# Optimization of Suspension System of Sport Car in Three Dimensional Reactions Space

**R. Kazemi***
Department of Mechanical Engineering,
K. N. Toosi University of Technology
E-mail: kazemi@kntu.ac.ir
*Corresponding author

**M. Raf'at & H. Zolghadr**
Department of Mechanical Engineering,
K. N. Toosi University of Technology
E-mail: fgh.rafat@gmail.com, sdsad9079@yahoo.com

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**Abstract:** Suspension system as one of the most important key elements of vehicle has always been considered by research centres and auto makers over the world. This is of higher importance when it comes to sports cars which work within especially sensitive conditions and their performances may be affected adversely even by the smallest defects. This study investigates optimization of effective handling parameters of a sport car with three optimization indices covering racetrack at the shortest possible time. For this purpose, a sport car model simulated in ADAMS/Car software has been used and all test steps have been implemented in this environment. To reach optimum solution in different racetrack conditions, Pareto solution set optimization method was used in which optimization indices have been examined covering distance in standard constant radius test with three different radii. Implementing these experiments at different levels of optimization parameters, all solution sets were collected in a three dimensional diagram called performance space whose coordinates axes are optimization indices values; then using LP norm idea, possible optimum solutions were selected among other ones. In order to find optimum solution in a special racetrack, a new idea was proposed through which weighted coefficients related to optimization indices for a special racetrack were determined and optimum solutions appropriate for that track were selected from Pareto solution set.

**Keywords:** Optimization, Pareto Solution Set Method, Three Dimensional Reactions Space, Sport Car

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**Biographical notes: R. Kazemi** received his BSc and MSc in mechanical engineering from Isfahan University of Technology (IUT), Iran, in 1993 and 1995, respectively, and his PhD in mechanical engineering from Amirkabir University of Technology, Iran, in 2001. He is an Associated Professor of Mechanical Engineering and head of the Advanced Vehicle Systems Research Center (AVSRC) at K.N.Toosi Univ. of Technology, Tehran, Iran. He is currently involved in modelling, design, and improvement of vehicle steering, suspension, and braking systems and also Vehicle Dynamics Control systems. **M. Raf'at** received his BSc and MSc in mechanical engineering from Semnan University and K.N.Toosi University of Technology (KNTU) Iran, in 2010 and 2013, respectively. He is interested in Vehicle Subsystem Design and Optimization, Vehicle Dynamic Control and Vehicle Safety System Design. **H. Zolghadr** received his BSc and MSc in mechanical engineering from K.N.Toosi University of Technology (KNTU) Iran, in 2005 and 2008, respectively. He is interested in nonlinear control and adaptive control for non predictable systems.

## 1 INTRODUCTION

General vehicle design, its tuning and also set up in different racetracks are up to engineering department of each team which provides optimum conditions for driver to cover racetrack faster than others. Any small change in vehicle basic design or at its set up time may contribute to increase vehicle efficiency and in the case of capable drivers, results in increased efficiency in traversing racetrack [1], [2]. In addition to general vehicle designing, computer softwares have been used as well for setting up simulation and race car optimization in different racetrack conditions. A sport car model simulated by ADAMS/Car software has been used in the current study to implement the relevant tests. Vehicle designing is a significant example of optimization designing problems in which different characteristics such as different dynamic and aerodynamic features, tires, engine, shock absorber, driver's decision, etc. may affect vehicle controlling approach and efficiency.

This research is aimed at finding optimum solutions for vehicle dynamic parameters subjected to different racetracks in various tracks. Along a racetrack, the driver faces with turns with different radii and also straight paths. Each turn requires related optimum characteristics of the vehicle. To cross total racetrack, thus, it is necessary to provide optimum conditions for all the mentioned settings [3]. In order to optimize sensitive parameters of suspension system that is previously determined by sensitive analysis method [4], handling characteristics of a sport car have been optimized in this paper through Pareto solutions set. A set of appropriate solutions were collected and via a method explained later on optimum solution selection, optimum indices of suspension system have been determined in different racetrack conditions.

