FA-ABC: A Novel Combination of Firefly Optimization Algorithm and Artificial Bee Colony for Mathematical Test Functions and Real-World Problems

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Abstract: In this research study, an attempt is made to present a new optimization scheme by combination of the firefly algorithm and artificial bee colony (FA-ABC) to solve mathematical test functions and real-world problems as best as possible. In this regard, the main operators of the two meta-heuristic algorithms are employed and combined to utilize both advantages. The results are compared with those of five prominent well-known approaches on sixteen benchmark functions. Moreover, thermodynamic, economic and environmental modeling of a thermal power plant known as the CGAM problem is represented. The proposed FA-ABC algorithm is used to reduce the total cost and increase the efficiency of the system as shown in the Pareto front diagrams.

Keywords: Artificial Bee Colony Algorithm, CGAM Problem, Firefly Algorithm, Hybrid Optimization Algorithm

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1 INTRODUCTION

The beginning of the twentieth century can be considered as the beginning of the widespread use of mathematical models and optimization fields. Mathematical optimization in engineering refers to select the best member from a set of achievable members. In the simplest form, an attempt is made to obtain the maximum and minimum values of a real function by systematically selecting data from an achievable set and calculating the value of a real function. Optimization consists of several sectors such as operational research, artificial intelligence and even computer science. It has been used in many different fields, such as energy, agriculture, industry, management, economy, business etc. which shows the importance of the issue of optimization.
These branches together can help us improve the efficiency of industrial systems [1]. Analytical optimization methods seek to solve problems accurately. Therefore, they include derivation of the objective function for finding the optimal solution [2]. The main advantage of this type of optimization methods is to guarantee the optimal solution, but it is difficult to use then for problems with high complexity or large number variables or discrete functions [3]. Therefore, the need for more efficient optimization methods seems necessary. Researches show that nature inspired meta-heuristic optimization methods have more ability in comparison with the classical methods to solve these kind of optimization problems [4]. These methods have been more widely used in the last two decades due to increasing the speed and power of the computers. They can be grouped in five main categories (see “Fig. 1”): evolution based, physics based, swarm intelligence based, human based and hybrid methods. Holland introduces Genetic Algorithm (GA) as one of the first meta-heuristic algorithms to solve complicated optimization problems [5].

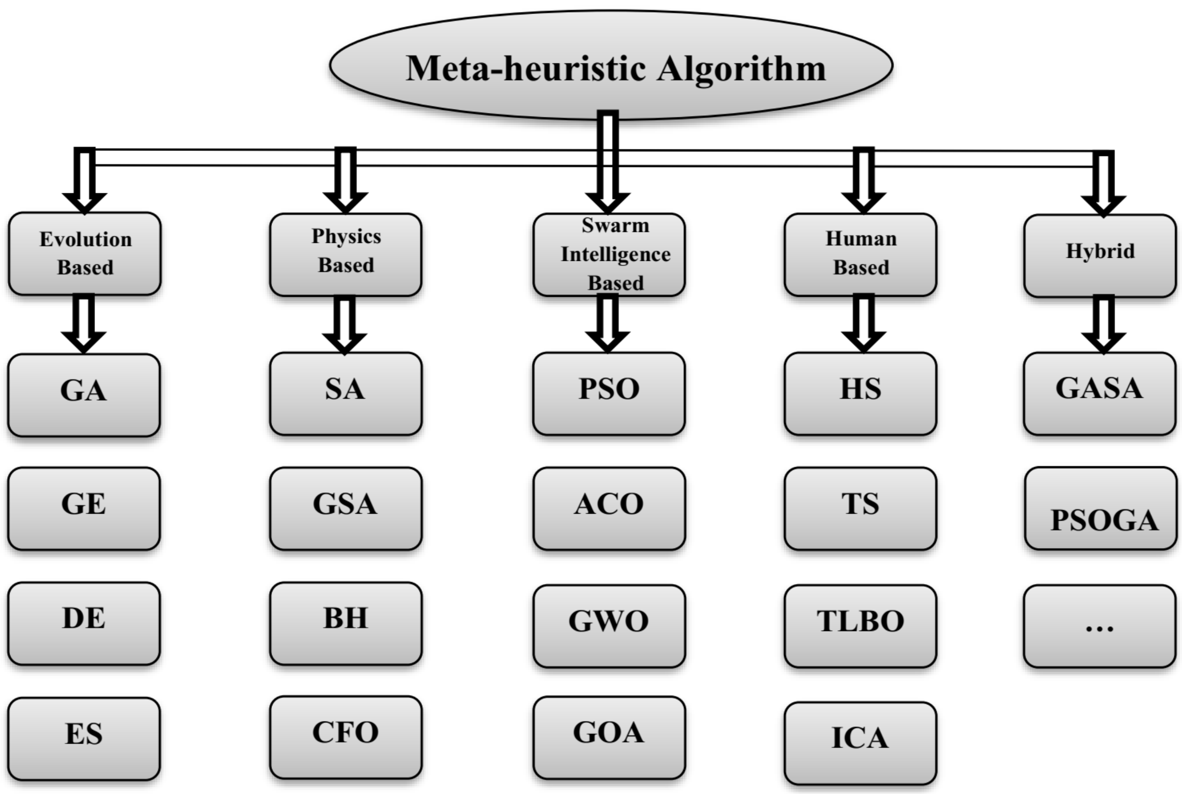


Fig. 1 Classification of the meta-heuristic algorithms.

It is a special type of evolutionary algorithms that uses some biological techniques such as crossover, mutation biology, and Darwin's principles to find the optimal solutions. In 1995, Eberhart and Kennedy originally proposed the particle swarm optimization algorithm inspired by the swarm movement of birds looking for food sources [6]. Ant colony algorithm was introduced through studies and observations on ant colonies [7]. The studies have shown that the ants are social insects that live in colonies and their behavior is more colonial than individual. This algorithm is generally based on the swarm intelligence of ants. In 2014, Mirjalili et al. introduced a meta-heuristic algorithm called Gray Wolf Optimization (GWO) [8]. Meta-heuristic methods are inspired by the hierarchical structure and social behavior of gray wolves while hunting. In 2019, the Harris Hawk Optimization (HHO) was presented by Heydari et al. [9] Inspired by the participatory behavior and pursuit style of the hawk in nature, which is known as surprise attack. In this smart strategy, several falcons collaboratively work to surprise a prey from different angles. The algorithms were some of the outstanding optimization approaches, while many other algorithms have been introduced in recent years, Such as: artificial fish swarm optimization algorithm [10], cat swarm optimization algorithm [11], imperialist competition optimization algorithm [12], magnetic optimization algorithm [13], artificial bee colony optimization algorithm [14], firefly optimization algorithm [15], bat optimization algorithm [16], lions optimization algorithm [17], whale optimization algorithm [18], dragonfly optimization algorithm [19], bacterial foraging optimization algorithm [20] etc. In recent decades, researchers have combined meta-heuristic algorithms together to achieve new techniques algorithms that have better performance and faster convergence speed [21-22]. For instance, Yu and Zhu developed a PSO-GA optimal model to predict energy demand in China using gross domestic product, population, economic structure, urbanization rate, and energy structure with linear, exponential and quadratic forms [23]. Kiran et al. presented a novel hybrid algorithm based on the PSO and ACO for energy demand forecasting in Turkey [24]. In another work, they also applied two new models in order to estimate electricity energy demand in Turkey using Artificial Bee Colony (ABC) and PSO algorithms [25]. Further, Yu and Zhu proposed a hybrid technique based on the PSO and GA to improve energy demand estimation in China by applying linear, exponential, and quadratic models and considering the GDP, population, economic structure, urbanization rate, and energy consumption structure [26]. Piltan et al. used the PSO and GA to attain the parameters of the energy demand forecasting model in Iranian metal industries [27].

