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Research Article

Double diffusive reaction-convection in viscous fluid layer

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Abstract

In the present study, the onset of double diffusive reaction-convection in a fluid layer with viscous fluid, heated and salted from below subject to chemical equilibrium on the boundaries, has been investigated. Linear and nonlinear stability analysis have been performed. For linear analysis normal mode technique is used and for nonlinear analysis minimal representation of truncated Fourier series is used. The effect of Lewis number, solute Rayleigh number, reaction rate and Prandtl number on the stability of the system is investigated. A weak nonlinear theory based on the truncated representation of Fourier series method is used to find the heat and mass transfer.

Keywords : Double diffusive convection; chemical reaction; viscous fluid.

1 Introduction

The study of convective instabilities in the presence of two bouyancy driven components caused by temperature and concentration with different diffusivities is known as double diffusive convection. The study of double diffusive convection is of considerable interest due to its implications in geothermal reservoirs, under ground disposal of nuclear wastes, migration of moisture in fibrous insulation, electrochemical, drying process, thermal insulation, nuclear waste repository, grain storage, food engineering, chemical engineering, oil reservoir modeling and bio-mechanics. Excellent works are investigated by Chandrasekhar [1], Ingham and Pop [2, 3], Vafai [4, 5], Malashetty [9] and Rudraiah and Malashetty [10].
Thermal convection plays important role in many practical cases a major mechanicm for the transfer and deposition of salts and other chemical reaction which occurs in sedimentary basins. A variety of chemical reactions can occur as fluid to carry various dissolved species which moves through a permeable matrix. The behavior of resulting mixture depends on the reaction kinetics, temperature variation, pressure and other parameter present in the governing model. The effect of chemical reactions on the convective motion is not fully discribed and has a little attention now. The first study on the effect of chemical reaction on the onset of convection saturated in porous medium was given by Stgeinberg and Brand [8]. Their study was restricted to the regime where the chemical reaction was sufficiently fast so that the the solute diffusivity could be neglected. The effect of exothermic- reaction on the stability of the porous medium has been studied by Gatica *et. al.* [10] and Viljoen *et. al.* [11]. There is a limitation on thermal and solutal diffusivities as if they are, over damped oscillations are not possible. Malashetty and Gaikwad [12] investigated the linear stability analysis for chemically driven instabilities in binary liquid mixtures with fast chemical reaction. They investigated the effect

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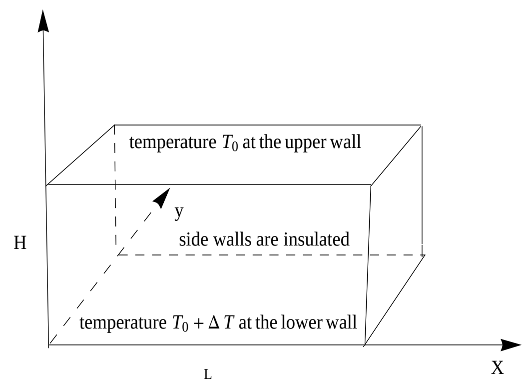
of chemical reaction by obtaining an analytical expression for onset of stationary and oscillatory convection. Pritchard and Richardson [13] studied an exhaustive case of the effect of temperature dependent solubility on the onset of thermosolutal convection in anisotropic porous medium. A linear stability analysis was used to depict how the dissolution of concentration affects the onset of convection and selection of an unstable wave number. Recently, Malashetty and Biradar [14] studied the double diffusive reaction convection in an anisotropic porous medium. They obtain an analytical expression for both linear and nonlinear stability analysis. A linear stability analysis is used to perform how the dissolution or precipitation of reactive components affects the onset of convection. They found that effect of chemical reaction may be stabilizing or destabilizing.

In present paper, we study the double diffusive reaction-convection in viscous fluid. The main aim of this study to analyze the linear and weak non-linear stability analysis of double diffusive reaction-convection with a viscous fluid in a finite fluid layer similar to that of Nield and Kuznetsov [15]. The fluid layer is assumed to be isotropic. Our aim is to study how the onset criterion for stationary, oscillatory convection and the heat and mass transfer depend the chemical reaction parameter, Prandtl number and Lewis number.

2 Mathematical Formulation

2.1 Basic equations

consider a fluid saturated a finite layer of depth d and width L, which is heated from below and cooled from above. The x-axis is taken along the lower boundary, and the z-axis vertically upward. The lower surface is held at temperature $T_0 + \triangle T$ and concentration $S_0 + \triangle S$, while the upper surface is at temperature T_0 and concentration S_0 respectively. A uniform adverse temperature gradient $\triangle T/d$ and a stabilizing concentration gradient $\triangle S/d$ are maintained between the lower and upper surfaces. Malashetty and Biradar[14] investigated the effect of chemical reaction in anisotropic porous medium. They considered the Darcy model for momentum equation in which there is no time derivative term. We have extended the study of Malashetty and Biradar[14] in finite fluid layer to see the effect of aspect ratio and fluid motion on stationary



and oscillatory convection. The continuity and momentum equations governing the motion of an incompressible viscous fluid are given by

$$\nabla.\mathbf{q} = 0 \tag{2.1}$$

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\frac{1}{\rho_0}\nabla p + \frac{\rho}{\rho_0}g + \frac{\mu}{\rho_0}\nabla^2 q \tag{2.2}$$

We assume that the solid and fluid phases of medium are in local thermal equilibrium. The energy and concentration equations are given by

$$\frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)T = \kappa_T \nabla^2.T \tag{2.3}$$

$$\frac{\partial S}{\partial t} + (\mathbf{q}.\nabla)S = \kappa_s \nabla^2 S + k(S_{eq}(T) - S) \tag{2.4}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)] \tag{2.5}$$

where q is the velocity of fluid in cavity medium, p the fluid pressure, $\mathbf{g}$ the gravitational acceleration, μ the dynamic viscosity. A Cartesian system (x, y, z) is used, with the z- axis vertically upward in the gravitational field. Further, κ_T and κ_S are the thermal and solute diffusivities, respectively. k is lumped effective reaction rate and $S_{eq}(T)$ is the equlibrium concentration of the solute at a given temperature. The quantities β_T and β_S are the thermal volume coefficient and solute density coefficient, respectively. As suggested by Pritchard and Richardson[13], we may write the equilibrium solute concentration is a linear function of temperature as $S_{eq}(T) = S_0 + \phi(T - T_0)$. If boundaries are kept at chemical equilibrium then we assume $\phi = \frac{\triangle S}{\triangle T}$. In general, ϕ may be positive or negative. Obviously, if $\phi > 0$, the solubility increases with temperature and if $\phi < 0$, the solubility decreases with temperature. Here, the case $\phi > 0$ is considered.

