ s_{x_{Mi}} \\ s_{y_{Mi}} \\ s_{z_{Mi}} \end{pmatrix} V_i' = \begin{bmatrix} s_{x_{11}} & s_{x_{12}} & \cdots & s_{x_{1N'}} \\ s_{y_{11}} & s_{y_{12}} & \cdots & s_{y_{1N'}} \\ s_{z_{11}} & s_{z_{12}} & \cdots & s_{z_{1N'}} \\ \vdots & \vdots & \ddots & \vdots \\ s_{x_{M1}} & s_{x_{M2}} & \cdots & s_{x_{MN'}} \\ s_{y_{M1}} & s_{y_{M2}} & \cdots & s_{y_{MN'}} \\ s_{z_{M1}} & s_{z_{M2}} & \cdots & s_{z_{MN'}} \end{bmatrix} \begin{pmatrix} V_1' \\ V_2' \\ \vdots \\ V_3' \end{pmatrix} \tag{6}$$

Equation (6) could be used for Cartesian, Cylindrical, and Spherical coordinate systems. Based on the chosen distribution for sources of variation, random deviations are generated, and the spring-back related to each deviation is calculated using the sensitivity matrix. Distribution of the assembly deviations could then be obtained by this collected data. When the sources of variation are independent and normal distribution is assumed (like statistical tolerance analysis), the distribution of assembly deviations could also be estimated using “Eqs. (7) and (8)”.

$$\{\mu_a\} = [S]\{\mu_p\} \tag{7}$$

$$\{\sigma_a^2\} = [S_{ij}^2]\{\sigma_p^2\} \tag{8}$$

In these equations, $\{\mu_a\}$, $\{\sigma_a^2\}$, $\{\mu_p\}$ *and* $\{\sigma_p^2\}$ are mean deviation and variance vector of control points and sources of variation, respectively. Finally, the tolerance of the assembly could be determined by assuming that the equal bilateral tolerance is equal to ±3σ. It is worthwhile to mention some advantages of MIC that might have been less noticed in papers:

1) This method could be employed when another distribution such as lognormal is valid for the sources of variation using MCS based on MIC.

2) Based on the desired assembly tolerances, the optimum tolerances of parts could be easily calculated by Eqs. “(7) and (8)”. It will be discussed in the following sections.

3) Considering the uniform distribution for deviation in sources of variation leads to the MIC similar to WC method which is examined in this paper. Based on this assumption, the vector of assembly's equal bilateral tolerance T_a could be estimated using “Eq. (9)”:

$$\{T_a\} = [S]\{T_p\} \tag{9}$$

Where, T_p is the equal bilateral tolerance vector of sources of variation in parts. Note that in this paper, whenever MIC is used with uniform distribution, the method is called uniform MIC, and when using the normal distribution, it is only referred to as MIC.

MCS

MCS is one of the common approaches for the variation simulation of flexible assemblies. In order to calculate the assembly deviations in this method, the random dimensions based on the statistical distribution of each source of variation are generated. The finite element simulation is created according to the FEM simulation box in "Fig. 1", except that the geometry of the model is based on these random deviations. The distribution of spring-back and deviations in the assembly is calculated by solving adequate finite element models. This method is very time-consuming. More information about MCS could be found by referring to Liu and Hu 1997, [7].

3 CASE STUDY

The case study of this paper is the assembly of two sheet metal parts in the form of quarter cylinders. As demonstrated in "Fig. 2", parts are joined together in six weld points.

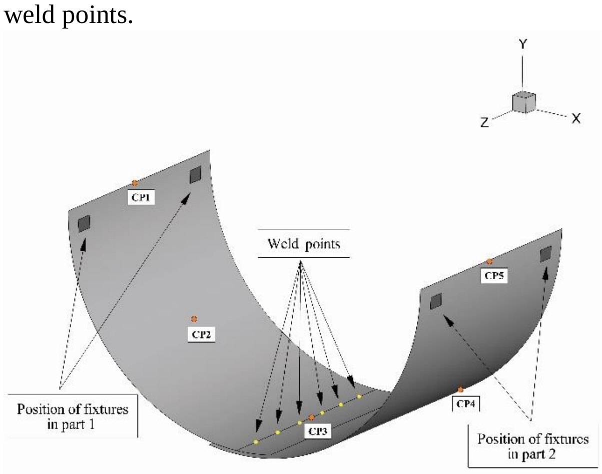

Fig. 2 Representative picture of the case study.

