Transient Analysis of Heat and Mass Transfer via Natural Convection over a Vertical Complex Wavy Surface

Morteza Ghadimi*
Young Researchers and Elites Club,
Science and Research Branch, Islamic Azad University, Tehran, Iran
E-mail: mortezaghadimi@ut.ac.ir
*Corresponding author

Amir Lotfi
Department of Mechanical Engineering,
K.N.Toosi University of Technology, Tehran, Iran
E-mail: amirlotfi61@yahoo.com

Cyrus Aghanajafi
Department of Mechanical Engineering,
K.N.Toosi University of Technology, Tehran, Iran
E-mail: aghanajafi@kntu.ac.ir

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Abstract: A numerical study of transient natural convection heat and mass transfer along a vertical complex wavy surface has been performed. A complex wavy surface was created from two sinusoidal functions, a fundamental wave and its first harmonic. The complex wavy surface is maintained at uniform wall temperature and constant wall concentration. An implicit finite-difference scheme is used for analysis. The numerical results demonstrate that the additional harmonic substantially alters the flow field, temperature and concentration distribution near the surface. Also the numerical results show that the local heat and mass transfer rate for a complex surface are smaller than of a flat plate. This decreased local heat and mass transfer rate seems to depend on the ratio of amplitude surface.

Keywords: Heat transfer, Mass transfer, Natural convection, Wavy surface

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Biographical notes: M. Ghadimi is PhD student of Materials engineering at the University of Tehran, Iran. His current research interest includes Nanotechnology and Analytical Modelling. **A. Lotfi** received his MSc in Mechanical engineering from K. N. Toosi University of Technology, Iran. His main research interest includes Heat and Mass transfer. **C. Aghanijafi** is Professor of Mechanical engineering at the K. N. Toosi University of Technology, Iran. His current research focuses on fluid mechanics and heat transfer.

1 INTRODUCTION

The analysis of natural convection has been of considerable interest to engineers and scientists. Most studies of natural convection are mainly concerned with heat convection solely. However, Gebhart and Pera [1] indicated that buoyancy effects from concentration gradients can be as important as those from temperature gradients. There are applications of interest in which combined heat and mass transfer by natural convection, such as design of chemical processing equipment, design of heat exchangers, formation and dispersion of fog, distributions of temperature and moisture over agricultural fields, pollution of the environments and thermoprotection systems.

Extensive studies of natural convection of heat and mass transfer have been investigated in the past decades and various extensions of the problem have been reported in the literature. Yan and Lin [2] studied numerically on natural convection heat and mass transfer with film evaporation and condensation in vertical concentric annular ducts. They found that the heat transfer is enhanced due to heat exchange with phase changing. In addition, the enhancement of heat transfer due to mass transfer is more significant with a higher wetted wall temperature. Chang et al. [3] investigated the combined buoyancy effects of thermal and mass diffusion on the natural convection flows in a vertical open tube. They found that the heat transfer augmentation through mass diffusion connected with film evaporation is considerable. Massive amount of works on heat and mass transfer have focused mainly on regular geometries, such as a vertical flat plate [4], flat plate with inclination [5], parallel-plate channel [6], and rectangular ducts [7], etc. However, it is necessary to study the heat and mass transfer for complex geometries because the prediction of heat and mass transfer for irregular surfaces is a topic of fundamental importance and irregular surfaces often appear in many applications, for examples, flat-plate solar collectors and flat-plate condensers in refrigerators. Few studies have considered the effects of complex geometries. Wang and Kleinstreuer [8] investigated the thermal convection on micropolar fluids passing a convex with suction/ injection. They developed a numerical model to study the effectiveness of dehydration media for wedge-shaped surface with mass and heat transfer. Yih [9] studied the heat and mass transfer characteristic in natural convection flow over a truncated cone subjected to uniform wall temperature and concentration or uniform heat and mass flux embedded in porous media. He first investigated the natural convection heat transfer from an isothermal vertical wavy surface and used an extended Prantdl's transposition theorem and a finite- difference scheme. He proposed a simple transformation to study the natural convection heat transfer for an isothermal vertical sinusoidal surface. Chiu and Chou [10] analyzed the transient forced and free thermal convection along a wavy surface in micropolar fluids, and investigated the natural convection heat transfer along a vertical wavy surface in micropolar fluids. Recently, the study of natural convection heat transfers along a wavy surface in a thermally stratified fluid saturated porous medium with the effects of wave phase was presented by Yih [11]. Besides, Yan and his colleagues [12] performed a series of studies about the natural convection heat transfer in porous enclosures. Cheng [13] studied coupled heat and mass transfer by natural convection flow along a wavy conical surface and vertical wavy surface in a porous medium. Jang and Yan [14] analyzed natural convection along a vertical wavy surface. Recently, Yan [15] has studied numerically on natural convection heat transfer along a vertical complex wavy surface with Newtonian fluids. However, this study only pertains to steady flow and heat transfer.