2.2 Basic state

The basic state of the fluid is quiescent and is given by,

$$\mathbf{q}_b = (0,0,0),\ p = p_b(z),\ T = T_b(z),$$
$$S = S_b(z),\ \rho = \rho_b(z) \tag{2.6}$$

Using (2.6), Eqs.(2.1) - (2.5) can be written as

$$\frac{dp_b}{dz} = \rho_b \mathbf{g},\quad \frac{d^2T_b}{dz^2} = 0,\quad \frac{d^2S_b}{dz^2} = 0,$$
$$\rho_b = \rho_0[1 - \beta_T(T_b - T_0) + \beta_S(S_b - S_0)] \tag{2.7}$$

2.3 Perturbed state

On the basic state, we suppose perturbations in the form

$$\mathbf{q} = \mathbf{q}_b + \mathbf{q}',\ T = T_b(z) + T',\ S = S_b(z) + S',$$
$$p = p_b(z) + p',\ \rho = \rho_b(z) + \rho' \tag{2.8}$$

where primes denotes perturbations. Substituting Eq. (2.8) into Eqs. (2.1) – (2.5) and using the basic state solutions, we obtain the equations governing the perturbations in the form,

$$\nabla.\mathbf{q}' = 0 \tag{2.9}$$

$$\frac{\partial \mathbf{q}'}{\partial t} + (\mathbf{q}'.\nabla)\mathbf{q}' = -\frac{1}{\rho_0}\nabla p' + \frac{\rho'}{\rho_0}\mathbf{g} + \frac{\mu}{\rho_0}\nabla^2\mathbf{q}' \tag{2.10}$$

$$\frac{\partial T'}{\partial t} + w'(\frac{dT_b}{dz}) + (\mathbf{q}'.\nabla)T' = \kappa_T \nabla^2 T' \tag{2.11}$$

$$\frac{\partial S'}{\partial t} + w'\frac{dS_b}{dz} + (\mathbf{q}'.\nabla)S' = \kappa_S \nabla^2 S' + k(S_{eq}(T') - S') \tag{2.12}$$

$$\rho' = -\rho_0[\beta_T T' - \beta_S S'] \tag{2.13}$$

2.4 Transverse rolls

we consider the case where

$$u^{'} = u^{'}(x,z,t),\ v^{'} = 0,\ w^{'} = w^{'}(x,z,t),$$
$$T^{'} = T^{'}(x,z,t),\quad S^{'} = S^{'}(x,z,t)$$

Now introducing the stream function ψ and $\mathbf{g}$ by

$$(u', w') = (\frac{\partial \psi}{\partial z}, -\frac{\partial \psi}{\partial x}),\ \mathbf{g} = (0,0,-g) \tag{2.14}$$

which satisfy the continuity Eq. (2.9).
Eliminating pressure term from (2.10), introducing the stream function ψ and non- dimensionalising the resulting equation as well as Eqs. (2.10) - (2.12) by using the following non-dimensional parameters

$$(x,z) = d(x^*, z^*)\ t = \frac{d^2}{\kappa_T}t^*,\ \psi = \frac{\kappa_T}{d}\psi^*$$
$$T' = (\triangle T)T^*,\ S' = (\triangle S)S^* \tag{2.15}$$

We obtain the following non- dimensional equations (on dropping the asterisks for simplicity),

$$(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2)\nabla^2\psi + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = \frac{\partial(\psi, \nabla^2\psi)}{\partial(x,z)} \tag{2.16}$$

$$(\frac{\partial}{\partial t} - \nabla^2)T = -\frac{\partial \psi}{\partial x} + \frac{\partial(\psi, T)}{\partial(x,z)} \tag{2.17}$$

$$(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2)S = -\frac{\partial \psi}{\partial x} + \frac{\partial(\psi, S)}{\partial(x,z)} + \chi(T - S) \tag{2.18}$$

where $Pr = \nu/\kappa_T$, the Prandtl number, $Ra_T = \frac{\beta_T(\triangle T)gdK}{\nu\kappa_T}$, the Rayleigh number, $Ra_S = \beta_S(\triangle S)gd/\nu\kappa_T$, the solute Rayleigh number coefficient, $Le = \kappa_T/\kappa_s$, the Lewis number, $\chi = kd^2/\kappa_T$, dimensional reaction rate measured by Damkohler number.
The Eqs. (2.16) - (2.18) are solved for stress - free, isothermal, boundary conditions given by

$$\psi = \frac{\partial^2\psi}{\partial z^2} = T = S = 0 \quad \text{at} \quad z = 0, 1 \tag{2.19}$$

$$\frac{\partial T}{\partial x} = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = Ar \tag{2.20}$$

3 Linear stability theory

In this section we discuss the linear stability analysis, which is very useful in the local non -linear stability analysis discussed in the next section. To make this study we neglect the non -linear term in Eqs. (2.16) – (2.18) and assume the solution to be periodic waves of the form

$$\begin{bmatrix} \psi \\ T \\ S \end{bmatrix} = e^{\omega t}\begin{bmatrix} \Psi\sin(\frac{\pi x}{Ar}) \\ \Theta\cos(\frac{\pi x}{Ar}) \\ \Phi\sin(\frac{\pi x}{Ar}) \end{bmatrix}\sin(\pi z) \tag{3.21}$$

where ω is the growth rate and in general a complex quantity ($\omega = \omega_r + i\omega_i$). Substituting

Eqs. (3.21) in the linearized form of Eqs. (2.16)-(2.18), we get

$$(\frac{1}{Pr}\frac{\partial}{\partial t}-\nabla^2)\nabla^2\psi+Ra_T\frac{\partial T}{\partial x}-Ra_S\frac{\partial S}{\partial x}=0 \quad (3.22)$$

$$(\frac{\partial}{\partial t}-\nabla^2)T+\frac{\partial \psi}{\partial x}=0 \quad (3.23)$$

$$(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2)S+\frac{\partial \psi}{\partial x}+\chi(S-T)=0 \quad (3.24)$$

where $\delta^2 = \pi^2(Ar^2+1)/Ar^2$. For non- trivial solution of Ψ, Θ and Φ, we have

$$Ra_{T,c}=\frac{Ar^2\delta^2(\frac{\omega}{Pr}+\delta^2)(\omega+\delta^2)}{\pi^2} + \frac{Ra_S(\chi+\omega+\delta^2)}{(\omega+\frac{\delta^2}{Le}+\chi)} \quad (3.25)$$

