Application of the Relevance Vector Machine for Modeling Surface Roughness in WEDM Process for Ti-6Al-4V Titanium Alloy

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Abstract: Cutting the Titanium alloys is a complicated task which cannot be performed by traditional methods and modern machining processes, such as Wire electro-discharge machining (WEDM) process which are mainly used for this purpose. As a result of the high price of the Ti-6Al-4V alloy, proper tuning of the input parameters so as to attain a desired value of the surface roughness is an important issue in this process. For this purpose, it is necessary to develop a predictive model of surface roughness based on the input process parameters. In this paper, The Taguchi method was used for the design of the experiment. According to their effectiveness, the input parameters are pulse-on time, pulse-off time, wire speed, current intensity, and voltage; and the output parameter is surface roughness. However, a predictive model cannot be defined by a simple mathematical expression as a result of the complicated and coupled multivariable effect of the process parameters on the surface roughness in this process. In this study, application of the relevance vector machine as a powerful machine learning algorithm for modeling and prediction of surface roughness in wire electro-discharge machining for Ti-6Al-4V titanium alloy has been investigated. The predicting result of model based on the root means square error (RMSE) and the coefficient of determination (R^2) statistical indices, prove that this approach provides reasonable accuracy in this application.

Keywords: Modeling, Relevance Vector Machine, Ti-6Al-4V Alloy, WEDM

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1 INTRODUCTION

In recent decades, the Ti-6Al-4V titanium alloy has widely been used in many industrial and medical applications due to its low density, high strength and corrosion resistance and excellent mechanical properties. It has applied in a wide range of applications from implants for dental and orthodontic wires to the chemical industry and gas turbines. As a result of special mechanical and metallurgical properties of this alloy, such as poor thermal conduction, extreme tendency to chemical reaction with cutting tool, and low elasticity modulus, Titanium alloys are difficult to cut by traditional methods and in some instances these methods may lead to several problems such as tolerances, tendency to weld onto tool's surface and rapid tool destruction, and occurrence of chatter phenomenon in long parts [1].

Wire electro-discharge machining (WEDM) and electrical discharge machining (EDM) are modern machining processes [2], [3] proven to be efficient for cutting the Titanium alloys. Surface roughness is an important factor in this process. As a result of the high price of this alloy and its special applications in sensitive parts such as implants, proper tuning of the WEDM parameters to achieve a desirable surface roughness has always been a crucial problem in this process. Therefore, it is important to investigate the mathematical relationship between surface roughness and the process parameters. However, no explicit mathematical expression has been proposed for this purpose.

Thanks to the advances in machine learning, many problems in engineering can now be solved easier. Supervised machine learning algorithms provide us with the possibility of generating models based on a limited set of observations. Therefore, application of supervised machine learning tools can be applied to generate a model from the dependency of surface roughness to WEDM parameters based on a limited set of measurements. Nevertheless, few studies have been performed for modeling surface roughness in wire electro-discharge machining for Ti-6Al-4V titanium alloy and investigation of supervised learning algorithms for this purpose can be helpful for the researchers in this field.

The performance of machine learning algorithms is largely dependent on the quantity and accuracy of databases used for training the models. For this purpose, Taguchi experiment design method was applied for designing the experiments to obtain a database of surface roughness and the corresponding WEDM parameters. Based on this database, the performance of both the relevance vector machine (RVM) and the support vector machine (SVM) algorithms are evaluated by means of standard statistical indices, which prove the RVM method to be efficient in this field based on the precision and generalization capability of the models. The main advantage of RVM-based modelling over the previous works is the acceptable accuracy and generalization capability of this approach, although the reduced number of measurements were obtained based on the experiments designed by the Taguchi method.

2 LITERATURE REVIEW

Tosun et al. [4] investigated the effect of the parameters, namely, wire speed, voltage, pulse-on time, and spray pressure of dielectric on kerf width and material removal rate using Taguchi experiments design. Based on the obtained signal per noise ratio, the voltage and pulse-on time parameters become more effective on output parameters. Muthu et al. [5] used a hybrid of Taguchi and GRA to achieve high optimized material removal rate, low surface roughness, and small kerf width simultaneously. According to the experiment results of titanium cutting, Arikatla et al. [6] figured out that the surface roughness increases with the rise of feed rate, pulse-on time, and current intensity of spark; on the other hand, the surface roughness decreases with the rise of the average gap voltage, wire speed, dielectric spray pressure, and wire tension.