## 2 OPTIMIZATION PARAMETERS

A sport sedan was considered in this study to be converted to a race car. For this purpose, some optimizations were implemented on its suspension system. Its front suspension system of Mcpherson type associated with anti roll bar and its back suspension system is multi link type with five arms and back anti roll bar. Good capability to improve journey comfort and handling are advantages of this system [5]. Also, its steering system is of rack and pinion type and other modelled elements such as tires, sub-chassis and body, engine and powertrain system are simulated according to original car characteristics.

Optimization parameters in front and back suspension system of the sample car, a tuned sedan simulated by ADAMS/Car software, are shown in Figs. 1 and 2. The mentioned parameters with applied changes during optimization are available in Table 1.

## 3 OPTIMIZATION INDICES

In order to analyze vehicle handling, many indices can be examined, including maximum roll angle, roll angle value (subgraph area), roll angle value in static status, lateral acceleration lunge, rising time of lateral acceleration, lateral acceleration value in static status, yaw rate lunge, rising time of yaw rate, yaw rate value in static status and time differences between 50% of steering wheel angle value and 50% of static status of lateral acceleration and also in frequency rang indices such as yam rate RMS and lateral acceleration RMS. These characteristics determine vehicle handling quality through affecting temporal solution area and system static status [6], [7].

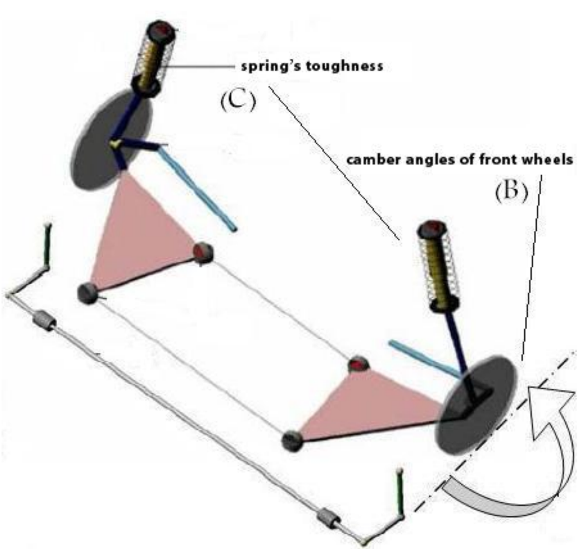


**Fig.1** used parameters of front suspension

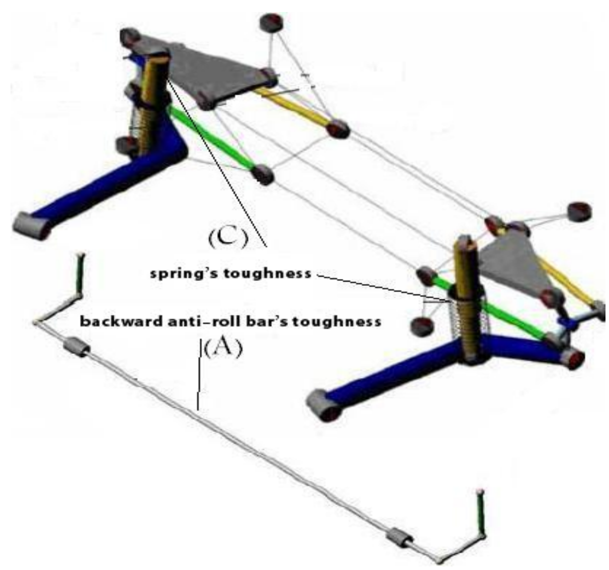


**Fig. 2 B**ack suspension parameters used

**Table 1** Optimization parameters

| Level -2 | Level -1 | Level 0 | Level 1 | Level 2 | levels / parameters |
|---|---|---|---|---|---|
| -3 | -2 | -1 | 0 | 1 | A (degree) |
| 108 | 114 | 120 | 126 | 132 | B (Nm/deg) |
| 0.7 | 0.8 | 0.9 | 1 | 1.1 | C (coefficient) |

**Table 2** Analysis characteristics

| Analysis characteristics | Turning radius (m) | Experiment time period (s) | Initial velocity (km/h) | Final velocity (km/h) |
|---|---|---|---|---|
| Analyze I | 30 | 11.5 | 35 | 60 |
| Analyze II | 80 | 18.5 | 60 | 90 |
| Analyze III | 160 | 22.25 | 90 | 140 |

An index must be chosen to fulfil the goal of this research that is to optimize a sport car and the concept of "velocity race", indicating traversing a special distance at the shortest possible time as the most important parameter to compare these vehicles. Therefore the index of "traversed distance value at a constant time along a special course" has been selected which indicates required time to cover a racetrack. In order to cover this index of all the above mentioned characteristics related to vehicle handling, the selected track for analyzing a standard track has different curves that a vehicle with certain radius turns inside it.

