Free Chattering Fuzzy Sliding Mode Controllers to Robotic Tracking Problem

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Abstract: Sliding Mode Control (SMC) is a powerful approach to solve the tracking problem for dynamic systems with uncertainties. However, the traditional SMCs introduce actuator chattering phenomenon which performs a desirable behavior in many physical systems such as servo control and robotic systems, particularly, when the zero steady state error is required. Many methods have been proposed to eliminate chattering from SMCs which use a finite DC gain controller. Although these methods provide a free chattering control but they only deal with the steady state error and are not able to reject input disturbances. This paper presents a fuzzy combined control (FCC) using appropriate PID and SMCs which presents infinite DC gain. The proposed FCC is a free chattering control which guarantees a zero steady state error and rejects the disturbances. The stability of the closed loop system with the proposed FCC is also proved using Lyapunov stability theorem. The proposed FCC is applied to a two degree of freedom robot manipulator to illustrate effectiveness of the proposed scheme.

Keywords: Chattering Phenomenon, Fuzzy Controller, Lyapunov Stability, Controllers, Sliding Mode Controllers

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1 INTRODUCTION

Sliding mode control (SMC) has been recognised as a powerful method for designing robust controllers for complex high-order nonlinear dynamic plants operating under various uncertainty conditions. The major advantage of SMC is low sensitivity to plant parameter variations and disturbances which relaxes the necessity of exact modelling [1].

In the traditional SMC theory, the control switches between two different sub-controllers repeatedly, leading to an undesirable phenomenon, the so-called chattering. However, in practice, this type of switching is impossible due to finite time delays and physical limitations. Many methods have been proposed for eliminating or reducing chattering including the boundary layer, continuous approximations and higher order SMC approaches. Using the boundary layer approach a continuous control associated with the original SMC is introduced to enforce the trajectories to remain nearby the sliding surface within a specified region, well-known as a boundary layer. If system uncertainties are large, an SMC design based on the boundary layer techniques requires a high switching gain with a thicker boundary layer to eliminate the influence of the chattering. However, using a boundary layer with large thickness, a sliding motion may not occur as the system trajectories may not be sufficiently close to the sliding surface. Therefore, the boundary layer SMC with large thickness may not guarantee the insensitivity with respect to the matched uncertainties. However, this method yields a finite steady state error due to finite non-switching gain of the controller in the steady state [2].

Another approach which prevents the occurrence of chattering is based on the generation of an ideal sliding mode in the auxiliary loop including using an observer. The observer-based chattering suppression obviously requires additional effort in control design; the plant parameters must be known to obtain a proper observer. However, including an observer in the control system may bring extra benefits such as identification of uncertainties and disturbances, in addition to its value in estimating unavailable states [1, 3]. Since the magnitude of chattering is proportional to the switching gain, some chattering reduction approaches are based on reducing the values of the switching gain to decrease the amplitude of chattering preserving the existence of sliding mode. Some of these approaches may benefit from the idea of proposing a state-dependent gain method [4, 5, 6]. In this method the amplitude of the discontinuous control input is significantly reduced as the states stabilise, however chattering arises in the presence of unmodelled dynamics. The switching gain also can be adjusted in other manners, e.g. it may be a function of the non-switching part of SMC (continuous part, say the equivalent control). This methodology also looks promising since the equivalent control decreases as the sliding mode occurs along the discontinuity surface. In this method, the input signal contains chattering although its amplitude decreases as the system trajectory is sufficiently nearby the sliding surface [1]. Some methods embed an SMC in a fuzzy logic controller (FLC). In these methods, a set of fuzzy linguistic rules based on expert knowledge are used to design the switching control law using either the output error or the change of output error as input of the fuzzy controller [7]. Many other methods, apply the distance between the states and the sliding surface as the input of FLC. In these methods, since the switching performance is not present, the system gain in the steady state is finite and the steady state error may still exist [8]. To remove this obstacle, an FLC is proposed in [9] that combines an SMC with a feedback linearization with integral action. This method assures the infinite gain of controller in the neighbourhood of the sliding surface so that the zero steady state error is achieved. This method deals only with the stability problem of SISO systems. Moreover, the method of stability proof is restricted to achieving the same phrase for the time derivative of the Lyapunov function when each control law is applied to the system, this prevents considering uncertainties in the theoretical results. In this paper, a combination of SMC and proportional plus integrator with derivation (PID) controller is proposed to solve the tracking problem of a class of nonlinear MIMO system. First, an SMC with integral action is studied in the presence of structured uncertainties of the system. Then it is shown that in the neighbourhood of the sliding surfaces, the system can be considered as a linear time variant system. A suitable PID control is proposed to guarantee the asymptotic tracking and input disturbance rejection of the closed-loop system in this region. These control laws are combined using suitable fuzzy rules and input variable to introduce the fuzzy combined controller (FCC). The system stability with FCC is shown using the Lyapunov stability theorem. This paper is organised as follows: Section 2 describes the class of nonlinear systems which is considered in this paper. The procedures of the SMC and PID control design are addressed in Section 3. Also this section introduces the fuzzy control system with FCC and provides analytical results on the stability of the closed-loop system. In Section 4, the proposed FCC is applied to a two degree of freedom robot manipulator

(2DOFRM) to support the theoretical results and prove the effectiveness of the proposed scheme. Conclusions are given in Section 5.

2 PROBLEM STATEMENT

Consider the nonlinear MIMO system in the following form:

$$\begin{cases} \dot{x}_{2i-1} = x_{2i} \\ \dot{x}_{2i} = f_{2i}(x) + \sum_{i=1}^{n} g_{2i,i}(x) u_i \qquad i = 1, 2, \ldots, n \\ y = \left[x_1, \cdots, x_{2n-1}\right]^T \end{cases} \tag{1}$$

where $x = \left[x_1, \cdots, x_{2n}\right]^T \in \Omega_x \subset R^{2n}$ is the state vector and the operating region Ω_x is a compact set. Also $u \in R^n$ and $y \in R^n$ are the control input and the system output, respectively. The mappings $f_{2i} : R^{2n} \to R$ and $g_{2i,i} : R^{2n} \to R$ are partially known as Lipschitz continuous functions of their arguments and $g_{2i,i}(x) \neq 0, \quad \forall x \in \Omega_x$, which means $g_{2i,i}(x)$ is either positive or negative on the compact set Ω_x. In fact, the system (1) presents the general form of n degree of freedom dynamics of robot manipulators with rigid links [10]. For the sake of simplicity and without loss of generality, here the proposed control method is derived for n=2, although the results can be straightforwardly extended to any system in the form of (1) with a higher order. For n=2 the system dynamics is:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_2(x) + g_{21}(x) u_1 + g_{22}(x) u_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_4(x) + g_{41}(x) u_1 + g_{42}(x) u_2 \\ y = \left[x_1, x_3\right]^T \end{cases} \tag{2}$$

The goal is to design a chattering free combined control such that the outputs track the desired reference signals $y_d = [y_{1d}, y_{3d}]$ with zero tracking error. Moreover, the closed-loop system eliminates the effect of the disturbance on the outputs. Next section presents various features of the proposed control method.