In the present paper, an attempt is made to combine two meta-heuristic algorithms to provide an algorithm that is able to improve the solutions of the optimization problems. As the novelty of the present study, the firefly optimization algorithm and the artificial bee colony are the two algorithms discussed in this paper. By combining the two algorithms, it can be seen that the proposed algorithm has much better performance than other optimization algorithms for solving mathematical benchmark problems. Next, thermodynamic, economic and environmental modeling of a power plant with a production capacity of 30MW and 14 $^{Kg}/_{s}$ of saturated steam at a pressure of 20bar known as the CGAM problem is discussed. Eventually, the proposed optimization algorithm (FA-ABC) is successfully employed to reduce the total cost and increase efficiency of the system, and the results are depicted in the form of Pareto front diagrams.

2 OPTIMIZATION ALGORITHMS

2.1. Firefly Algorithm

The firefly algorithm was initially designed through modeling the luminosity of fireflies and their behavior in nature by Xin-She Yang in 2008 at the University of Cambridge [28]. Most of fireflies produce short and rhythmic lights with unique and particular patterns. The main task of these lights is attracting for hunting. The attractiveness of each firefly is related to the power of its light radiation. In fact, for each pair of fireflies, the one with less light is attracted to the other having side, which has lighter [28]. The motion of firefly Ith to ward firefly Jth is simulated via the following Equation.

$$x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r_{ij}^2}\left(x_j^t - x_i^t\right) + \alpha\epsilon_i^t \quad (1)$$

Where, x_i and x_j denote the position of firefly ith and jth, respectively. t represents the iteration number. The second part of the right-hand side is employed to simulate the attractiveness of the fireflies, while the third part randomly changes the position to escape from local minimum points. Moreover, α is a random coefficient and ϵ_i is a vector of random numbers having Gaussian or uniform distributions. For simplicity, $\in_i$ can be replaced by random number with the uniform distribution in range of [0, 1]. In the actual implementation, attractiveness function $\beta(r)$ can be any function with the uniform decreasing. For example, the following general Equation could be considered for computing attractiveness function:

$$\beta(r) = \beta_0 e^{-\gamma r^m}(m \geq 1) \quad (2)$$

Where, β_0 is the attractiveness in $r = 0$. For length scale Γ in an optimization example, parameter γ can be regarded as a typical initial value having the following relation.

$$\gamma = \frac{1}{\Gamma^m} \quad (3)$$

2.2. Artificial Bee Colony Algorithm

The artificial bee colony optimization algorithm was originally introduced in 2005 by Dervis Karaboga to solve unconstrained problems [29]. Based on the searching behavior of the bee to find food source, they are divided into three parts: scout bee, employed bee and onlooker bee [30]. Initially, a set of random solutions $\left(P_1, \ldots, P_{N_e}\right)$ are generated by the scout bee.

$$x_i^j = x_{min}^j + r(x_{max}^j - x_{min}^j) \quad (4)$$

Where, $j\epsilon\{1,2,\dots,D\}$ is a dimension-based value, D represents the number of design variables and N_e represents the number of the food sources. When the scout bees return to the hive, they interact the employed bee by performing an especial dance. At this stage, depending on the richness of each food source, the number of employed bees sent to each source are different. Moreover, by applying Equation (5), the employed bees try to improve the found solutions.

$$V_i^j = x_i^j + \varphi_i^j(x_i^j - x_k^j) \quad (5)$$

Where, $V_i \epsilon\ \Omega$ represents a new solution generated in vicinity of existing solution P_i. Further $j\epsilon\{1,2,\dots,D\}$ and $k\epsilon\{1,2,\dots,N_e\}$ are randomly selected, while $k \neq i$. φ_i is also a real random number between 1 and -1. At this stage, according to the richness of each food source (solution), the Probability of each source is obtained by using the following Equation:

$$p_i = \frac{fit_i}{\sum_{n=1}^{N_e} fit_n} \quad (6)$$

Which, in this Equation, fit_i would be calculated from the following Equation:

$$fit_i = \begin{cases} \dfrac{1}{1+\sqrt{\sum_{n=1}^{N_e} f_i}} & if\ f_i \geq 0 \\ 1+\sqrt{\left|\sum_{n=1}^{N_e} f_i\right|} & if\ f_i < 0 \end{cases} \quad (7)$$

Where, f_i is the value of the cost function related to bee i. When the employed bee returned to the hive and shared their information with the onlookers, according to the suitability of each food source obtained from Equation (7), and a probabilistic selection algorithm such as the roulette wheel method, one of those sources is selected. As the last step, if one of the food sources could not be improved after a certain number of cycles, the employed bee leaves that food source (answer) and randomly replaces it by a new source.

3 FA-ABC OPTIMIZATION ALGORITHM

In this section, the proposed optimization algorithm based on the firefly algorithm and artificial bee colony is discussed, in more details. As it was mentioned in the previous sections, in the firefly optimization algorithm, the fitness of a firefly depends on the power of its light, mathematically formulated by applying Equation (1).