The transient natural heat and mass transfer along a vertical complex wavy surface, especially in Newtonian fluid, has not been well investigated. The objective of the present investigation is to analyze the transient natural convection heat and mass transfer in Newtonian fluid flow along a vertical complex wavy surface numerically by using Prandtl's transposition theorem.

2 PROCEDURE FOR PAPER SUBMISSION

2.1. Problem statement

The geometry of this problem as schematically shown in Fig. 1 is a vertical wavy surface. $\bar{u}$ and $\bar{V}$ velocity are the velocity components in the $\bar{x}$ and $\bar{y}$ directions respectively. The flow is transient, laminar and incompressible with simultaneous heat and mass along a semi-infinite vertical wavy surface. The thermo-physical properties are assumed to be constant except the buoyancy term in the $\bar{x}$ momentum equation. The Boussinesq approximation is used to characterize the buoyancy effect. The wavy surface of the plate is described in the function below:

$$\bar{y} = \bar{\sigma}(\bar{x}) = \bar{a}_1 Sin(\frac{2\pi\bar{x}}{l}) + \bar{a}_2 Sin(\frac{4\pi\bar{x}}{l}) \quad (1)$$

Which is the fundamental wavelength. The origin of the coordinate system is placed at the leading edge of the vertical surface. Initially, i.e. $\bar{t}$ < 0, the fluid oncoming to the surface is still quiescent and both the fluid and the wavy surface have constant temperature T_∞ and concentration C_∞. At time $\bar{t}$ = 0, the temperature and the concentration of the wavy surface are suddenly changed to new levels, T_W and C_W, respectively. Due to the temperature and concentration differences between the wavy surface and ambient, the combined buoyancy forces are then generated, which in turn, induce the fluid motion in the ambient.

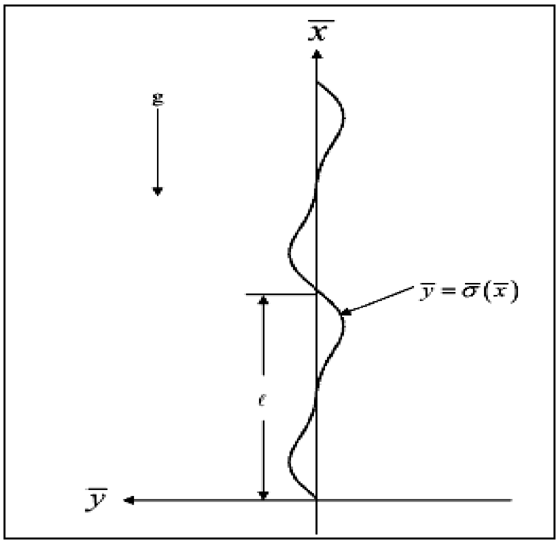


Fig. 1 Physical model and coordinates

2.2. Governing equations

The governing equations for an unsteady, laminar, and incompressible flow along a semi-infinite vertical wavy surface with boussinesq approximation may be written as:

Continuity equation:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{V}}{\partial \bar{y}} = 0 \tag{2}$$

Momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{V}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + g + \upsilon\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \tag{3}$$

$$\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{V}}{\partial \bar{x}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + \upsilon\left(\frac{\partial^2 \bar{V}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{V}}{\partial \bar{y}^2}\right) \tag{4}$$

Energy equation:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{V}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) \tag{5}$$

Concentration equation:

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{C}}{\partial \bar{x}} + \bar{V}\frac{\partial \bar{C}}{\partial \bar{y}} = D\left(\frac{\partial^2 \bar{C}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{y}^2}\right) \tag{6}$$

In non-dimensionalizing the governing equations, the following dimensionless variables were introduced:

$$y = \frac{\bar{y} - \bar{\sigma}(\bar{x})}{l} Gr^{1/4}; \qquad x = \frac{\bar{x}}{l};$$

$$y = \frac{\bar{y} - \bar{\sigma}(\bar{x})}{l} Gr^{1/4}; \qquad u = \frac{\bar{u} l}{\upsilon Gr^{1/2}};$$

$$V = \frac{\bar{V} - \sigma'(\bar{x})}{\upsilon Gr^{1/4}} l; \qquad p = \frac{\bar{p} l^2}{\rho \upsilon^2 Gr};$$

$$\sigma' = \frac{\partial \sigma(\bar{x})}{\partial \bar{x}} = \frac{\partial \bar{\sigma}(\bar{x})}{\partial \bar{x}}; \qquad \sigma(x) = \frac{\bar{\sigma}(\bar{x})}{l};$$

$$\theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}; \qquad Gr = \frac{g\beta_t(\bar{T}_w - \bar{T}_\infty)\rho^2 l^3}{\mu^2};$$