The nominal radius of each sheet is 120 mm with a thickness of 1 mm. Each part is fixed with two fixtures, as shown in "Fig. 2". The fixtures constrain the motion of the area in all directions. The Young's modulus, Poisson's ratio, and friction factor are equal to 206 GPa, 0.3, and 0.1, respectively. As shown in "Fig. 2", five control points (CPi, i=1 to 5) are chosen to determine the spring-back and deviations of the assembly in each analysis. Note that the changes in the results along the height (z-direction in "Fig. 2") are negligible. Therefore, the deviations in the middle line of assembly are considered for spring-back calculation.

In order to verify the finite element analysis, the case study of Liao X and Wang GG 2007[10] was simulated in which two rectangular parts with dimensions of 166.37mm×212.85mm are joined together in six weld points. The results show that there is a good agreement with the experimental results of the mentioned reference. In this paper, the deviation in the radius of parts is considered as the sources of variation, and it is assumed that the curved parts are ideal cylinders. Figure 3.a illustrates the cross-section of the two quarter cylinders with the same centerline. Figure 3.b depicts the sheets fixed by fixtures in their ideal positions.

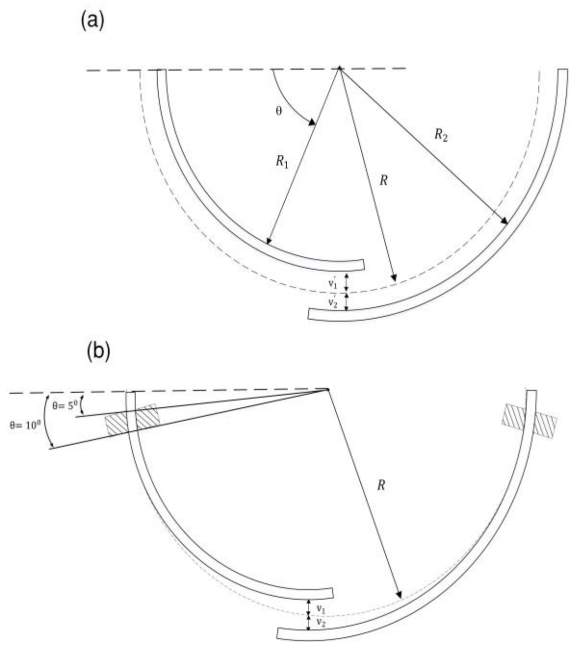

Fig. 3 Symbolic model of the case study: a) geometry of the parts. b) parts in the fixture position.

It is assumed that the radius of part 1 varies between 119 to 120mm (119.5±0.5mm), and the radius of part 2 varies between 120 to 121mm (120.5±0.5mm). Thus, in the IWC method, four extreme cases exist, which are described below:

Case I	Part 1 with a radius of 119 mm Part 2 with a radius of 120 mm
Case II	Part 1 with a radius of 120 mm Part 2 with a radius of 121 mm
Case III	Part 1 with a radius of 119 mm Part 2 with a radius of 121 mm
Case IV (ideal case)	Part 1 with a radius of 120 mm Part 2 with a radius of 120 mm

Since the radius of both parts in case IV is equal to 120 mm, it does not need any FEM simulation. In order to determine the radius of the assembly, after releasing the fixture of part 1, the center of two edges is selected as the center of the half-cylinder. The case study is also analyzed with MIC and MCS. In MIC, two cases of uniform distribution and normal distribution are assumed. In MCS, the normal distribution is considered for sources of variation.

4 RESULTS AND DISCUSSION

Figure 4 depicts the deviation of radius versus the angle for the middle line passing through the control points for all the extreme cases.

As can be seen, the largest and the smallest radiuses are 120.496 and 119.504 mm, related to the case I and II, respectively. The largest deviation from circularity which shows the profile tolerance of the middle line, happened at case III between CP2 and CP4, with the value of 0.828 mm.

Table 1 Distance between two edges (IWC method)

	Case I	Case II	Case III	Cas e IV	Mean value	Equal bilater al toleran ce
Dista nce (mm)	239.0 069	240.9 812	240.0 751	240	239.9 940	0.9872

The tolerance of distance between two edges of the half-cylinder assembly is an important parameter, especially for further assemblies in multi-station assembly processes. The tolerance of this distance is listed in "Table 1", which is calculated by means of the IWC method. The mean value and standard deviation of half-cylinder radius, calculated by the MIC and MCS methods, are listed in "Table 2" for the control points. In order to obtain the satisfactory distribution, 230 simulations are done in the MCS method by scripting the FEM software with Python. Figure 5 illustrates the normal distributions of the half-cylinder radius for CP1 to CP5.