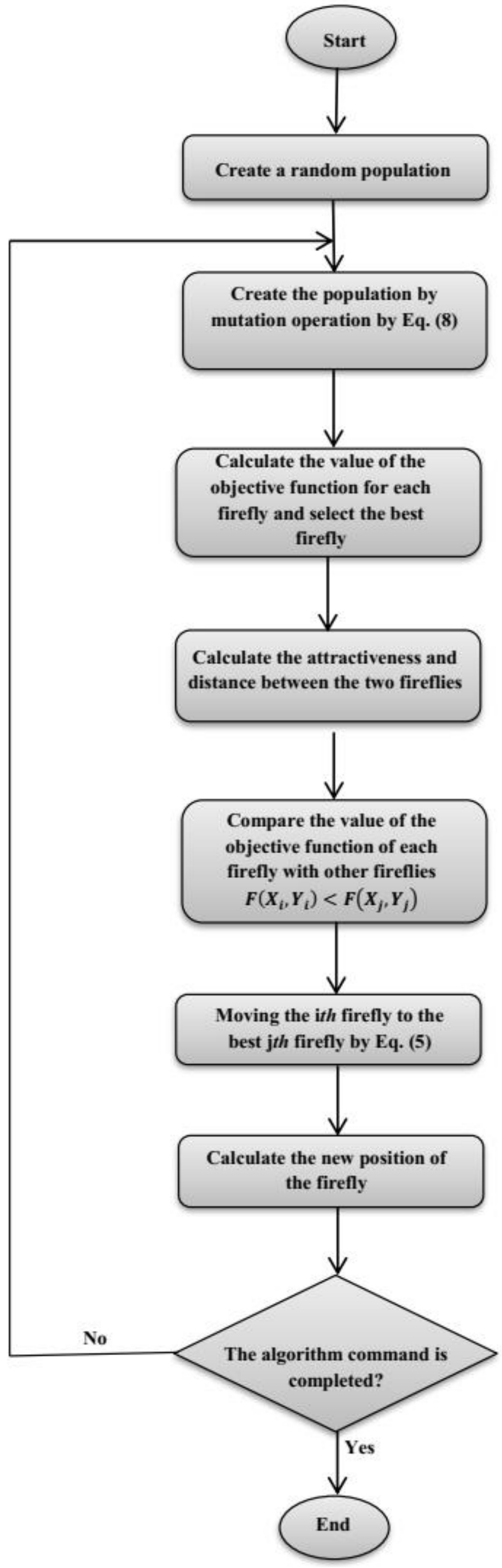


Fig. 2 Flowchart of the by FA-ABC algorithm.

On the other hand, in the artificial bee colony algorithm, after generating the initial solutions by the scout bee, share this information with the employed bee. According to the richness of resources, the employed bee would be sent to food sources in mathematical terms Formulated as Equation (5). Based on these distributions, the FA-ABC optimization algorithm is established in two steps. In the first step, if the fitness of the j*th* firefly is higher than that of the i*th* firefly, the i*th* firefly is attracted to the j*th* Firefly by applying the formulation that is used to send the employed bee to the food sources (Equation (5)).

Define the objective function, design variables and algorithm parameters
Starting the population of fireflies x_i (i =1,2,…,n)
***While** (it < Maximum iterations)*
Starting the population of mutation x_k (k = 1,2,…,k)
***For** k=1 to number of mutants*
Mutation operation by Eq. 8
***End For** k*
***For** i = 1 to all n fireflies*
***For** j = 1 to all n fireflies*
f (x_i) is used to ascertain the light intensity I_i at x_i
***If** ($I_j > I_i$)*
The firefly approaches firefly in all dimensions by Eq. 5
End If
Attractiveness fluctuates within the distance r by exp [-γr]
Assess new solution and update the intensity of the light
***End For** j*
***End For** i*
Devoting ranks to fireflies and obtaining the current best
End while
Analyzing the results and visualization

Fig. 3 Pseudo-code proposed by the algorithm FA-ABC.

In the second one, a mutation operator is used to escape the populations from the local minimum points. In fact, the mutation operator randomly changes the position of a number of the fireflies in the population. This number is determined as $P_m \times nPop$, where, P_m and $nPop$ are the probability of the mutation and the population size, respectively. This operator guarantees variety of the population and reduces the probability of the algorithm converging to the local optima. Let x_j represent a randomly selected firefly, then the mutation formulation is defined as:

$$x(new) = x(j) + \sigma \times randn \quad (8)$$

Where, σ is determined as $\frac{(x_{max}-x_{min})}{10}$, x_{max} and x_{min} are respectively the upper and lower bounds of the related domain and randn produces a random number with normal distribution. Figures 2 and 3 show the flowchart and the pseudo-code of the proposed algorithm, respectively.

4 COMPARISONS ON MATHEMATICAL TEST FUNCTIONS

In order to evaluate the accuracy and convergence speed of the proposed algorithm, several test functions with different properties illustrated in "Table 1" are used. Initially, the performance of the proposed algorithm in term of the accuracy of the solutions, is evaluated through comparison method, with the Particle Swarm Optimization (PSO), Firefly Algorithm (FA), Artificial Bee Colony (ABC), Imperialist Competition Algorithm (ICA) and Ant Colony Optimization (ACO), is evaluated. This evaluation is performed for the five algorithms under exactly the same conditions on the population size 25, maximum number of iterations 10000, and problem dimensions 10, 20 and 30. "Table 2" shows the results in terms of mean and standard deviation for 100 runs performed on the unimodal and multimodal functions.

Table 1 Description of the unimodal, multimodal and fixed-dimension multimodal benchmark functions

Function	Formulation	Domain	f_{min}
F1	$\sum_{i=1}^{n} X_i^2$	$[-100, 100]^n$	0
F2	$\sum_{i=1}^{n} iX_i^2$	$[-100, 100]^n$	0
F3	$\sum_{i=1}^{n} \lvert X_i \rvert + \prod_{i=1}^{n} \lvert X_i \rvert$	$[-10, 10]^n$	0
F4	$\sum_{i=1}^{n} \lvert X_i sin(X_i) + 0.1X_i \rvert$	$[0, 10]^n$	0
F5	$\sum_{i=1}^{n-1} \left[100\left(X_{i+1} - X_i^2\right)^2 + (X_i - 1)^2\right]$	$[-10, 10]^n$	0

F6	$\sum_{i=1}^{n}([X_i - 0.5])^2$	$[-100, 100]^n$	0
F7	$\sum_{i=1}^{n} iX_i^4 + random[0,1]$	$[-1.28, 1.28]$	0
F8	$\sum_{i=1}^{n-1}\left[X_i^2 - 10COS(2\pi X_i) + 10\right]$	$[-5.12, 5.12]$	0
F9	$-20exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} X_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi X_i)\right) + 20 + exp$	$[-32, 32]^n$	0
F10	$\left(\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j + \sum_{j=1}^{2}(x_i - a_{ij})^6}\right)^{-1}$	$[-65, 65]^n$	1
F11	$4X_1^2 - 2.1X_1^4 + \frac{1}{3}X_1^6 + X_1X_2 - 4X_2^2 + 4X_2^4$	$[-5, 5]^n$	−1 .0316
F12	$\left(X_2 - \frac{5.1}{4\pi^2}X_1^2 + \frac{5}{\pi}X_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)cosX_1 + 10$	$[-5,5]^n$	0.389
F13	$[1 + (X_1 + X_2 + 1)^2(19 - 14X_1 + 3X_1^2 - 14X_2 + 6X_1X_2 + 3X_2^2)] \times [30 + (2X_1 - 3X_2)^2 \times (18 - 32X_1 + 12X_1^2 + 48X_2 - 36X_1X_2 + 27X_2^2)]$	$[-2, 2]^n$	3
F14	$-\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{6} a_{ij}(X_j - p_{ij})^2\right)$	$[0, 1]^n$	-3.32
F15	$-\sum_{i=1}^{5}[(X - a_i)(X - a_i)^T + c_i]^{-1}$	$[0, 10]^n$	-10.1532
F16	$-\sum_{i=1}^{7}[(X - a_i)(X - a_i)^T + c_i]^{-1}$	$[0, 10]^n$	-10.1532