In general growth number ω is complex quantity such that $\omega=\omega_r+i\omega_i$. the system is stable if $\omega_r<0$ and unstable if $\omega_r>0$. For the neutral stability state $\omega_r=0$. Therefore, now we set $\omega = i\omega_i$ in above expression and after simplification we have

$$Ra_T=\wedge_1+i\omega_i\wedge_2, \quad (3.26)$$

where

$$\wedge_1=\frac{Ar^2\delta^2}{\pi^2}(\delta^4-\frac{\omega_i^2}{Pr}) + \frac{Ra_S[(\delta^2+\chi)(\frac{\delta^2}{Le}+\chi)+\omega_i^2]}{(\frac{\delta^2}{Le}+\chi)^2+\omega_i^2}, \quad (3.27)$$

$$\wedge_2=\frac{Ar^2\delta^2}{\pi^2}(\frac{\delta^2}{Pr}+\delta^2) + \frac{Ra_S(\delta^2Le^{-1}-\delta^2)}{(\delta^2Le^{-1}+\chi)^2+\omega_i^2}, \quad (3.28)$$

3.1 Marginal state

If ω is real, then marginal stability occurs when $\omega=0$. Then Eq. (3.25) gives the stationary Rayleigh number Ra_T^{st} at the margin of stability, in the form

$$Ra_T^{st}=\frac{\delta^6Ar^2}{\pi^2}+\frac{Ra_S(\chi+\delta^2)}{\delta^2Le^{-1}+\chi} \quad (3.29)$$

For a single component system, i.e. when $Ra_S=0$, Eq.(3.29) reduces to

$$Ra_T^{st}=\frac{\delta^6Ar^2}{\pi^2}, \quad (3.30)$$

which is eqivalent to classical result obtained by Horton and Rogers[16] and Lapwood[17]. Further if we set $\chi=0$ Eq. (3.29) gives

$$Ra_T^{st}=\frac{\delta^6Ar^2}{\pi^2}+LeRa_S, \quad (3.31)$$

which is equivalent to result obtained by Malashetty and Biradar[14] in case of isotropic porous medium.

3.2 Oscillatory convection

We set $\omega=i\omega_i$ in Eq (3.25) and clear the complex quantities from denominator to get the oscillatory Rayleigh number Ra_T^{osc} at the margin of stability, Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (3.26) it follows that either $\omega_i=0$ (steady onset) or $\wedge_2=0$ ($\omega_i\neq0$, oscillatory onset). For the oscillatory onset $\wedge_2=0$ ($\omega_i\neq0$) and this gives a dispersion relation of the form

$$\omega^2=\frac{Ra_S(1-Le^{-1})}{Ar^2(Pr^{-1}+1)}-(\delta^2Le^{-1}+\chi)^2 \quad (3.32)$$

Now substituting $\wedge_2=0$ in Eq. (3.26), we get

$$Ra_T^{Osc}=\frac{Ar^2\delta^2}{\pi^2}(\delta^4-\frac{\omega_i^2}{Pr}) + \frac{Ra_S[(\delta^2+\chi)(\frac{\delta^2}{Le}+\chi)+\omega_i^2]}{(\frac{\delta^2}{Le}+\chi)^2+\omega_i^2} \quad (3.33)$$

The expression for the frequency and oscillatory Rayleigh number given in Eqs. (3.32) and (3.33), respectively, coincides with the one obtained by Malashetty and Biradar[14] in the limit when the dimensionless number $Pr\to\infty$ and $\xi=\eta=1$. For oscillatory convection ω^2 should be positive. Since Eq. (3.32) is a quadratic equation in ω^2, it can give rise to more than one positive root for the fixed values of chosen parameters. We find the oscillatory neutral stability solutions from Eq. (3.33). We proceed this as follows: First find the solution of Eq. (3.32). If there are no solution then oscillatory instability is not possible. If there are more than one solution then minimum value (over Ar^2) of Eq. (3.33) with ω^2 given by Eq. 3.32 gives the oscillatory neutral Rayleigh number $Ra_{T,c}^{Osc}$ with the critical aspect ratio a_c and the critical frequency of oscillatory instability ω^2. The analytical expression for oscillatory Rayleigh number given by Eq. (3.33) is calculated at $Ar=Ar_c$ and $\omega^2=\omega_c^2$ for different values of physical parameters to find their effect on the oscillatory convection.

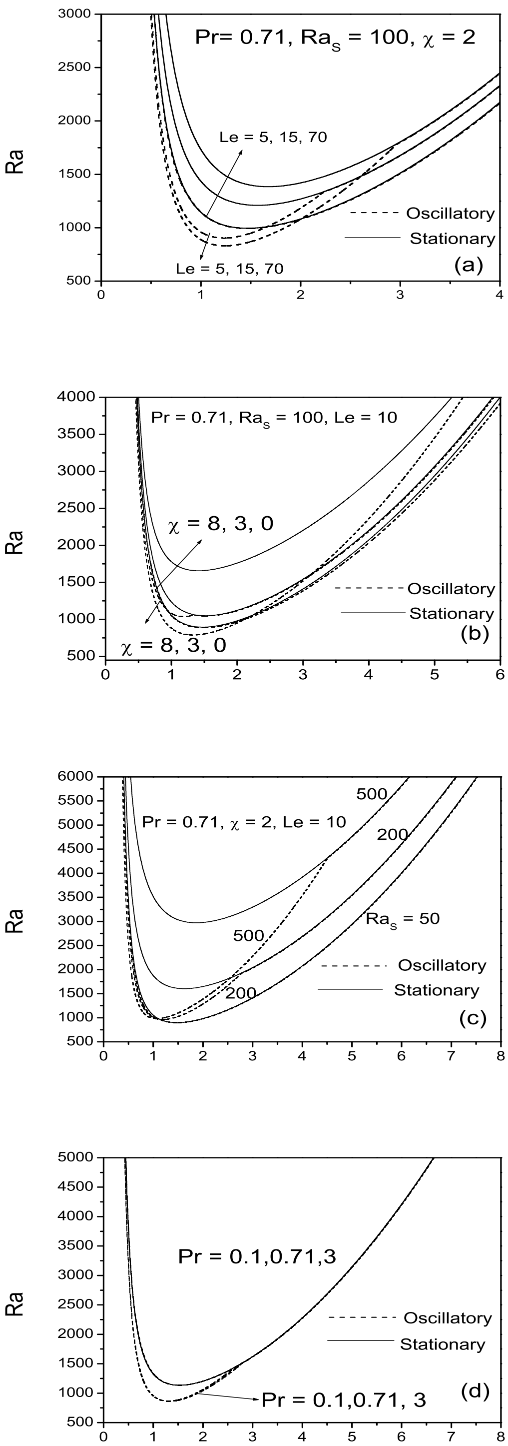


Figure 1: Variation of Rayleigh number with Ar for different values of (a)Le, (b)χ, (c)Ra_S and (d) Pr.