Jangra et al. [7] proceeded to simultaneously optimize the material removal rate and surface roughness, employing a hybrid of Taguchi method, gray surface analysis, and entropy measurement. The entropy measurement was used for determining the weights of the machining parameters. Foorginejad et al. [8] used the neural network to model the electro-discharge process. They applied the obtained model for optimization of material removal rate and tool wear by means of firefly algorithm.

Shabgard et al. [9] utilized the fuzzy logic to successfully predict the parameters of electro-discharge machining. According to their study, the surface roughness rises with the increase of current and spark occurrence time. Ugrasen et al. [10] considered four input parameters, namely, pulse-on time, pulse-off time, current, and speed, and investigated their impact on material removal rate, wire wear, and surface roughness. In order to approximate the target parameters such as surface roughness, tool wear, and material removal rate, they used multiple linear regression (MRA) and group method of data handling (GMDH) models.

In the present study, Taguchi experiment design method was applied for designing the experiments. By operating the Taguchi table's experiments and collecting data, the modeling of data was carried out using relevance vector machine. The results from the implementation suggested that any of the presented methods are efficient, despite the simplicity, for modeling the surface roughness in the cutting process of Ti-6Al-4V titanium alloy.

3 WIRE ELECTRO-DISCHARGE MACHINING

The process of wire electro-discharge machining (WEDM) uses a source of thermo-electrical energy for material removal. In this process, repetitive controlled sparks which occur between the electrode and work piece are used to cut it. The electrode is a thin wire that is fed by a spool and passes through the part, and is wound by another mechanism from the other side. There is a small distance between the wire and the work piece that is called ‘gap’ which is filled by the dielectric liquid during the machining. The electric discharge occurs between the wire and the work piece with appropriate voltage, and the produced sparks locally evaporate it. Then, the dielectric liquid washes them off and the material removal process is done. The schematic view of the electro-discharge wire cut process is depicted in “Fig. 1” [11].

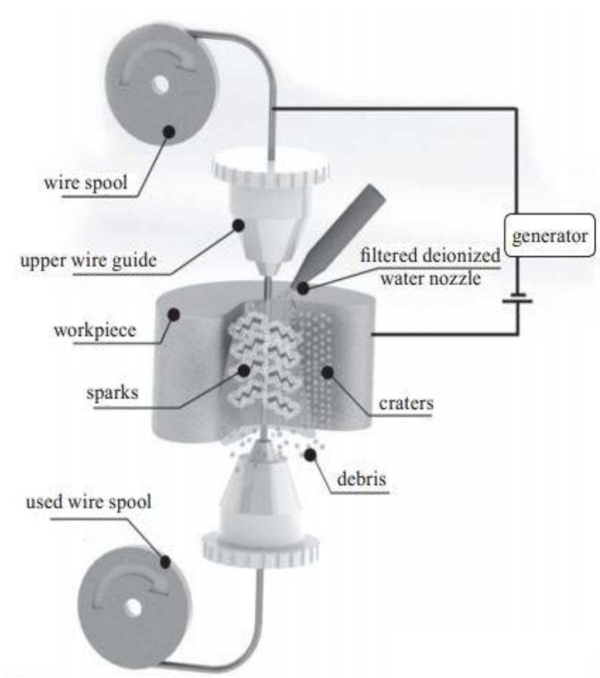

Fig. 1 Schematic of the WEDM process [11].

The ability to produce parts with high dimensional precision, controllable surface quality and the possibility to create sharp edges are the advantages of electro-discharge wire cut machining [12]. In addition, the manufacturing process is not affected by the strength and hardness of the machined part.
As depicted in “Fig. 2”, many parameters affect the electro-discharge wire cut process, and proper tuning of these parameters contributes to the improvement of the material removal rate and surface roughness [13]. Several studies have been performed to determine the proper parameters in order to achieve the desirable surface quality.

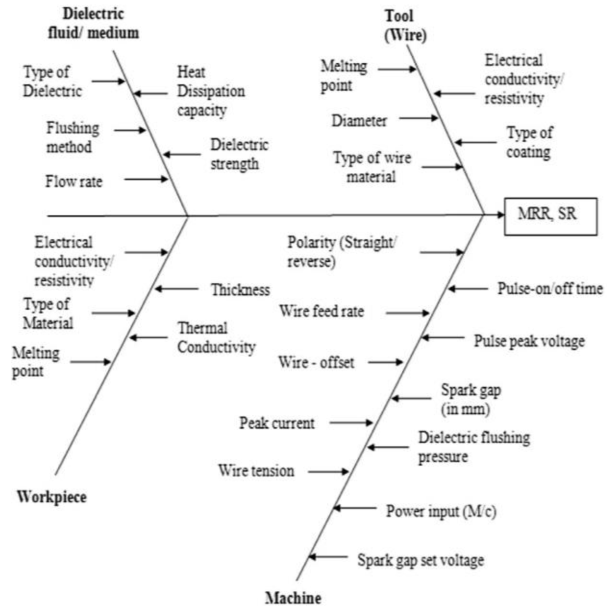

Fig. 2 The Ishikawa cause and effect diagram [13].

