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Research Article

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A two-stage model for ranking DMUs using DEA/AHP

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Abstract

In this paper, we present a two-stage model for ranking of decision making units (DMUs) using interval analytic hierarchy process (AHP). Since the efficiency score of unity is assigned to the efficient units, we evaluate the efficiency of each DMU by basic DEA models and calculate the weights of the criteria using proposed model. In the first stage, the proposed model evaluates decision making units, and in the second stage it establishes pair-wise comparison matrix then ranks all DMUs by AHP. Finally, a numerical example and an application of the proposed model in 23 universities are provided.

Keywords : Data envelopment analysis; Analytic hierarchy process; Ranking.

1 Introduction

DAta envelopment analysis (DEA) is a non-parametric method for evaluating relative efficiency in decision making units (DMUs), which was represented by Charnes et al. [3]; the presented model by them is usually denoted as CCR model. It was further extended by Banker et al. [2]. Since, more than one efficient DMU is evaluated in DEA, so ranking of efficient DMUs is very important question and many DEA researchers and practitioners have studid about it. First, Andersen and Petersen were introduced AP model for ranking efficient DMUs [1]. The AP model is not always feasible and in some cases it is unstable. Whereas, there was the necessity of a strong technic for ranking. Analytic hierarchy process (AHP) was introduced by Saaty [5] for the first time. This method has used in economic and social issued and management decision making. In general, AHP uses pair-wise comparisons between criteria and alternatives, decision maker judgments, to rank the alternatives overall. AHP developed by many researchers (e.g., Sinuany-Stern et al. [7], Saaty [6]). Sinuany-Stern et al. [7] has presented a method for ranking of decision making units by using AHP and DEA models. In theirs model, other units are not involved in comparison with pair-wise decision making units, while in our propose model the evaluation of each pair-wise decision making units is provided from comparing with the other decision making units performance. This paper is organized as follows: In Section 2, data envelopment analysis is discussed. In Section 3, the two-stage ranking model AHP/DEA is reviewed. The proposed approach with new production possibility set is presented in Section 4. A numerical example is used to comparing the proposed model with Sinuany-Sterns model in Section 5. An application is used to illustrate the proposed models in section 6. Finally, Section 7 includes conclusions.

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2 Overview of DEA

Production technology is a transformation that take the inputs x into outputs y. Theoretically, production possibility set can be shown as following:

$$T = \{(x, y)|\text{input (x) can produce the output}(y)\}$$

Assume n decision making units (DMUs) that are under evaluation with m inputs and s outputs Such as $x_j = (x_{1j}, ..., x_{mj})$ and $y_j = (y_{1j}, ..., y_{sj})$ respectively and assume production possibility set, T^{CCR}, which was defined by Charns et al. [3] as following:

$$T^{CCR} = \{(x, y)|x \geq \sum_{j=1}^{n} \lambda_j x_j$$

$$, y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0, j = 1, 2, \cdots, n\}.$$

Data envelopment analysis (DEA) model to evaluating DMUs in T^{CCR} is as following:

$$\min \quad \theta_o - [\sum s_i^- + s_r^+]$$

s.t.

$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta_o x_{io} \ i = 1, 2, \cdots, m,$

$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad r = 1, 2, \cdots, s,$

$\lambda_j \geq 0, \quad j = 1, 2, \cdots, n,$

s˙iˆ- $\geq 0, \quad i = 1, 2, \cdots, m,$

s˙rˆ+ $\geq 0, \quad r = 1, 2, \cdots, s.$

(2.1)

where $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)^t$ and ε is a infinitesimal constant (usually 10^{-8}). $s_i^-, \{i = 1, 2, \cdots, m\}$, and $s_r^+, \{r = 1, 2, \cdots, s\}$, are slack variables expressing the difference between virtual inputs/outputs and appropriate inputs/outputs of the evaluated decision making unit (DMUo). The above-mentioned model is known as CCR model the optimum answer of the CCR model is related to the efficiency rate of DMUo . If$\theta_o^* = 1$ then this decision making unit is efficient. The dual model is as following:

$$\max \quad \sum_{r=1}^{s} u_r y_{ro}$$

s.t.