Assessing vehicle reaction in different radii needs to be carried out in three different situations reflecting low, intermediate, and high limit racetrack radii. Since racetrack is designed according to the type and performance of participating vehicles, selected radii for this research experiments are calculated by considering wheel steering roll limitation on one hand, and calculating maximum available velocity and lateral acceleration that the vehicle can bear on the other hand [8]. The minimum and maximum radii, accordingly, were considered.

An optimization index in Pareto method was adopted for constant radius experiments of 30, 80, and 160m and traversed distance value in each. Analysis index (I) with 30m radius was considered as X axis index, and analysis indices of (Π) with 80m radius and (III) with 160m radius as Y and Z axis indices respectively. The mentioned analyses characteristics are shown in table 2. Tests of this analysis are implemented in ADAMS/Car software and by driver control file (DCF) - driven constant radius cornering analytic method. This is a standard test to specify vehicle handling characteristics in stable status.

## 4 PARETO SOLUTION SET OPTIMIZATION METHOD

Pareto solution set have been used as a border line of efficient solution in performance space regarding the optimization problems of indices proposed in this research. Advantage of using this method is to achieve a series of solutions; each of its constituents may be counted as an optimum solution in a special racetrack condition. So, in facing with different racetracks, optimum parameters can be determined among few options via a new method which will be explained later. Similar to all selected methods, this one consists of two basic steps: potential solutions determination and selection from among them [9].

In Pareto analysis method, there are rules and conditions for a solution to be included in Pareto selected solution series which must be fulfilled. These conditions are mathematically expressed as in Eq. (1). A x* design vector is a Pareto optimum point if and only if for each x and I, Eq. (1) exists [3]:

$$f_j(x) \leq f_j(x^*) \ , j = 1, \dots, m \qquad (1)$$
$$j \neq i \ ; \quad f_i(x) \geq f_i(x^*)$$

### 4.1. Pareto reactions space

Pareto solution sets are usually analyzed in a performance space which by considering designing indices is shown as coordinate axes, and includes performance of the system under study in all possible designs. To this aim, a point has been drawn in each design, which is related to a certain value in any index. After selecting optimization parameters and their variable levels, the vehicle optimization solutions are separately obtained for each index through analyzing constant radius at 30, 80 and 160m that for each analysis, the combination of all parameters levels must be examined. Because the number of parameters are 3 and each of them has 5 levels, for each analysis ((53) =125) test must be conducted. Therefore, by implementing 375 testes for 3 indices, 125 points can be drawn in performance space that its longitudinal, latitudinal, and vertical positions, all determined by traversed distance, are demonstrated in (I), (Π), and (III) analyses for each combination of parameters. Pperformance space, thus, has been obtained from optimization parameters as shown in Fig. 3.