3 CONTROLLER DESIGN METHOD

In this section, first an SMC with integral action is studied in the presence of modelling errors. Then a suitable PID controller is proposed to stabilise the error dynamics when the system trajectories are near the sliding surface. Finally, to remove the chattering phenomenon without loosing the advantages of SMC, a fuzzy control is designed using the PID and SMCs.

3.1. Sliding mode controls

Let the uncertainties on f_2 and f_4 be in additive form as follows:

$$\left|\hat{f}_{2i} - f_{2i}\right| \le F_{2i} \qquad i = 1, 2 \tag{3}$$

where $\hat{f}_2(x)$ and $\hat{f}_4(x)$ are estimates of $f_2(x)$ and $f_4(x)$ respectively. Also, consider the uncertainties on the input matrix in the multiplicative form as:

$$\begin{aligned} & G(x) - \hat{G}(x) = \Delta \cdot \hat{G}(x); \\ & \left|\Delta_{ij}\right| \le \gamma_{ij}; \ i = 1, 2, \ j = 1, 2 \end{aligned} \tag{4}$$

where $G = \begin{bmatrix} g_{21} & g_{22} \\ g_{41} & g_{42} \end{bmatrix}$ and G is an estimate of G.

Assumption 1: The maximum eigenvalue of the matrix $\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$ is less than 1, that is $\lambda_{max}(\Gamma) < 1$. Note that since Γ is a matrix with nonnegative real entries, then it has a nonnegative real eigenvalue which dominates the absolute value of other eigenvalues of Γ.

Consider the system (2), and define the error state vector as:

$$e = \left[e_1, e_2, e_3, e_4\right]^T := \left[x_1 - y_{1d}, x_2 - \dot{y}_{1d}, x_3 - y_{3d}, x_4 - \dot{y}_{3d}\right]^T \tag{5}$$

Then the error dynamics can be written in the following form:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = f_2(e, y_d) + g_{21}(e, y_d) u_1 + g_{22}(e, y_d) u_2 - \ddot{y}_{1d} \\ \dot{e}_3 = e_4 \\ \dot{e}_4 = f_4(e, y_d) + g_{41}(e, y_d) u_1 + g_{42}(e, y_d) u_2 - \ddot{y}_{3d} \end{cases} \tag{6}$$

where $y_d := \left[y_{1d}, \dot{y}_{1d}, y_{3d}, \dot{y}_{3d}\right]^T$, in which y_{1d} and y_{3d} are the desired trajectories x_1 and x_3, respectively.

Now let the sliding surfaces σ_1 and σ_3 be defined as follows:

$$\begin{aligned}\sigma_1 &= \dot{e}_1 + \lambda e_1 + \lambda^2 \int_0^t e_1(\tau)\,d\tau \\ \sigma_3 &= \dot{e}_3 + \lambda e_3 + \lambda^2 \int_0^t e_3(\tau)\,d\tau\end{aligned} \tag{7}$$

where λ is a positive constant. Sufficient conditions for enforcing the error trajectories reaching the sliding surfaces in finite times and remaining on it afterwards, are that the SMC, $u = [u_1, u_2]^T$ is designed such that the following condition is fulfilled:

$$\frac{1}{2}\frac{d}{dt}\left(\sigma_1^2 + \sigma_3^2\right) \le -\eta_1 |\sigma_1| - \eta_3 |\sigma_3| \tag{8}$$

where η_1 and η_3 are small positive numbers.

Proposition 1. Consider the SMC

$$u_{SM} = \hat{G}^{-1}\begin{bmatrix} -\hat{f}_2 - E_1 - k_{s1}\,\mathrm{sgn}(\sigma_1) \\ -\hat{f}_4 - E_3 - k_{s3}\,\mathrm{sgn}(\sigma_3)\end{bmatrix} \tag{9}$$

with $E_1 := \lambda\dot{e}_1 + \lambda^2 e_1 - \ddot{y}_{1d}$, $E_3 := \lambda\dot{e}_3 + \lambda^2 e_3 - \ddot{y}_{3d}$, then there exists the vector gain $k_s = [k_{s1}, k_{s3}]^T$, which satisfies the reaching conditions (8).

Proof: Consider the following Lyapunov function:

$$V = \frac{1}{2}\sigma_1^2 + \frac{1}{2}\sigma_3^2 \tag{10}$$

Using (2) and (7), the time-derivative of (10) becomes:

$$\begin{aligned}\dot{V} &= \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\begin{bmatrix}\dot{\sigma}_1 \\ \dot{\sigma}_3\end{bmatrix} = \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\begin{bmatrix}\ddot{x}_1 + E_1 \\ \ddot{x}_3 + E_3\end{bmatrix} \\ &= \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\left(\begin{bmatrix}f_2 \\ f_4\end{bmatrix} + Gu + \begin{bmatrix}E_1 \\ E_3\end{bmatrix}\right)\end{aligned} \tag{11}$$

Applying the control law (9) yields:

$$\dot{V} = \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\left(\begin{bmatrix}f_2 \\ f_4\end{bmatrix} + G\hat{G}^{-1}\begin{bmatrix}-\hat{f}_2 - E_1 - k_{s1}\,\mathrm{sgn}(\sigma_1) \\ -\hat{f}_4 - E_3 - k_{s3}\,\mathrm{sgn}(\sigma_3)\end{bmatrix} + \begin{bmatrix}E_1 \\ E_3\end{bmatrix}\right) \tag{12}$$

Substituting $G\hat{G}^{-1} = I + \Delta$ into (12) and using (4), the time-derivative of V is:

