Buckling Analysis of Orthotropic Annular Graphene Sheet with Various Boundary Conditions in an Elastic Medium

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Abstract: In this study, axisymmetric buckling of annular orthotropic graphene sheet embedded in a Winkler–Pasternak elastic medium is scrutinized for different boundary conditions based on non-local elasticity theory. With the aid of principle of virtual work, the non-local governing equations are derived based on First-order Shear Deformation Theory (FSDT). Differential Quadrature Method (DQM) is also used to solve equilibrium equations. Edges of Nano-plate might be restrained by different combinations of free, simply supported or clamped boundary conditions. To confirm results, comparison of studies is made between results obtained and available solutions in the literature. Finally, a detailed parametric study is conducted to investigate the impact of small scale effects, surrounding elastic medium, boundary conditions and geometrical parameters on critical buckling load. The main goal of this work is to study the effect of various non-local parameters on the buckling load of annular Nano-plate for different boundary conditions, Winkler and shear foundation parameters, annularity and thickness-to-radius ratios. It is seen that for Nano-plates without an elastic foundation, the impact of thickness on buckling load does not depend on values of non-local parameter and annularity. Results also show that impact of elastic basis on the buckling load is independent of small scale effects.

Keywords: Buckling, DQ Method, Elastic Medium, Graphene Sheet, Non-Local Elasticity

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1 INTRODUCTION

For mechanical, electrical, chemical, and thermal properties of the nanostructures, they are centre of attention of researchers. One type of the structures of Nano-systems is two-dimensional Nano-plate with feature superior mechanical characteristics compared to conventional engineering material [1]. Among Nano-plates under theoretical and experimental studies is Graphene Sheet (GS). Because of unique properties of Nano-plate, such as GS, it has a wide range of potential applications such as in Micro- Electromechanical System (MEMS) or Nano-Electromechanical System (NEMS) [2], composite materials [3], Nano actuators [4], biosensors [5], transistors [6], and biomedicine [7]. Given accurate mechanical analysis of discrete nanostructures, experiments and molecular dynamic simulations are of more importance. However, conducting experiments with nanoscale size specimens are both demanding and costly. Besides, atomic simulation is computationally expensive and limited to systems with a small number of molecules and atoms and this simulation is therefore restricted to the study of small-scale modelling. Thus, theoretical modelling of nanostructures is an important issue regarding their approximate analysis. So, modelling of nanostructures as an equivalent continuum structure is an interest in research since it provides a balance between accuracy and efficiency [8]. However, classic continuum model is unable to account for influence of inter-atomic and inter-molecular cohesive forces on small scales such as “Nano” scale. In order to overcome this problem, various size-dependent continuum theories such as strain gradient [9-10], couple stress [11], surface elasticity [12], and non-local elasticity [13-14] are developed. Among theories mentioned, it is seen that non-local elasticity theory can produce well-matched results with lattice dynamics [14]. Recently, the application of non-local elasticity in micro and Nano-materials has received much attention among Nano-technology community for its reliable and quick analysis of Nano-structures [15]. Since understanding of stability response of graphene sheets as MEMS or NEMS components under in-plane load is of great importance from a designing perspective, numerous studies focused on studying the buckling behaviour of Nano-plates based on non-local elasticity theory.

Using Navier’s approach, Pradhan [16] studied buckling characteristics of Single-Layered Graphene Sheets (SLGS) with all edges under simply supported boundary conditions based on higher-order-plate theory and non-local elasticity. Murmu and Pradhan [17] considered elastic buckling behaviour of simply supported orthotropic small scale plates under uniaxial and biaxial compression based on Classical Plate Theory (CPT). Pradhan and Murmu [18-19] used non-local elasticity theory and Differential Quadrature Method (DQM) for buckling analysis of rectangular isotropic SLGS under biaxial compression with and without surrounding elastic medium based on non-local CPT. Using Navier type solution for simply supported plates and Levy type method for plates with two opposite edges of simply supported and remaining others arbitrary, Aksencer and Aydogdu [20] investigated buckling and vibration of nano-plates based on non-local CPT. Using an explicit solution, Samaei et al. [21] studied buckling behaviour of anisotropic single layered rectangular graphene sheet embedded in a Pasternak elastic medium based on non-local Mindlin plate theory. Using Navier’s method, buckling analysis of isotropic simply supported rectangular nano-plates is done by Narendar [22] based on two variables refined plate theory and non-local small scale effects. Farajpour et al. [23] investigated stability behaviour of bi-axially compressed isotropic rectangular Nano-plates with variable thickness employing non-local CPT. They [23] solved non-local governing equation of simply supported Nano-plate by Galerkin method. Farajpour et al. [24] considered buckling response of orthotropic rectangular SLGS with various boundary conditions using non-local elasticity theory and DQM. Murmu et al. [25] presented an analytical solution for buckling of double Nano-plate-system subjected to biaxial compression using non-local elasticity theory. Zenkour and Sobhy [26] studied thermal buckling of SLGS lying on an elastic medium based on various plate theories along with non-local continuum model. Using DQM method and non-local plate theory, Mohammadi et al. [27] investigated shear buckling of orthotropic rectangular Nano-plate in elastic medium and thermal environment.

So, there is a strong desire to realize stability behaviour of circular and annular Nano-plates in design of Nano-devices. By considering the reported works mentioned in the latter section [28-37], the effect of various non-local parameters on the buckling load of annular Nano-plate for different boundary conditions, Winkler and shear foundation parameters, annularity and thickness-to-radius ratios has not been investigated yet. In this study, in order to fill this gap, axisymmetric buckling of annular orthotropic SLGS embedded in a Pasternak elastic medium under uniform in-plane loading is investigated for various boundary conditions and geometries based on non-local Mindlin plate theory. Using principle of virtual work, non-local governing equations are derived for annular Nano-plates. Differential quadrature method is used to solve governing equations for free, simply supported and clamped boundary conditions and various combinations of them. The presented formulation and method of solution are validated by comparing results, in limited cases with those available in the open literature. Excellent agreement between obtained and available results could be observed. Finally, small scale impact on bucking loads of graphene sheets are examined by considering different parameters like boundary conditions, Winkler and Pasternak elastic foundations that are ratio of outer-to-inner radius and ratio of thickness-to-radius.

2 LITERATURE REVIEW

According to the literature, some studies have been presented for buckling analysis of rectangular graphene sheets using Non-local elasticity theory. However, in comparison to rectangular graphene sheets, studies about Nano-plates of circular geometry are limited [28]. Using an exact analytical approach, free vibration analysis of thick circular/annular Functionally Graded (FG) Mindlin Nano-plates is investigated by Hosseini-Hashemi et al. [29] based on Eringen non-local elasticity theory. Sobhy [30] studied analytically free vibration, mechanical buckling and thermal buckling behaviour of Multilayered Graphene Sheets (MLGS) based on Eringen’s Nonl-ocal elasticity model and new two-variable plate theory. Using a closed form solution, Mohammadi et al. [31] considered free vibration behaviour of circular and annular graphene sheet based on non-local CPT. Sobhy [32] presented an explicit solution for natural frequency and buckling of orthotopic Nano-plates resting on Pasternak's elastic foundations using sinusoidal shear Deformation Plate Theory (SPT) in conjunction with non-local elasticity theory. Farajpour et al. [33] analysed axisymmetric buckling of circular isotropic graphene sheet under uniform radial compression using non-local elasticity and classical plate theory. They [33] obtained critical buckling loads based on explicit expressions for clamped and simply supported boundary conditions. Using Galerkin method, Asemi et al. [28] investigated axisymmetric buckling of isotropic circular graphene sheets subjected to uniform in-plane edge loads under a thermal environment based on non-local CPT. Karamooz Ravari and Shahidi [34] studied buckling behaviour of isotropic circular Nano-plates under uniform compression for several combinations of boundary conditions using finite difference method and non-local CPT. Farajpour et al. [35] used non-local elasticity theory and DQM for axisymmetric buckling analysis of circular single-layered graphene sheets in thermal environment. Bedroud et al. [36] presented an analytical solution for axisymmetric and asymmetric buckling analysis of isotropic moderately thick circular and annular Mindlin Nano-plates under uniform radial compressive in-plane load based on non-local elasticity theory of Eringen. Mohammadi et al. [37] presented a closed-form solution for frequency vibration of circular graphene sheets under in-plane preload using the non-local elasticity theory and Kirchhoff plate theory. Despite significant contributions to investigation of buckling behaviour of SLGS in previous years, the effect of various non-local parameters on the buckling load of annular orthotropic Nano-plate for different boundary conditions, Winkler and shear foundation parameters, annularity and thickness-to-radius ratios has not been studied yet. Thus, the main goal of this work is to present a detailed parametric study on the buckling behaviour of the orthotropic annular Nano-plate with different geometries, small scale effect and boundary conditions in presence and absence of elastic foundations.