$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq o \ j = 1, 2, \cdots, n,$

$\sum_{j=1}^{m} v_i x_{io} = 1,$

v˙i$\geq \varepsilon, \quad i = 1, 2, \cdots, m,$

v˙r $\geq \varepsilon, \quad r = 1, 2, \cdots, s.$

(2.2)

where u_r and v_i are dual variables of model. They can be interpreted as normalized shadow price. Therefore, these input and output prices of DMU under evaluation which is shown to be the most optimal possible price.

3 Sinuany-Stern's model

AHP is one of the most efficient analytical hierarchy process decision-making techniques which were first introduced by Tomas in 1980[5]. This technique is based on paired comparison which allows managers to study various scenarios. In the science of decision, in which choosing one solution from the solutions at hand or prioritizing those solutions are considered, methods of MCDM (Multi Criteria Decision Making) especially AHP were rendered in recent years. Hybrid models AHP/DEA integrate two well-known models, DEA and AHP. Now, we discussed the AHP/DEA ranking model, that is presented by Sinuany-Stern et al. [7] for ranking decision making units. In its first stage, it compares and evaluates decision making unit pairs to each other by data envelopment analysis, and in its second stage pair-wise comparison matrix is established by use of done pair-wise comparison, and the decision making units are ranked by making use of analytic hierarchy process.

3.1 The first stage

For each pair unit$A$ and$B$, the DEA model is considered as following:

E˙AA=max $\sum_{r=1}^{s} u_r y_{rA}$

s.t.

$\sum_{r=1}^{s} u_r y_{rB} - \sum_{i=1}^{m} v_i x_{iB} \leq o,$

$\sum_{r=1}^{s} u_r y_{rA} \leq 1$

$\sum_{i=1}^{m} v_i x_{iA} = 1,$

v˙i $\geq \varepsilon, \quad i = 1, 2, \cdots, m,$

u˙r $\geq \varepsilon, \quad r = 1, 2, \cdots, s.$

(3.3)

In order to cross evaluate unit B, using the optimal weights of unit A, there is:

$$E_{BA} = \frac{\sum_{r=1}^{s} u_r y_{rB}}{\sum_{i=1}^{m} v_i x_{iB}} \tag{3.4}$$

it can be seen $E_{AB} = E_{AA} and E_{BA} = E_{BB}$. Thus, the evaluation of A over$B$ isE_{AA}/E_{BB}. For cross evaluation of unit B by using optimum weights of unit A, it is possible for the time that$v_i \geq \varepsilon$ and$u_r \geq \varepsilon$ bear more than one optimal solution for the optimal weight. Therefore, by carrying the optimal solution for unit$A$ in order to guarantee the most optimal cross evaluation of unit B, we solve the problem according to proposed model by Oral et al. [4], as following:

E˙BA=max $\sum_{r=1}^{s} u_r y_{rB}$

s.t.

$\sum_{r=1}^{s} u_r y_{rA} - E_{AA} \sum_{i=1}^{m} v_i x_{iA} = o,$

$\sum_{r=1}^{s} u_r y_{rB} \leq 1$

$\sum_{i=1}^{m} v_i x_{iB} = 1,$

v˙i $\geq \varepsilon, \quad i = 1, 2, \cdots, m,$

u˙r $\geq \varepsilon, \quad r = 1, 2, \cdots, s.$

(3.5)

Actuality, E_{BA} is the optimal cross evaluation of unit B, we obtain E_{BB} and E_{AB} symmetrically.