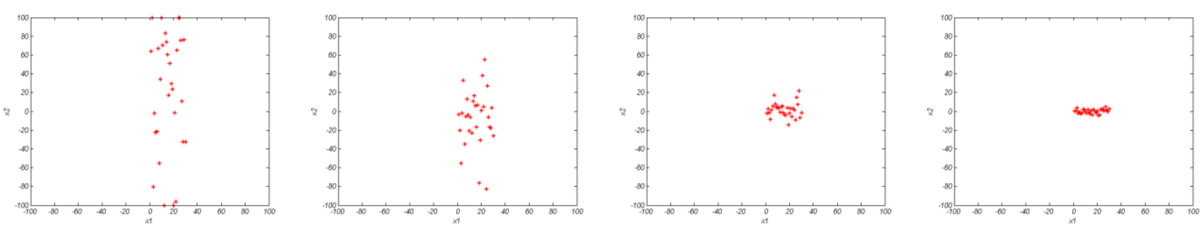
Table 2 Comparison of mean and standard deviation of the results related to the unimodal and multimodal test function found by the five optimization algorithms

		PSO [6]		FA [15]		ABC [14]		ICA [12]		ACO [7]		FA-ABC	
	D	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F1	10	0	0	1.3e-177	0	1.3e-277	0	1.1e-37	4.5e-37	0	0	0	0
	20	5.5e-268	0	4.8e-177	0	8.4e-68	8.4e-67	2.3e-31	9.0e-31	3.8e-175	0	0	0
	30	5.7e-160	3.8e-59	7.2e-146	7e-145	3.1e-05	1.1e-04	7.4e-28	3.2e-27	4.1e-56	1.3e-55	1.6e-220	0
F2	10	0	0	9.8e-180	0	1.6e-280	0	4.6e-39	1.8e-38	0	0	0	0
	20	1.1e-269	0	1.6e-178	0	5.6e-207	0	2.3e-33	5.9e-33	1.6e-176	0	0	0
	30	1.1e-157	1e-156	6.4e-46	6.4e-45	3.6e-104	3.6e-103	6.4e-30	1.5e-29	1.1e-57	2.9e-57	6.4e-258	0
F3	10	1.5e-245	0	9.2e-91	9.2e-90	1.2e-221	0	1.1e-20	2.5e-20	0	0	0	0
	20	2.9e-106	2.9e-105	3.3e-90	3.3e-89	5.2e-110	5.2e-109	1.3e-17	2.5e-17	4.2e-111	1.9e-110	0	0
	30	7.4e-47	7.4e-46	8.6e-58	8.6e-57	9.8e-74	9.8e-73	5.4e-16	7.9e-16	2.4e-37	9.2e-37	8.7e-170	0
F4	10	7.7e-15	2.4e-15	1.3e-15	1.4e-15	0	0	4.94e-15	3.2e-15	0	0	0	0
	20	1.1e-11	1.1e-10	5.1e-15	3.3e-15	0.05411	0.04365	1.19e-14	4.7e-15	6.1e-18	6.1e-17	0	0

	30	4.7e-11	3.8e-10	3.7e-08	3.7e-07	0.41390	0.14541	1.6e-14	5.5e-15	0.00536	0.0105	0	0
F5	10	0.5990	1.4302	1.1920	1.9956	0.2393	0.1189	0.5868	1.0894	0.03751	0.2647	0.0926	0.298
	20	1.1171	1.8048	1.2370	2.2187	1.8269	0.9763	2.0789	2.5173	0.25561	0.6873	0.0065	0.039
	30	4.3200	3.1267	2.9043	16.836	6.9863	3.3057	3.5983	9.6728	20.3192	21.5355	0.4705	1.247
F6	10	0	0	0.05	0.297	0	0	0	0	0	0	0	0
	20	0.21	0.795	0.94	1.391	0.41	0.4943	0	0	0	0	0	0
	30	3.61	6.607	3.97	6.963	3.36	1.534	0	0	0	0	0	0
F7	10	4.2e-04	4.1e-04	2.6e-05	1.7e-05	0.0209	0.0074	0.0011	6.9e-04	1.8e-04	9.6e-05	3.8e-05	2.7e-05
	20	0.0014	9.1e-4	3.2e-04	4.3e-04	0.1524	0.0365	0.0043	0.0017	0.0016	6.1e-04	4.8e-04	8.5e-04
	30	0.0039	0.0024	0.00151	0.002362	0.3961	0.0841	0.0091	0.0033	0.0087	0.0022	0.0029	0.0032
F8	10	11.660	6.0116	9.9595	4.9404	4.3e-14	1.3e-13	0	0	5.16003	7.11467	0	0
	20	27.4707	10.8402	34.2298	15.3010	0.0025	0.0198	0	0	95.6027	9.2307	2.7e-06	2.4e-05
	30	48.1459	15.9637	69.1164	23.1419	1.0387	1.0748	0	0	202.5606	12.8040	3.7e-05	1.6e-04
F9	10	0.02310	0.1625	6.8e-15	1.8e-15	2.1e-14	6.2e-15	1.2e-14	4.7e-15	3.7e-15	1.4e-15	6.3e-15	1.8e-15
	20	0.5668	0.8001	0.09583	0.39255	3.4e-13	2.4e-13	3.2e-14	7.8e-15	4.4e-15	0	1.6e-14	6.4e-15
	30	1.5397	0.9358	0.69655	0.88198	2.8e-12	1.9e-12	6.1e-14	1.1e-14	6.9e-15	1.6e-15	2.2e-14	3.6e-15

Table 3 Comparison of mean and standard deviation of five optimization algorithms for fixed-dimension multimodal functions