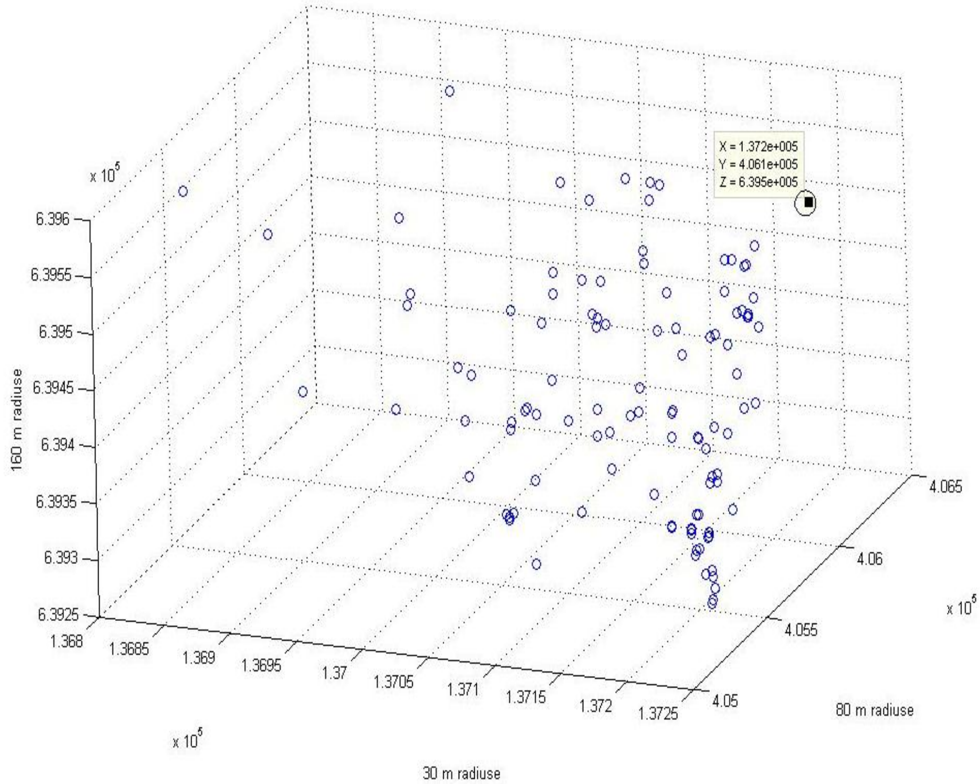


**Fig. 3** Pareto performance space and optimum solution in common weighted indices condition

**Table 3** Parameters characteristics related to Pareto solution set

| Number of experiments | **28** | **103** | **3** | **61** | **70** | **2** | **26** | **10** | **101** |
|---|---|---|---|---|---|---|---|---|---|
| Camber angle of front wheel (deg) | -3 | -3 | -3 | -1 | -3 | -3 | -3 | -2 | -3 |
| Rear Anti-roll bar stiffness (Nm/deg) | 114 | 132 | 108 | 120 | 114 | 108 | 114 | 108 | 132 |
| Stiffness coefficient of front and rear springs | 0.9 | 0.9 | 1 | 0.7 | 0.8 | 0.8 | 0.7 | 1.1 | 0.7 |
| Traversed distance in analyze I (mm) | 137220 | 137214 | 137225 | 137227 | 137160 | 137161 | 137100 | 137230 | 136944 |
| Traversed distance in analyze II (mm) | 639534 | 639535 | 639507 | 639498 | 639558 | 639547 | 639572 | 639422 | 639588 |

### 4.2. Pareto solution set

Pareto solution set property is that for each point in the mention set there is no corresponding point in performance space which simultaneously has better performance in both indices axes [3]. Since the considered index covers distance value during the experiment, the research is aimed at maximizing indices of this problem; therefore, for selected points to be included in Pareto solution set, there must be no point with simultaneously greater value in both axes. Table (3) shows parameters value related to Pareto solution set.

In Fig. 3, with regard to weighted coefficients of each index considered as 1, covered distance value in each point of performance space is equal to sum of covered distance in each related analysis of that solution.

**4.3. Optimum solutions selection**

LP norms is one the most applied methods in Pareto solution set to determine an optimum point. In this method that was developed by Eschenauer in 1990, minimum distance to Pareto solution set with respect to an ideal solution (ideal point) to get optimum solution is obtained as Eq. (2) [6]:

$$Minimize\ (\sum_{i=1}^{m}(f_i\ (\mathrm{x})\ -\ f_i^{*})^{p})^{1/\mathrm{p}} \quad (2)$$

Current applications of LP norms are L1, L2, L∞ where P is equal to 1, 2, and infinite (∞) respectively.

This approach and L2 norms (p=2) are also used similarly in this research to provide an appropriate method to find required solution to optimize vehicle handling performance when covering racetrack under all different track conditions. The Optimum solution is a combination of optimization parameters that by using them, the vehicle could traverse the most possible distance in a special course.

Using distance functions such as L2 norms makes the concept of ideal point very important, because this point is the best one to be obtained theoretically. According to L2 norm method, optimum point has the shortest geomantic distance to the ideal in Pareto solution set, where this distance is calculated as a vector in performance space.