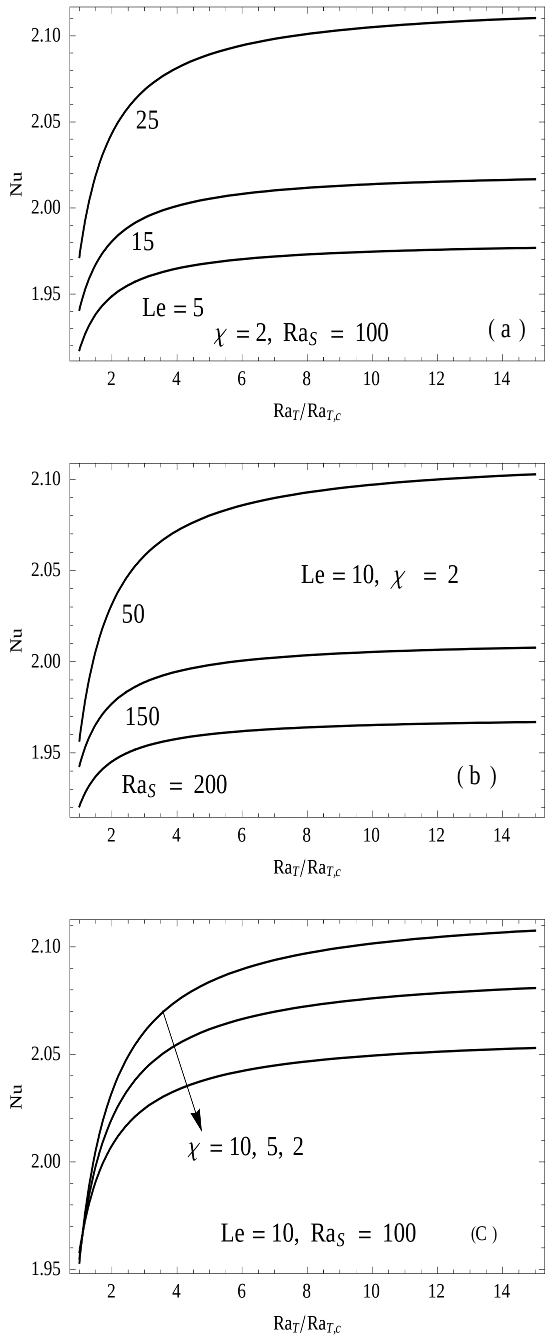


Figure 2: Variation of Nusselt number with the critical Rayleigh number for different values of (a) Le, (b) Ra_S, (c) χ

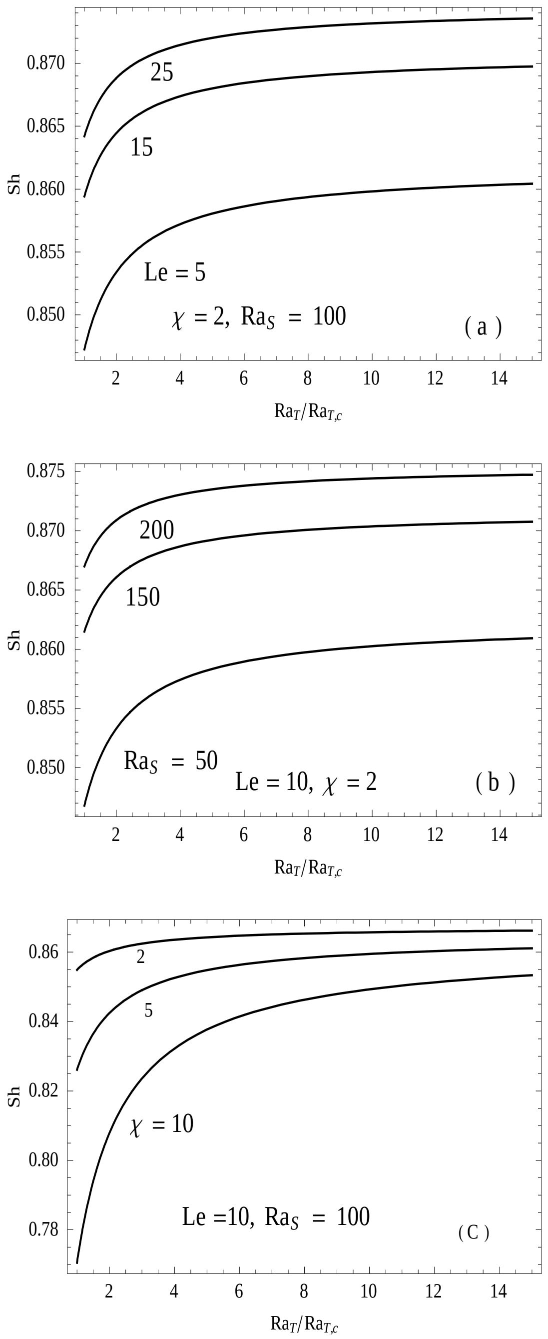


Figure 3: Variation of Sherwood number with the critical Rayleigh number for different values of (a) Le, (b) Ra_S, (c) χ