		PSO [6]		FA [15]		ABC [14]		ICA [12]		ACO [7]		FA-ABC	
	D	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
F10	2	4.3903	3.8734	1.1171	0.4519	0.9980	4.3e-16	0.9980	9.8e-16	1.1364	0.7154	0.9980	4.6e-13
F11	2	-1.0316	1.5e-15	-1.0316	1.3e-15	-1.0316	1.5e-15	-1.0316	1.5e-15	-1.0316	1.5e-15	-1.0316	1.3e-15
F12	2	0.3979	1.1e-15	0.3979	1.1e-15	0.3979	1.1e-15	0.3979	1.1e-15	0.3979	1.1e-15	0.3979	1.1e-15
F13	2	3.54	3.799	3	1.4e-15	3	2.02e-5	3	1.2e-15	3	1.2e-15	3	1.2e-15
F14	6	-3.2732	0.588	-3.2661	0.0596	-3.3220	2.2e-15	-3.2828	0.0562	-3.2768	0.0580	-3.2761	0.0542
F15	4	-4.8733	3.1092	-8.7465	2.6504	-10.1532	1.9e-14	-7.2145	2.7e-10	-5.2322	3.5507	-9.7023	1.6825
F16	4	-6.6937	3.5730	-9.9012	1.7263	-10.4029	1.4e-14	-7.1314	3.3715	-8.2558	3.3090	-10.0206	1.5353



Iteration = 1 Iteration = 30 Iteration = 60 Iteration = 90

Fig. 4 Positions of the fireflies observed in 4 stages of the FA-ABC process for Sphere function.

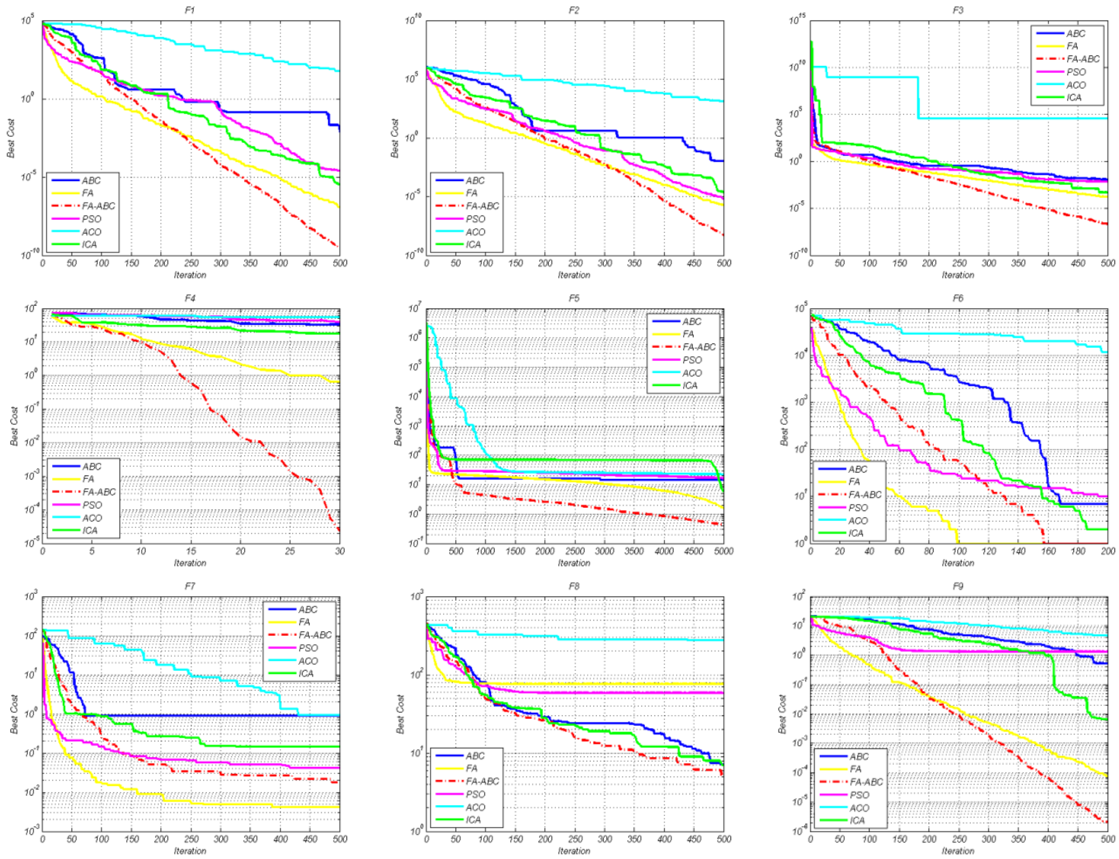


Fig. 5 Comparison of the convergence process of the five algorithms applied on the test functions for 30 dimensions.

"Table 3" shows the results obtained by the six algorithms in terms of mean and standard deviation for the fixed-dimension multimodal functions. In "Fig. 4", the positions of the fireflies are observed in 4 steps of the FA-ABC process with population size 25 for the sphere function having dimension 30. This figure shows that the population is distributed in the initial iterations. After 60 iterations, the particles gather round to the position of the best particle. At the end, in about 90 iterations, all particles can find the best solution. Figure 5 shows a comparison of the convergence process of the five algorithms (PSO, FA, ABC, ICA and ACO) on the test functions having dimension 30. This figure shows that the proposed algorithm performs better for most functions than other algorithms.

In order to make further investigation, the shifted test functions are employed for comparison of the five algorithms. In "Tables 4", the test functions mentioned in "Tables 1" have been shifted one unit. Therefore, the shifted test functions have a minimum value of 0 at $x_i = 1$, except for G5 which has a minimum of 0 at $x_i = 2$. Table 5 shows the results in terms of mean and standard deviation for these Shifted unimodal and multimodal functions found in 100 runs. In this evaluation, the dimensions of the problem are 30, the maximum number of iterations is set at 10000, and the number of populations is considered as 25.

Table 4 Shifted unimodal and multimodal test functions

Function	Formulation	Domain	f_{min}
G1	$\sum_{i=1}^{n}(X_i - 1)^1$	$[-100, 100]^n$	1
G2	$\sum_{i=1}^{n} i(X_i - 1)^2$	$[-100, 100]^n$	1
G3	$\sum_{i=1}^{n}\lvert X_i - 1\rvert + \prod_{i=1}^{n}\lvert X_i - 1\rvert$	$[-100, 100]^n$	1

G4	$\sum_{i=1}^{n}\lvert(X_i-1)sin(X_i-1)+0.1(X_i-1)\rvert$	$[0,10]^{n}$	1
G5	$\sum_{i=1}^{n-1}\left[100\left((X_{i+1}-1)-(X_i-1)^2\right)^2+\left((X_i-1)-1\right)^2\right]$	$[-10,10]^{n}$	2
G6	$\sum_{i=1}^{n}([(X_i-1)-0.5])^2$	$[-100,100]^{n}$	1
G7	$\sum_{i=1}^{n}i(X_i-1)^4+random[0,1]$	$[-1.28,1.28]^{n}$	1
G8	$\sum_{i=1}^{n-1}\left[(X_i-1)^2-10COS\left(2\pi(X_i-1)\right)+10\right]$	$[-5.12,5.12]^{n}$	1
G9	$-20exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i-1)^2}\right)-exp\left(\frac{1}{n}\sum_{i=1}^{n}cos\left(2\pi(X_i-1)\right)\right)+20+exp$	$[-32,32]^{n}$	1