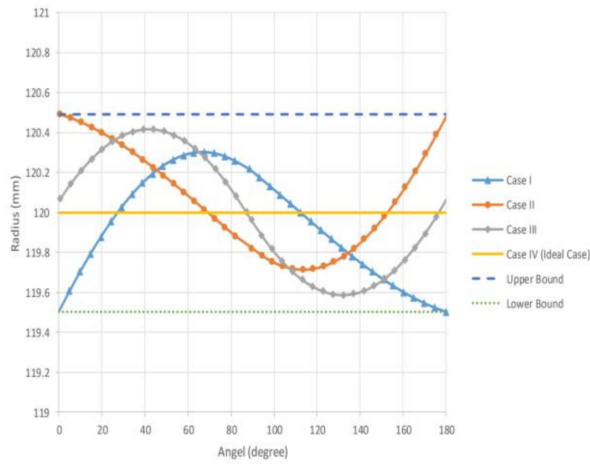

Fig. 4 Radius of the middle line in each case (the diagram of IWC method).

Table 2 Normal distribution of radius in the control nodes

control point	Mean value (MCS)	Standard deviation (MCS)	Mean value (MIC)	Standard deviation (MIC)	MIC/MCS difference (Mean value)	MIC/MCS difference (Standard deviation)
CP1/CP5	119.9910	0.1147	119.9924	0.1163	0.0014	0.0015
CP2	120.2090	0.0529	120.2080	0.0490	0.0010	0.0038
CP3	120.0140	0.0549	120.0114	0.0468	0.0026	0.0081
CP4	119.7980	0.0499	119.7988	0.0474	0.0008	0.0025

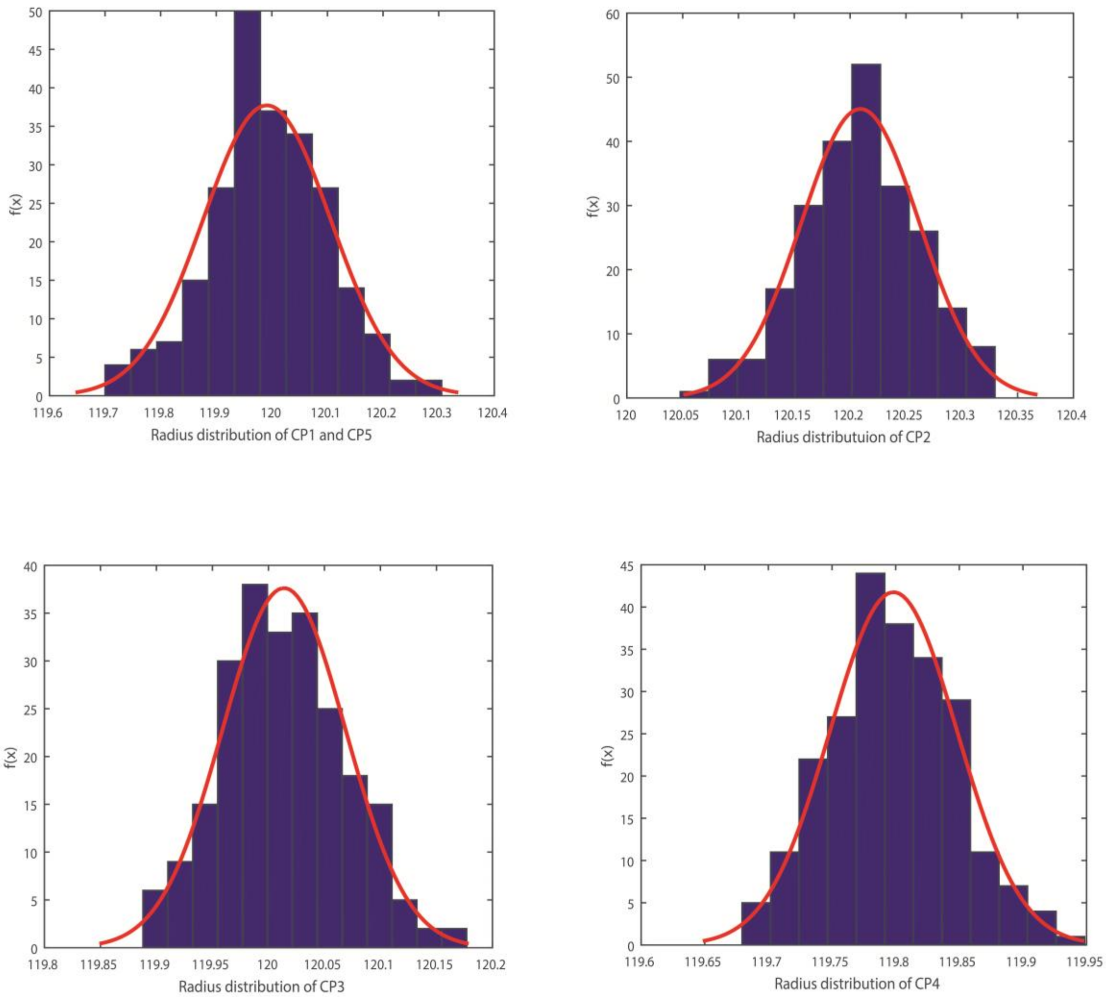

Fig. 5 Statistical distribution of radius in the control points.