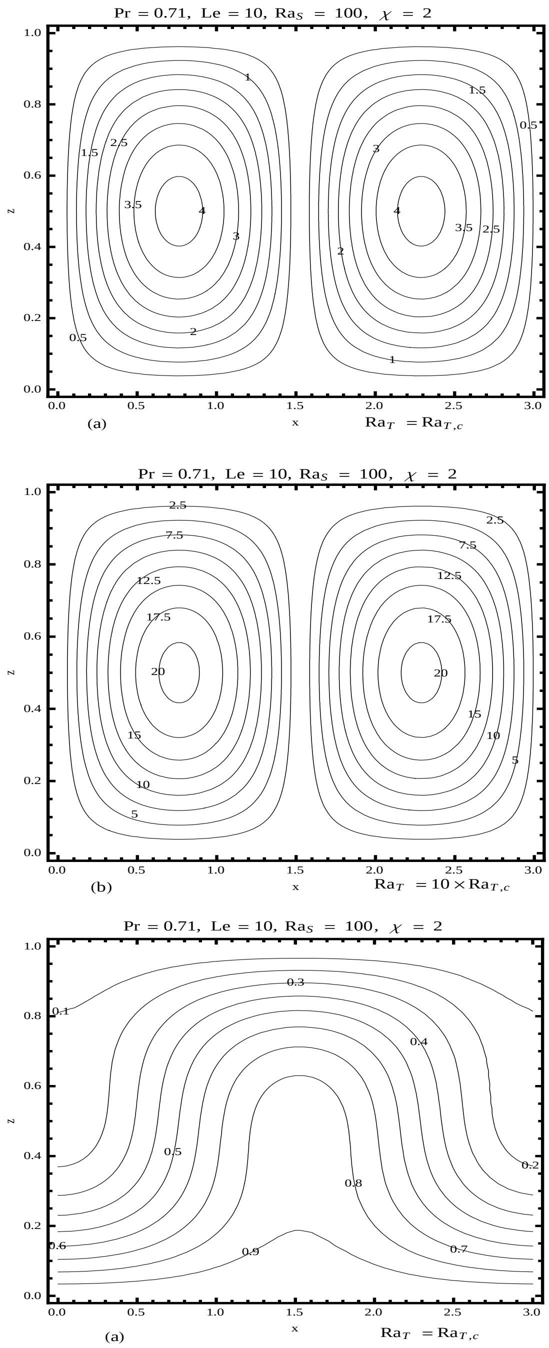


Figure 4: Streamlines for steady nonlinear case

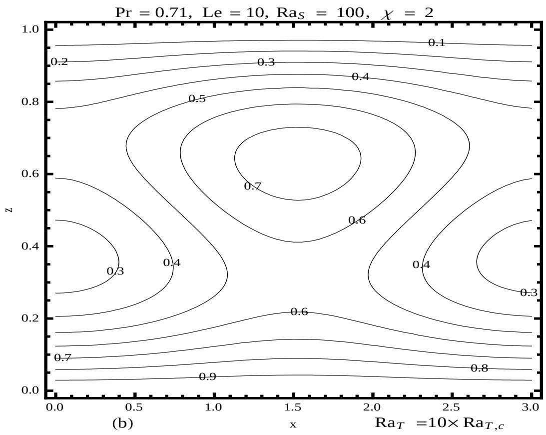


Figure 5: Isotherms for steady nonlinear case

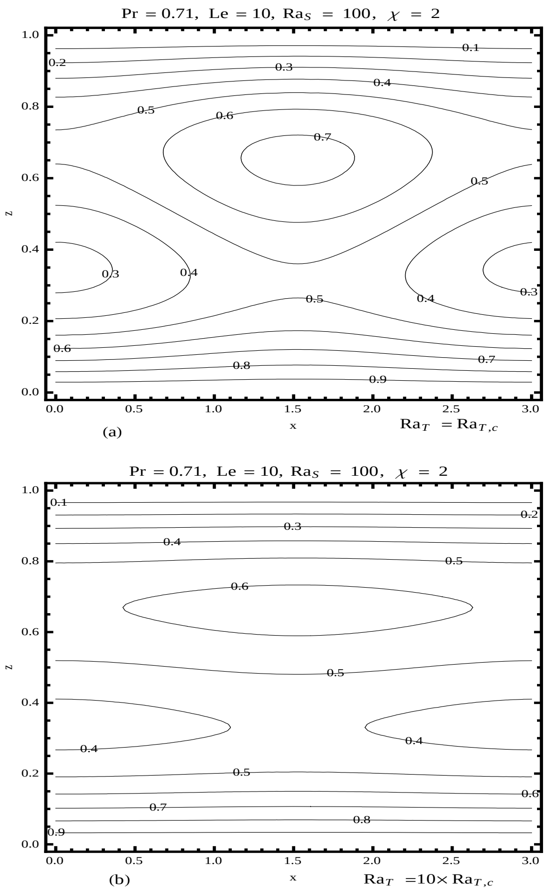


Figure 6: Isohalines for steady nonlinear case

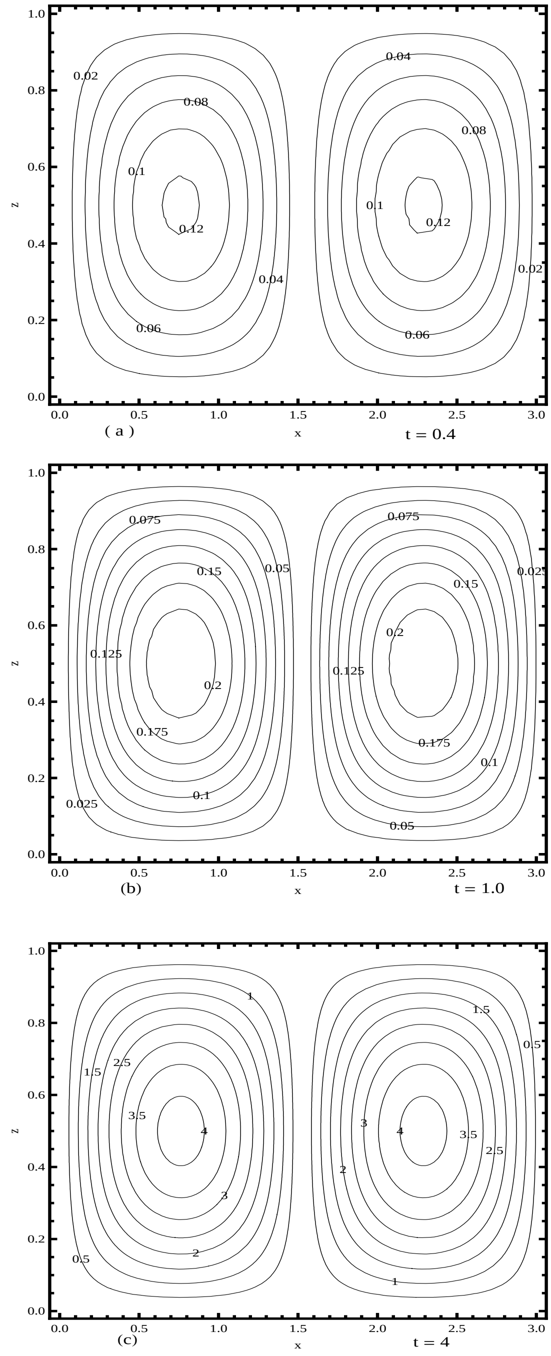


Figure 7: Streamlines for unsteady nonlinear case

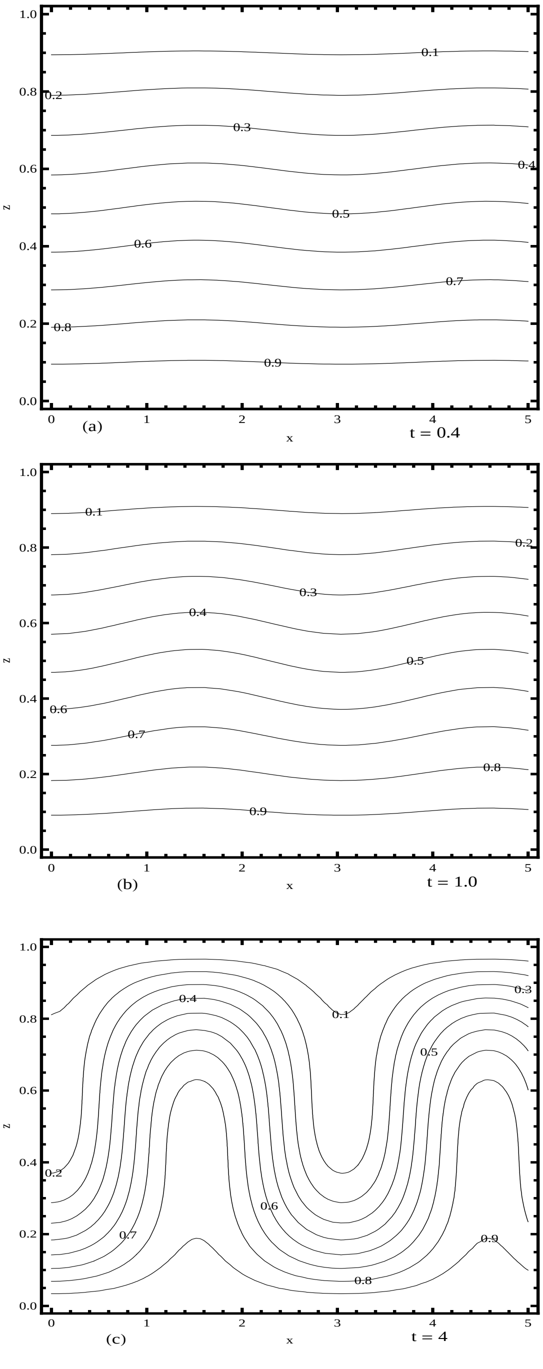


Figure 8: Isotherms for unsteady nonlinear case

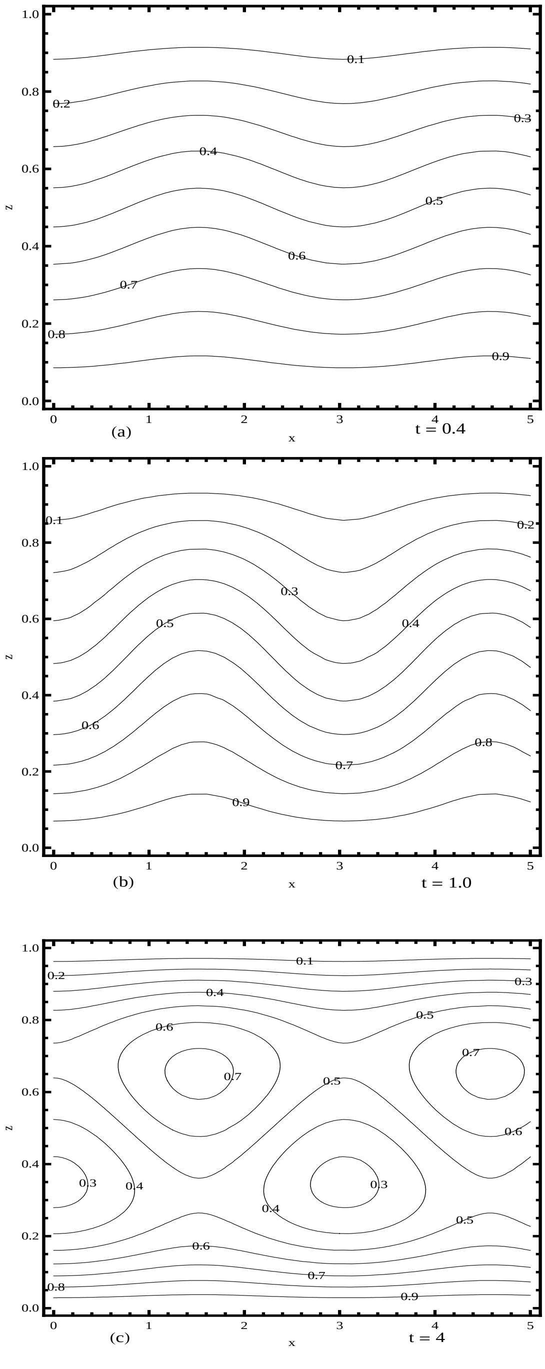


Figure 9: Isohalines for unsteady nonlinear case

4 Finite Amplitude Analysis

In the previous section, we investigated the linear stability analysis using normal mode technique. Although the linear stability analysis is sufficient for study of stability condition of the motionless solution describing the convective flow. However, since this analysis cannot provide the information about the convection amplitudes and hence the rate of heat and mass transfer. In this section, a local nonlinear stability analysis shall be performed by using minimal representation of Fourier series for stream function ψ, temperature T and concentration S.