Table 5 Comparison of mean and standard deviation of five optimization algorithms for shifted unimodal and multimodal functions

		PSO [6]		FA [15]		ABC [14]		ICA [12]		ACO [7]		FA-ABC	
	D	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
G1	30	1.6e-31	7.9e-31	3.7e-31	1.6e-30	5.6e-05	1.7e-04	3.1e-28	8.9e-28	4.1e-32	4.4e-32	3.2e-32	4.2e-32
G2	30	8.3e-29	6.6e-28	3.7e-30	5.3e-30	0.0022	0.0027	1.5e-27	3.0e-27	6.72e-31	7.8e-31	4.1e-31	7.3e-31
G3	30	1.0e-11	8.0e-11	5.1e-13	5.1e-12	3.1e-15	1.6e-15	8.7e-16	2.1e-15	1.9e-17	6.1e-17	9.9e-18	3.8e-17
G4	30	1.0e-11	6.8e-11	0.2726	0.6511	0.016	0.0101	9.9e-15	2.2e-15	0.0027	0.0078	8.6e-16	8.6e-16
G5	30	4.853	7.155	10.29	89.14	9.70	3.94	4.15	12.006	19.53	12.69	6.4145	7.8381
G6	30	6.22	17.21	5.38	8.20	3.14	1.38	0	0	0	0	0	0
G7	30	0.0038	0.0023	0.0358	0.069	0.53	0.085	0.10	0.0037	0.0090	0.0027	7.3e-04	0.0010
G8	30	58.3	18.7	71.737	21.2	1.02	0.95	0	0	2.01e+02	11.44	0.0089	0.0357
G9	30	1.45	0.08	0.67	0.89	2.0e-12	2.5e-12	3.9e-14	1.7e-14	2.7e-15	9.6e-16	1.3e-14	2.9e-15

5 CGAM PROBLEM MODELING

5.1. CGAM Problem

Conventional mathematical optimization methods are not efficient for large systems having many design variables. Moreover, economic and exergy analysis can be used to help optimization of these problems [31]. The word CGAM is derived from the first letters of the names of four researchers who have worked in this field (C. Frangopoulos, G. Tsatsaronis, A. Valero and M. Von Spakovsky). In 1990, a group of experts in the field of the large thermal systems have compared different solving methods for optimization of the CGAM [32]. In this study, by examining the economic, environmental and thermodynamic analysis of the CGAM problems and considering the system costs and efficiency as the objective functions, the system is optimized by using the FA-ABC algorithm. This system includes the air compressor, combustion chamber, air preheater, gas turbine and heat recovery. In this problem, environmental conditions are defined as $P_0 = 1.013\ bar$ and $T_0 = 298.15K$. The fuel injected into this system is pure methane with a Lower Heating Value (LHV) of 50000 $^{Kj}/_{Kg}$. The system design parameters include compressor pressure ratio (P_{AC}), isentropic compressor efficiency (η_{AC}), turbine isentropic efficiency (η_{GT}), gas turbine inlet temperature (T_4) and air preheater efficiency (ε_{ap}) [33].

5.2. Economical Modeling

To calculate the investment costs, which include the cost of equipment purchase and maintenance, the following relation is proposed.

$$\dot{Z}_K = Z_K CRF\varphi/(N*3600) \quad (11)$$

Where, Z_K is the cost of purchasing for the K component in dollars. As we know, the global energy price does not have much fluctuation in the dollar, and the results in this article will be usable for a longer period of time. CRF is the annual capital recovery factor used to estimate the equipment life (its value in this article is equal to 0.182). N stands for the system operating hours per year (8000 hours). Parameter φ is the maintenance coefficient that its value is determined according to the type of the power plant, considered as 1.6 in this article. The fuel cost flow rate ($\dot{C}_f$) is given by:

$$\dot{C}_f = \dot{m}_f * C_f * LHV \quad (12)$$

Where, C_f is the fuel cost (in this paper is considered to be 0.000004) and m_f is the mass flow rate of the fuel, and LHV is the Lower Heating Value, which is equivalent to 50000 $^{Kj}/_{Kg}$ for methane [34]. Equations (13 to 17) are related to the cost of equipment, including compressor air preheater, gas turbine, combustion chamber and heat recovery, respectively. Table 6 shows the constants for Equations (13 to 17).

$$Z_{AC} = \left(c_{11} * \frac{\dot{m}_a}{c_{12}-\eta_{AC}}\right) * r_{cp} * \log(r_{cp}) \quad (13)$$

$$Z_{APH} = C_{41}\dot{m}_g * \frac{(h_5 - h_6)}{U\Delta T_{lmAPH}} \quad (14)$$

$$Z_{cc} = \left(c_{21} * \frac{\dot{m}_a}{c_{22}-\frac{P_3}{P_4}}\right) * [1 + \exp(c_{23}T_4 - C_{24})] \quad (15)$$

$$Z_{GT} = \left(C_{31} * \frac{m_g}{c_{32} - \eta_{GT}}\right) * \log\left(\frac{P_4}{P_3}\right) * [1 + \exp(c_{33}T_4 - C_{34})] \quad (16)$$

$$Z_{HRSG} = c_{51} * \left(\left(\frac{\dot{Q}_{EC}}{\Delta T_{lmEC}}\right)^{0.8} + \left(\frac{\dot{Q}_{EV}}{\Delta T_{lmEV}}\right)^{0.8} \right)\dot{m}_{st} + c_{52}\dot{m}_{st} + c_{53}\dot{m}_g^{1.2} \quad (17)$$

Table 6 Numerical values of the variables related to the economic model

Air compressor	Combustion chamber	Gas turbine	Air preheater	Heat recovery
$c_{11} = 39.5\$/(^{kg}/_s)$ $c_{12} = 0.9$	$c_{21} = 25.6\$/(^{kg}/_s)$ $c_{22} = 0.995$ $c_{23} = 0.018K^{-1}$ $c_{24} = 26.4$	$c_{31} = 266.3\$/(^{kg}/_s)$ $c_{32} = 0.92$ $c_{33} = 0.036K^{-1}$ $C_{34} = 54.4$	$c_{41} = 2290\$/(m^{1/2})$ $c_{42} = 0.018(\frac{KW}{m^2k})$	$c_{51} = 3650\$/(KW/K)^{.8}$ $c_{52} = 11820\$/(^{kg}/_s)$ $c_{53} = 658\$/(^{kg}/_s)^{.5}$

5.3. Thermodynamical Modeling

The CGAM problem refers to a cogeneration plant which provides 30MW electricity and 14 $^{Kg}/_s$ saturated steam at 20bar pressure. Figure 6 shows a schematic of the problem, and the temperature changes of the air preheater and heat recovery. In order to avoid forming the sulfuric acid and corrosion of the tubes, the temperature of exhaust gases must be kept more than 400 K. Other specifications and operating conditions of the problem are regarded according to Ref. [34]. Moreover, the following assumptions are considered in this work. (A) All processes are steady-state, (B) The law of the ideal gas is used for the air and combustion gases, (C) The pressure drop of the combustion chamber, air preheater and HRSG are given and (D) Heat loss in the combustion chamber is 2% of LHV and the other processes are considered to be adiabatic.