$$C = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty} \tag{7}$$

It is noted that when N is equal to zero, there is no mass diffusion body force and the problem reduces to pure heat convection, when N becomes infinite, there is no thermal diffusion. After ignoring small order terms in Gr, the dimensionless governing equations become:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \sigma' Gr^{1/4}\frac{\partial p}{\partial y} + \theta + NC + (1+\sigma'^2)\frac{\partial^2 u}{\partial y^2} \tag{8}$$

$$\sigma'' u^2 + \sigma'(\theta + NC) = \sigma'\frac{\partial p}{\partial x} - (1+\sigma'^2)\frac{\partial p}{\partial y} Gr^{1/4} \tag{9}$$

$$\sigma'' u^2 + \sigma'(\theta + NC) = \sigma'\frac{\partial p}{\partial x} - (1+\sigma'^2)\frac{\partial p}{\partial y} Gr^{1/4} \tag{10}$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + V\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}(1+\sigma'^2)\frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y}=\frac{1}{SC}(1+\sigma'^2)\frac{\partial^2 C}{\partial y^2} \quad (12)$$

It is worth noting that σ' and σ'' indicate the first and second differentiations of σ with respect to x. Eq. (13) shows that when $N<0$, the mass diffusion buoyancy forces oppose those of thermal diffusion, and when $N>0$, the mass diffusion buoyancy forces aid those of thermal diffusion. For the current problem, $\partial p/\partial x$, the pressure gradient is zero. Therefore, eliminating $\partial p/\partial y$ in Eq. (9) and (10) results in the following equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+V\frac{\partial u}{\partial y}= \frac{1}{1+\sigma'^2}(\theta-u^2\sigma'\sigma''+NC)+(1+\sigma'^2)\frac{\partial^2 u}{\partial y^2} \quad (13)$$

Finally Eqs. (14), (15), (16) and (17) in the parabolic coordinates (x , y) become:

$$\frac{\partial u}{\partial x}+\frac{\partial V}{\partial y}=0 \quad (14)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+V\frac{\partial u}{\partial y}= \frac{1}{1+\sigma'^2}(\theta-u^2\sigma'\sigma''+NC(1+\sigma'^2)\frac{\partial^2 u}{\partial y^2} \quad (15)$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+V\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}(1+\sigma'^2)\frac{\partial^2 \theta}{\partial y^2} \quad (16)$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y}=\frac{1}{SC}(1+\sigma'^2)\frac{\partial^2 C}{\partial y^2} \quad (17)$$

2.3. Initial and boundary conditions

The appropriate initial condition can be written as:

$$\bar{t}<0 \quad \bar{T}=\bar{T}_\infty,\ \ \bar{C}=\bar{C}_\infty,\ \ \bar{u}=\bar{V}=0 \quad (18)$$

For $\bar{t}=0$, the boundary conditions for the problem are:

At the wavy surface;

$$\bar{u}=0,\quad \bar{V}=0,\ \bar{T}=\bar{T}_W,\ \bar{C}=\bar{C}_W \quad (19)$$

Matching with the quiescent free stream:

$$\bar{u}=0,\ \bar{V}=0,\ \ \bar{T}=\bar{T}_\infty,\ \ \bar{C}=\bar{C}_\infty \quad (20)$$

Substituting dimensionless parameters into the Eq. (18)-(20), the corresponding initial and boundary condition are:

$$t<0; \quad u=V=\theta=c=0 \quad (21)$$

$$t\geq 0;\ \ y=0; \quad u=V=0,\ \theta=1;\ C=1 \quad (22)$$

$$y\to\infty; \quad u=0; \quad \theta\to 0;\ \ C\to 0 \quad (23)$$

2.4. Governing parameters

After obtaining the velocity, temperature and concentration fields along the complex wavy surface, the computations of local friction coefficient, Nusselt number and Sherwood number are of practical interest. The local heat and mass transfer rates are large when the normal velocity is approaching surface; they are small when the convective stream moves away from the surface. The heat and mass transfer mechanism along a complex wavy surface is different from that along a flat surface, and is modified by the fluid motion normal to the surface. The local Nusselt number and Sherwood number are defined respectively as:

$$\frac{hl}{k}=Nu_x=-Gr^{1/4}(1+\sigma'^2)^{1/2}\frac{\partial\theta}{\partial y}\Big|_{y=0} \quad (24)$$

$$\frac{h_D l}{D}=Sh_x=-Gr^{1/4}(1+\sigma'^2)^{1/2}\frac{\partial C}{\partial y}\Big|_{y=0} \quad (25)$$

The shearing stress on the wavy surface is:

$$\tau_W=\mu(\frac{\partial\bar{u}}{\partial\bar{y}}+\frac{\partial\bar{V}}{\partial\bar{x}})\Big|_{\bar{y}=0} \quad (26)$$