$$\psi = A_{11}(t)\sin(\frac{\pi x}{Ar})\sin(\pi z) \tag{4.34}$$

$$T = B_{11}(t)\cos(\frac{\pi x}{Ar})\sin(\pi z) + B_{02}(t)\sin(2\pi z) \tag{4.35}$$

$$S = E_{11}(t)\cos(\frac{\pi x}{Ar})\sin(\pi z) + E_{02}(t)\sin(2\pi z) \tag{4.36}$$

where the amplitudes $A_{11}(t)$, $B_{11}(t)$, $B_{02}(t)$, $E_{11}(t)$ and $E_{02}(t)$ are to be determined from the dynamics of the system. Substituting Eqs. (4.34)-(4.36) into Eqs. (2.16)-(2.18) and equating the coefficients of like terms we have the following non-linear system of differential equations

$$\frac{dA_{11}}{dt} = -Pr\delta^2 A_{11} - \frac{\pi Pr Ra_T}{\delta^2 Ar}B_{11} + \frac{\pi Pr Ra_S}{\delta^2 Ar}E_{11} \tag{4.37}$$

$$\frac{dB_{11}}{dt} = -\frac{\pi}{Ar}A_{11} - \delta^2 B_{11} - \frac{\pi^2}{Ar}A_{11} \tag{4.38}$$

$$\frac{dB_{02}}{dt} = \frac{\pi^2}{2Ar}A_{11}B_{11} - 4\pi^2 B_{02} \tag{4.39}$$

$$\frac{dE_{11}}{dt} = -\frac{\pi}{Ar}A_{11} - (\frac{\delta^2}{Le} + \chi)E_{11} + \chi B_{11} - \frac{\pi^2}{Ar}A_{11}E_{02} \tag{4.40}$$

$$\frac{dE_{02}}{dt} = \frac{\pi^2}{2Ar}A_{11}E_{11} - (\frac{4\pi^2}{Le} - \chi)E_{02} + \chi B_{02} \tag{4.41}$$

The non-linear system of differential equation can't be solved analytically for the general time dependent variable and we have to solve it using numerical method. In the case of steady motions, setting left hand side of Eqs. (4.37) -(4.41) equal to zero, we get

$$\delta^2 A_{11} + \frac{\pi Ra_T}{\delta^2 Ar}B_{11} - \frac{\pi Ra_S}{\delta^2 Ar}E_{11} = 0 \tag{4.42}$$

$$\frac{\pi}{Ar}A_{11} + \delta^2 B_{11} + \frac{\pi^2}{Ar}A_{11}B_{02} = 0 \tag{4.43}$$

$$\frac{\pi}{Ar}A_{11} + (\frac{\delta^2}{Le} + \chi)E_{11} + \frac{\pi^2}{Ar}A_{11}E_{02} - \chi B_{11} = 0 \tag{4.44}$$

$$\frac{\pi^2}{2Ar}A_{11}E_{11} - (\frac{4\pi^2}{Le} + \chi)E_{02} + \chi B_{02} = 0 \tag{4.45}$$

The steady state solutions are useful because they suggest that a finite amplitude solution of the system is possible. Eliminating all amplitudes except A_{11} yields an equation with $\frac{A_1^2}{8} = x$ as

$$x_1 x^2 + x_2 x + x_3 = 0 \tag{4.46}$$

where

$$x_1 = \frac{4\pi^6\delta^2}{Ar^4} \tag{4.47}$$

$$x_2 = \frac{4\pi^4\delta^4}{Ar^2} + \frac{\pi^2\delta^2}{Ar^2}(\delta^2 Le^{-1} + \chi)\left(4\pi^2 Le^{-1} + \chi\right) + \frac{4\pi^6}{\delta^2 Ar^4}(Ra_S Le^{-1} - Ra_T) \tag{4.48}$$

$$x_3 = (4\pi^2 Le^{-1})[\delta^4(\delta^2 Le^{-1} + \chi) - \frac{\pi^2}{\delta^2 Ar^2} \times \{Ra_T(\delta^2 Le^{-1} + \chi) - Ra_S(\delta^2 + \chi)\}] \tag{4.49}$$

The required root of equation Eq. (4.46) is given by,

$$x = \frac{1}{2x_1}[-x_2 + \sqrt{x_2^2 - 4x_1x_3}] \tag{4.50}$$

If we let the radical term in Eq. (4.50) to vanish, we obtain the expression for the finite amplitude Rayleigh number Ra_T^f which characterizes the onset of finite amplitude steady motions. The finite amplitude Rayleigh number can be obtained in the form,

$$Ra_T^f = \frac{1}{2A_1}[-A_2 + \sqrt{A_2^2 - 4A_1A_3}] \tag{4.51}$$

where

$$A_1 = \frac{16\pi^4}{\delta^4 Ar^4} \tag{4.52}$$

$$A_2 = \frac{8\pi^2}{Ar^2}\left[(2\pi^2 - 1)(\frac{4\pi^2}{Le} + \chi)(\frac{\delta^2}{Le} + 1) - \frac{4\pi^4 Ras}{\delta^4 Ar^2 Le} - 4\pi^2\delta^2\right] \tag{4.53}$$

$$A_3 = \left[\delta^2(\frac{4\pi^2}{Le} + \chi)(\frac{\delta^2}{Le} + 1) + \frac{4\pi^4 Ras}{\delta^2 Ar^2 Le} + 4\pi^2\delta^4\right]^2 - 16\pi^2\delta^2(\frac{4\pi^2}{Le} + 1) \times [\delta^4(\frac{\delta^2}{Le} + 1) + \frac{\pi^2 Ra_S}{\delta^2 Ar^2}(\delta^2 + \chi)] \tag{4.54}$$

5 Heat and mass transport

In the study of convection in fluids, the quantification of heat and mass transfer is important. This is because the onset of convection, as Rayleigh number increased, is more readily detected by its effect on heat and mass transfer. In the basic state, heat and mass transport is by conduction only.

If $\bar{H}$ and $\bar{J}$ are the rate of heat and mass transport per unit area respectively, then

$$\bar{H} = -\kappa_T \left\langle \frac{\partial T_{Total}}{\partial z} \right\rangle_{z=0} \tag{5.55}$$

$$J = -\kappa_s \left\langle \frac{\partial S_{Total}}{\partial z} \right\rangle_{z=0} \tag{5.56}$$

where angular bracket stands for horizontal average and

$$T_{Total} = T_0 - \triangle T \frac{z}{d} + T(x,z,t) \tag{5.57}$$

$$S_{Total} = S_0 - \triangle S \frac{z}{d} + S(x,z,t) \tag{5.58}$$

Substituting Eqs.(4.35) and (4.36) in Eqs. (5.57) and (5.58) respectively and using the resultant equations in (5.55) and (5.56), we get

$$\bar{H} = \frac{\kappa_T \triangle T}{d}(1 - 2\pi B_{02}) \tag{5.59}$$

$$J = \frac{\kappa_s \triangle S}{d}(1 - 2\pi E_{02}) \tag{5.60}$$

The thermal and solute Nusselt numbers are given by

$$Nu = \frac{H}{\kappa_T \triangle T/d} = 1 - 2\pi B_{02} \tag{5.61}$$

$$Sh = \frac{J}{\kappa_s \triangle S/d} = (1 - 2\pi E_{02}) \tag{5.62}$$

Now substituting the values of B_{02} and E_{02} in terms of A_{11} from Eqs.(4.42) - (4.45) in Eqs. (5.61) and (5.62) respectively, we get

$$Nu = 1 + \frac{2\pi^2 x}{\delta^2(\pi^2 + Ar^2 x)} \tag{5.63}$$

$$\begin{aligned} Sh = 1 + 2 \big[& \{4\pi^2 Le^2 x(\chi Ar^2 + \delta^2 Ar^2 + x) \\ & + Le\chi Ar^2 x(\delta^2 + Le\chi) \} \\ & \div \{(\delta^2 Ar^2 + x)(Ar^2(\delta^2 + Le\chi) \\ & \times (4\pi^2 + Le\chi) + 4\pi^4 Le^2 x)\}\big] \end{aligned} \tag{5.64}$$