$$\begin{aligned}\dot{V} &= \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\left(\begin{bmatrix}f_2 \\ f_4\end{bmatrix} + \begin{bmatrix}1+\Delta_{11} & \Delta_{12} \\ \Delta_{21} & 1+\Delta_{22}\end{bmatrix}\begin{bmatrix}-\hat{f}_2 - E_1 - k_{s1}\,\mathrm{sgn}(\sigma_1) \\ -\hat{f}_4 - E_3 - k_{s3}\,\mathrm{sgn}(\sigma_3)\end{bmatrix}\right) \\ &\quad + \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\left(\begin{bmatrix}E_1 \\ E_3\end{bmatrix}\right) = \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\begin{pmatrix}(f_2 - \hat{f}_2) - \Delta_{11}(\hat{f}_2 - E_1) \\ (f_4 - \hat{f}_4) - \Delta_{21}(\hat{f}_2 - E_1)\end{pmatrix} \\ &\quad - \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\begin{pmatrix}\Delta_{12}(\hat{f}_4 - E_3) - k_{s1}(1+\Delta_{11})\mathrm{sgn}(\sigma_1) - k_{s3}\Delta_{21}\mathrm{sgn}(\sigma_3) \\ \Delta_{22}(\hat{f}_4 - E_3) - k_{s1}\Delta_{21}\mathrm{sgn}(\sigma_1) - k_{s3}(1+\Delta_{22})\mathrm{sgn}(\sigma_3)\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\dot{V} &= \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\left(\begin{bmatrix}f_2 \\ f_4\end{bmatrix} + \begin{bmatrix}1+\Delta_{11} & \Delta_{12} \\ \Delta_{21} & 1+\Delta_{22}\end{bmatrix}\begin{bmatrix}-\hat{f}_2 - E_1 - k_{s1}\,\mathrm{sgn}(\sigma_1) \\ -\hat{f}_4 - E_3 - k_{s3}\,\mathrm{sgn}(\sigma_3)\end{bmatrix}\right) \\ &\quad + [\sigma_1 \quad \sigma_3]\left(\begin{bmatrix}E_1 \\ E_3\end{bmatrix}\right) \\ &= \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\begin{pmatrix}(f_2 - \hat{f}_2) - \Delta_{11}(\hat{f}_2 - E_1) - \Delta_{12}(\hat{f}_4 - E_3) \\ (f_4 - \hat{f}_4) - \Delta_{21}(\hat{f}_2 - E_1) - \Delta_{22}(\hat{f}_4 - E_3)\end{pmatrix} - \\ &\quad \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}\begin{pmatrix}-k_{s1}(1+\Delta_{11})\mathrm{sgn}(\sigma_1) - k_{s3}\Delta_{21}\mathrm{sgn}(\sigma_3) \\ -k_{s1}\Delta_{21}\mathrm{sgn}(\sigma_1) - k_{s3}(1+\Delta_{22})\mathrm{sgn}(\sigma_3)\end{pmatrix}\end{aligned}$$

Then the bound defined in (3) and (4) gives:

$$\begin{aligned}\dot{V} &\le \left(F_2 + \gamma_{11}|\hat{f}_2 - E_1| + \gamma_{12}|\hat{f}_4 - E_3|\right)|\sigma_1| \\ &\quad - \left((1-\gamma_{11})k_{s1} + k_{s3}\gamma_{12}\right)|\sigma_1| + \left(F_4 + \gamma_{21}|\hat{f}_2 - E_1|\right)|\sigma_2| \\ &\quad + \left(\gamma_{22}|\hat{f}_4 - E_3| + \gamma_{11}k_{s1} - (1-\gamma_{22})k_{s3}\right)|\sigma_2|\end{aligned} \tag{13}$$

If the following conditions are satisfied

$$\begin{aligned}(1-\gamma_{11})k_{s1} &\ge F_2 + \gamma_{11}|\hat{f}_2 - E_1| + \gamma_{12}|\hat{f}_4 - E_3| \\ &\quad + \gamma_{12}k_{s3} + \eta_1 \\ (1-\gamma_{22})k_{s3} &\ge F_4 + \gamma_{21}|\hat{f}_2 - E_1| + \gamma_{22}|\hat{f}_4 - E_3| \\ &\quad + \gamma_{11}k_{s1} + \eta_3\end{aligned} \tag{14}$$

then the sliding mode reaching conditions (6) are verified:

$$\dot{V} \le -\eta_1|\sigma_1| - \eta_3|\sigma_3| \tag{15}$$

Note that in general, if $u = \alpha u_{SM}$ with $0 < \alpha \le 1$ is applied to the plant the above conditions are replaced with:

$$\left(1-\gamma_{11}\right)k_{s1} \geq \frac{F_2}{\alpha} + \left(\gamma_{11} + \frac{1-\alpha}{\alpha}\right)\left|\hat{f}_2 - E_1\right| + \gamma_{12}\left|\hat{f}_4 - E_3\right| + \gamma_{12}k_{s3} + \frac{\eta_1}{\alpha}$$
$$\left(1-\gamma_{22}\right)k_{s3} \geq \frac{F_4}{\alpha} + \gamma_{21}\left|\hat{f}_2 - E_1\right| + \left(\gamma_{22} + \frac{1-\alpha}{\alpha}\right)\left|\hat{f}_4 - E_3\right| + \gamma_{11}k_{s1} + \frac{\eta_3}{\alpha} \tag{16}$$

Using the Frobenius-Perron Theorem, it can be shown that there exists the positive vector $[k_{s1} \quad k_{s3}]$ such that inequalities (16) hold. Consider the above inequalities as follows:

$$\Bigg(I - \underbrace{\begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}}_{\Gamma}\Bigg)\begin{bmatrix} k_{s1} \\ k_{s3} \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_3 \end{bmatrix} \tag{17}$$

where $\beta_1 > 0$ and $\beta_3 > 0$ are chosen such that:

$$\beta_1 \geq \frac{F_2}{\alpha} + \left(\gamma_{11} + \frac{1-\alpha}{\alpha}\right)\left|\hat{f}_2 - E_1\right| + \gamma_{12}\left|\hat{f}_4 - E_3\right| + \frac{\eta_1}{\alpha}$$
$$\beta_3 \geq \frac{F_4}{\alpha} + \gamma_{21}\left|\hat{f}_2 - E_1\right| + \left(\gamma_{22} + \frac{1-\alpha}{\alpha}\right)\left|\hat{f}_4 - E_3\right| + \frac{\eta_3}{\alpha} \tag{18}$$

Since Γ is a positive definite matrix, according to the Frobenius-Perron Theorem, if $\lambda_{max}(\Gamma) < 1$ (see. Assumption 1) then there exists some unique positive k_{s1} and k_{s3} which satisfy the equation (17). Hence from (17) and (18) it can be concluded that there exist the gains $[k_{s1} \quad k_{s3}]$ which satisfy (16).

When the system trajectory is near the sliding surface, the SMC normally deals with the chattering phenomenon. So a control should be designed to eliminate the chattering. Based on this fact, a PID controller is proposed to control the system in the neighbourhood of the sliding surface.

3.2. PID control design

As stated before, the chattering phenomenon occurs when the system trajectory is in the neighbourhood of the sliding surfaces, that is $|\sigma| \leq \delta_1$ where δ_1, is a small positive value.

To eliminate chattering, the SMC should be designed such that switching control performance is avoided. Therefore, a suitable set of fuzzy rules is applied to activate an appropriate PID controller when the system trajectory is in the vicinity of the sliding surfaces. This importance is addressed in the next section. Using the following lemma it can be shown that under condition of $\|\sigma\| \leq \delta_1$ where $\sigma := \begin{bmatrix}\sigma_1 & \sigma_3\end{bmatrix}^T$, the nonlinear error dynamic (6) can be approximated as a linear time variant system.

Lemma 1: let $\|\sigma\| \leq \delta_1$ then $\|e\| \leq 4\left(2 + \frac{1}{\lambda}\right)\delta_1$.