3 FORMULATION

In this section, a non-local continuum model is presented for the axisymmetric buckling analysis of an annular graphene sheet. As shown in “Fig. 1”, an annular single-layered graphene sheet whose thickness is (h), inner radius is (r_i) and outer radius is (r_o) resting on Winkler springs (k_w) and shear layer (k_g) is subjected to uniform radial compression load (N). Axial symmetry in geometry and loading as well as cylindrical coordinates (r,θ,z) are considered.

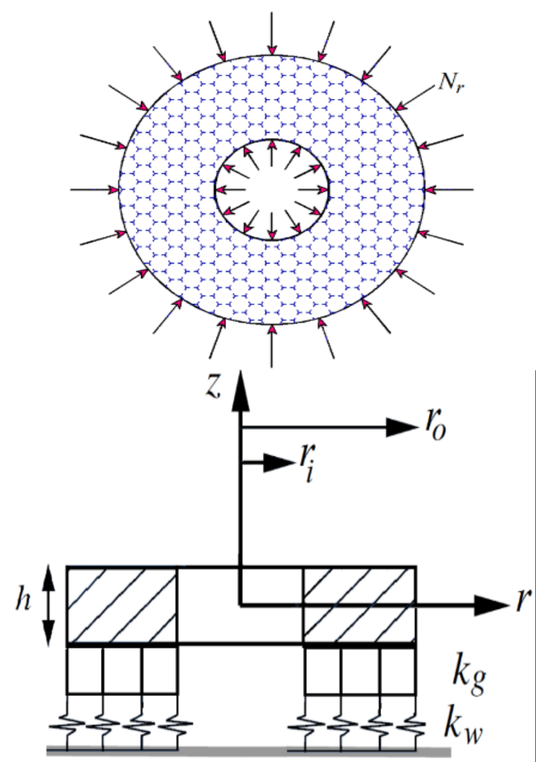


Fig. 1 A model of an annular Nano-plate in an elastic medium under uniform radial compression.

According to the non-local continuum theory of Eringen [13-14], for the small scale effects by having stress at a reference point as a function of strain field at all neighbor points in the continuum body, constitutive behavior of a Hookean solid can be presented as follows:

$$(1-\mu\nabla^2)\sigma^{nl} = \sigma^{l} \tag{1}$$

Where, σ^{nl} and σ^{l} denote the non-local and local stress tensors. $\mu=(e_0 a)^2$ is non-local parameter which incorporates small scale effect into formulation, a is an internal characteristic length and e_0 is Eringen's Non-local elasticity constant. Moreover, ∇^2 is Laplacian operator that in axisymmetric polar coordinate could be expressed by $\nabla^2=\frac{d^2}{dr^2}+\frac{d}{rdr}$. Local stress tensor σ^{l} is related to strain tensor by Hooke's law:

$$\sigma^{l} = C:\varepsilon \tag{2}$$

Where, C and ε are the stiffness and local strain tensors; and symbol ':' indicates double dot product. As reported by Ref. [38], nano-plates such as graphene sheets possess orthotropic properties. In this study, to accurately predict the behavior, orthotropic material properties of graphene sheets are taken into account. So, using "Eqs. (1) and (2)", plane stress in Non-local constitutive relation of an orthotropic annular graphene sheet in polar coordinates can be expressed as:

$$\begin{Bmatrix}\sigma_r^{nl}\\ \sigma_\theta^{nl}\\ \sigma_{rz}^{nl}\end{Bmatrix} - \mu\nabla^2\begin{Bmatrix}\sigma_r^{nl}\\ \sigma_\theta^{nl}\\ \sigma_{rz}^{nl}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & C_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma_{rz}\end{Bmatrix} \tag{3}$$

Where, stiffness coefficients for orthotropic layer are defined as bellow:

$$\begin{aligned}&Q_{11}=\frac{E_1}{1-\nu_{12}\nu_{21}}\quad Q_{22}=\frac{E_2}{1-\nu_{12}\nu_{21}}\\ &Q_{12}=\frac{\nu_{12}E_{12}}{1-\nu_{12}\nu_{21}}\quad C_{55}=G_{13}\end{aligned} \tag{4}$$

E1 and E2 are the Young's modulus in directions r and, and G_{13} is shear modulus. ν12 and ν21 are Poisson's ratios. Furthermore, ε_r and ε_θ are normal strains and γ_{rz} expresses shear strain. Regarding Axisymmetric buckling analysis and based on the Mindlin plate theory (FSDT), displacement field of an annular plate is expressed as:

$$\begin{aligned}&U(r,\theta,z)=u(r,\theta)+z\phi\\ &V(r,\theta,z)=0\\ &W(r,\theta,z)=w(r)\end{aligned} \tag{5}$$

Where, (U, V, W) are displacement components of an arbitrary point (x, θ, z) of a plate, and u and w are displacements of mid-plane in r and z directions. ϕ is rotation around θ direction. Based on FSDT and nonlinear von Karman theory, strain-displacement relations can be written as:

$$\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma_{rz}\end{Bmatrix} = \begin{Bmatrix}\frac{du}{dr}+\frac{1}{2}(\frac{dw}{dr})^2+z\frac{d\phi}{dr}\\ \frac{u}{r}+z\frac{\phi}{r}\\ \phi+\frac{dw}{dr}\end{Bmatrix} \tag{6}$$

Force, moment and shear stress resultants (Ni (i = r, θ), Mi (i = r, θ) and Qr) of non-local elasticity are defined as:

$$\begin{aligned}&(N_r,N_\theta,Q_r)^{nl}=\int_{-h/2}^{h/2}(\sigma_r,\sigma_\theta,k\,\sigma_{rz})^{nl}dz\\ &(M_r,M_\theta)^{nl}=\int_{-h/2}^{h/2}(\sigma_r,\sigma_\theta)^{nl}zdz\end{aligned} \tag{7}$$

In which, k is transverse shear correction coefficient and it is considered 5/6. Using "Eqs. (3), (6), and (7)", stress resultants can be expressed in terms of displacements as follows:

$$\begin{Bmatrix}N_r\\ N_\theta\\ Q_r\\ M_r\\ M_\theta\end{Bmatrix}^{nl} - \mu\nabla^2\begin{Bmatrix}N_r\\ N_\theta\\ Q_r\\ M_r\\ M_\theta\end{Bmatrix}^{nl} = \begin{Bmatrix}A_{11}\frac{du}{dr}+A_{11}\frac{1}{2}(\frac{dw}{dr})^2+A_{12}\frac{u}{r}\\ A_{12}\frac{du}{dr}+A_{12}\frac{1}{2}(\frac{dw}{dr})^2+A_{22}\frac{u}{r}\\ H_{55}\phi+H_{55}\frac{dw}{dr}\\ D_{11}\frac{d\phi}{dr}+D_{12}\frac{\phi}{r}\\ D_{12}\frac{d\phi}{dr}+D_{22}\frac{\phi}{r}\end{Bmatrix} \tag{8}$$

Where, Aij (i , j=1, 2), Dij (i, j=1, 2) and H55 are extensional stiffness, flexural stiffness and shear stiffness of the sheet shown as below:

$$(A_{ij}, D_{ij}) = (h, \frac{h^3}{12})Q_{ij} \quad i, j = 1, 2$$
$$H_{55} = kC_{55}h \tag{9}$$

The equilibrium equations of an annular nano-plate on an elastic foundation under uniform radial compression load is derived based on the principle of virtual work and it is as written below:

$$\delta \prod = \delta(U + V_f + V_g) = 0 \tag{10}$$

Where, Π, U and Vf and Vg are total potential energy, strain energy, potential energy of the foundation, and potential energy of the applied compressive load, N, of a plate. Variation of strain energy, δU , is expressed as:

$$\delta U = \int_{r_i}^{r_o}\int_0^{2\pi}\int_{-h/2}^{h/2}(\sigma_r\delta\varepsilon_r + \sigma_\theta\delta\varepsilon_\theta + \sigma_{rz}\delta\varepsilon_{rz})rdzd\theta dr \tag{11}$$

Variations of potential energy of foundation (δV_f) and applied compressive load (δV_g) are defined as follows:

$$\delta V_f = \int_{r_i}^{r_o}\int_0^{2\pi}(k_w w\delta w + k_g \frac{\partial w}{\partial r}\delta(\frac{\partial w}{\partial r}))rd\theta dr \tag{12}$$

$$\delta V_g = \frac{N}{h}\int_{r_i}^{r_o}\int_0^{2\pi}\int_{-h/2}^{h/2}\frac{\partial(r\delta u)}{\partial r}dzd\theta dr \tag{13}$$

Using the principle of virtual work, the following equilibrium equations can be obtained:

$$\begin{aligned}
&\frac{dN_r}{dr} + \frac{N_r - N_\theta}{r} = 0\\
&\frac{dM_r}{dr} + \frac{M_r - M_\theta}{r} - Q_r = 0\\
&\frac{dQ_r}{dr} + \frac{Q_r}{r} + (1-\mu\nabla^2)N_r\frac{d^2w}{dr^2}\\
&+(1-\mu\nabla^2)N_\theta(\frac{1}{r}\frac{dw}{dr})\\
&-k_w(1-\mu\nabla^2)w + (1-\mu\nabla^2)k_g(\nabla^2 w) = 0
\end{aligned} \tag{14}$$