5.4. Environmental Modeling

Since determining the pollutant emissions is essential to organize an environmental objective function, an extra model, based on semi-analytical correlations, is added here for thermo economical modeling of the plant. The adiabatic flame temperature at the primary zone of the combustion chamber is derived from the expression introduced in [35] as follows:

$$T_{PZ} = A\sigma^x * \exp(\beta(\sigma+\lambda)^2 * \pi^x * \theta^y * \psi^z \quad (18)$$

Where, π is a dimensionless pressure (p/p_{ref} that p being the combustion pressure p_3, and $p_{ref} = 1atm$). θ is a dimensionless temperature T/T_{ref} (T being the inlet temperature T_3, and $T_{ref} = 300K$); ψ is the $^H/_C$ atomic ratio (ψ = 4); θ is the dimensionless temperature. The factors consisting of A, β, λ, a_i, b_i and c_i are constants presented in [35-36]. In order to have an accurate prediction, four sets of constants for the following ranges are used.

$$0.3 < \emptyset < 1 \ \& \ 0.92 < \theta < 2.0 \quad (19)$$

$$0.3 < \emptyset < 1 \ \& \ 2 < \theta < 3.2 \quad (20)$$

$$1 < \emptyset < 1.6 \ \& \ 0.92 < \theta < 2.0 \quad (21)$$

$$1 < \emptyset < 1.6 \ \& \ 2 < \theta < 3.2 \quad (22)$$

Also, x, y and z are polynomial functions of σ introduced by:

$$x = a_1 + b_1\sigma + c_1\sigma \quad (23)$$

$$y = a_2 + b_2\sigma + c_2\sigma \quad (24)$$

$$z = a_3 + b_3\sigma + c_3\sigma \quad (25)$$

The adiabatic flame temperature is used in the semi-empirical correlations proposed by Rizk and Mongia [36] to determine the pollutant emissions in grams per kilogram of fuel as follows:

$$\dot{m}_{NOx} = \left(\frac{0.5\times10^{16}\times(\tau)^{0.5}\times\exp(\frac{-7100}{T_{pz}})}{P_3^{0.05}(\frac{\Delta P_3}{P_3})^{\,0.5}} \right) \quad (26)$$

$$\dot{m}_{co} = \left(\frac{0.179\times10^{9}\times exp(\frac{-7800}{T_{PZ}})}{P_3^{2}\times\tau\times(\frac{\Delta P_3}{P_3})^{\,0.5}} \right) \quad (27)$$

Where, τ is the residence time in the combustion zone (τ is assumed constant and equal to 0.002s), T_{pz} is the temperature in the primary zone combustion; p_3 is the pressure in the combustor inlet, $\Delta p_3/p_3$ is the non-dimensional pressure drop in the combustor ($\Delta p_3/p_3$ = 0.05 is assumed for the considered CGAM problem). Note that the temperature of the primary zone temperature is used in the NO_x correlation instead of the stoichiometric temperature, since the maximum attainable temperature in premixed flames is T_{pz}, as pointed out by Lefebvre [37]. C_{CO_2} and C_{NO_x} are equal to 0.02086 $^{\$}/_{Kg} - fule$ and 6.853 $^{\$}/_{Kg} - fule$, respectively.

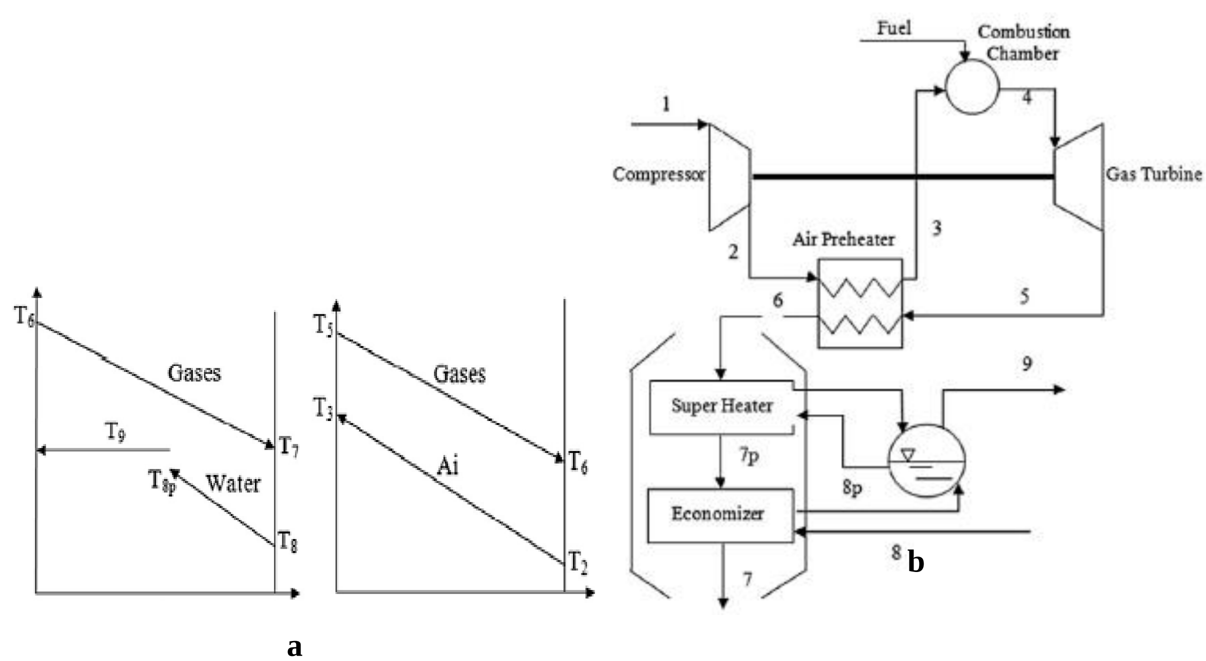


Fig. 6 (a): Schematic of the CGAM problem, and (b): its temperature profiles for the air preheater and heat recovery steam generator.