Since the local skin-friction coefficient C_{fx} is defined by:

$$Cf_x=\frac{2\tau_W}{\rho\tilde{U}^2} \quad (27)$$

Which is a characteristic velocity. $\tilde{U}=(\mu Gr^{1/2})/(\rho l)$ Substituting Eq. (26) into Eq. (27) in terms of the non-dimensional quantities, we have:

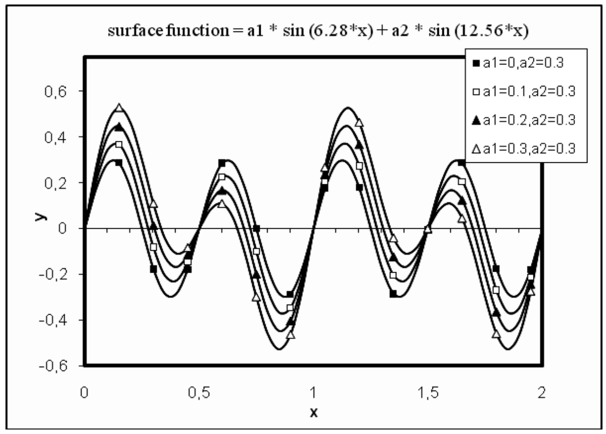
$$Cf_x = \frac{2(1-\sigma'^2)}{Gr^{1/4}} \frac{\partial u}{\partial y}\Big|_{y=0} \quad (28)$$

3 NUMERICAL APPROACH

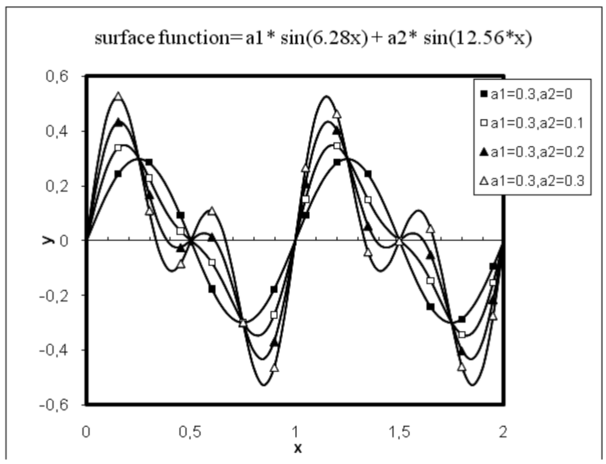
Because of the non-linear interactions among the momentum equations, the energy equation and the concentration equation, solution for the problem can be solved by numerical finite-difference procedures. The governing equations are parabolic in x and t. Hence the solution can be marched at time and the downstream direction. For the purpose of the numerical stability, a fully implicit formulation at time is adopted. The unsteady terms are approximated by backward difference. The axial convection is approximated by the upstream difference and the transverse convection and diffusion terms by the central difference is used to transform the governing equations into the finite-difference equations. The resulting system of algebraic equations can be cast into a tri-diagonal matrix equation, which can be efficiently solved by the Thomas algorithm [38]. During each transient and axial step, the numerical evaluation is iterated until the relative errors of the velocity, temperature and concentration at sequential iterations are less than 10^{-5}. If not, repeat the iterations for the current axial location. If yes, apply the above procedures from $x = 0$ to the desired downstream location ($x = 4$). Then march the solution from the onset of the transient to the final steady state. The steady-state criteria for the relative deviations of the variables, u, V, θ and C, between two time intervals are less than 10^{-6}. The detailed numerical procedures are similar to those of Ref. [9]. In the study, 251 non-uniform grid points were employed in the transverse direction (y). Some of the calculations were tested using 501 grid points in the y-direction, but no significant improvement over the 251 grid points was found. Additionally, there are 401 grid points in the marching direction. In the program test, a finer axial step size was tried and found to give acceptable accuracy. For the time interval, the first time interval is set to be 10^{-6}. The sequential time interval is then enlarged by 1%. To further check the adequacy of the numerical scheme used in this work, the results for the limiting case of natural convection heat transfer in a wavy surface were first obtained. Excellent agreement between the present predictions and those of Yan [3] was found. Besides, the predicted results of natural convection heat and mass transfer at steady state agree with the related work [1]. Through these program tests, it was found that the present numerical method is suitable for this study.

4 RESULT AND DISCUSSION

In the present study, numerical calculations are performed for the wavy surface described by $\bar{\sigma}(\bar{x}) = \bar{a}_1 Sin(\frac{2\pi\bar{x}}{l}) + \bar{a}_2 Sin(\frac{4\pi\bar{x}}{l})$ or dimensionless $\bar{\sigma}(x) = a_1 Sin(\frac{2\pi\bar{x}}{l}) + a_2 Sin(\frac{4\pi\bar{x}}{l})$ which is an equation. In Fig. 2 the geometry of this problem is shown for two cases.