Stability equations are derived from adjacent equilibrium criterion [39]. Displacement components of a neighboring configuration of stable state can be expressed as [40]:

$$\begin{aligned}
&u = u^0 + u^1\\
&w = w^0 + w^1\\
&\phi = \phi^0 + \phi^1
\end{aligned} \tag{15}$$

In which over-script 0 denotes pre-buckling configuration of a nano-plate, and u1, w1, and ϕ^1 are small increments from stable configuration. Using “Eq. (15)”, Non-local resultants can be defined as follows [36-41]:

$$\begin{aligned}
&N_r = N_r^0 + N_r^1 \qquad M_r = M_r^0 + M_r^1\\
&N_\theta = N_\theta^0 + N_\theta^1 \qquad M_\theta = M_\theta^0 + M_\theta^1\\
&Q_r = Q_r^0 + Q_r^1
\end{aligned} \tag{16}$$

Stability equations will be obtained for Mindlin nano-plate under in-plane radial loads acting on plane z = 0, so deflection and rotations in pre-buckling configuration disappear (u^0, w^0, and ϕ^0). Finally, by substituting “Eqs. (15), (16)” into “Eqs. (14)” and eliminating quadratic and cubic incremental terms, stability equations in non-local form can be obtained as follows [36-42]:

$$\begin{aligned}
&\frac{dN_r^1}{dr} + \frac{N_r^1 - N_\theta^1}{r} = 0\\
&\frac{dM_r^1}{dr} + \frac{M_r^1 - M_\theta^1}{r} - Q_r^1 = 0\\
&\frac{dQ_r^1}{dr} + \frac{Q_r^1}{r} + (1-\mu\nabla^2)N_r^0\frac{d^2w^1}{dr^2}\\
&+(1-\mu\nabla^2)N_\theta^0(\frac{1}{r}\frac{dw^1}{dr}) - k_w(1-\mu\nabla^2)w^1\\
&+(1-\mu\nabla^2)k_g(\nabla^2 w) = 0
\end{aligned} \tag{17}$$

Where, N_r^0 and N_θ^0 are pre-buckling in-plane stress resultant defined as follows for uniform radial compression:

$$N_\theta^0 = N_r^0 = -N \tag{18}$$

By substituting “Eqs. (8)” in “Eqs. (17)”, local stability equations of axisymmetric nano-plates in terms of the displacements can be acquired as follows:

$$\begin{aligned}
&A_{11}\frac{d^2u^1}{dr^2}+A_{11}\frac{1}{r}\frac{du^1}{dr}-A_{22}\frac{u^1}{r^2}=0\\
&D_{11}\frac{d^2\phi^1}{dr^2}+D_{11}\frac{1}{r}\frac{d\phi^1}{dr}-D_{22}\frac{\phi^1}{r^2}-H_{55}\phi^1-H_{55}\frac{dw^1}{dr}=0\\
&H_{55}\frac{d\phi^1}{dr}+H_{55}\frac{d^2w^1}{dr^2}+H_{55}\frac{\phi^1}{r}+H_{55}\frac{1}{r}\frac{dw^1}{dr}-N(1-\mu\nabla^2)(\nabla^2w^1)\\
&-K_w(1-\mu\nabla^2)w^1+K_g(1-\mu\nabla^2)(\nabla^2w^1)=0
\end{aligned}\tag{19}$$

It is clearly seen that in-plane (u^1) and out-of-plane (w^1 and ϕ^1) governing equations are uncoupled. Thus, the first relation of "Eqs. (19)" can be eliminated from the stability equation.

4 SOLUTION METHODOLOGY

In this section, Differential Quadrature Method (DQM) is employed in order to solve equilibrium equations. Since DQM provides a simple formulation and low computational cost in contrast with other numerical methods, many researchers utilized DQM for solving governing equations of nanostructures [35], [37], [43-46]. DQM is based on a simple concept: partial derivative of a function could be approximated regarding a space variable at a discrete mesh point in domain by taking a weighted linear sum of functional values at all grid points in the domain [24]. Therefore, every partial differential equation can be easily simplified to a set of algebraic equations using these coefficients [47]. Partial derivatives of a function f(x) at the point (xi) can be expressed by [49]:

$$\frac{\partial^s f(x_i)}{\partial x^s}=\sum_{j=1}^{n}C_{ij}^{s}f(x_j)\qquad i=1,2,\mathrm{K},n\tag{20}$$

In which, n is the number of grid points in x-direction and C_{ij}^{s} is weighting coefficient of the derivative and is obtained as follows [49]:
If s = 1, namely, for the first order derivative:

$$\begin{aligned}
&C_{ij}^{1}=\frac{R(x_i)}{(x_i-x_j)R(x_j)}\quad i\neq j\quad i,\ j=1,2,...,N\\
&C_{ii}^{1}=-\sum_{j=1,i\neq j}^{n}C_{ij}^{s}\qquad i=1,2,...,N
\end{aligned}\tag{21}$$

Where, $R(x)$ is defined as:

$$R(x_i)=\prod_{j=1,\neq i}^{n}(x_i-x_j)\tag{22}$$

Moreover, for higher order partial derivatives, weighting coefficients are gained by:

$$\begin{aligned}
&C_{ij}^{2}=\sum_{k=1}^{n}C_{ik}^{1}C_{kj}^{1}\qquad C_{ij}^{3}=\sum_{k=1}^{n}C_{ik}^{1}C_{kj}^{2}\\
&C_{ij}^{4}=\sum_{k=1}^{n}C_{ik}^{1}C_{kj}^{3}\qquad i,\ j=1,2,...n
\end{aligned}\tag{23}$$

In order to obtain suitable number of discrete grid points and a better mesh point distribution, Gauss-Chebyshev-Lobatto technique has been employed as follows:

$$x_i=\frac{1}{2}\left(1-\cos\frac{i-1}{n-1}\pi\right)\qquad i=1,2,...,n\tag{24}$$

As mentioned earlier, the first relation of "Eqs. (19)" plays no role in determining critical loads of a plate. So, through implementation of DQM into the two last relations of "Eq. (19)", discretized stability equations can be obtained as in equation (25).
The complete set of boundary conditions is extracted within the process of virtual work. In the following, three sets of DQM discretized forms of boundary conditions suitable for buckling analysis are given.
So, outer or inner edge of a plate may take one of the three sets of DQM as mentioned below [42-43]:

$$
\begin{aligned}
& D_{11}\sum_{j=1}^{n} C_{ij}^{2}\phi_j + \frac{D_{11}}{r_i}\sum_{j=1}^{n} C_{ij}^{1}\phi_j \\
& -D_{22}\frac{\phi_i}{r_i^{2}} - A_{66}\phi_i - A_{66}\sum_{j=1}^{n} C_{ij}^{1} w_j = 0 \\
& \sum_{j=1}^{n} C_{ij}^{1}\phi_j + \sum_{j=1}^{n} C_{ij}^{2} w_j + \frac{1}{r_i}\sum_{j=1}^{n} C_{ij}^{1} w_j + \frac{\phi_i}{r_i} \\
& \quad + \frac{N\mu}{H_{55}}\left\{\sum_{j=1}^{n} C_{ij}^{4} w_j + \frac{2}{r_i}\sum_{j=1}^{n} C_{ij}^{3} w_j + \frac{1}{r_i^{2}}\sum_{j=1}^{n} C_{ij}^{2} w_j + \frac{1}{r_i^{3}}\sum_{j=1}^{n} C_{ij}^{1} w_j\right\} \\
& \quad - \frac{N}{H_{55}}\left\{\sum_{j=1}^{n} C_{ij}^{2} w_j + \frac{1}{r_i}\sum_{j=1}^{n} C_{ij}^{1} w_j\right\} - \frac{k_w}{H_{55}} w_i \\
& \quad + \frac{k_w\mu}{H_{55}}\left\{\sum_{j=1}^{n} C_{ij}^{2} w_j + \frac{1}{r_i}\sum_{j=1}^{n} C_{ij}^{1} w_j\right\} + \frac{k_g}{H_{55}}\left\{\sum_{j=1}^{n} C_{ij}^{2} w_j + \frac{1}{r_i}\sum_{j=1}^{n} C_{ij}^{1} w_j\right\} \\
& \quad - \frac{k_g}{H_{55}}\left\{\sum_{j=1}^{n} C_{ij}^{4} w_j + \frac{2}{r_i}\sum_{j=1}^{n} C_{ij}^{3} w_j + \frac{1}{r_i^{2}}\sum_{j=1}^{n} C_{ij}^{2} w_j + \frac{1}{r_i^{3}}\sum_{j=1}^{n} C_{ij}^{1} w_j\right\} = 0
\end{aligned}
\tag{25}
$$

(a): For simply supported edges:

$$
\begin{aligned}
& w_i = 0 \\
& (M_r^{1})_i = 0 \quad : \\
& D_{11}\sum_{j=1}^{n} C_{ij}^{1}\phi_j + D_{12}\frac{\phi_i}{r_i} = 0 \quad i = 1, n
\end{aligned}
\tag{26}
$$