4 SURFACE ROUGHNESS

Surface roughness is an important criterion in determining the surface quality of machined parts. Surface roughness can be defined as an indicator of the material tissue, which represents the unevenness of a surface in comparison with a smooth surface. Various methods are used for measuring the surface roughness, which is divided into two groups, namely, contact and non-contact. In the present research, contact method has been used for measuring the cut surface roughness. For evaluation of the surface roughness, three different indices have been proposed by researchers which are arithmetical mean roughness Ra, ten-point mean roughness Rz, and root mean square roughness Rq, from which the arithmetical mean roughness is mainly used in the literature. This index is calculated using the relations (1) and (2) [11], as shown in “Fig. 3”.

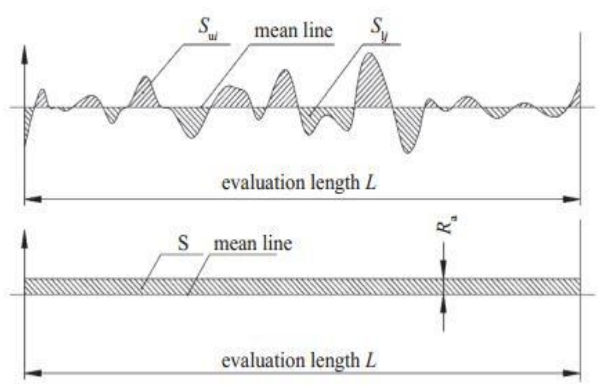

Fig. 3 Scheme of surface roughness Ra [11].

5 EXPERIMENTAL DETAILS

In the electro-discharge wire cut process, a series of sparks have to be generated between the wire and work piece via a constant electrical field in a dielectric environment, leading to the cutting of the work piece. The spark is generated in the presence of dielectric liquid with proper setting of the applied voltage. The machine utilized to conduct the experiments was a CNC 5-axis electro-discharge wire cut machine of Charmilles Robofill 310F model with two generators, i.e. iso-pulse and iso-frequency, and the experiments were carried out by the iso-pulse generator. The surface roughness was considered as the process efficiency parameter.
As Ishikawa effects diagram shows, the surface roughness is related to process input parameters such as pulse-on time (Ton), pulse-off time (Toff), wire speed, current, and voltage. Proper setting of the machining parameters can improve the surface quality by reducing its roughness value. In order to run the experiment, a 9-mm-thick titanium alloy specimen was cut by the electro-discharge wire cut process. “Table 1” shows the chemical composition of the titanium alloy used.
Surface roughness was measured by the Taylor-Hobson surface roughness tester as shown in “Fig. 4”. For each specimen, the surface roughness was measured 3 times based on Ra, and the average roughness value was calculated and considered as the surface roughness value.

Table 1 The chemical composition of Ti-6Al-4V

Component	Al	Fe	Sn	V	Ti
Weight %	6.22	0.187	0.56	3.35	89.6

6 PROCESS CONTROL PARAMETERS AND TAGUCHI STANDARD ORTHOGONAL ARRAY SELECTION

Taguchi method is a powerful approach for designing experiments to obtain the optimized values of the process parameters and has been proven as one of the most effective methods for manufacturing products with high quality and low cost. Unlike the full factorial design of experiments [14] reduction of experiments is one of the main benefits of using this method [15]. As it was indicated, in this study, the input parameters are pulse-on time, pulse-off time, wire speed, current intensity, and voltage, and the output parameter is surface roughness. The ranking of control parameters is shown in “Table 2”.
Taguchi orthogonal array is selected based on the different levels for the entire input parameters. The orthogonal array is a special matrix which includes different levels of the input parameters. Each row of the matrix shows a different unique condition of the experiment, while each column contains the entire levels of a specific input parameter which are equally iterated.

Fig. 4 Measurement of the surface roughness.