Proof: Appling the Laplace transform to (7) gives:

$$\Sigma_1(s) = sE_1(s) + 2\lambda E_1(s) + \lambda^2 \frac{E_1(s)}{s} \tag{19}$$

where $E_1(s) = L\left[e_1(t)\right]$ and $\Sigma_1(s) = L\left[\sigma_1(t)\right]$. Equation (19) yields:

$$E_1(s) = \frac{s}{\left(s+\lambda\right)^2}\Sigma_1(s) \tag{20}$$

From $\|\sigma\| \leq \delta_1$ it can be concluded that $|\sigma_1| \leq \delta_1$ and $|\sigma_3| \leq \delta_1$. Also let define the auxiliary variable $z_1(t) := L^{-1}\underbrace{\left[\frac{1}{\left(s+\lambda\right)}\Sigma_1(s)\right]}_{:=Z_1(s)}$, which is bounded as:

$$\left|z_1(t)\right| \leq \int_0^t \left|e^{-\lambda(t-\tau)}\right|\left|\sigma_1(\tau)\right| d\tau \leq \delta_1 \int_0^t e^{-\lambda(t-\tau)} d\tau \leq \frac{\delta_1}{\lambda}\left(1 - e^{-\lambda(t-\tau)}\right) \leq \frac{\delta_1}{\lambda} \tag{21}$$

Now the bounds on e_1 and $\dot{e}_1$ can be derived using (20) and (21) as follows:

$$e_1(t) = L^{-1}\left[E_1(s)\right] = L^{-1}\left[\frac{s}{\left(s+\lambda\right)^2}\Sigma_1(s)\right] = L^{-1}\left[\frac{-\lambda}{\left(s+\lambda\right)}Z_1(s) + Z_1(s)\right] = \int_0^t -\lambda e^{-\lambda(t-\tau)} z_1(\tau)d\tau + z_1(t)$$

Hence,

$$\left|e_1(t)\right| \leq \frac{\lambda|z|}{\lambda}\left(1 - e^{-\lambda(t-\tau)}\right) + \left|z_1\right| \leq 2\frac{\delta_1}{\lambda} \tag{22}$$

Also the bound on $\dot{e}_1$ can be calculated as:

$$\dot{e}_1(t) = L^{-1}\left[sE_1(s)\right] = L^{-1}\left[\frac{s^2}{(s+\lambda)^2}\Sigma_1(s)\right] =$$

$$L^{-1}\left[\frac{\lambda^2}{(s+\lambda)}Z(s) - 2\lambda Z(s) + \Sigma_1(s)\right]$$

$$= \int_0^t \lambda^2 e^{-\lambda(t-\tau)} z_1(\tau)d\tau - 2\lambda z_1(t) + \sigma_1(t)$$

Thus, using $|\sigma_1| \le \delta_1$ and (21), it gives:

$$\left|\dot{e}_1(t)\right| \le \frac{\lambda^2 |z|}{\lambda}\left(1 - e^{-\lambda(t-\tau)}\right) + 2\lambda|z| + |\sigma_1| \le 4\delta_1 \tag{23}$$

Using similar procedures, the bounds on e_2 and $\dot{e}_2$ can be obtained. Now (5), (22) and (23) yields:

$$\|e\| \le |e_1| + |\dot{e}_1| + |e_2| + |\dot{e}_2| \le 4\left(2 + \frac{1}{\lambda}\right)\delta_1 \tag{24}$$

The trajectories eventually enters into a small ball $\|\mathbf{e}\| \le 4\left(2 + \frac{1}{\lambda}\right)\delta_1$, and if δ_1 is selected to be sufficiently small, then e-trajectories move within this ball close to the origin. To stabilise the error system when the trajectories are inside this ball, a PID sub-controller is designed. Since the behaviour of the nonlinear system (6) and its linearised counterpart are the same, to prove the system stability the following associate linear system is considered.

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = a_2^T e + b_2^T u \\ \dot{e}_3 = e_4 \\ \dot{e}_4 = a_4^T e + b_4^T u \end{cases} \tag{25}$$

where

$$a_2^T(y_d) = \left.\frac{\partial f_2}{\partial e}\right|_{e=0}, \quad b_2^T = \left[g_{21}(0, y_d), g_{22}(0, y_d)\right]$$

$$a_4^T(y_d) = \left.\frac{\partial f_4}{\partial e}\right|_{e=0} \text{ and } b_4^T = \left[g_{41}(0, y_d), g_{42}(0, y_d)\right].$$

To stabilise the above controllable system the following PD controller may be used.

$$u_{PD} = K_e(t)e = \begin{bmatrix} k_{11} & k_{12} & 0 & 0 \\ 0 & 0 & k_{23} & k_{24} \end{bmatrix} e$$

In order to guarantee the asymptotic tracking and input disturbance rejection of the closed-loop system, an integral action is added to the controller. Let the new state variables be defined as $p_1 := \int_0^t e_1(\tau)d\tau$ and $p_3 := \int_0^t e_3(\tau)d\tau$. Now the augmented open-loop plant can be presented as:

$$\frac{d}{dt}\underbrace{\begin{bmatrix} p_1 \\ e_1 \\ e_2 \\ p_3 \\ e_3 \\ e_4 \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & a_{21} & a_{22} & 0 & a_{23} & a_{24} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & a_{41} & a_{42} & 0 & a_{43} & a_{44} \end{bmatrix}}_{:=A_p} \underbrace{\begin{bmatrix} p_1 \\ e_1 \\ e_2 \\ p_3 \\ e_3 \\ e_4 \end{bmatrix}}_{:=e_p} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_{21} & b_{22} \\ 0 & 0 \\ 0 & 0 \\ b_{41} & b_{42} \end{bmatrix}}_{:=b_p} u \tag{26}$$

A new PID controller is proposed to stabilise the closed- loop error dynamics.

$$u_{PID} = K_p(t)e_p = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \end{bmatrix} e_p \tag{27}$$

Since (A_p, b_p) is controllable, it is always possible to place the eigenvalues of $A_p + b_p K_p$ in the desired locations. Here the control gain $K_p(t)$ is obtained such that all eigenvalues of the following matrix are negative

$$\begin{bmatrix} A_p + b_p K_p & 0_{6\times6} \\ 0_{6\times6} & \underbrace{\Lambda^T \hat{G}(y_d) K_p}_{:=-Q_G} \end{bmatrix} < 0 \tag{28}$$

where $\Lambda = \begin{bmatrix} \lambda^2 & 2\lambda & 1 & 0_{1\times3} \\ 0_{1\times3} & & \lambda^2 & 2\lambda & 1 \end{bmatrix}$. Note that there are sufficient free parameters k_{ij} $i = 1,2$ $j = 1,\ldots,6$ to hold the condition (28). Hence the stable closed loop dynamics can be presented as:

$$\dot{e}_p = \underbrace{\left(A_p + b_p K_p\right)}_{A_{cl}} e_p \tag{29}$$

Fig. 1 shows the closed-loop system with the added integral control.