To select optimum solution from among Pareto solution set as mentioned above, L2 norm method was used to select the ideal point. Since this research indices cover distance value in constant radius experiment, infinite point must be selected as the theoretical ideal point for both indices. However, because this point is geometrically meaningless in performance space, it is impossible to use it. Thus, a new suggested way is selecting zero point as the worst possible solution for all three indices. Thus, the optimum point would be a spot with uttermost distance from this point. The ideal point based on Pareto solution set is determined by drawing a sphere with Pareto performance space coordinates origin as its centre; the sphere radius increases and thus, the last spot which sphere crosses would be the optimum point. The optimum point solution obtained by this method with both indices having equally weighted coefficients is shown in Fig. 3.

In this section, indices with significant weighted values are assumed to be fixed and equal to 1. In other words, as observed in Fig. 3, both indices have the same certain relative significance; here solution number 3 is counted as the optimum solution. The performance space transformation due to significance change of each designing index will be discussed in the paragraphs to come.

**4.4. Scaling performance space axes**

Assuming Fig. 3 has been drawn by considering significance equal to 1 for both indices. Now if analysis (I) index significance is multiplied by 2, that means x axis index significance would be twice z and y axes indices; then related diagram would be elongated on x axis direction. Therefore, diagram shape varies and possibly another design point would be considered as the optimum solution.

It is worthy of note that the existing design point in Pareto solution set would not differ with respect to the prior case. In other words, Pareto solution set points are independent of significance value of design indices and the only difference is their drawing approach in performance space. Also in this stage, use of an accurate weighted ratio has no significance, and the important issue in this connection is the quality of performance space transformation due to relative significance alteration of indices weight.

The objective of this section is to determine required weighted coefficient for each Pareto performance space indices calculated considering racetrack conditions. Since each index indicates vehicle behaviour in a certain turn radius, the importance of these indices depends on using turns radii in different racetrack.

As seen in Fig. 4, the racetrack, as an example of a course in the US, is a composition of turns with different radii and also straight paths. Vehicle optimum composition in a turn with radius of 160m observed in right corner of path is different from another one with radius of 30m observed in the centre of picture. So, to design a vehicle with the best performance in all turns of a racetrack, optimum selection from among all existing parameters is needed.

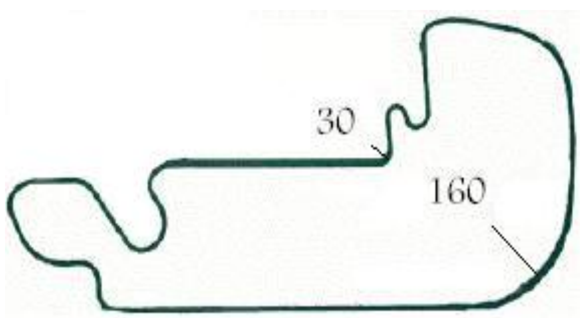


**Fig. 4** An example of a racetrack

In this step in order to introduce each index significance effect, its axis must be multiplied by a weighted coefficient to get the appropriate performance space of this special problem. Here, the goal is to provide an appropriate method to calculate the abovementioned coefficient. As seen in Table 2, experiments radii are selected so that vehicle velocity in these 3 radii includes all vehicle efficiency to traverse racetrack. So, the vehicle crosses a turn with radius of 30m by ultimate velocity of 60 km/h and a turn with 80m radius by 60 to 90 km/h velocity and a turn with 160m radius by 90 to 140 km/h. Thus, racetrack different radii considering maximum velocity by which a vehicle could cross, could be assigned to one of 3 intervals and be considered equal to one of source radii. Thus, the racetrack is assumed to have turns just with 30, 80, 180 m radii. Moreover, weighted coefficient of each index would be determined by distance value a vehicle traverses in each turn. In other words, in addition to the number of turns, another factor indicating course length of each turn curve affects indices with significant value.

Since constant radius experiments are implemented in approximately the same turn numbers, covered course length in each experiment, therefore, it is different from others and depends on curve radius ($L=r\Theta$). For this reason, since in experiments course length effect on turn is introduced with respect to its radius, to introduce turn length effect during the racetrack, an angle size is used in which the turn rolls with constant radius. So, the index weighted coefficient related to each turn is equal to turn angle value. Multiplying each turn weighted percent by its angle value and summing up these values for all turns, total coefficient related to each index is calculated. To simplify the calculation, "π" can be eliminated from coefficients of all turns and to apply changes upon just one of performance space axis, higher coefficient can be divided by the lower, and the resulted coefficient will be applied on the related axis. So, applying indices weighted coefficients, optimum solution may be obtained by using L2 norm method as discussed in previous section.