Mean, maximum and minimum values of assembly parts radius, as well as radius tolerances of the assembly, are presented in “Table 3”, calculated by various methods. Note that when the normal distribution is assumed for sources of variation, equal bilateral tolerance is equal to $\pm 3\sigma$. It can be seen that the results of MIC and MCS with normal distribution are particularly close. Therefore, MIC can be used for curved sheet metal assemblies. The results of the IWC and uniform MIC are also very close. Thus, IWC could be used as an accurate and independent approach for tolerance analysis. It can be noted that the mean value in all the used methods has the same value with a negligible difference. The radius tolerance in the MIC and the MSC is derived based on the results in CP2 and CP4 while in the IWC and the uniform MIC, CP1 and CP5 are used.

Table 3 Radius tolerances of the assembly

Method	Mean value	Equal bilateral tolerances	Maximum value	Minimum value	Upper limit from 120 mm	Lower limit from 120 mm
IWC	119.9980	0.4933	120.4913	119.5047	0.4913	0.4953-
Uniform MIC	119.9924	0.4970	120.4857	119.4992	0.4857	-0.5008
MCS	120.0079	0.3597	120.3676	119.6482	0.3676	-0.3518
MIC	120.0058	0.3492	120.355	119.6565	0.3550	-0.3435

"Table 4" compares the assembly's profile tolerances derived from various methods derived from CP2 and CP4. "Table 5" Compares the tolerances of the distance between two edges of the assembly determined by various methods. As can be seen, the tolerance ranges of IWC and uniform MIC are greater than MIC and MCS because of the proposed distribution.

Table 4 Profile tolerance of the assembly

Method	Control point	Mean value	Bilateral tolerance	Maximum value	Minimum value	Profile tolerance
IWC	CP2	120.2123	0.2123	120.4246	120	0.8277
	CP4	119.7985	0.2015	120	119.5970	
Uniform MIC	CP2	120.2080	0.2080	120.4159	120	0.8197
	CP4	119.7988	0.2012	120	119.5976	
MCS	CP2	120.2090	0.1586	120.3676	120.0504	0.7194
	CP4	119.7980	0.1498	119.9478	119.6482	
MIC	CP2	120.2080	0.1471	120.3550	120.0609	0.6985
	CP4	119.7988	0.1423	119.9411	119.6565	

Table 5 Tolerances of distance of the two edges in the assembly

Method	Mean value	Bilateral tolerance	Upper limit from 240 mm	Lower limit from 240 mm
IWC	239.9940	0.9872	0.9812	-0.9931
Uniform MIC	239.9849	0.9865	0.9714	-1.0016
MCS	239.982	0.6884	0.6704	-0.7064
MIC	239.9849	0.6977	0.6826	-0.7128

The tolerances of parts could be determined using MIC linear equations based on the desired assembly tolerances. In order to obtain the desired tolerance for the distance between two edges, "Eqs. (7) and (8)", as well as the sensitivity matrix given in "Table A1" (Appendix) are considered, and "Eqs. (10) and (11)" are derived for the case study:

$$-1.00163\mu_{p1} + 0.9714\mu_{p2} = \mu_a \tag{11}$$

$$1.003263\sigma_{p1}{}^2 + 0.94371\sigma_{p2}{}^2 = \sigma_a{}^2 \tag{12}$$

Where, $\sigma_{p1}{}^2$ and $\sigma_{p2}{}^2$ are the variance of absolute radius deviation in part 1 and 2, respectively, and $\sigma_a{}^2$ is the variance of distance between edges deviation in the assembly. μ_{p1} and μ_{p2} are mean values of absolute radius deviation in part 1 and 2, respectively, and μ_a is the mean value deviation of distance between the two edges. For this case study, since the two parts are similar, it is reasonable to assume that the mean and standard deviation of the parts radius are approximately equal. Therefore, $\mu_a \cong 0$ would be true.