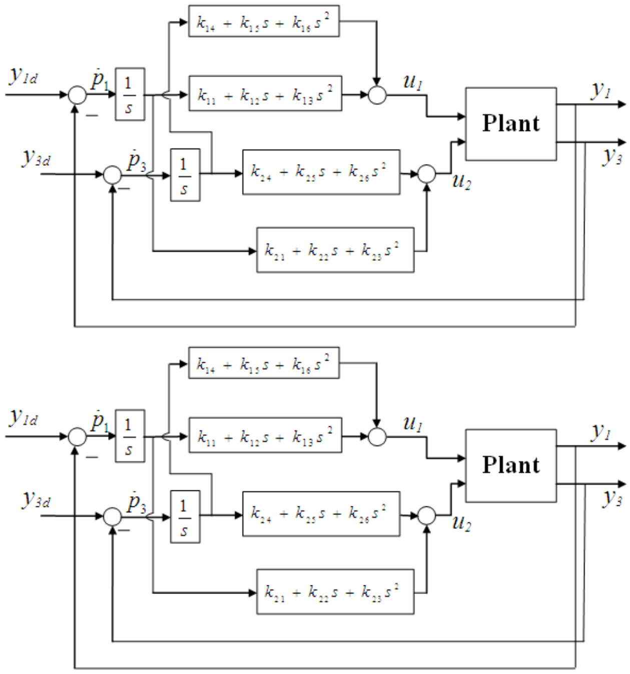

Fig. 1 Closed-loop system with an integral action

Now it is shown that the system trajectories (29), asymptotically tend to the origin along the sliding surface $\sigma_1 \cap \sigma_3$.

Proposition 2: The time derivative of the Lyapunov function (10) is negative on closed loop dynamic (29).

Proof: The Lyapunov function (8) can be rewritten as:

$$V = \frac{1}{2}\sigma^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \sigma \tag{30}$$

where $\boldsymbol{\sigma} = [\sigma_1 \quad \sigma_3]^T$, in which σ_1 and σ_3 are defined in (7) and can be represented as:

$$\begin{aligned} \sigma_1 &= \begin{bmatrix} \lambda^2 & 2\lambda & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} \lambda^2 & 2\lambda & 1 & 0_{1\times 3} \end{bmatrix} e_p, \\ \sigma_3 &= \begin{bmatrix} \lambda^2 & 2\lambda & 1 \end{bmatrix} \begin{bmatrix} p_3 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} 0_{1\times 3} & \lambda^2 & 2\lambda & 1 \end{bmatrix} e_p \end{aligned} \tag{31}$$

Substituting (31) into (30), yields:

$$V = \frac{1}{2} e_p^T \underbrace{\Lambda^T \Lambda}_{P} e_p \tag{32}$$

where P is a symmetric positive definite matrix. Using (29), the time-derivative of (32) becomes:

$$\dot{V} = \frac{1}{2}\dot{e}_p^T P e_p + \frac{1}{2} e_p^T P \dot{e}_p = \frac{1}{2} e_p^T A_{cl}^T P e_p + \frac{1}{2} e_p^T P A_{cl} e_p = \frac{1}{2} e_p^T \left(A_{cl}^T P + P A_{cl} \right) e_p$$

The stability of A_{cl} ensures existence of the symmetric positive-definite solution P of the following algebraic Riccati equation.

$$PA_{cl} + A_{cl}^T P = -Q \tag{33}$$

where Q is an arbitrary symmetric positive-definite matrix. Hence,

$$\dot{V} = -\frac{1}{2} e_p^T Q e_p \le -\frac{1}{2}\lambda_{min}(Q) \left| e_p \right|^2 \le 0 \tag{34}$$

3.3. Fuzzy combination of PID and SMC

To remove chattering without loosing the advantages of SMC such as asymptotic tracking and robustness, a fuzzy combination of PID and SMC is used. In this approach, based on the rules, a fuzzy system decides about the activation of the PID and SMCs. In this method, a continuous fuzzy switch makes smooth changes between these two controllers based on the following fuzzy IF-THEN rules:

$$\begin{aligned} &\text{Rule 1: IF } |\boldsymbol{\sigma}| \text{ is S, THEN } u = u_{PID} \\ &\text{Rule 2: IF } |\boldsymbol{\sigma}| \text{ is L, THEN } u = u_{SM} \end{aligned} \tag{35}$$

Where S and L are fuzzy sets defined on input fuzzy variable $|\boldsymbol{\sigma}|$, which is applied to the fuzzy controller. Also u_{SM} and u_{PID} are the outputs of the fuzzy inference engine for the above fuzzy rules. In the above fuzzy rules, there exists at least one nonzero degree of membership $\mu_S(\sigma),\ \mu_L(\sigma) \in [0,1]$ corresponding to each rule as depicted in Fig. 2 in which δ_2 is a positive small number.

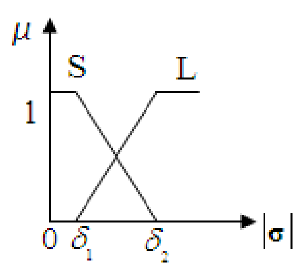

Fig. 2 Membership function of fuzzy variable $\|\boldsymbol{\sigma}\|$

Appling the weighted sum defuzzification method, the overall output of the fuzzy controller can be written as:

$$u = \frac{\mu_S(\sigma)u_{PID} + \mu_L(\sigma)u_{SM}}{\mu_S(\sigma) + \mu_L(\sigma)} \tag{36}$$

Using the following property of input matrix of robot manipulator system, G, the stability of the closed-loop system over the entire operation range of the fuzzy logic control system is studied.

Lemma 2: Consider the estimated input matrix of a 2DOFRM system as $\hat{G}(x) = \hat{H}^{-1}$ where

$$\hat{H}(x) = \begin{bmatrix} c_{11} + d_{11}\cos x_3 & c_{12} + d_{12}\cos x_3 \\ c_{12} + d_{12}\cos x_3 & c_{22} \end{bmatrix}$$

and x_3 is the angle of second joint of manipulator as depicted in Fig. 3. If $\|\boldsymbol{\sigma}\| < \delta_2$ and $\frac{\delta_2}{\lambda}1$, then the input matrix can be considered as:

$$\hat{G}(x) = \hat{G}(y_d) + \Delta_G(y_d, e) \qquad \|\Delta_G\| \le 2w\frac{\delta_2}{\lambda} \tag{37}$$

where w is a positive constant.