(b): For clamped edges:

$$
w_i = \phi_i = 0 \tag{27}
$$

(c): For free edges:

$$
\begin{aligned}
& \{N_r^{0}(1-\mu\nabla^2)\frac{dw^{1}}{dr} \\
& + k_g(1-\mu\nabla^2)\frac{dw^{1}}{dr} + Q_r^{1}\}_i = 0 \\
& \quad : -N\left\{\sum_{j=1}^{n} C_{ij}^{1} w_j - \mu\sum_{j=1}^{n} C_{ij}^{3} w_j - \frac{\mu}{r_i}\sum_{j=1}^{n} C_{ij}^{2} w_j\right\} \\
& \quad + kg\left\{\sum_{j=1}^{n} C_{ij}^{1} w_j - \mu\sum_{j=1}^{n} C_{ij}^{3} w_j - \frac{\mu}{r_i}\sum_{j=1}^{n} C_{ij}^{2} w_j\right\} \\
& \quad + H_{55}\phi_i + H_{55}\sum_{j=1}^{n} C_{ij}^{1} w_j = 0 \\
& (M_r^{1})_i = 0 \quad : D_{11}\sum_{j=1}^{n} C_{ij}^{1}\phi_j + D_{12}\frac{\phi_i}{r_i} = 0 \quad i = 1, n
\end{aligned}
\tag{28}
$$

To have buckling loads of a plate, equations of stability, “Eqs. (25)”, for $(2N-4)\times(2N-4)$ inner points of a plate along with boundary condition equations, “Eqs. (26-28)”, for edge points should be simultaneously solved. The assembly of boundary conditions and stability equations yields a set of following algebraic equations [49]:

$$
\begin{aligned}
& \begin{bmatrix} [K_{bb}] & [K_{bi}] \\ [K_{ib}] & [K_{ii}] \end{bmatrix} \begin{Bmatrix} d_b \\ d_i \end{Bmatrix} = \\
& N\begin{bmatrix} 0 & 0 \\ [KN_{ib}] & [KN_{ii}] \end{bmatrix} \begin{Bmatrix} d_b \\ d_i \end{Bmatrix}
\end{aligned}
\tag{29}
$$

Where, elements of sub-matrices $[K_{bb}]$ and $[K_{bi}]$ represent weighting coefficients related to boundary conditions while $[K_{ib}]$, $[K_{ii}]$, $[KN_{ib}]$, and $[KN_{ib}]$ denote sub-matrices of stability equations. In addition, displacement vectors of inner points of a plate and edge points are defined by $d_i = \{w_2, \ldots, w_{n-1}, \phi_2, \ldots, \phi_{n-1}\}$ and $d_b = \{w_1, w_n, \phi_1, \phi_n\}$. In order to transform “Eq. (29)” into standard eigenvalue equation, vector $\{d_b\}$ must be eliminated. Thus, following standard eigenvalue problem can be obtained [49]:

$$
([K_{total}] - N[I])\{d_i\} = 0 \tag{30}
$$

Where:

$$[K_{total}]=\Big[[KN_{ib}][K_{bb}]^{-1}[K_{bi}]+[KN_{ii}]\Big]^{-1} \Big[-[K_{ib}][K_{bb}]^{-1}[K_{bi}]+[K_{ii}]\Big] \quad (31)$$

And i denotes identity matrix. Moreover, N is critical buckling load which can be calculated from "Eq. (30)" using a standard eigenvalue solver.

5 RESULT AND DISCUSSION

For numerical investigations of our analysis, buckling of orthotropic annular nano-plate is scrutinized under different boundary conditions, namely those Clamped–Clamped (CC), Clamped–Simply (CS), Simply–Clamped (SC), Clamped–Free (CF), Free–Clamped (FC) and Simply–Simply (SS) supports at inner and outer edges. Orthotropic material properties of nano-plate are taken as that of E1=1765 Gpa, E2=1588 Gpa, ν12=0.3, ν21=0.27 [1]. Outer radius of an annular graphene sheet is assumed r0=20 nm, unless stated otherwise. Furthermore, thickness of a nano-plate is 0.34 nm. The value of small scale effect (e0a) can be calculated by molecular dynamics and must be less than 2.0 nm [50]. Results are defined and presented in terms of following non-dimensional quantities, $\Omega = N_r r_0^2/D$, $R = r_i/r_o$, $\mathrm{K_w} = k_w r_0^4/D$, $\mathrm{K_g} = k_g r_0^2/D$ which are critical buckling load, annularity, Winkler and shear foundations, and $D = E_1 h^3/12(1-\nu_{12}\nu_{21})$. Moreover, in order to measure influence of small scale effect on buckling, buckling load ratio is defined as:

$$\text{Buckling load ratio} = \frac{\text{Buckling load by nonlocal theory}}{\text{Buckling load by local theory}}$$

To verify accuracy of formulation and results, DQM solutions for buckling of isotropic annular Nano-plate under uniform compression load are compared to those reported by Karamooz Ravari and Shahidi [34] and Bedroud et al. [36] in "Tables 1 and 2" for different boundary conditions, annularity and non-local parameters. As seen in Tables 1 and 2, the present results are in good agreement with those obtained by Refs. [34-36]. After verifying results, it was necessary to carry out a convergence test because results of DQM procedure rely on the number of grid points [48]. Thus, non-results are in good agreement with those obtained by Refs. [34-36]. After verifying results, it was necessary to carry out a convergence test because results of DQM procedure rely on the number of grid points [48]. Thus, non-dimensional buckling loads of CC and SS orthotropic annular graphene sheet are tabulated in Table 3 for different numbers of grid points and non-local parameters. As indicated in "Table 3", ten grid points along the radial direction are enough to have converge solution.

For parametric study on effects of different parameters such as small scale effects, boundary conditions, surrounding elastic medium and geometry are investigated on buckling behavior of annular SLGS based on FSDT. To achieve this goal, non-dimensional buckling loads of annular nano-plate with various non-local parameters are illustrated for different boundary conditions, Winkler and shear foundation parameters, annularity and thickness-to-radius ratios.

Figures 2(a-f) illustrate non-dimensional critical buckling load in terms of non-local parameters with and without presence of elastic medium and different values of R for various types of boundary conditions. As you can see, the greatest to the smallest effects of non-local parameter on buckling load are put to order as $CF \langle FC \langle SS \langle CS \langle SC \langle CC$ with respect to type of boundary condition. Furthermore, by increasing non-local parameter, critical buckling load reduces for all types of boundary conditions.

This reduction is higher for larger values of R so results are approximately independent of non-local parameter for lower ratios of R in annular Nano-plates with r_0=20 nm.

Moreover, impact of elastic foundations on buckling load is independent of small scale effects. Maximum and minimum effects of elastic foundations depends on CF and CC boundary conditions, respectively.

In order to investigate independence of buckling load from non-local parameter for different geometry of annular nano-plate, outer radiuses are assumed 20nm in "Fig. 2" and non-dimensional critical buckling load is in terms of non-local parameters with different annularity (R) and outer radiuses (10nm, 15nm and 30nm) for boundary conditions of CC, SS and CF in "Figs. 3, 4 and 5" with and without presence of elastic medium.

Table 1 Comparison of DQM results with those calculated by Karamooz Ravari and Shahidi [34] for non-dimensional buckling load of isotropic annular nano-plate under uniform compression load