5.5. Decision Variables and Constraints

The following parameters in this paper are considered as the decision variables for the optimization process: compressor pressure ratio (P_{AC}), compressor isentropic efficiency (η_{AC}), turbine isentropic efficiency (η_{GT}), recuperator efficiency (ε_{ap}), and gas turbine inlet temperature (T_4). Moreover, the feasible ranges of these decision variables mentioned in "Table 7" are applied as the constraints of the problem and implemented through the final approach for the proposed FA-ABC algorithm.

Table 7 Feasible ranges for the design variables

Cause	constraint
Metallurgical temperature limitations	$1400 < T_4 < 1650$
Available in the Market	$7 < P_{AC} < 16$
Available in the Market	$\eta_{AC} = 0.8468$
Available in the Market	$0.6 < \varepsilon_{ap} < 0.9$
Available in the Market	$\eta_{GT} = 0.8786$

5.6. Objective Function

Thermodynamical, economic and environmental objectives are considered to design the CGAM problem. The pollution costs are also added directly to the investment costs and the fuel costs. Therefore, the following objective functions are defined for multi-objective optimization.

$$\eta_H = \frac{\dot{W}_{net} + \dot{m}_g(e_9 - e_8)}{\dot{m}_f e_f} \quad (28)$$

$$\dot{C}_{fuel\&investment} = \dot{C}_f + \sum_i \dot{Z}_i + \dot{C}_{env} \quad (29)$$

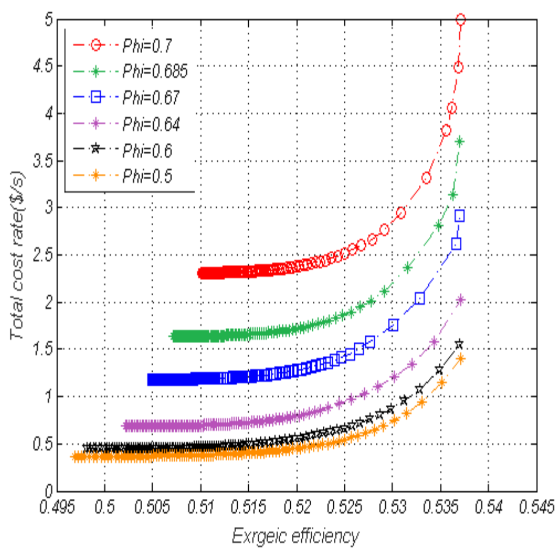


Fig. 7 Pareto fronts for different equivalence ratios (C_f = 0:004 $/kg-fuel and NO_x =0.05 g/kg-fuel).

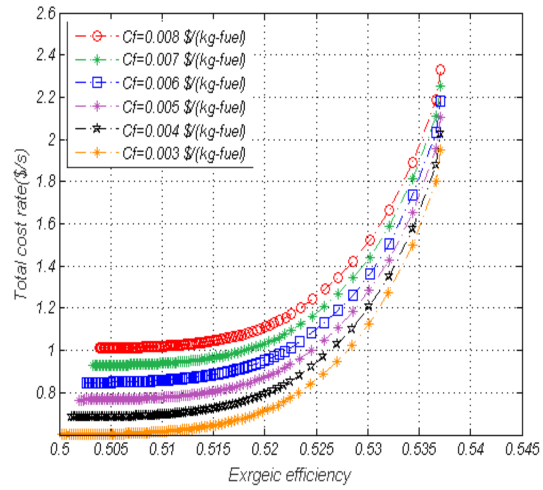


Fig. 8 Pareto fronts for different unit costs of fuel (∅ = 0.64 and NOx = 0.05 g/kg-fuel).

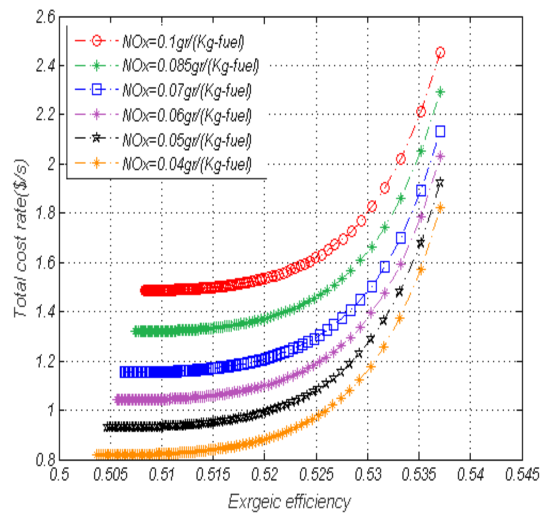


Fig. 9 Pareto fronts for different emissions (∅ =0 .64 and C_f= 0.004 $/kg-fuel).

Pareto Front is shown in "Figs. (7–9)". Figure 7 obviously shows that increasing the equivalence ratio leads to higher Pareto fronts for C_f = 0.004 $^{\$}/_{Kg} - fule$ and NO_x = 0.05 $^{g}/_{Kg} - fule$. That is due to inevitable increasing of the total cost rate and definitely is not desirable. This behavior is perhaps due to the reduction of the equivalence ratio resulted in decreasing the adiabatic flame temperature. Thus, it reduces harmful emissions and leads to reduction of the environmental cost rate.

Figure 8 depicts the obtained Pareto fronts for the different unit costs of the fuel for ∅ = 0.64 and NO_x = 0.05 $^{g}/_{Kg} - fule$. As it could be seen, raising the unit fuel-costs leads to increasing in the cost rate and drives the Pareto fronts higher which is not desired. At lower values of the exergetic efficiency, the difference between the Pareto fronts is very clear; however, for the higher values, the Pareto fronts converge together. This affirms the qualified investment for compensating the rising of the unit cost of the fuel. Figure 9 illustrates the effect of the different emissions on the Pareto front diagrams for $\emptyset$ = 0.64 and C_f= 0.004 $^{\$}/_{Kg} - fule$. Increasing the emission makes rising in the environmental and, total cost rate.

6 CONCLUSIONS

In the present paper, it was tried to provide a novel algorithm by combining two optimization approaches to solve single and multi-objective problems. By combining the firefly algorithm and artificial bee colony, the proposed FA-ABC method was far more successful than other algorithms for solving mathematical test functions. Finally, thermodynamical, economic and environmental modeling of a power plant was discussed and optimized by the proposed algorithm. Results illustrated that the fewer equivalence ratios have more desirable effects on performance of the system economically, exergetically, and environmentally. Moreover, rising in the unit fuel-cost and the emission is harmful for the performance of the system; but it can be compensated by utilizing high qualified investments.

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