Proof: Using (5) and expanding $\hat{H}$, gives:

$$\hat{H}(x) = \begin{bmatrix} c_{11} + d_{11}\cos\left(e_3 + y_{3d}\right) & c_{12} + d_{12}\cos\left(e_3 + y_{3d}\right) \\ c_{12} + d_{12}\cos\left(e_3 + y_{3d}\right) & c_{22} \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} + d_{11}\cos e_3 \cos y_{3d} & c_{12} + d_{12}\cos e_3 \cos y_{3d} \\ c_{12} + d_{12}\cos e_3 \cos y_{3d} & c_{22} \end{bmatrix}$$

$$- \begin{bmatrix} d_{11}\sin e_3 \sin y_{3d} & d_{12}\sin e_3 \sin y_{3d} \\ d_{12}\sin e_3 \sin y_{3d} & 0 \end{bmatrix}$$

Since $\frac{\delta_2}{\lambda}1$, $\cos e_3 \approx 1$ and $\sin e_3 \approx e_3$, therefore,

$$\hat{H}(x) \approx \hat{H}(y_d) - \underbrace{\begin{bmatrix} d_{11}\sin y_{3d} & d_{12}\sin y_{3d} \\ d_{12}\sin y_{3d} & 0 \end{bmatrix}}_{T} e_3$$

Applying the inverse of sum of matrix formula, and $\hat{G}(x) = \hat{H}^{-1}$, yields $\hat{G}(x) = \hat{G}(y_d) + \Delta_G(y_d, e_3)$

Where $\Delta_G(y_d, e_3) = \underbrace{\left(\hat{G}(y_d)\left(I - T\hat{G}(y_d)e_3\right)^{-1} T\hat{G}(y_d)\right)}_{\Delta_1} e_3$

As it was shown in Lemma 1, $|\boldsymbol{\sigma}| < \delta_2$ ensures that $|e_3| < \frac{2\delta_2}{\lambda}$. So $\|\Delta_G\| = \|\Delta_1\|\|e_3\| \le 2w\frac{\delta_2}{\lambda}$ where w is defined as the upper bound of $\|\boldsymbol{\Delta}_1\|$.

Theorem 1: Consider the fuzzy control (36), with the membership function as depicted in Fig. 2, and let $\lambda_{\max}(\Gamma)$ be sufficiently small then the closed-loop system is asymptotically stable if $\frac{\delta_2}{\lambda}$ is selected such that:

$$\lambda_{\min}\left(Q_G\right) > \left(\lambda_{\max}(\Gamma)\left\|\hat{G}(y_d)\right\| + 2\|\Delta_1\|\left(1 + \lambda_{\max}(\Gamma)\right)\frac{\delta_2}{\lambda}\right)$$
$$\|\Lambda\|\|K_p\|$$

Proof: It can be seen from Fig. 2 that, for any value of $\|\sigma\|$, only one of the following two cases will occur:

1) Either Rule 1 or Rule 2 is active. In this case $\|\sigma\| \le \delta_1$ or $\|\sigma\| \ge \delta_2$.

If $\|\sigma\| \le \delta_1$ then $u = u_{PID}$. According to Proposition 1,

$$\dot{V} \le -\frac{1}{2}\lambda_{\min}(Q)\left\|e_p\right\|^2 \tag{38}$$

Also from (30) and (31), $|\sigma|^2 = e_p^T P e_p$, so

$$\lambda_{\min}\left(P\right)\left\|e_p\right\|^2 \le \|\sigma\|^2 \le \lambda_{\max}\left(P\right)\left\|e_p\right\|^2 \tag{39}$$

Then (37) and (38) yields:

$$\dot{V} \le -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}\left(P\right)}\|\sigma\|^2 \tag{40}$$

$\|\boldsymbol{\sigma}\| \le \delta_1$ guarantees that $\dot{V} < 0$ except when $\|\boldsymbol{\sigma}\| = 0$ which implies $\dot{V} \le 0$. In this case from (7), it can be concluded that $\mathbf{e}_p = 0$. On the other hand, when $\|\sigma\| \ge \delta_2$, $u = u_{SM}$. Therefore from (15)

$$\dot{V} \le -\eta_1\left|\sigma_1\right| - \eta_3\left|\sigma_3\right| \le -\eta_{\min}\|\sigma\| \tag{41}$$

where $\eta_{\min} = \min\{\eta_1, \eta_3\}$. Since $\|\sigma\| \ne 0$, (40) shows $\dot{V} < 0$

2) Either Rules 1 and 2 are active simultaneously, i.e. $\delta_1 < \|\sigma\| < \delta_2$.

In this case, the overall control is the convex combination of the PID and SMCs which is applied to the system.

$u = \alpha u_{SM} + (1-\alpha) u_{PID}$ where $0 < \alpha = \dfrac{\mu_L}{\mu_S + \mu_L} < 1$.

Consider the Lyaponuv function (10). Using (2) and (7), the time-derivative of V becomes:

$$\dot{V} = \begin{bmatrix} \sigma_1 & \sigma_3 \end{bmatrix} \begin{bmatrix} \dot{\sigma}_1 \\ \dot{\sigma}_3 \end{bmatrix} = \begin{bmatrix} \sigma_1 & \sigma_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 + E_1 \\ \ddot{x}_3 + E_3 \end{bmatrix} =$$
$$\left(\begin{bmatrix} f_2 \\ f_4 \end{bmatrix} + \alpha G u_{SM} + \begin{bmatrix} E_1 \\ E_3 \end{bmatrix} \right) + (1-\alpha) \begin{bmatrix} \sigma_1 & \sigma_3 \end{bmatrix} G u_{PID}$$

Appling the SMC (9) with the conditions (16), yields:

$$\dot{V} \le -\eta_1 |\sigma_1| - \eta_3 |\sigma_3| + (1-\alpha) e_p^T \Lambda^T G u_{PID} \tag{42}$$

Substituting from (4), (37) and (27):

$$\dot{V} \le -\eta_1 |\sigma_1| - \eta_3 |\sigma_3| + (1-\alpha) e_p^T \Lambda^T \left(\hat{G}(x) + \Delta \cdot \hat{G} \right) \times$$
$$K_p e_p \le -\eta_1 |\sigma_1| - \eta_3 |\sigma_3| + (1-\alpha) e_p^T \Lambda^T \times$$
$$\left(\hat{G}(y_d) + \Delta_G + \Delta \cdot \hat{G}(y_d) + \Delta_G \Delta \right) K_p e_p$$

Using (28), $Q_G = -\Lambda^T \hat{G}(y_d) K_p$, where Q_G is a positive matrix and the bounds defined in (4) and (7), gives:

$$\dot{V} \le -\eta_1 |\sigma_1| - \eta_3 |\sigma_3| - (1-\alpha) \eta_e \|e_p\|^2 \tag{43}$$

where

$$\eta_e = \lambda_{min}(Q_G) - \left(\lambda_{max}(\Gamma) \|\hat{G}(y_d)\| + 2\|\Delta_1\| \left(1 + \lambda_{max}(\Gamma)\right) \frac{\delta_2}{\lambda} \right) \|\Lambda\| \|K_p\|$$

Assume that the uncertainty $\boldsymbol{\Delta}$, defined in (4), and respectively $\lambda_{max}(\Gamma)$ is sufficiently small. Moreover let δ_2/λ be selected small enough such that the following condition holds:

$$\lambda_{min}(Q_G) > \left(\lambda_{max}(\Gamma) \|\hat{G}(y_d)\| + 2\|\Delta_1\| \left(1 + \lambda_{max}(\Gamma)\right) \frac{\delta_2}{\lambda} \right) \|\Lambda\| \|K_p\| \tag{44}$$

Then, (43) guarantees the stability of the closed-loop system over the entire operational region of the fuzzy logic control system.