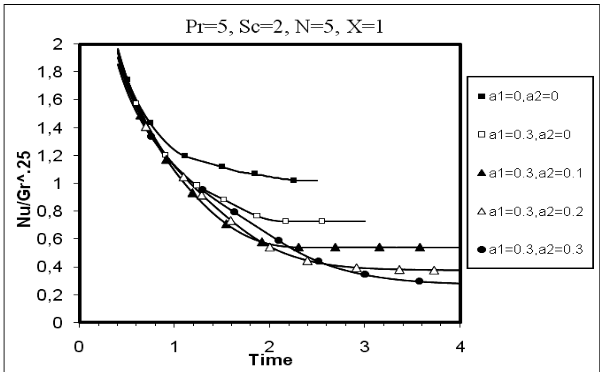
(a)



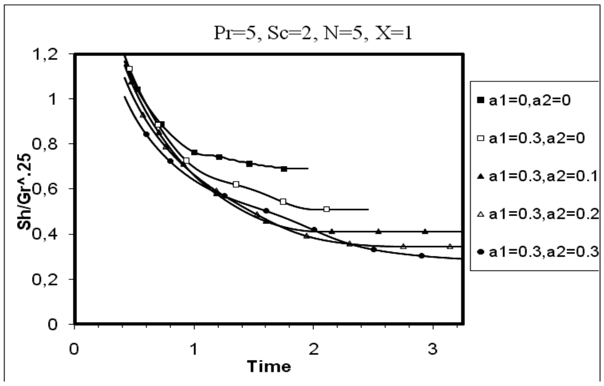
(b)

Fig. 2 (a): Surface curvature for variable a2 and a1 = 0.3 & (b): Surface curvature for variable a2 and a1 = 0.3

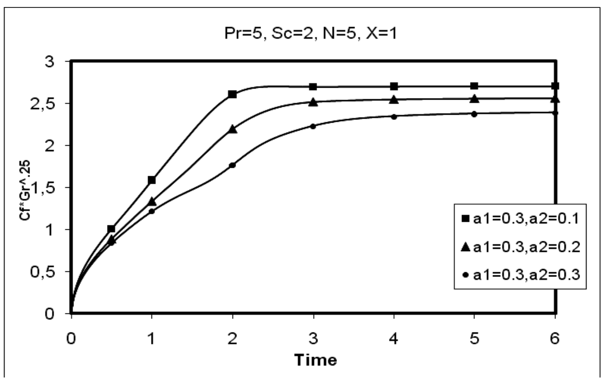
In case 1 (Fig. 2.a), amplitude of the harmonic wave (a_2) increases, while amplitude of the fundamental wave (a_1) is fixed at 0.3. In case 2 (Fig. 2.b), the amplitude of fundamental wave increases while the amplitude of the harmonic wave is fixed at 0.3. Numerical results show that in case 1, by increasing the amplitude of harmonic wave, the rate of local heat and mass transfer and the friction coefficient on the surface generally decreases (Fig. 3).



(a)



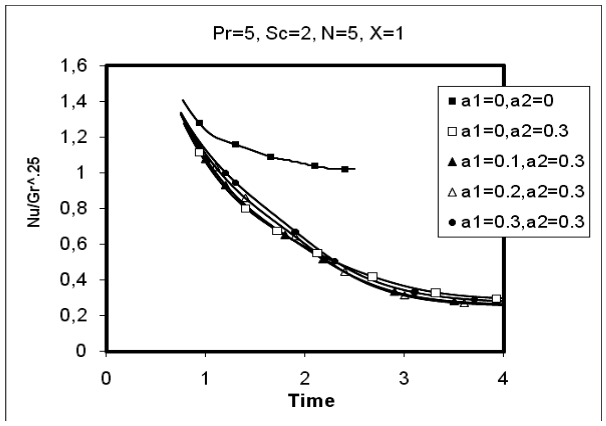
(b)



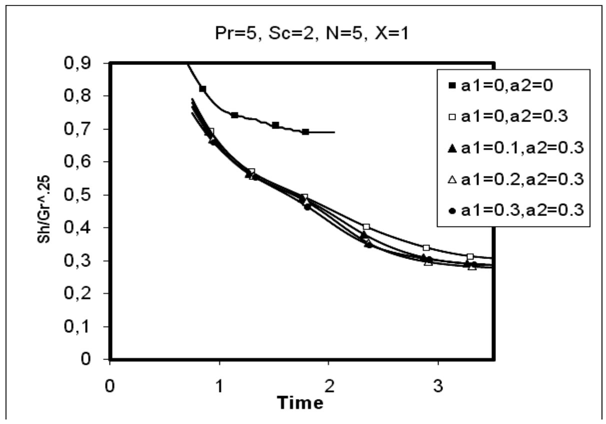
(c)

Fig. 3 (a): Local heat transfer rate per unit wetted area for variable a2 and a1 = 0.3, (b): Local mass transfer rate per unit wetted area for variable a2 and a1 = 0.3 & (c): Local friction coefficient per unit wetted area for variable a2 and a1 = 0.3