Table 2 The Process parameters and their levels

Process parameter	Unit	Level 1	Level 2	Level 3	Level 4
Gap voltage	v	35	45	55	65
Discharge current	A	6	8	10	12
Pulse on time (Ton)	µs	0.6	0.8	1	1.2
Pulse off time (Toff)	µs	4	6	8	10
Wire feed rate	m/s	1	1.5	2	2.5

7 SUPPORT VECTOR MACHINE REGRESSION

In recent decades, the need for systems that can learn from limited information and solve complex decision problems has become more important as a result of the rapid advances in information processing systems. The study and construction of algorithms, capable of learning from and making predictions based on a limited set of observed data are explored in a subfield of computer science known as machine learning [13]. In supervised learning, given a set of N input vectors $\{x_n\}_{n=1}^{N}$ and the corresponding targets $\{t_n\}_{n=1}^{N}$, we want to learn a model of the dependency of the targets on the inputs in order to predict the targets in case of inputs which have not been observed [15].
Support vector machines (SVMs) are supervised learning models with associated learning algorithms that analyse data and recognize patterns, used for classification and regression analysis. The initial form of the support vector machines was a generalization of the Generalized Portrait algorithm, developed in the 1960s. However, the present form of the SVMs, was developed by Vapnik and his coworkers at the AT&T Bell

Laboratories in the 1990s [16]. In SVM-based regression, in order to estimate a function in form of "Eq. (1)" based on a limited set of observations, the input space is mapped into a high dimensional feature space via the kernel function $\phi(x)$ and then a linear optimal regression is performed in this space.

$$y = f(x) = w^T\phi(x) + w_0 \tag{1}$$

The vector of weights w and the bias w_0 are estimated based on structural risk minimization principles [17], by solving the following optimization problem:

$$\min\left\{R(w) = \frac{1}{2}\|w\|^2 + C\,.\sum_{i=1}^{N} \xi_i + \xi_i^*\right\}$$

$$\text{s.t.}\begin{cases} y_i - (\boldsymbol{w^T}\phi(x_i) + b) \le \varepsilon + \xi_i \\ (\boldsymbol{w^T}\phi(x_i) + b) - y_i \le \varepsilon + \xi_i^* \\ \quad \xi_i \text{ و } \xi_i^* \ge 0 \ \ , \ \ i = 1, \ldots, N \end{cases} \tag{2}$$

Where C is the regularization factor, ε is the insensitivity parameter and $\xi_i$and ξ_i^* are slack variables, calculated based on the Vapnik's ε-insensitive loss function, as:

$$\xi = |y - f(x)|_\varepsilon = \max\{0, |y - f(x)| - \varepsilon\} \tag{3}$$

Figure 5 depicts the concept of ε -insensitivity in SVM-based regression. The optimization problem can be solved via quadratic programming optimization and the estimated function is expressed based on the optimal values as:

$$f(x) = y(x_i, w) = \sum_{i=1}^{N} w_i K(x, x_i) + w_0 \tag{4}$$

Where N is the number of training samples and $K(x, x_i)$ is calculated as:

$$K(x_k, x_i) = \langle\varphi(x_k), \varphi(x_i)\rangle \quad (k, i = 1, \ldots, N) \tag{5}$$

The training samples associated with non-zero weights are called the support vectors, which determine the number of necessary kernel functions for estimating a function. The Gaussian radial basis function (RBF) kernel is the most popular kernel function in SVM and other kernel methods, expressed as:

$$K(x, x_i) = \exp\left(-\frac{\|x - x_i\|^2}{2\sigma^2}\right) \tag{6}$$

Where σ^2 is the kernel function's parameter. The key feature of the SVMs is the minimization of the structural risk besides the empirical risk, resulting in model sparseness besides accuracy and state-of-the-art results have been reported on many tasks where SVMs have been applied [18].

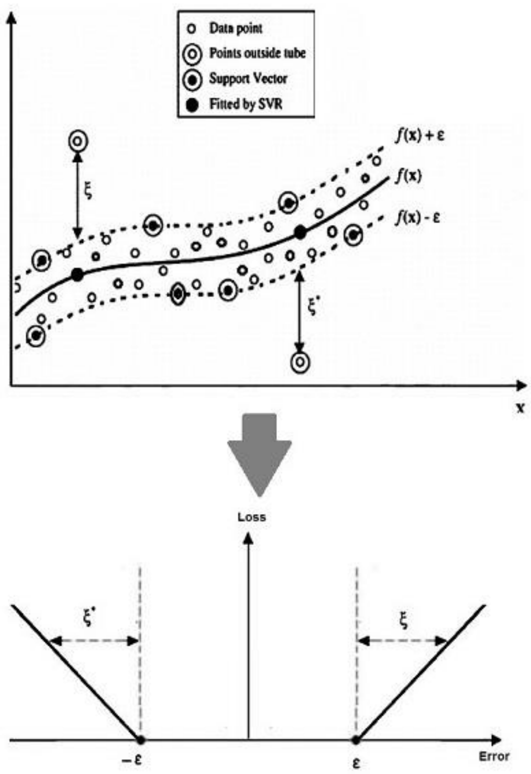

Fig. 5 SVM-based regression and the concept of $\boldsymbol{\varepsilon}$ – insensitivity.