4 EXAMPLE

The performance of the proposed controller is shown through simulations using a two degree of freedom robot manipulator (2DOFRM) (see Fig. 3). The dynamics of this system is described by the following differential equations [2]:

$$H(q)\ddot{q} + M(q,\dot{q})\dot{q} + N(q) = \tau$$

with $q = [q_1, q_2]^T$ being the two joint angle and $\boldsymbol{\tau} = [\tau_1, \tau_2]^T$ being the joint inputs. H is the mass matrix, and M is the vector associated with the Coriolis and centrifugal forces. The elements of H and M are as follows:

$$H_{11} = I_1 + I_2 + m_1 l_{c1}^2 + m_2 l_1^2 + m_2 l_{c2}^2 + 2 m_2 l_1 l_{c2} \cos q_2$$

$$H_{22} = m_2 l_{c2}^2 + I_2, \quad H_{12} = H_{21} = m_2 l_{c2}^2 + I_2 +$$
$$m_2 l_1 l_{c2} \cos q_2$$

$$M_{11} = -m_2 l_1 l_{c2} \dot{q}_2 \sin q_2, \quad M_{12} = -m_2 l_1 l_{c2} \left(\dot{q}_1 + \dot{q}_2 \right) \sin q_2$$

$$M_{21} = m_2 l_1 l_{c2} \dot{q}_1 \sin q_2, \quad M_{22} = 0$$

$$N_1 = m_1 l_{c1} g \cos q_1 + m_2 g \left(l_{c2} \cos(q_1 + q_2) + l_1 \cos q_1 \right)$$

$$N_2 = m_2 l_{c2} g \cos(q_1 + q_2)$$

Where m_1, m_2, l_1, l_2, I_1, I_2 are masses, lengths and inertia moments of the arms, respectively, and $l_{c1} = 0.5 l_1, l_{c2} = 0.5 l_2$.

Defining the state vector as $x := [q_1, \dot{q}_1, q_2, \dot{q}_2]^T$ and the input vector as $u := [\tau_1, \tau_2]^T$ the above system can be rewritten as $\dot{x} = f(x) + g_1(x) u_1 + g_2(x) u_2$ where

$$f(x) = \begin{pmatrix} x_2 \\ -h_{11}(M_{11}x_2 + N_1) - h_{12}(M_{21}x_2 + N_2) - h_{11}M_{12}x_4 \\ x_4 \\ -h_{12}(M_{11}x_2 + N_1) - h_{22}(M_{21}x_2 + N_2) - h_{12}M_{12}x_4 \end{pmatrix}$$

$$g_1(x) = \begin{bmatrix} 0 \\ h_{11} \\ 0 \\ h_{12} \end{bmatrix}, \quad g_2(x) = \begin{bmatrix} 0 \\ h_{12} \\ 0 \\ h_{22} \end{bmatrix}$$

With $H^{-1} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

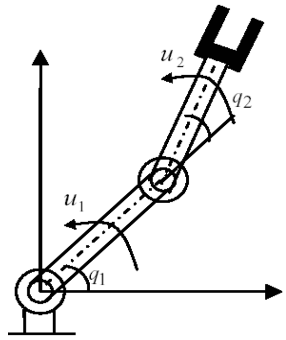

Fig. 3 A two degree of freedom robot manipulator model

The simulations have been carried out using the following parameters and initial conditions.

$m_1 = 1.2\text{kg},\ m_2 = 0.82\text{kg},\ l_1 = 0.73\text{m},\ l_2 = 0.65\text{m},$

$I_1 = 0.0833\text{kgm}^2,$

$I_1 = 0.0652\text{kgm}^2\ x_1(0) = -0.1\,\text{rad},\ x_2(0) = 0\,\text{rad / s},$

$x_3(0) = -0.8\,\text{rad},\ \ x_4(0) = -0.1\,\text{rad / s}$. Also the parameter uncertainties are considered as $\hat{m}_1 = 1.05m_1,\ \hat{m}_2 = 1.05m_2,\ \hat{l}_1 = 1.04l_1,\ \ \hat{l}_2 = 0.97l_2,$ $\hat{I}_1 = 1.05I_1,\ \hat{I}_2 = 0.95I_2$. $\lambda = 10$ is selected and the fuzzy controller parameters are chosen as $\delta_1 = 0.1, \delta_2 = 1$.

First the SMC is applied to the system with $k_{s1} = k_{s2} = 1$ as Figs. 4 and 5 show the tracking error exists due to plant uncertainties. In spite of the control signal (see Fig. 6) chattering is free. To compensate the tracking error, the sliding gains are increased to $k_{s1} = k_{s2} = 5$. In this case, the asymptotic tracking is achieved, but the control signals contain chattering.