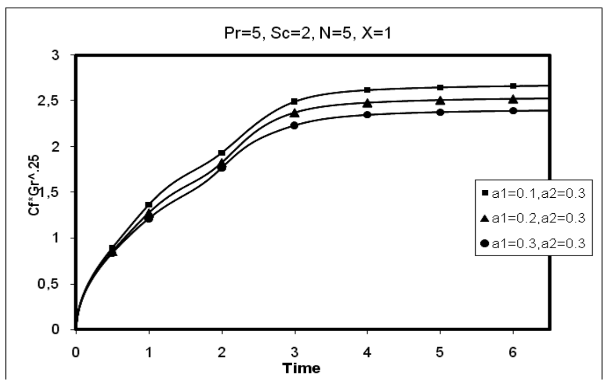
This decrease is sensitive to the amplitude that is less than 0.3. In this situation the fundamental wave amplitude is fixed at 0.3. For case 2 by increasing the amplitude of fundamental wave, the rate of local heat and mass transfer and the friction coefficient on the surface decrease (Fig. 4), this decrease is insensitive to the amplitude. This is due to the fact that the flow velocity near the wavy surface determines the rate of local heat and mass transfer and the friction coefficient on the surface.



(a)



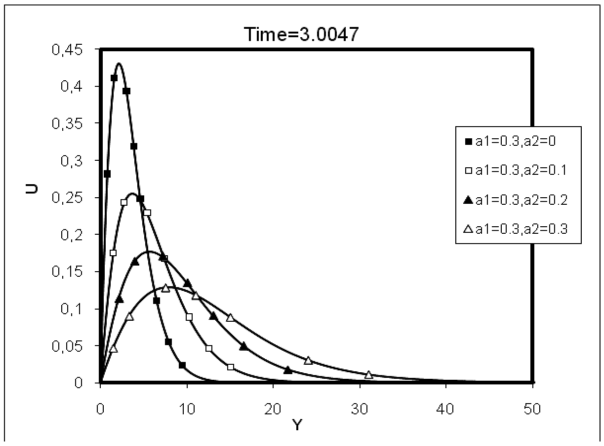
(b)



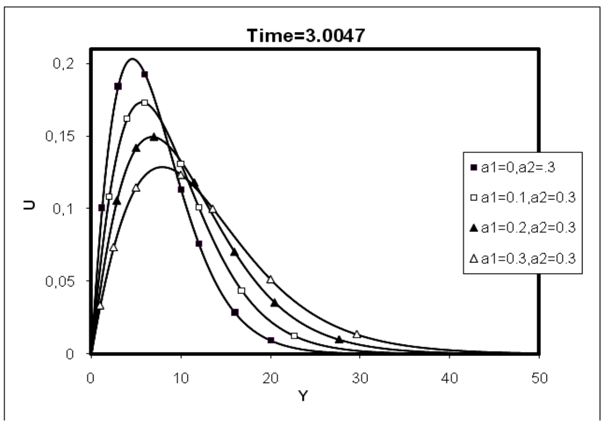
(c)

Fig. 4 (a): Local heat transfer rate per unit wetted area for variable a1 and a2 = 0.3, (b): Local mass transfer rate per unit wetted area for variable a1 and a2 = 0.3 & (c): Local friction coefficient per unit wetted area for variable a1 and a2 = 0.3

This can be seen clearly with plot of velocity near the surface in Fig 5. For case 1, by increasing the amplitude of harmonic wave, the velocity near the surface decreases intensely. While for case 2, this decrease is lower than case 1. With decrease in velocity in case 1, the rate of local heat and mass transfer and the friction coefficient suddenly decrease. While in the case 2, this decrease is lesser. In fact in the case 2 by increasing a_1, the effect on the ratio of amplitude to wavelength is low. While for the case 1 by increasing a_2, this effect is greater. Otherwise, if the amplitude of the harmonic wave becomes dominant, the fundamental wave is lesser factor in determining the surface curvature, consequently, the amplitude of the fundamental wave plays a lesser role in determination the velocity near the surface (shown in Fig. 5).



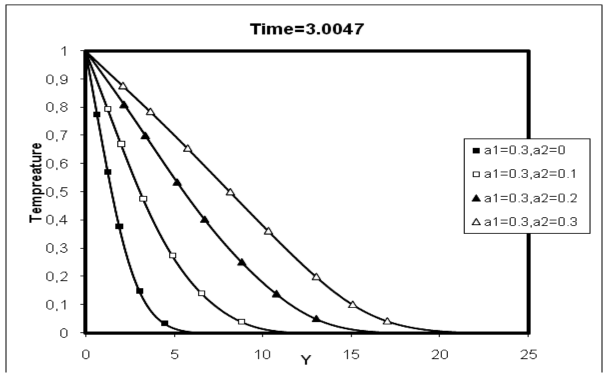
(a)