For instance, to achieve a tolerance of 240±2mm for the distance between two edges in the assembly, the tolerances that come into the designer's mind are 119.5±0.5 (120^{0}_{-1}) and 120.5±0.5 (120^{+1}_{0}) for the radius of part 1 and 2, respectively. However, by solving the equations, the tolerances of 118.4±1.6 and 121.6±1.6 mm are obtained for parts 1 and 2, respectively which results in a standard deviation of 0.56. The results indicate that the tolerance ranges of the radius for parts could be up to three times larger than the initial choice. "Eqs. (12) to (17)" are obtained based on the rows of CP2 and CP4 in the sensitivity matrix given in Table A2 (Appendix), as well as "Eqs. (7) and (8)". These equations are used to determine the radius tolerances of parts in order to achieve the desired radius tolerance for assembly.

$$+0.2045\mu_{p1} + 0.2114\mu_{p2} = \mu_{a2} \tag{12}$$

$$0.04182\sigma_{p1}{}^2 + 0.04469\sigma_{p2}{}^2 = \sigma_{a2}{}^2 \tag{13}$$

$$-0.2045\mu_{p1} + 0.19782\mu_{p2} = \mu_{a4} \tag{14}$$

$$0.04185\sigma_{p1}{}^2 + 0.03913\sigma_{p2}{}^2 = \sigma_{a4}{}^2 \tag{15}$$

$$\mu_{a2} + 3\sigma_{a2} = A \quad (16)$$

$$\mu_{a4} - 3\sigma_{a4} = B \quad (17)$$

Where, A and B are upper and lower limit of radius tolerance in the assembly, respectively. σ_{a2} and σ_{a4} are the standard deviation of radius in CP2 and CP4. μ_{a2} and μ_{a2} are the mean values of radius deviation for CP2 and CP4. Same as before, it could be assumed that: $\mu_{p1} = \mu_{p2}$ and $\sigma_{p1} = \sigma_{p2}$. Therefore, the following equations are obtained from Eqs. (12 to 17):

$$+0.4159\mu_{p1} + 0.8823774\sigma_{p1} = A \quad (18)$$

$$-0.40238\mu_{p2} - 0.85371\sigma_{p2} = B \quad (19)$$

The assembly has equal bilateral tolerance, so A=B. These two equations have no appropriate solution together. Therefore, these equations should be solved separately. In this case study, it is considered that $3\sigma_p$ is equal to μ_{p1} for each part. The minimum value calculated for the standard deviation from "Eqs. (18) and (19)" is chosen as the final result.
For instance, if the tolerance of 120±1mm is considered for the radius of the assembly, the mean value of radius deviation and standard deviation of the radius in each part from "Eq. (18)" will be 1.4084 and 0.4695, respectively, and from "Eq. (19)" will be 1.4557 and 0.4852. Therefore, the designer could choose a tolerance of 121.4±1.4 for the radius of part 2 and 118.6±1.4 for part 1. This wide range of tolerance could lower the product cost.

5 CONCLUSIONS

The Interactive Worst Case (IWC) method, in this paper, is introduced for tolerance analysis of flexible sheet metal assembly. By using finite element analysis in this method, it is possible to calculate the tolerance of assembly for each extreme case in which sheet metals have their maximum or minimum size limits. Although the proposed method is very simple, it could be helpful in the tolerance analysis. For complicated cases, the upper and lower limits of assembly tolerances could be determined by plotting the values of specific characteristics at various points along the assembly parts. The diagram provides a general overview of the effects of deviations in parts on the deviations of the assembly. The accuracy of the method was verified by comparing the results of the case study in which two quarter cylinders are joined together by spot welding, with ones obtained from the uniform MIC.
In order to use MIC for curved sheet metal parts, the MIC equations were expanded to 3-D equations in this paper. The results were compared to the results of the MCS method, and according to it, the MIC method keeps its accuracy when the parts are curved. The best tolerances of the parts could be determined according to the given assembly tolerance by means of MIC equations. This approach helps to have wider tolerance ranges and reduces the cost of parts product. Therefore, this article proves that the MIC could be very helpful during the tolerance allocation process.

8 APPENDIX

Table A1 The sensitivity matrix for calculating the distance between the two edges

	The deviation in the radius of part 2	The deviation in the radius of part 1
The distance between the two edges	-0.97145	1.00163

Table A2 The sensitivity matrix for calculating radius in the control points

	The deviation in the radius of part 2	The deviation in the radius of part 1
The radius of CP1	-0.48572	0.500815
The radius of CP2	-0.2114	-0.2045
The radius of CP3	0.18684	-0.209584
The radius of CP4	0.197818	0.204561
The radius of CP5	-0.48572	0.500815

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