6 Result and Discussion

In this study, we have extended the work of Malashetty and Biradar [14] in finite fluid layer to examine the effect of aspect ratio, chemical reaction parameter, Lewis number and solute Rayleigh number on stationary and oscillatory convection and heat and mass transfer in the presence of fluid motion. The neutral stability curves in the Ra_T–Ar plane for the various parameters are depicted in Figs. 1(a)-(d). The linear stability criterion is expressed as it becomes stable below the critical Rayleigh number and becomes unstable above the critical Rayleigh number. The effect on critical Rayleigh number of aspect ratio Ar is observed in figures 1(a)-(d) by changing each parameter one by one and keeping others constant for stationary and oscillatory convection. The values of different parameters is taken $Le = 10, \chi = 2, Ra_S = 100, Pr = 0.71$. From these figures we observe that as Ar increases the oscillatory mode of convection is dominated by stationary mode of convection. The effect of Lewis number Le on the neutral stability curves are established in Fig. 1(a). From this figure we find that on increasing Lewis number Le, it decreases the critical Rayleigh number for oscillatory mode which indicates that effect of Lewis number is to destabilize the system. Fig. 1(b) shows the neutral stability curves for different values of the Damkohler number χ and for the fixed values of other parameters. We observe form this figure that increasing value of Damkohler number, it increases the critical Rayleigh number. This indicates that chemical reaction parameter stabilizes the system in the case of oscillatory mode. Malashetty and Biradar [14] investigated that the effect of chemical reaction parameter is either stabilizing or destabilizing. They observed that in an infinite horizontal porous layer the chemical reaction parameter has stabilizing effect but we observe that in viscous fluid saturated in finite fluid layer, the chemical reaction parameter has also stabilizing effect. Fig. 1(c) indicates the effect of the solute Rayleigh number Ra_S on the neutral stability curves for the fixed values of the other parameters. We observe that on increasing the value of solute Rayleigh number Ra_S, there is increase in the critical Rayleigh number for oscillatory mode. Therefore, solute Rayleigh number has stabilizing effect. From Fig. 1(a)-(d), we observe that there is no effect of Prandtl number Pr

on the neutral stability curves.

In Figs. 2(a)-(c) and 3(a)-(c), we display effect of various parameters on the rate of heat transfer i.e. Nusselt number(Nu) and mass transfer i.e. Sherwood number (Sh) for the fixed values of $Le = 10, Ra_S = 100, \chi = 2, Pr = 0.71$ respectively. These figures indicate that the rate of heat and mass transfer across is bounded. This limitation is due to the double Fourier series representation used for the stream function, temperature and concentration.

Figs. 2(a) and 3(a) show that increase in the Lewis number Le increase both heat and mass transfer. We display the effect of solute Rayleigh number on heat and mass transfer in Figs. 2(b) and 3(b) respectively. We find the similar result for solute Rayleigh number Ra_S as for Lewis number Le. We find from Fig. 2(c) that the Nusselt number increases with increase in chemical reaction parameter, while the Sherwood number decreases with the increase in chemical reaction parameter as shown in Fig. 3(c).

In Figs. 4, 5, 6, we draw streamlines, isotherms and isohalines respectively for the steady-state at the fixed values of $Pr = 0.71, Le = 10, Ra_S = 100, \chi = 2$. Figs. 4, 5, 6(a) are drawn at $Ra_T = Ra_{T,c}$, while Figs. 4, 5, 6(b) are drawn at $Ra_T = 10 \times Ra_{T,c}$. From Figs. 4(a),(b), we observe that on increasing the values of Rayleigh number, the magnitude of stream function increases which shows that convection occurs much faster at higher values of Ra_T. From Figs. 5(a),(b), we observe that isotherms are flat near to the boundaries and becomes contours in the middle while they become more flat on increasing the value of Ra_T. From Figs. 6(a),(b), we found similar results for isohalines as shown in Figs. 5(a),(b) for isotherms.

In Figs. 7, 8, 9, we draw streamlines, isotherms and isohalines respectively for the unsteady-state case at the fixed values of $Pr = 0.71, Le = 10, Ra_S = 100, \chi = 2$. From the Figs. 7(a)-(c), for ψ, the sense of motion in the subsequent cells is alternative identical and opposite to that of adjoining cells. We observe that streamlines contracts and their magnitude increases as time increases. In Figs. 8(a)-(c), we found that as temperature increases with time, isotherms change their nature from slow convection to faster convection. Also isotherms become more contour with increase in time. In case of isohalines, we found qualitatively similar results as observed for isotherms in Figs. 9(a)-(c).

7 Conclusion

In this article, we performed the effect of chemical reaction on double diffusive convection in a viscous fluid saturated in finite fluid layer. The problem has been solved analytically, linear and non-linear stability analysis has been done. The graphs for streamlines, isotherms and isohalines have been drawn for steady and unsteady cases. The following conclusions are drawn:

(i) From the neutral stability curves, it is found that the effect of increase in Le is to advance the onset of convection while the effect of Ra_S and χ is to sustain the stability of the system.

(ii)Critical value of Ra_T decreases with the increase in value of Le, while it increases with increase in χ, Ra_S.

(iii) The effect of increasing Le, Ra_S is to increase the rates of heat and mass transfers, whereas increase in value of χ increases the rate of heat transfer but decreases the rate of mass transfer.

(iv) The magnitude of stream functions increases with increase in Ra_T, isotherms and isohalines become more contour as the value of Ra_T increases.

(v) For unsteady case, magnitude of streamlines increseas with increase in time. Further isotherms and isohalines attain contour patterns with increase in time.

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