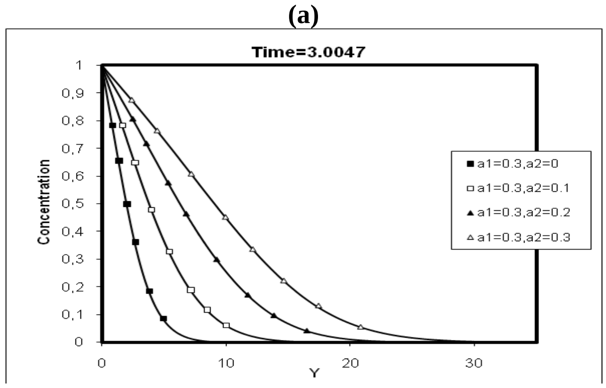
(b)

Fig. 5 (a): Axial velocity distribution for variable a2 and a1 = 0.3 at time=3.0047 & (b): axial velocity distribution for variable a2 and a1 = 0.3 at time=3.0047

Fig. 6 and Fig. 7 shows the concentration and the temperature distribution for the case 1 and 2. For the case 1 the thermal and concentration boundary layer thickness is thinner than case 2. In addition, for the first case, the mass and heat boundary layer grows more than second case in the same increase in amplitude. These results mention this fact that the rate of local heat and mass transfer for case 1 is larger than case 2. As mentioned before this effect is because of the shape of the surface.

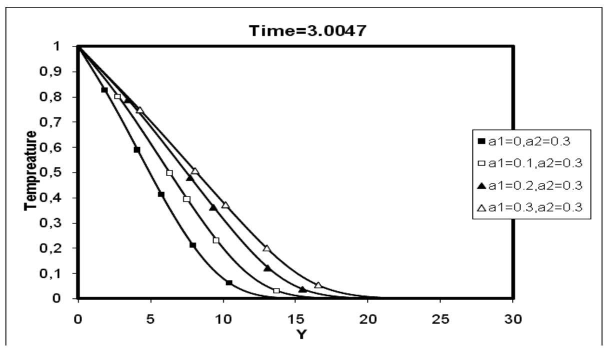


(a)

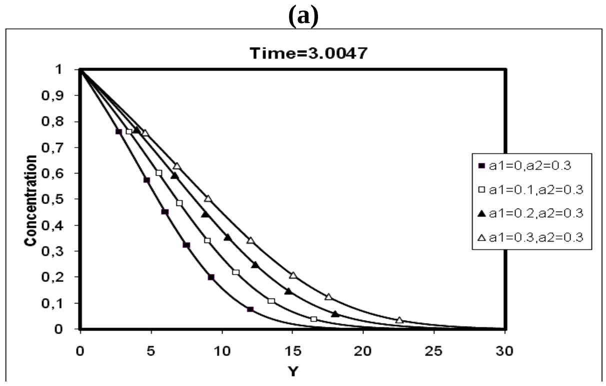


(b)

Fig. 6 (a) Temperature distribution for variable a2 and a1 = 0.3 at time=3.0047 & (b): Concentration distribution for variable a2 and a1 = 0.3 at time=3.0047



(a)



(b)

Fig. 7 (a): Temperature distribution for variable a1 and a2 = 0.3 at time=3.0047 & (b): Concentration distribution for variable a1 and a2 = 0.3 at time=3.0047

7 CONCLUSION

A numerical research of transient natural convection heat and mass transfer along a vertical complex wavy surface has been accomplished. The numerical consequences determine that the additional harmonic substantially alters the flow field, temperature and concentration distribution near the surface. Moreover, the numerical results demonstrate that the local mass and heat transfer rate for a complex surface are smaller than of a flat plate. This decreased local mass and heat transfer rate seem to depend on the ratio of amplitude surface.

8 APPENDIX OR NOMENCLATURE

a	amplitude of the wavy surface (m)	$\tilde{U}$	characteristic velocity
$\bar{a}$	amplitude-wavelength ratio, a/l	u, V	dimensionless velocity
$\bar{c}$	concentration	$\bar{x}, \bar{y}$	coordinate system (m)
C	dimensionless concentration	x, y	dimensionless coordinate system
Cf_x	skin-friction coefficient	**Greek symbols**	
D	mass diffusivity ($m^2 s^{-1}$ m2 s-1)	υ	kinematic coefficient of viscosity $(m^2 s^{-1})$
Gr	Grashof number	β_T	thermal expansion coefficient
g	gravitational acceleration (ms^{-2})	β_C	concentration expansion coefficient
K	conductivity ($Wm^{-1}K^{-1}$)	σ	dimensionless coordinate of the wavy surface
L	wavelength of the wavy surface (m)	$\bar{\sigma}$	coordinate of the wavy surface
N	buoyancy ratio	μ	viscosity ($kgm^{-1}s^{-1}$)
		ρ	fluid density ((kgm^{-3})
P	dimensionless pressure	θ	dimensionless temperature
$\bar{p}$	pressure (Nm^{-2})	τ_w	Shearing stress
Pr	Prandtl number		
Sc	Schmidt number		
Sh	Sherwood number	**Subscripts**	
$\bar{t}$	time (s)	∞	ambient temperature
t	dimensionless time	c	caused by concentration
$\bar{T}$	temperature (K)	w	surface condition
$\bar{u}, \bar{V}$	velocity components in the x and y directions, respectively (m s^{-1})		