3.2 The second stage

In analytic hierarchy process elements are compared pair by pair and are formed, and the local priority is computed with this matrix . In this stage we make pair-wise compari- son matrix, that is needed for AHP from the DEA pair results, which was discussed in the previous stage, therefore for every decision making unit pair:

$$a_{jk} = \frac{E_{jj} + E_{jk}}{E_{kk} + E_{kj}} \tag{3.6}$$

Note that in AHP, the pair-wise comparison matrix A on the diagonal has a rank of 1, and the elements a_{jk} reflect the evaluation of unit j over unit k. If $a_{jk} \leq 1$, it means that unit j is evaluated less than unit k. Obviously:

$$a_{jk} = \frac{1}{a_{kj}}$$

This matrix has not been evaluated subjectively by a decision maker. The objective evaluations are calculated from the DEA pair-wise runs, as every unit receives the most favorable value relative to any other unit. By using this pair-wise comparison matrix A, we can get the overall priorities as vector w, that its every component introduces the introducer of overall priority corresponding decision making unit. Ranking decision making units is based on these overall priorities. For deriving priorities, the following least squares method is used. In this method w_i and w_j are calculated such that the difference between $\frac{w_i}{w_j}$ and a_{ij} be minimizes as follows:

min $z = \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij} w_j - w_i)^2$

s.t.

$\sum_{i=1}^{n} w_i = 1$

w˙i $\geq o, \quad i = 1, 2, \cdots, n.$

(3.7)

For solving model (*3.2*), we consider its Lagrangian equation as follows

$$\lambda = \sum_{i=1}^{n} \sum_{i=1}^{n} (a_{ij} w_j - w_i)^2 + 2\lambda(\sum_{i=1}^{n} w_i - 1). \tag{3.8}$$

If we derivate equation(3.8), we will have:

$$\sum_{i=1}^{n} (a_{ik} w_k - w_i) a_{ik} - \sum_{j=1}^{n} (a_{kj} w_j - w_k) + \lambda = 0. \tag{3.9}$$

In the equation (3.9)there is $(n + 1)$non-homogeneous linear equation and$(n + 1)$ unknown variables. By solving this linear equation w_j is obtained.

4 Proposed AHP/DEA ranking model

In this section, we propose a two-stage method for ranking of decision making units, this model use model's Sinuany-Stern et al. [7]. Dissimilarly, the Sinuany-Stern's model obtain pair-wise comparison of Decision Making Units without using other decision making units , while in our propose model the evaluation of each decision making unit pair is obtained by comparing to the function of all the decision making units.

4.1 The first stage:Pair-wise comparison by DEA model

by assuming production possibility set, $T^{p,q}$, we mean a set as following:

$$T^{p,q} = \{(x, y) | x \geq \sum_{j=1, j \neq p,q}^{n} \lambda_j x_j,$$

Table 1: Futures of the decision making units in the example

DMUs	x˙1	x˙2	y
A	3	1	1
B	2	2	1
C	2	3	1
D	1	5	1
E	2	5	1
F	3	4	1
G	5	1	1

Table 2: Results of ranking by new method.

DMUs	ˆ*	w	Ranking
A	1	0.17	2
B	1	0.15	3
C	0.89	0.12	5
D	1	0.23	1
E	0.73	0.10	6
F	0.62	0.09	7
G	1	0.14	4

Table 3: Results of ranking by old method.

DMUs	θ^*	w
A	1	1
B	1	0.95
C	0.89	0.96
D	1	1
E	0.73	1
F	0.62	1.33
G	1	1

Table 4: Comparison of the results of the new method with AP and MAJ.

DMUs	θ^*	EFF	RANK	EFF	RANK	w	Ranking
	-CCR	-AP	-AP	-MAJ	- MAJ	-new method	-new method
A	1	1.3	2	1.1	2	0.17	2
B	1	1.17	3	1.07	3	0.15	3
C	0.89	0.89	-	0.95	-	0.12	5
D	1	2.00	1	1.2	1	0.23	1
E	0.73	0.73	-	0.85	-	0.10	6
F	0.62	0.62	-	0.75	-	0.09	7
G	1	1	4	1	4	0.14	4

Consider the following Linear Programming model:

$$y \leq \sum_{j=1, j\neq p,q}^{n} \lambda_j y_j, \lambda_j \geq 0, j = 1, \cdots, n, j \neq p, q\}$$

Table 5: All inputs and outputs of 23 universities.