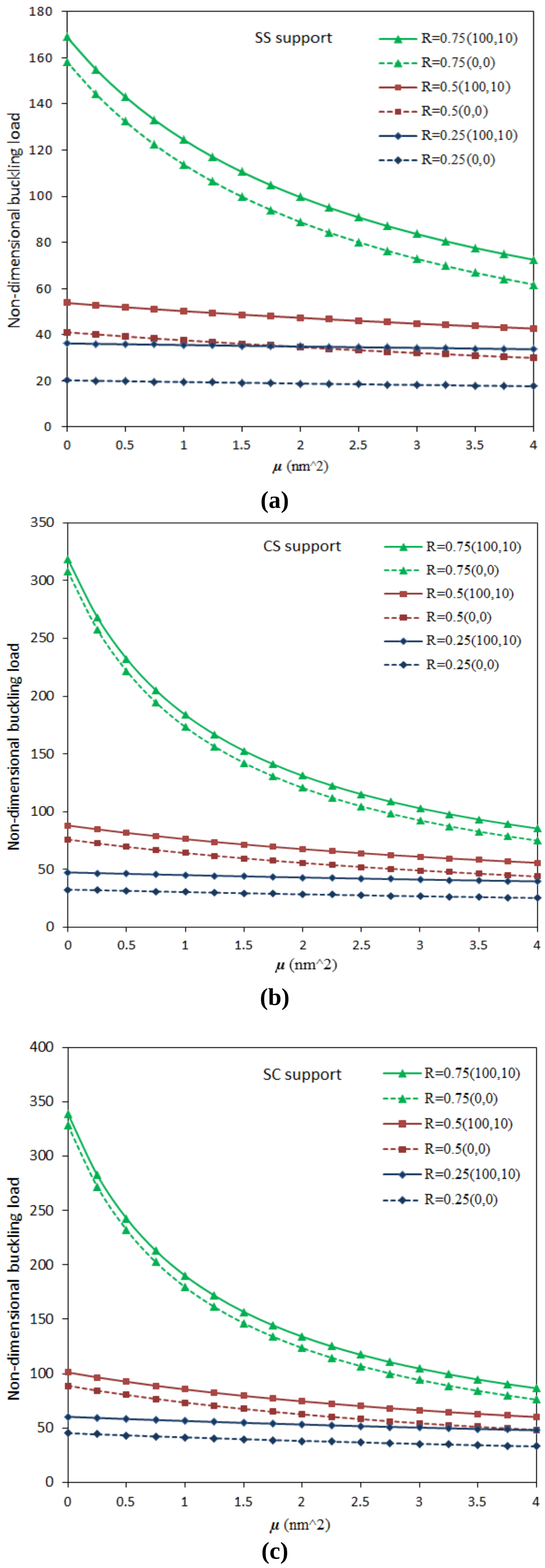
(r_o, r_i) nm	boundary condition		Nonlocal parameter μ (nm^2)						
			0	0.04	0.25	1	1.44	2.25	4
(20,10)	SS	Present Study	41.05	40.92	40.12	37.56	36.19	33.93	29.90
		Ref. [34]	41.35	41.15	39.94	37.32	35.71	33.50	29.06
	CS	Present Study	75.68	75.12	72.39	63.99	59.93	53.64	43.75
		Ref. [34]	71.46	71.02	68.36	60.73	56.85	50.99	41.69
	SC	Present Study	88.10	87.37	83.71	72.78	67.62	59.80	47.85
		Ref. [34]	94.33	93.32	88.89	76.00	70.36	61.50	48.40
	CC	Present Study	155.56	153.19	141.84	112.17	99.89	83.17	61.06
		Ref. [34]	157.38	154.76	143.08	112.86	100.17	83.45	60.89
(30,10)	SS	Present Study	20.32	20.30	20.18	19.78	19.56	19.15	18.33
		Ref. [34]	24.58	24.68	24.69	24.09	23.75	23.32	22.38
	CS	Present Study	42.18	42.10	41.73	40.46	39.71	38.43	36.04
		Ref. [34]	38.34	38.25	37.90	36.66	36.13	34.99	32.70
	SC	Present Study	54.01	53.86	53.34	51.32	50.19	48.32	44.65
		Ref. [34]	59.01	58.84	57.97	55.26	53.86	51.50	46.68
	CC	Present Study	89.75	89.45	87.73	82.03	79.04	74.09	65.18
		Ref. [34]	88.86	88.42	86.68	81.06	77.81	72.72	63.67

Table 2 Comparison of non-dimensional buckling load obtained in the present study with results obtained by Bedroud et al. [36] for isotropic annular Nano-plate with different boundary condition

boundary condition		Nonlocal parameter μ (nm^2)						
		0	0.04	0.25	1	1.44	2.25	4
CS	Present Study	25.91	24.74	23.18	20.97	20.20	19.15	13.40
	Ref. [36]	25.99	24.60	22.85	20.22	19.47	18.18	12.26
FC	Present Study	10.58	10.58	10.59	10.60	10.60	10.61	10.60
	Ref. [36]	10.66	10.52	10.38	10.24	10.02	9.95	8.75
CC	Present Study	52.98	47.64	41.39	33.94	31.66	28.75	15.51
	Ref. [36]	53.08	47.22	40.41	32.66	30.46	27.46	15.13

Table 3 Convergence study of DQM results for the non-dimensional buckling loads of CC and SS orthotropic annular Nano-plate

	μ (nm^2)	number of grid point						
		7	8	10	13	15	18	20
SS	0	39.4341	40.2690	41.1039	41.1039	41.0397	41.0397	41.0397
	0.25	38.5992	39.3699	40.1406	40.1406	40.1406	40.1406	40.1406
	1	36.2229	36.8652	37.5716	37.5716	37.5716	37.5716	37.5716
	2.25	32.8833	33.3328	33.9751	33.9751	33.9109	33.9109	33.9109
	4	29.0941	29.4794	29.9290	29.9290	29.9290	29.9290	29.9290
CC	0	189.1400	188.1767	157.6061	157.8630	157.7345	157.7345	157.7345
	0.25	169.3591	168.5884	143.7337	143.9264	143.7979	143.7979	143.7979
	1	129.1549	128.5769	113.6127	113.7412	113.7412	113.7412	113.7412
	2.25	92.6115	92.2904	84.2624	84.3266	84.3266	84.3266	84.3266
	4	66.4081	66.2155	61.8482	61.8482	61.9125	61.9125	61.9125



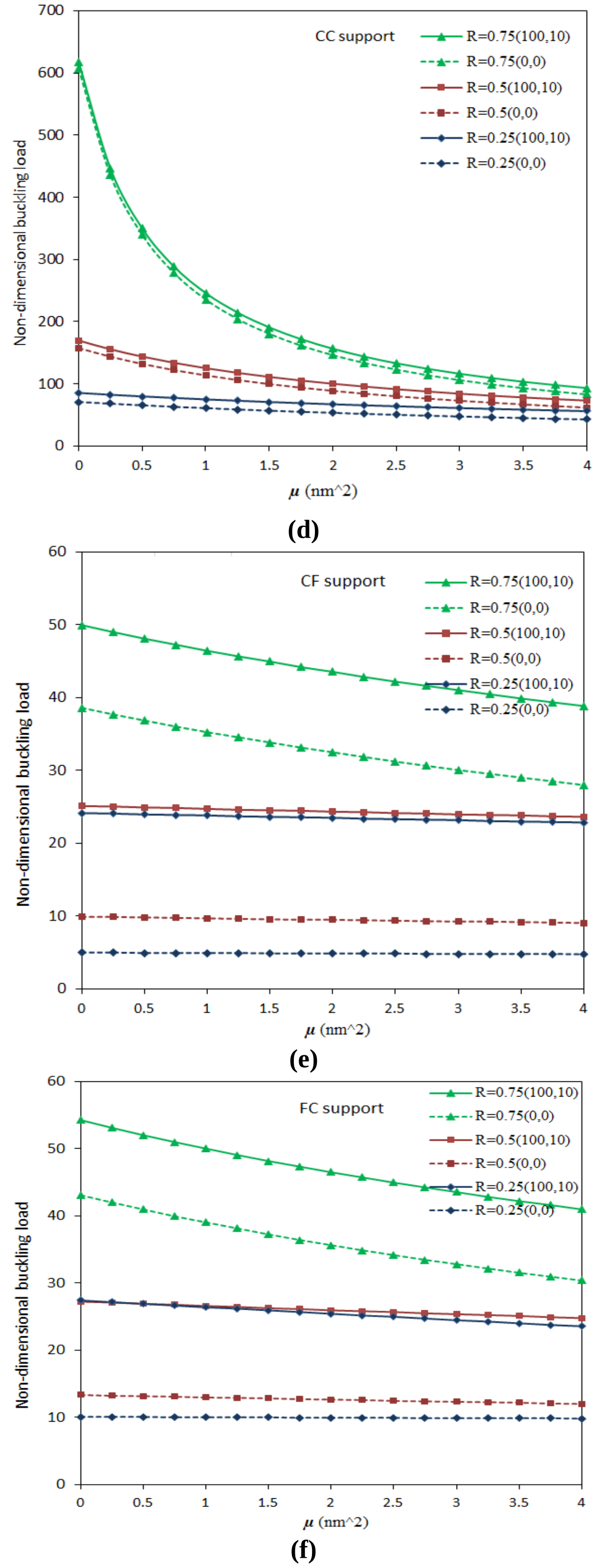
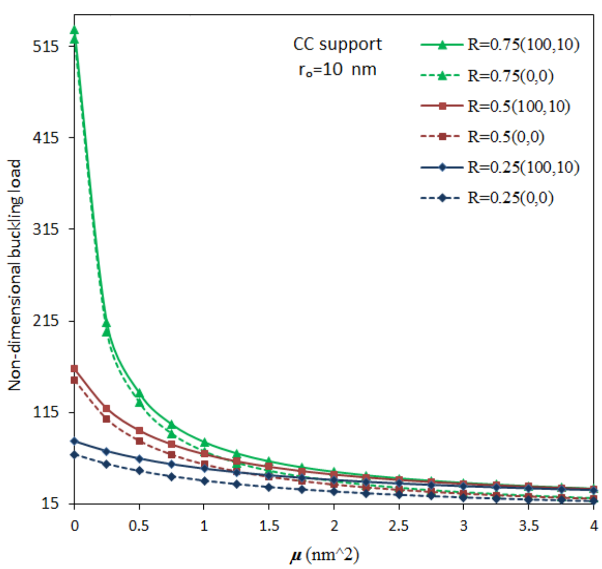


Fig. 2 Non-dimensional critical buckling load in terms of non-local parameter with and without presence of elastic medium (Kw, Kg) and different values of annularity (R), for: (a): SS, (b): CS, (c): SC, (d): CC, (e): CF and (f): FC, annular Nano-plate (r_0=20 nm).