Despite its widespread success, the SVM methodology suffers from some important disadvantages [19]:

- SVM makes point predictions and the predictions are not probabilistic. Ideally, estimation of a conditional distribution of the outputs $p(t|x)$ is desired in order to capture uncertainty in prediction. Although, posterior probability estimates have been coerced from SVMs via post-processing, they have been argued to be unreliable.
- Although relatively sparse, SVMs make liberal use of kernel functions, the requisite number of which grows steeply with the size of the training set.
- It is necessary to estimate the regularization parameter 'C', which trades off between the error and margin, and the insensitivity parameter ε, as the margin of tolerance in function estimation. For this purpose, a cross-validation procedure is mainly necessary, which is wasteful both data and computation.

The kernel function $K(x, x_i)$ must satisfy the mercer's condition. That is, it must be the continuous symmetric kernel of a positive integral operator.

8 RELEVANCE VECTOR MACHINE REGRESSION

To overcome the shortcomings of support vector machines, a fully probabilistic framework termed relevance vector machine (RVM) has been used. RVM is a nonlinear pattern recognition model with a simple structure based on Bayesian Theory and Marginal Likelihood. In addition to the improving the inadequacies of SVM, RVM utilizes a very fewer number of kernel functions. Therefore, it has been used in wide variety of applications, such as wind power grouping forecast, power system disturbance classification, load forecasting, canal flow prediction, EEG signal characterization and pneumatic actuator fault diagnosis. But its application has not yet been investigated for modeling the wire electro-discharge machining process. In RVM-based regression, in order to predict a function based on a set of N input-target pairs$\{x_n, t_n\}_{n=1}^{N}$, each target is modeled as a function of the corresponding inputs with additive white Gaussian noise to accommodate measurement error on the target:

$$t_i = y(x_i, w) + \varepsilon_i \tag{7}$$

ε_i is assumed to be mean-zero Gaussian with variance σ^2 and similar to the SVM, $y(x, w)$ is considered as a linear combination of N kernel functions centered at the training samples inputs, in form of "Eq. (1)". Therefore, with the assumption that we know y(xn), each target is independently distributed as Gaussian with the mean $y(x_n)$ and variance σ^2, expressed as:

$$p(t_n|x) = N(t_n|y(x_n), \sigma^2) \tag{8}$$

Due to the assumption of independence of the targets, the likelihood function of the whole samples can be written as:

$$p(\boldsymbol{t}|\boldsymbol{w},\sigma^2) = \frac{e^{\left\{\frac{-\|\boldsymbol{t}-\boldsymbol{\varphi}\, \boldsymbol{w}\|^2}{2\pi\sigma^2}\right\}}}{(2\pi\sigma^2)^{\frac{N}{2}}} \tag{9}$$

Where:

$$\boldsymbol{t} = (t_1 \dots t_N)^T \tag{10}$$

$$\boldsymbol{w} = (w_0 \dots w_N)^T \tag{11}$$

And $\boldsymbol{\varphi}$ is an N*(N+1) matrix, defined as:

$$\boldsymbol{\varphi} = [\, \varphi(x_1), \varphi(x_2), \dots, \varphi(x_N)]^T \tag{12}$$

In which the vector $\varphi(x_n)$ is calculated as:

$$\varphi(x_n) = \begin{bmatrix} 1, K(\boldsymbol{x}_n, \boldsymbol{x}_1), K(\boldsymbol{x}_n, \boldsymbol{x}_2) \\ , \dots, K(\boldsymbol{x}_n, \boldsymbol{x}_N) \end{bmatrix}^T, N = 1, \dots, N \tag{13}$$

It is expected that maximum likelihood estimation of w and σ^2 from (9) would lead to over-fitting. Therefore, additional constraints must be imposed on the parameters. For this purpose, a 'prior' zero-mean Gaussian probability distribution is assumed for the weights as follows:

$$p(\boldsymbol{w}|\boldsymbol{\alpha}) = \prod_{i=0}^{N} N(w_i|0, \alpha_i^{-1}) \tag{14}$$