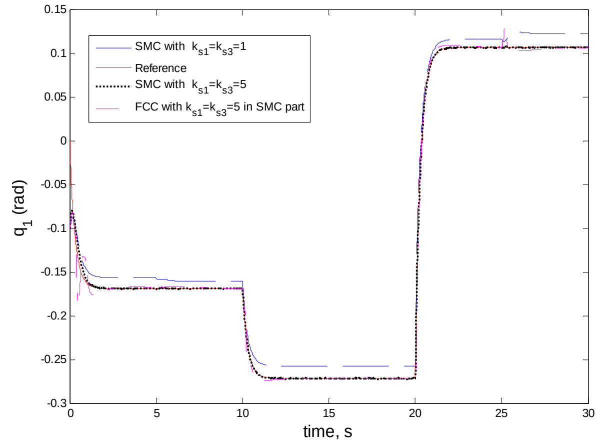

Fig. 4 The first joint tracking performance using the SMC and the proposed FCC

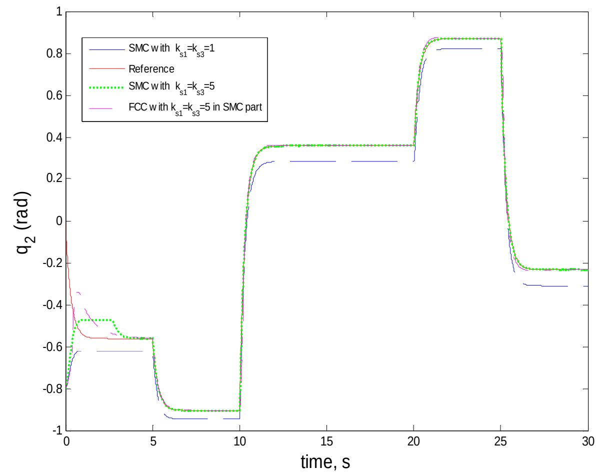

Fig. 5 The second joint tracking performance using the SMC and the proposed FCC

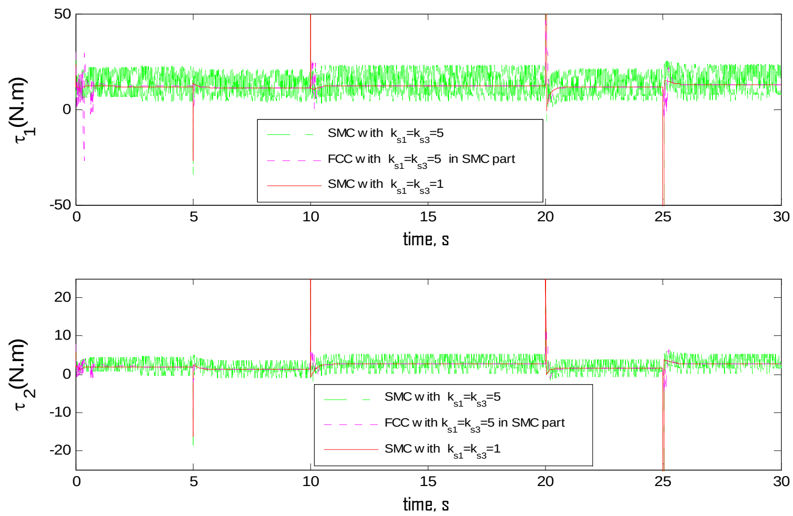

Fig. 6 SMC signals with various gains and the proposed FCC action

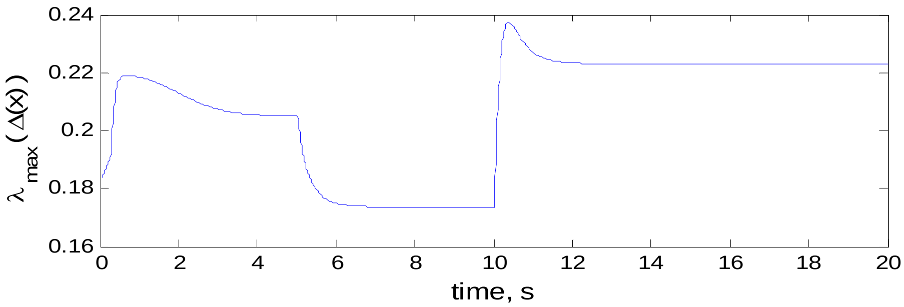

Fig. 7 Validation of condition $\lambda_{\max}\left(\boldsymbol{\Gamma}\right)<1$

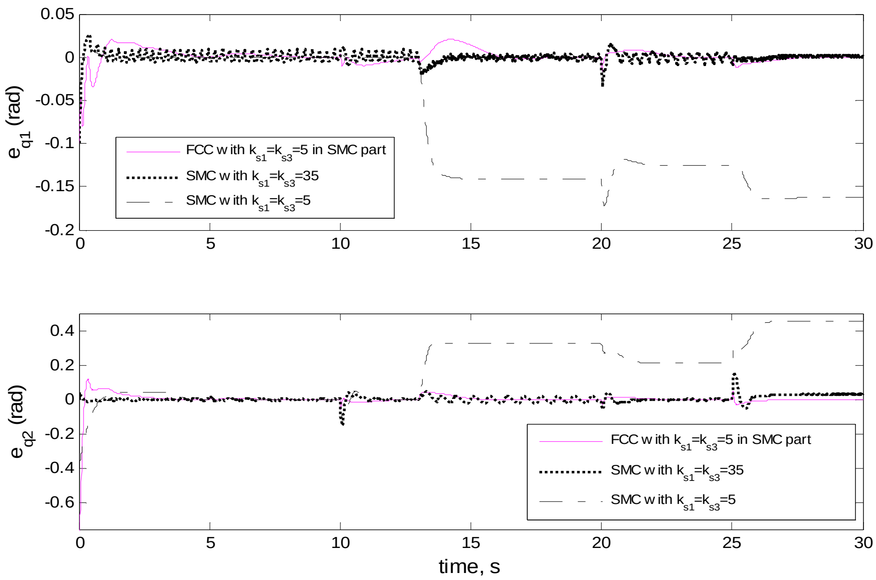

Fig. 8 Comparison of tracking errors using SMC and FCC in the presence of disturbance

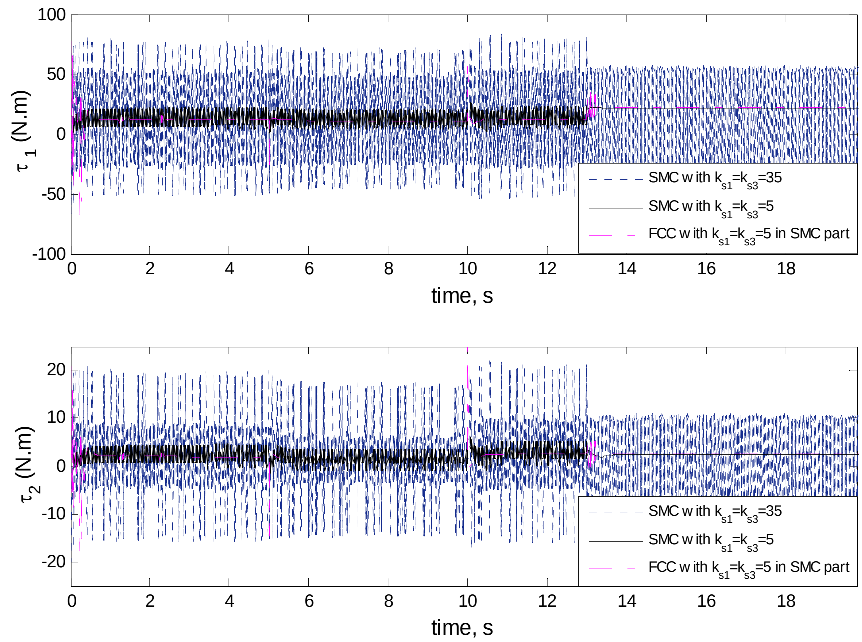

Fig. 9 SMC signals with various gains and the proposed FCC action in the presence of disturbance

To remove the chattering the fuzzy combined controller is applied. The results show both goals are achieved, zero tracking error is obtained, and the control signal is chatter free (see Fig. 6).

The validity of the necessary condition stated in Assumption 1, is verified in Fig. 7. The disturbance rejection ability of the proposed combined controller is examined by applying a step disturbance with amplitude of 10 N/m at $t = 13\,\text{sec}$. As Fig. 8 shows, the SMC with the gain $k_{s1} = k_{s2} = 5$, cannot reject the disturbance and so the tracking error is increased, this causes the system trajectories to move further from the sliding surfaces. To achieve proper disturbance rejection under the SMC, it is required to increase the control gains to $k_{s1} = k_{s2} = 35$. These high sliding mode gains guarantee perfect tracking, but if chattering of the control signal is very large and cannot be neglected (see Fig. 9), using the proposed FCC with $k_{s1} = k_{s2} = 5$, as the sliding mode gains for the SMC part would reject the disturbance and produce chattering free control signals.

5 CONCLUSIONS

A fuzzy combined control for a class of nonlinear MIMO system was proposed in this paper. The proposed method relays on the combination of conventional PID and SMCs. The proposed controller has the advantages of both controllers including the robustness of SMC and the smooth control signal, zero steady state error and the disturbance rejection property of PID control. The overall stability of FCC has been shown using the Lyapunov direct method. Simulations carried out for the 2DOFRM in the presence of parameter uncertainties, illustrate good performance of the proposed method.

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