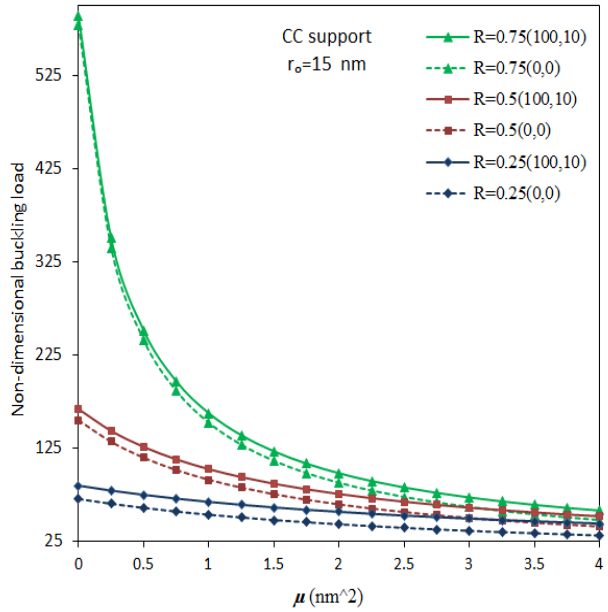
Independence of buckling load from non-local parameter happens with the increase of outer radius of annular nano-plate (r_0 >20 nm) and decrease of inner-to-outer radius ratios (R<0.25) for all cases of boundary conditions, as shown in “Figs. 3, 4 and 5”.

It should be noted that presence or absence of elastic foundation has no influence on the results of CC boundary condition. However, as seen in “Figs. 4 and 5”, unlike CC annular nano-plate, presence or absence of elastic foundation has influence on the state of independence or dependence of buckling load from non-local parameter at specified geometry of SS and CF annular nano-plate. So, at a defined inner-to-outer radius ratio (R), when the elastic foundation is absent, buckling load is independent of non-local parameter at smaller outer radiuses. Therefore, despite the fact that R value is low (R=0.25nm) in “Figs. 3-5 (a and b)”, buckling load depends on non-local parameter in annular Nano-plates with r0=10nm and 15nm for all cases of CC, SS and CF boundary conditions. However, as shown in “Fig. 3-5c” with the same amount of R, buckling load is independent of non-local parameter as a result of increasing outer radius from 20 nm for both cases of with and without presence of elastic medium and various boundary conditions. As a conclusion, for buckling load of annular Nano-plates to be independent of Non-local parameter, values of both inner and outer radiuses are of great importance. So, when outer radius is 20nm or greater and inner-to-outer radius ratios (R) is 0.25 or lower, buckling is independent of non-local parameter for different boundary conditions.

This reduction is higher for larger values of R so results are approximately independent of non-local parameter for lower ratios of R in annular Nano-plates with r_0=20 nm.



(a)



(b)

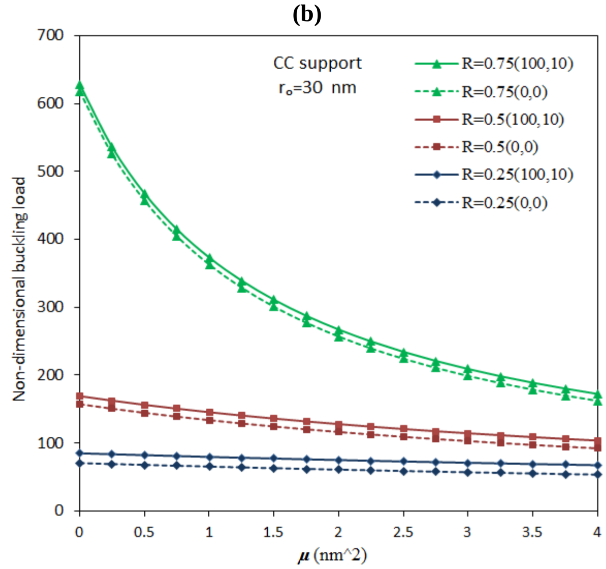
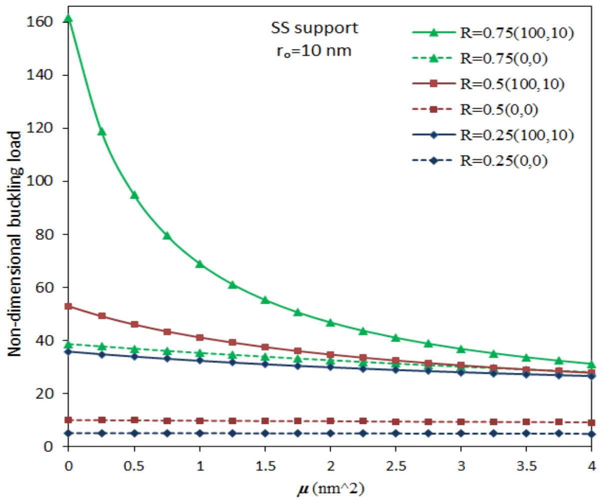


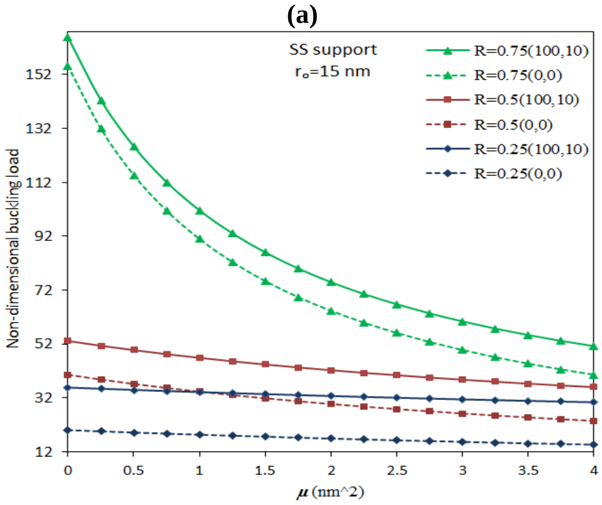
Fig. 3 Non-dimensional critical buckling load in terms of non-local parameter for different values of annularity (R) in absence and presence of elastic medium (Kw, Kg) with CC boundary conditions: (a) r_o=10nm, (b) r_0=15nm and (c) r_o=30nm.

Moreover, impact of elastic foundations on buckling load is independent of small scale effects. Maximum and minimum effects of elastic foundations depends on CF and CC boundary conditions, respectively.

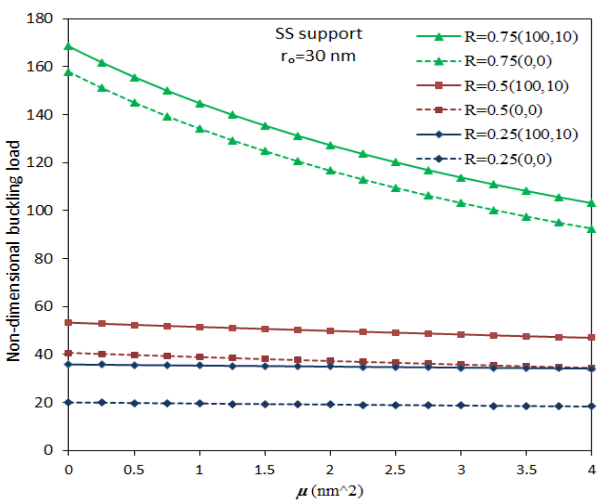
In order to investigate independence of buckling load from non-local parameter for different geometry of annular nano-plate, outer radiuses are assumed 20nm in “Fig. 2” and non-dimensional critical buckling load is in terms of non-local parameters with different annularity (R) and outer radiuses (10nm, 15nm and 30nm) for boundary conditions of CC, SS and CF in “Figs. 3, 4 and 5” with and without presence of elastic medium.