Where $\boldsymbol{\alpha}$ is a vector of N+1 hyper-parameters. The variance of this Gaussian probability distribution, α_i^{-1} controls how far from zero each weight is allowed to deviate, and a very large value for α_i means that the corresponding weight, w_i is estimated to be zero. Using Bayesian posterior inference, the posterior over w is computed as:

$$p(\boldsymbol{w}|\boldsymbol{t},\boldsymbol{\alpha},\sigma^2) = \frac{p(t|w,\sigma^2)\ p(w|\alpha)}{p(t|\alpha,\sigma^2)} = (2\pi)^{-\frac{N}{2}}\ |\Sigma|^{-\frac{1}{2}}\ e^{\left\{-\frac{(w-\mu)^T\, \Sigma^{-1}\,(w-\mu)}{2}\right\}} \tag{15}$$

Where $\sum$ and μ are calculated as:

$$\sum = (\sigma^{-2}\boldsymbol{\varphi}^T\boldsymbol{\varphi} + \boldsymbol{A})^{-\mathbf{1}} \tag{16}$$

$$\mu = \boldsymbol{\sigma}^{-2}\Sigma\boldsymbol{\varphi}^{T}\,\boldsymbol{t} \tag{17}$$

Where in A is a diagonal matrix formulated as:

$$A = diag(\alpha_0, \alpha_1, \dots, \alpha_N) \tag{18}$$

Integrating $p(\boldsymbol{w}|\boldsymbol{t},\boldsymbol{\alpha},\sigma^2)$ over the weights w, it can be concluded that:

$$p(\boldsymbol{t}|\boldsymbol{\alpha},\sigma^2) = \int p(\boldsymbol{t}|\boldsymbol{w},\sigma^2)\, p(\boldsymbol{w}|\boldsymbol{\alpha})\, d\boldsymbol{w} \tag{19}$$

The integral above is a convolution of Gaussians, and can be calculated as:

$$p(\boldsymbol{t}|\boldsymbol{\alpha},\sigma^2) = (2\pi)^{-\frac{N}{2}}\ |\Omega|^{-\frac{1}{2}}\ e^{\left\{-\frac{\boldsymbol{t}^T\,\Omega^{-1}\,\boldsymbol{t}}{2}\right\}} \tag{20}$$

Where Ω is a matrix defined as:

$$\Omega = \sigma^2\boldsymbol{I} + \boldsymbol{\varphi}\,\boldsymbol{A}^{-1}\boldsymbol{\varphi}^T \tag{21}$$

Learning process of RVM can be described as a search for the parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\sigma^2}$ which maximize the marginal likelihood of $p(\boldsymbol{t}|\boldsymbol{\alpha},\sigma^2)$ based on the training dataset. The optimal parameters cannot be obtained in closed form, and they are estimated using an iterative re-estimation procedure. Following the approach of MacKay, the following iterative relationship can be obtained for estimating the hyper-parameters α_i by differentiation of $p(\boldsymbol{t}|\boldsymbol{\alpha},\sigma^2)$ in "Eq. (20)" with respect to $\log(\alpha_i)$ and equating it to zero.

$$\alpha_i^{new} = \frac{1 - \alpha_i \sum_{ii}}{\mu_i^2} \tag{22}$$

Where μ_i is the i-th element of the vector μ in "Eq. (17)", and the i-th diagonal element of the matrix $\sum$ in "Eq. (16)". For the noise variance σ^2, the following update formula is obtained by setting the derivative of the marginal likelihood with respect to $\log(\alpha_i)$ to zero:

$$(\sigma^2)^{new} = \frac{\|\boldsymbol{t} - \boldsymbol{\varphi\mu}\|^2}{N - \sum_{i=0}^{N}(1 - \alpha_i \sum_{ii})} \tag{23}$$

Iterative calculation of the parameters α_i and σ^2 from "Eq. (22)", "Eq. (23)" concurrent with updating of the posterior statistics $\sum$ and μ from "Eq. (16)", "Eq. (17)" is repeated until some suitable convergence criteria have been satisfied. In this procedure, many of the hyper-parameters α_i tend to infinity, which means that the probability distribution of the corresponding weights, w_i is peaked at zero and they are estimated to be zero, thus pruning many of the kernel functions used in "Eq. (1)", which results in the sparseness of the model. The training set, which associates with the remaining nonzero weights is called the relevance vector.