DMUs	I1	I2	I3	I4	I5	O1	O2
1	593	8	2.75	16.731	17129300	285	0.8848
2	741	8	2	18.999	8903705	95	0.8597
3	600	7	2.75	19.437	15864760	307	0.9226
4	593	8	2.75	19.326	14802089	260	0.8928
5	746	7	2	20.125	8398300	154	0.812
6	992	9	2.75	21.821	19330020	254	0.8642
7	775	8	2.75	13.333	17182320	292	0.9109
8	1852	14	3.25	21.696	30126900	473	0.8632
9	625	5	2	16.285	7638220	106	0.8898
10	673	6	2	16.789	8659940	148	0.8668
11	423	6	2	13.304	10799980	151	0.9435
12	1292	18	3.25	18.333	47102720	782	0.9571
13	1300	8	2.75	17.73	17451040	288	0.899
14	582	8	2.75	19.178	15850628	260	0.9054
15	620	8	2	16.056	7938560	124	0.8744
16	1256	10	2.75	21.516	23034560	378	0.8465
17	765	10	2.75	19.145	15692740	303	0.8945
18	842	7	2.25	16.927	8029240	153	0.9074
19	1011	4	2.25	17.692	7702609	57	0.8764
20	1128	9	2.75	21.927	22143650	357	0.9028
21	3456	18	3.5	20.217	24892550	393	0.9195
22	1008	3	2.25	10.213	7405200	36	0.8611
23	910	4	2.25	12.941	8839280	72	0.7735

E(p,Tˆp,q)=min θ

s.t:

$\sum_{j=1,j\neq p,q}^{n} \lambda_j x_j - \theta x_p \leq 0$

$\sum_{j=1,jy\neq p,q}^{n} \lambda_j y_j \geq y_p$

$\lambda_j \geq 0, j = 1, \cdots, n, j \neq p, q.$
(4.10)

In the method $E(p, T_{p,q})$ is a relative evaluation of decision making unit (x_p, y_p) to production possibility set (i.e. $T^{p,q}$). Similarly, we can define $E(q, T^{p,q})$ model, as follows

E(p,Tˆp,q)=min θ

s.t:

$\sum_{j=1,j\neq p,q}^{n} \lambda_j x_j - \theta x_q \leq 0$

$\sum_{j=1,j\neq p,q}^{n} \lambda_j y_j \geq y_q$

$\lambda_j \geq 0, j = 1, \cdots, n, j \neq p, q.$
(4.11)

In fact we have evaluated DMU_p andDMU_q to the production possibility set,$T^{p,q}$, which is obtained from subtraction of the decision making units p and q. ,

4.2 The second stage: Ranking by AHP model

We define the pairwise comparison matrix AHP from the results of DEA for each decision making unit p to

Table 6: The results of ranking the efficient DMUs by the different ranking Methods.

Models	D3	D4	D7	D9	D10	D11	D12	D15	D18	D22
AP model	4	8	7	5	9	2	1	10	6	3
MAJ model	4	6	5	7	9	3	1	10	8	2
Reformable MAJ model	4	8	6	5	9	3	1	10	7	2
Auxiliary variable models	4	8	7	5	9	2	1	10	6	3
1 norm model	2	4	3	8	7	10	1	6	5	9
Infinite norm model	4	7	5	9	8	2	1	10	6	3
model CSW	3	5	8*	11*	6*	2	1	9*	4	22*
Gradient vector model	4	8	7	5	9	3	1	10	6	2
Ratio preferable model	13*	15*	4*	5*	8*	3	2	7*	6*	1
SBM Model	4	8	6	5	9	3	1	10	7	2
Monte carlo model	5	3	1	6	10	2	4	9	8	7
proposed Model	4	15	7	5	8	3	1	9	6	2

q.

$$A = [a_{p,q}]_{n\times n}$$
$$a_{p,q} = \frac{E(p, T^{p,q})}{E(q, T^{p,q})} \quad p, q = 1, 2, \cdots, n$$

We take the evaluation given to unit p by the model of unit$E(p, T^{p,q})$ and divide it by the evaluation given to unit q by the model of unit $E(q, T^{p,q})$.