As an instance of the resulted changes in the Pareto solution set and also obtained optimum solutions due to changes in optimization indices weighted coefficient, the rest part focuses on the optimum solution finding given that each weighted coefficient is 5 fold to other indices. For each case, the diagrams as shown in Figs 5, 6 and 7 are obtained.

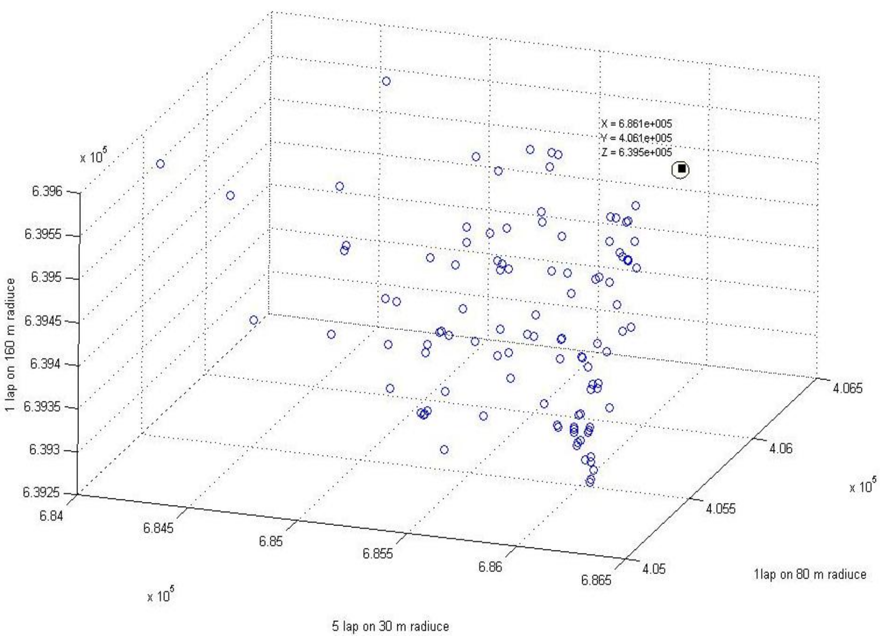


**Fig. 5** Optimum point determination with weighted coefficient 5 fold to x axis

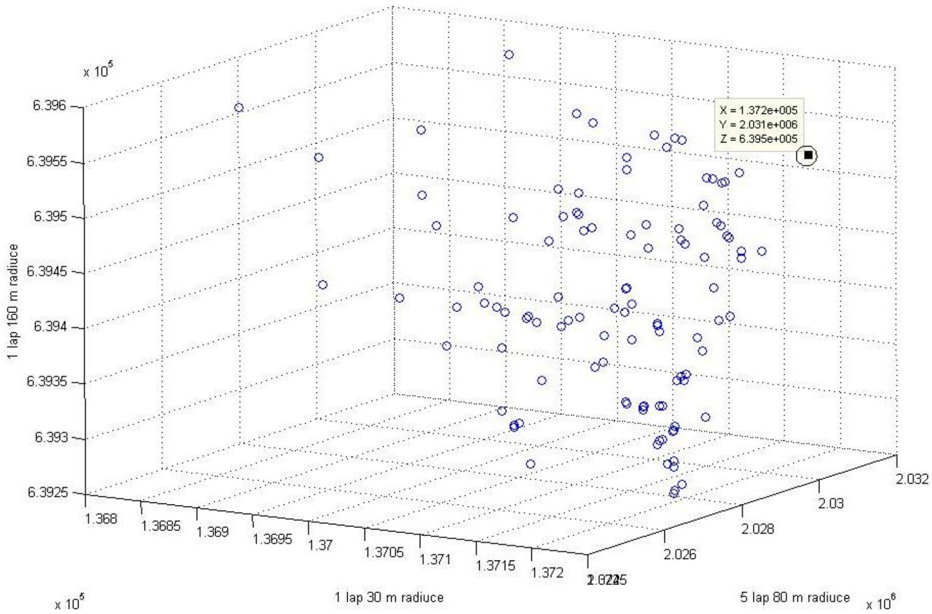


**Fig. 6** Optimum point determination with weighted coefficient 5 fold to y axis

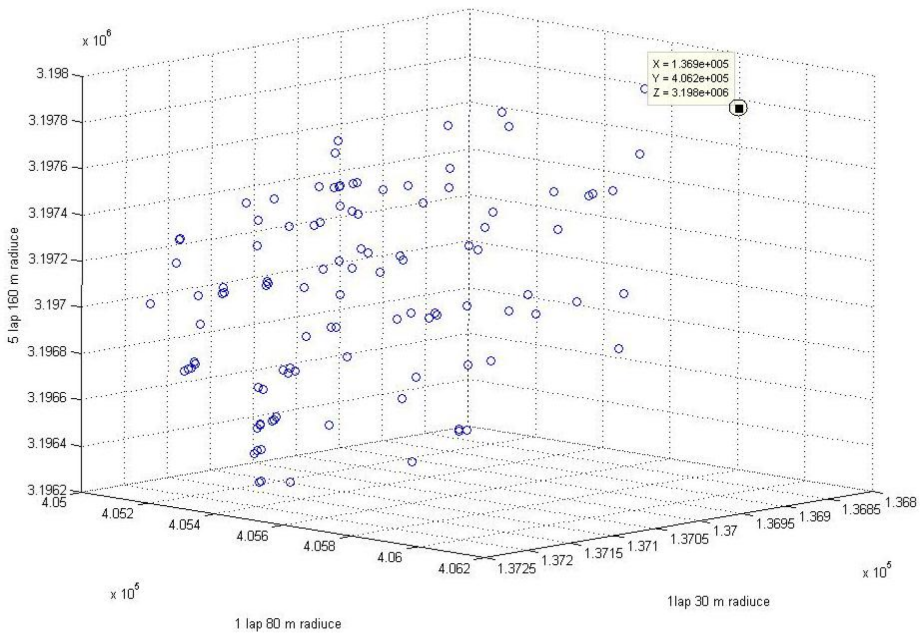


**Fig. 7** Optimum point determination with weighted coefficient 5 fold to z axis

Fig. 5 indicates Pareto solution set in such a status that x axis is multiplied by 5 and Fig. 6 shows Pareto solution set while y axis is multiplied by 5 and Fig. 7 shows it when z axis index is multiplied by 5. In these three diagrams, as seen axis related to indices with weighted coefficients, Pareto solution set is exposed to elongation and shape of Pareto solution set in these diagrams are different from each other as well as Fig. 4. Therefore, sphere showing optimum solution will pass different points on Pareto solution set, as obtained optimum points in the first, second and third statuses are 3, 3 and 70 respectively.

## 5 CONCLUSION

This paper attempted to provide an appropriate method which racing teams could be able to prepare their vehicles for different races at the best situation relevant to each racetrack, and make drivers capable of covering racetracks in the shortest possible time via a simulated model of sport car in ADAMS/Car software.

Accordingly, after determination of vehicle suspension system sensitive parameters which have great influence on desired handling indices, Pareto optimization method was elaborated. Considering advantage of this optimization method including capability of optimizing more than one index and also a semi-optimum solution set for different conditions, its each member could be an optimum solution in a special condition of different racetracks, where the method is adopted to achieve sport car optimum suspension system.

In this research and according to problem demanded solution type which is vehicle optimum parameters determination to cover racetrack in the shortest possible time, "covered distance value in a certain course with constant radices" as our index was used. So, the standard constant radius experiment implementation in 30, 80 and 160m radii that includes all velocity ranges a racetrack considering discussed vehicle efficiency range, provides three optimization indices.

By carrying out 375 experiments, obtained values are drawn in Pareto performance space diagram and then by using L2 norms method, a set as optimum solution was selected. Considering different racetrack conditions and race turn characteristics, a certain weighted coefficient was used for each index. This weighted coefficient is determined considering turn radius and its roll angle and use of explained method, then it is multiplied by performance space. Hence, solution set form changes according to racetrack condition and therefore, the optimum solution related to each automotive road race is determined.

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