After convergence of the hyper-parameter estimation procedure and obtaining the maximizing values $\boldsymbol{\alpha}_{MP}$ and σ_{MP}^2, the predictions are made based on the posterior distribution over the weights conditioned on them. It has been proven in that the predictive distribution for a new input sample x^* has a Gaussian distribution, expressed as:

$$p(t^*|t) = N(t^*|y^*, \sigma_*^2) \tag{24}$$

Where y^* and σ_*^2 are the predicted mean and variance values, calculated as:

$$y^* = \boldsymbol{\mu}^T \boldsymbol{\varphi}(\boldsymbol{x}^*) \tag{25}$$

$$\sigma_*^2 = \sigma_{MP}^2 + \boldsymbol{\varphi}(\boldsymbol{x}^*)^T \sum \boldsymbol{\varphi}(\boldsymbol{x}^*) \tag{26}$$

$$\varphi(\boldsymbol{x}^*) = \begin{bmatrix} 1, K(\boldsymbol{x}^*, \boldsymbol{x}_1), K(\boldsymbol{x}^*, \boldsymbol{x}_2), \\ \dots, K(\boldsymbol{x}^*, \mathbf{x}_N) \end{bmatrix}^T, N = 1, \dots, N \tag{27}$$

9 RESULTS AND DISCUSSION

In order to obtain a database for training the models, Taguchi method was employed to design the experiments and the mean surface roughness was measured for different process parameters set based on the Taguchi's L-16 standard array. From the obtained database shown in "Table 3", twelve measurements were used for generating the model and four of them, indicated by *, were applied to test the model accuracy. To improve the accuracy, all the input and target values were normalized between −1 and +1 as:

$$pn = 2 * \frac{p - \left(\frac{max + min}{2}\right)}{(max - min)} \tag{28}$$

Where, *max* and *min* are respectively the maximum or minimum value of the input or the output among the whole dataset, p is the input or output and pn is the corresponding normalized value. Based on the normalized dataset, RVM and SVM models were implemented by the SparseBayes package for MATLAB and the SVM-km toolbox and predicted outputs were scaled to their original range based on "Eq. (29)", as:

$$\hat{y} = y_n * \left(\frac{max - min}{2}\right) + \left(\frac{max + min}{2}\right) \tag{29}$$

Where $\hat{y}$ is the predicted output in the original range and y_n is the normalized predicted output. The accuracy of the final models was evaluated based on the root means square error (RMSE) and the coefficient of determination (R2) statistical indices, defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{N}} \tag{30}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2} \tag{31}$$

In these equations, y_i and $\hat{y}_i$ are the measured and the predicted outputs, respectively, N is the number of training samples, y_{max} and y_{min} are the maximum and minimum values of the measured outputs and $\bar{y}$ is the mean value of the measured output, calculated as:

$$\bar{y} = \frac{\sum_{i=1}^{N} y_i}{N} \tag{32}$$

Table 3 The 4 different levels of the input parameters, set based on the Taguchi's L-16 standard array

Expt. No.	Gap voltage (A)	Discharge current (V)	Pulse on time (µs)	Pulse off time (µs)	Wire feed rate (m/s)	Mean Surface Roughness (µm)
1	1	1	1	1	1	3.0000
2	1	2	2	2	2	2.3933
3	1	3	3	3	3	2.6467
4	1	4	4	4	4	3.3733
5	2	1	2	3	4	2.6333
6	2	2	1	3	4	2.0533
7	2	3	4	1	2	2.9333
8	2	4	3	2	1	2.7133
9	3	1	3	4	2	2.5000
10	3	2	4	3	1	2.9600
11	3	3	1	4	4	2.0200
12	3	4	2	1	3	2.6067
13	4	1	4	2	3	2.7467
14	4	2	3	1	4	2.4133
15	4	3	2	4	1	2.1733
16	4	4	1	3	2	2.0200

The calculated value of indices is listed in "Table 4". For the purpose of comparison, the ratio of RMSE to the maximum value of the measured outputs (y_{max}) is also added to the table. The RVM and SVM kernel and model parameters, obtained by minimizing the training root mean square error are also listed in Table 5.

Table 4 The Statistical indices for evaluation of the RVM and SVM

Database	Method	RMSE	RMSE/y_max	R2
Training	RVM	1.095×10-4	6.9×10-8	1
	SVM	2.7×10-7	6.9×10-8	1
Testing	RVM	0.1694	0.0502	0.8756
	SVM	0.2237	0.0663	0.783

As it can be observed, the RVM method has a good performance for the training data and a better performance for the test data and therefore a better generalization capability than the SVM method. The measured outputs together with the outputs predicted by the RVM method are depicted in "Fig. 6", showing a good agreement between them.

Table 5 The RVM and SVM parameters

RVM kernel parameter	2.2437
SVM kernel parameter	2.03
SVM regularization factor (C)	100
SVM insensitivity parameter (ε)	10-9

In this survey, the surface roughness was evaluated in terms of the arithmetical mean roughness Ra index with four different levels of the input parameters, including the gap voltage, discharge current, pulse on-time, pulse off-time and wire feed rate. With respect to the uncertainty in measurements, Ra is calculated three times for each sample. The results are shown in "Table 6".