REFERENCES

[1] Gebhart, H., Pera, L., “The Nature of Vertical Natural Convection Flows Resulting from the Combined Buoyancy Effects of Thermal and Mass Diffusion”, Int. J. Heat Mass Transfer, Vol. 14, No. 8, 1971, pp. 2025-2050.

[2] Parvin, S., Hossain, N. F., “Finite Element Simulation of MHD Combined Convection through a Triangular Wavy Channel Convection”, Int. Communication. Heat. Mass. Transfer, Vol. 39, No. 6, 2012, pp. 811- 817.

[3] Turkilmazoglu, M., “Multiple Solution of Hydromagnetic Permeable Flow and Heat for Viscoelastic Fluid”, J. Thermophysics. Heat. Transfer, Vol. 25, No. 4, 2011, pp. 595- 605.

[4] Al-amir, A., Khanafar, K., “Fluid-Structure Interaction Analysis of Mixed Convection Heat Transfer in a Lid Driven Cavity with a Flexible Bottom Wall”, Int. J. Heat Mass Transfer, Vol. 54, No. 17-18, 2011, pp. 3826- 3836.

[5] Yan, W. M., Lin, D., “Natural Convection Heat and Mass Transfer in Vertical Annuli with Film Evaporation and Condensation”, Int. J. Heat Mass Transfer, Vol. 44, No. 7, 2001, pp. 1143- 1151.

[6] Chang, C. J., Lin, T. F., and Yan, W. M., “Natural Convection Flow in a Vertical Open Tube Resulting from Combined Buoyancy Effects of Thermal and Mass

Diffusion", Int. J. Heat Mass Transfer, Vol. 29, No. 10, 1986, pp. 1543- 1552.

[7] Bottemanne, F. A., "Theoretical Solution of Simultaneous Heat and Mass Transfer by Free Convection about a Vertical Flat Plate", Appl. Sci. Res. 25, Vol. 29, No. 1, 1971, pp. 137- 149.

[8] Kleinstreuer, C., Wang, T. Y., "Thermal Convection on Micropolar Fluids Past Two-Dimensional or Axisymmetric Bodies with Suction/Injection", Int. J. Eng. Sci. 25, Vol. 26, No. 3, 1988, pp. 1266- 1277.

[9] Chiu, C. P., Chou H. M., "Transient Analysis of Natural Convection Along a Vertical Wavy Surface in Micropolar Fluids", Int. J. Eng. Sci, Vol. 32, No. 2, 1994, pp. 19- 33.

[10] Yih, K. A., "Uniform Transpiration Effect on Combined Heat and Mass Transfer by Natural Convection Over a Cone in Saturated Porous Media: Uniform Wall Temperature/Concentration or Heat/Mass Flux", Int. J. Eng. Sci. Vol. 42, No. 8, 1999, pp. 3533- 3537.

[11] Mustafa, I., Javed, T., and Ghaffari, A., "Hydromagnetic Natural Convection Flow of Water-Based Nanofluid along a Vertical Wavy Surface with Heat Generation", J. Molecular. Liquids. Vol. 229, No. 4, 2017, pp. 246- 254.

[12] Oosthuizen, P. H., "External Natural Convective Heat Transfer from Bodies Having a Wavy Surface for Conditions Under Which Laminar, Transitional, and Turbulent Flow Can Exist", Adv. Heat. Transfer. Vol. 48, No. 1, 2016, pp. 261- 317.

[13] Muthtamilselvan, M., Prakash, D., and Doh, D. H., "Effect of Thermal Non-Equilibrium on Transient Hydromagnetic Flow Over a Moving Surface in a Nanofluid Saturated Porous Media", J. Mech. Sci. Tech. Vol. 28, No. 9, 2014, pp. 3709- 318.

[14] Umavathi, J. C., Shekar, M., "Effect of Thermal Non-Equilibrium on Transient Hydromagnetic Flow Over a Moving Surface in a Nanofluid Saturated Porous Media", Heat. Trans. Eng. Vol. 36, No. 1, 2015, pp. 1- 20.

[15] Karami, A., Yousefi, T., Harsini, I., Maleki, E., and Mahmoudinezhad, S., "Neuro-Fuzzy Modeling of the Free Convection Heat Transfers from a Wavy Surface", Heat. Trans. Eng. Vol. 36, No. 9, 2015, pp. 847- 855.