(a)

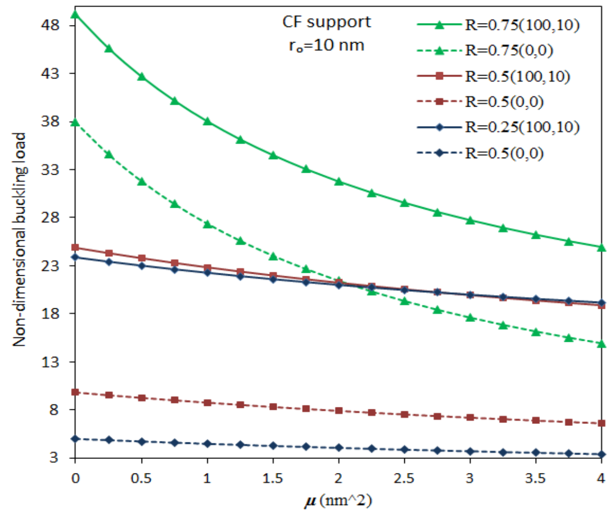


(b)

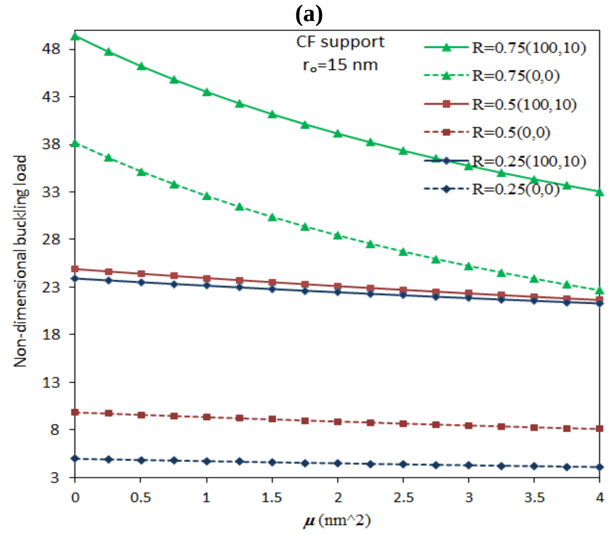


(c)

Fig. 4 Non-dimensional critical buckling load in terms of non-local parameter for different values of annularity (R) in absence and presence of elastic medium (Kw, Kg) with SS boundary conditions: (a) r_o=10nm, (b) r_0=15nm, (c) r_o=30nm.



(a)



(b)

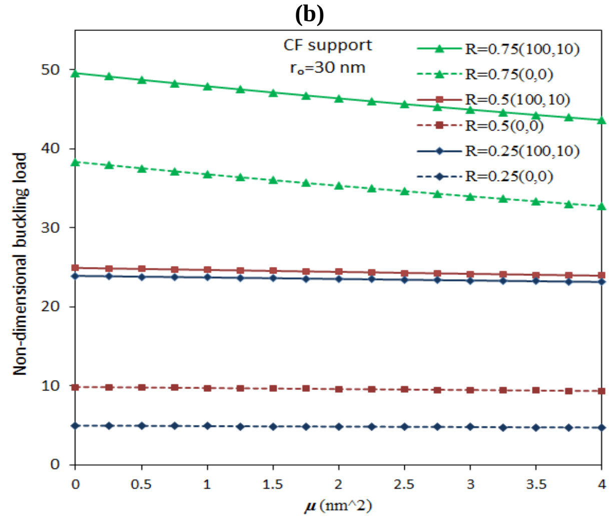
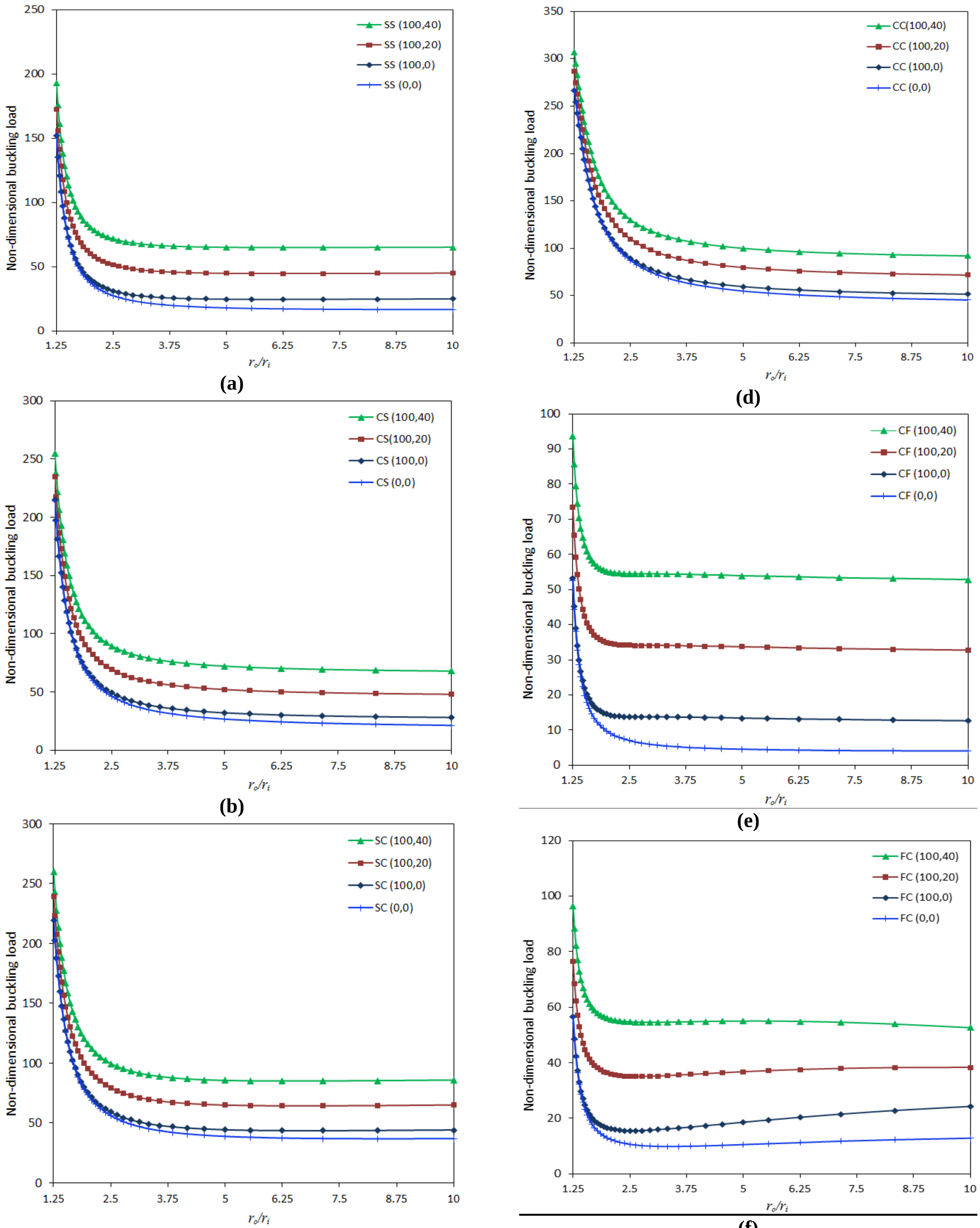


Fig. 5 Non-dimensional critical buckling load in terms of non-local parameter for different values of annularity (R) in absence and presence of elastic medium (Kw, Kg) with CF boundary conditions: (a) r_o=10nm, (b) r_0=15nm, (c) r_o=30nm.



(c)

Fig. 6 Non-dimensional buckling load versus the ratios of outer to inner radius for (a) SS, (b) CS, (c) SC, (d) CC, (e) CF and (f) FC annular nano-plate (r_0=20 nm) with and without presence of elastic medium (Kw, Kg).

"Fig. 7(a-b)" illustrate Non-local parameter impact on critical buckling load ratio of a nano-plate with and without elastic foundations for different boundary conditions.

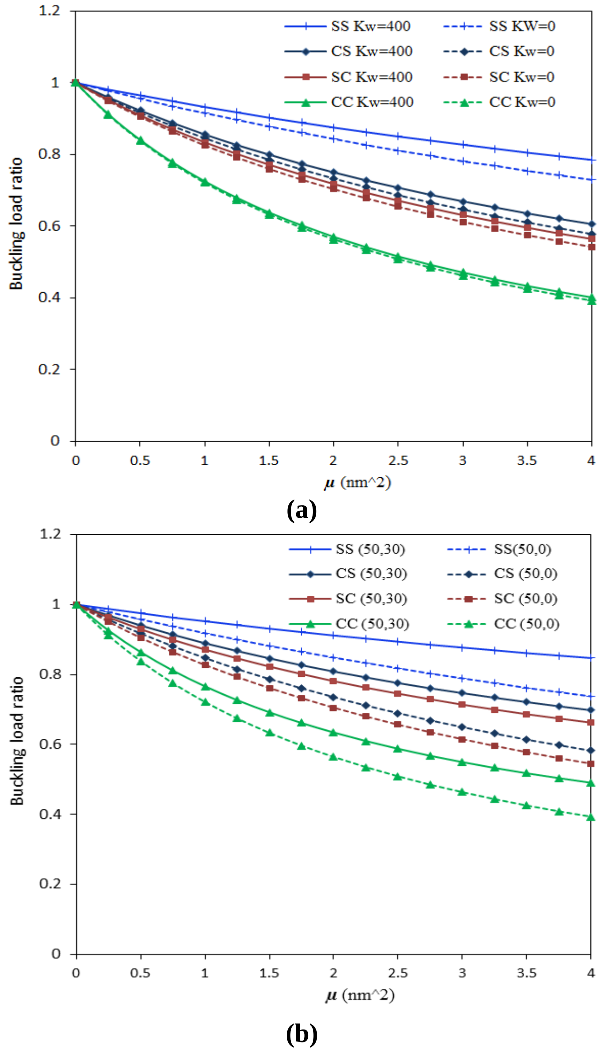


Fig. 7 Critical buckling load ratio versus the non-local parameter for various boundary conditions of annular nano-plate (R=0.5, r_0=20 nm) with (a) Winkler and (b) Pasternak elastic foundations.