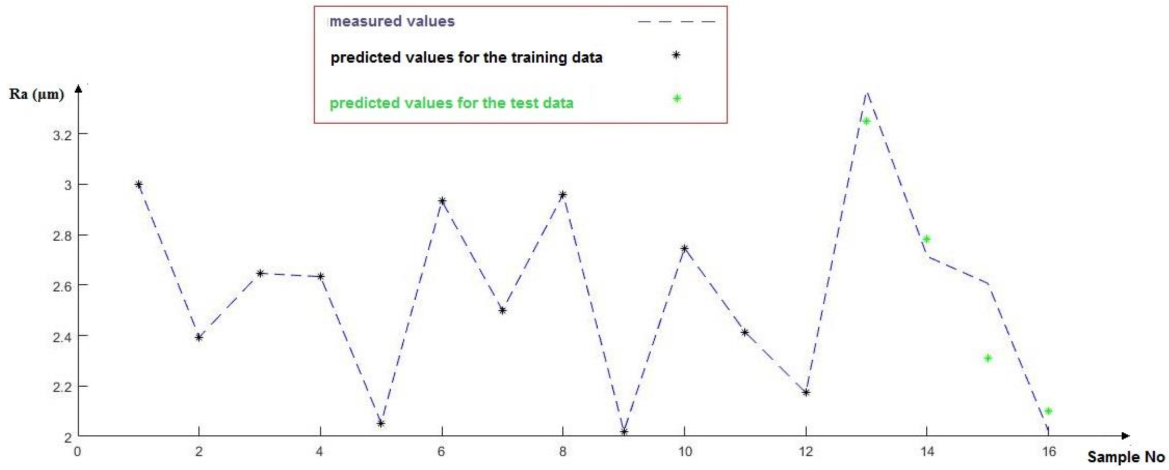

Fig. 6 The measured outputs and the outputs predicted by RVM.

Table 6 The mean, standard deviation and uncertainty of samples

Expt. No.	Ra1 (µm)	Ra2 (µm)	Ra3 (µm)	Mean Surface Roughness	Standard deviation	Uncertainty
1	2.84	2.04	3.12	3.0000	0.1178	0.1183
2	2.3	2.42	2.46	2.3933	0.0680	0.0690
3	2.46	2.72	2.76	2.6467	0.1330	0.1335
4	2.9	3.58	3.64	3.3733	0.3356	0.3358
5	2.64	2.56	2.7	2.6333	0.0573	0.0585
6	2.1	2.04	2.02	2.0533	0.0340	0.0359
7	3.4	2.58	2.82	2.9333	0.3442	0.3444
8	2.84	2.7	2.6	2.7133	0.0984	0.0991
9	2.52	2.22	2.76	2.5000	0.2209	0.2212
10	3.02	2.88	2.98	2.9600	0.0589	0.0600
11	1.84	2.08	2.14	2.0200	0.1296	0.1301
12	2.9	2.42	2.5	2.6067	0.2100	0.2103
13	2.78	2.72	2.74	2.7467	0.0249	0.0275
14	2.62	2.3	2.32	2.4133	0.1464	0.1468
15	2	2.36	2.16	2.1733	0.1473	0.1477
16	2.04	1.92	2.1	2.0200	0.0748	0.0757

The uncertainty in measurements is inevitable and is calculated based on "Eq. (33)", in which S and R are the standard deviation and resolution, respectively. The Roughness meter used, has a resolution of 0.02µm. "Table 6" shows the mean, standard deviation and uncertainty for the samples of "Table 3".

$$u = \sqrt{S^2 + \left(\frac{R}{\sqrt{3}}\right)^2} \quad (33)$$

10 CONCLUSION

Due to the high price of Ti-6Al-4V titanium alloy and its special applications in sensitive parts, proper tuning of the parameters of the wire electro-discharge machining process, as the most common process for machining this alloy, is an important issue which provides the possibility of achieving a desirable surface roughness. To this end, it is greatly important to the development of a global model of surface roughness based on the process parameters, which are pulse-on time, pulse-off time, wire speed, current, and voltage. For this purpose, based on the experiments designed by the Taguchi method, a database of the parameters and the corresponding measurements of the surface roughness is obtained. Using this database, the surface roughness is modeled by both the support vector machine and the relevance vector machine, as advanced machine learning techniques and the accuracy and generalization capability of the models is evaluated. Results show that modeling this process based on the relevance vector machine provides reasonable accuracy besides generalization capability and can be used to tune the process parameters to achieve a desirable amount of surface roughness.

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