Elastic foundation implemented in "Fig. 7a" is of Winkler type and the one in "Fig. 7b" is the two-parameter Pasternak foundation. Obviously, with an increase of non-local parameter, critical buckling load ratio faces a decrease and also the effect of Winkler and Pasternak foundations increases. As indicated in "Fig. 7(a-b)", when non-local parameter is constant at a certain value, rise of modules of elastic foundations leads to an increase in buckling load ratio which shows a difference between results of local and non-local theories. Hence, it is to conclude that shear module of Pasternak model makes a more significant effect on critical buckling load ratio than the one Winkler does. It is notable that growth of non-local parameter reduces the effect of elastic foundations kw and kg on critical buckling load ratio. In addition, unlike Winkler elastic foundation, influence of Pasternak foundation on the critical buckling load ratio is independent of the type of boundary conditions. Figure 8(a-b) show the Non-dimensional critical buckling load of SS annular nano-plate (R=0.5, r_0=20 nm) versus the (a) Winkler module and (b) Pasternak elastic foundation (Kw=50) for different values of non-local parameter.

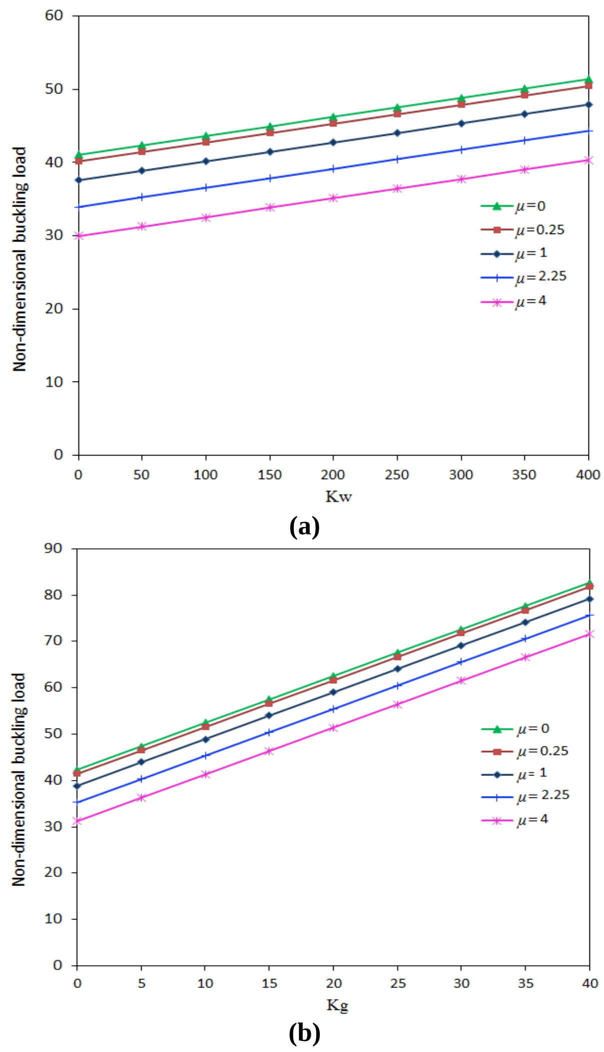


Fig. 8 Non-dimensional critical buckling load of SS annular nano-plate (R=0.5, r_0=20 nm) versus the (a) Winkler module and (b) Pasternak elastic foundation (Kw=50) for different values of non-local parameter.

To consider effects of Nano-plate thickness, values of non-dimensional critical buckling load of annular SLGS with and without Winkler elastic foundation are presented in "Table. 4" for different boundary conditions, outer-to-inner radius and outer radius-to-thickness ratio. Unlike Nano-plate without Winkler elastic foundation when Nano-plate rests on elastic foundation, the largest and slightest increases depend on CC and CF boundary conditions. Moreover, the effect of thickness on buckling load is completely dependent on the values of non-local parameter and annularity. So in Nano-plates resting on elastic foundation, increase of outer-to-thickness ratio (r_o/h) leads to reduction of non-local buckling load and the effect of this increase is stronger when outer-to-inner radius ratio (r_o/r_i) decreases and Non-local parameter increases.

Table 4 Critical buckling load (N/m) of annular Nano-plate with and without Winkler elastic foundation for different boundary conditions, outer-to-inner radius and outer radius-to-thickness ratio.

r_o/h	μ (nm^2)	r_o/r_i	kw=0					kw=0.1Gpa/nm				
			CC	SC	CS	SS	CF	CC	SC	CS	SS	CF
100	0	1.5	1.1373	0.6141	0.5606	0.2889	0.0697	1.4736	0.9787	0.9191	0.7454	0.4060
		3	0.2879	0.1727	0.1333	0.0778	0.0192	0.9444	0.8171	0.7696	0.7464	0.3545
		6	0.1859	0.1293	0.0838	0.0566	0.0131	0.9029	0.8106	0.7510	0.7300	0.3503
	1	1.5	0.6040	0.4182	0.3919	0.2374	0.0657	0.8282	0.7201	0.6989	0.6919	0.3293
		3	0.2374	0.1545	0.1222	0.0737	0.0182	0.7100	0.6686	0.6454	0.6363	0.3091
		6	0.1646	0.1202	0.0788	0.0556	0.0131	0.6647	0.6395	0.6256	0.6243	0.3060
	2.25	1.5	0.3808	0.2980	0.2848	0.1939	0.0616	0.5141	0.5050	0.5030	0.4949	0.2667
		3	0.1949	0.1364	0.1101	0.0697	0.0182	0.5050	0.5020	0.4969	0.4969	0.2616
		6	0.1444	0.1101	0.0747	0.0535	0.0131	0.5043	0.4878	0.4888	0.4880	0.2596
50	0	1.5	8.8385	4.8339	4.4127	2.2998	0.5575	9.1799	5.2076	4.7824	2.7553	1.3201
		3	2.2856	1.3787	1.0686	0.6232	0.1535	3.6259	2.8331	2.4462	2.4068	1.0423
		6	1.4837	1.0322	0.6696	0.4575	0.1121	3.4552	2.8815	2.3745	2.2321	0.9959
	1	1.5	4.6965	3.2896	3.0876	1.8887	0.5292	4.9258	3.6108	3.4158	2.3462	1.1959
		3	1.8857	1.2332	0.9757	0.5939	0.1515	3.0502	2.5796	2.2766	2.2735	0.9827
		6	1.3140	0.9615	0.6353	0.4444	0.1111	2.8563	2.5311	2.1796	2.0705	0.9494
	2.25	1.5	2.9613	2.3503	2.2452	1.5433	0.4979	3.1441	2.6391	2.5462	2.0018	1.0635
		3	1.5473	1.0898	0.8797	0.5616	0.1485	2.5422	2.2644	2.0624	2.0069	0.9161
		6	1.1504	0.8848	0.5959	0.4303	0.1101	2.3291	2.1927	1.9635	1.9049	0.8959

6 CONCLUSION

In this study, axisymmetric buckling of annular orthotropic SLGS embedded in a Pasternak elastic medium is investigated under uniform in-plane loading. Using the principle of virtual work, equilibrium equations are obtained through Mindlin orthotropic plate models, and Eringen non-local elasticity theory was applied to small scale effect parameter. Differential Quadrature Method (DQM) is employed to solve governing equations for free, supported and clamped boundary conditions or combinations of them. The proposed formulation and method of solution are validated by comparing results with those available in the literature. Finally, a detailed parametric study is carried out to investigate the influence of small scale effect, surrounding elastic medium, boundary conditions and geometrical parameters on non-dimensional buckling load. Some general impacts are mentioned below:

- In contrast to Winkler and Pasternak elastic foundations, the greatest to the smallest effect of non-local parameter on the buckling load is $(CF \langle FC \langle SS \langle CS \langle SC \langle CC)$ with respect to the type of boundary conditions.
- For annular Nano-plates in which their outer radiuses are 20nm or greater and inner-to-outer radius ratios (R) are 0.25 or lower, buckling is independent of non-local

parameter for various boundary conditions. So buckling happens with enlargement of outer radius in higher inner-to-outer radius ratios.
- Shear module of the Pasternak model makes a more significant effect on critical buckling load ratio than that of Winkler parameter.
- The variations rate of the critical buckling load is independent of small scale values with the change of both Winkler and Pasternak elastic foundations
- Unlike Nano-plate resting on Winkler elastic foundation, for nano-plate without elastic foundation, the largest and slightest effects of non-local parameter on buckling load depends on CC and CF boundary conditions, respectively.
- For Nano-plates without an elastic foundation, the effect of thickness on the buckling load is independent of values of non-local parameter and annularity.
- In Nano-plates resting on elastic foundation, increase of r_o/h leads to reduction of non-local buckling load specially when r_o/r_i decreases and non-local parameter increases.
- Unlike Winkler elastic foundation, influence of Pasternak foundation on critical buckling load ratio is independent of the type of boundary conditions.

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