We have:

$$a_{p,q} = \frac{1}{a_{q,p}}, p, q = 1, 2, \cdots, n$$

Matrix A is evaluated by DEA model in pairs, as each decision making unit attend the most optimum evaluation in comparison with other units. Based on pairwise comparison matrix A by computing vector w_j , is indicator of emphasized ratio to the unit$j$. Therefore, we rank decision making units by these priorities.

5 Numerical example

Consider seven decision making units by two inputs for producing a normalized output in the first level as given in Table 1. Respect to the propose method , Linear Programming model for evaluating DMU_A in set $T^{A,B}$, is as following:

E(A,Tˆ A,B)=min θ

s.t:

$$2\lambda_c + 1\lambda_D + 2\lambda_E + 3\lambda_F + 5\lambda_G - 3\theta \leq 0$$

$$3\lambda_c + 5\lambda_D + 5\lambda_E + 4\lambda_F + 1\lambda_G - 1\theta \leq 0$$

$$1\lambda_c + 1\lambda_D + 1\lambda_E + 1\lambda_F + 1\lambda_G \geq 1$$

$$\lambda_c, \lambda_D, \lambda_E, \lambda_F, \lambda_G \geq 0(5.12)$$

So Linear Programming model for evaluating DMU_B in set $T^{A,B}$, is as following:

E(B,Tˆ A,B)=min θ

Table 7: The results of ranking by proposed Methods.

DMUs	*The relative weight (w)*	*Ranking with the proposed method*
1	0.039767	13
2	0.038504	16
3	0.047898	4
4	0.038656	15
5	0.040886	11
6	0.034423	23
7	0.044412	7
8	0.03674	20
9	0.046077	5
10	0.041793	8
11	0.058816	3
12	0.078348	1
13	0.038192	19
14	0.038287	17
15	0.041785	9
16	0.038254	18
17	0.041035	10
18	0.045284	6
19	0.040367	12
20	0.039421	14
21	0.036121	21
22	0.059044	2
23	0.035887	22

s.t:

$2\lambda_c + 1\lambda_D + 2\lambda_E + 3\lambda_F + 5\lambda_G - 2\theta \leq 0$

$3\lambda_c + 5\lambda_D + 5\lambda_E + 4\lambda_F + 1\lambda_G - 2\theta \leq 0$

$1\lambda_c + 1\lambda_D + 1\lambda_E + 1\lambda_F + 1\lambda_G \geq 1$
$\lambda_c, \lambda_D, \lambda_E, \lambda_F, \lambda_G \geq 0$(5.13)

$E(A, T^{A,B}) = 1.30$ and $E(B, T^{A,B}) = 1.44$ are the optimal answers for two Linear Programming models, then:

$$a_{A,B} = \frac{E(A, T^{A,B})}{E(B, T^{A,B})} = \frac{(1/44)}{(1/30)} = 1/11$$

(5.14)

$$a_{B,A} = \frac{E(B, T^{A,B})}{E(A, T^{A,B})} = \frac{(1/30)}{(1/44)} = 0/90 \qquad (5.15)$$

It means DMU_A is better than decision making unit B on scale of$a_{AB} = 1.11$; in other words, the performance of DMU_A to DMU_B is equal to $a_{BA} = 0.9$. By comparing the decision making unit pairs, their pair-wise comparison matrix is formed. So: $A_1 =$

$$\begin{pmatrix} 1 & 1.11 & 1.51 & 0.66 & 1.85 & 2.16 & 1 \\ 0.90 & 1 & 1.17 & 0.59 & 1.51 & 1.66 & 1.17 \\ 0.66 & 0.59 & 1 & 0.50 & 1.22 & 1.44 & 0.89 \\ 1.51 & 1.17 & 2 & 1 & 2 & 2.56 & 2 \\ 0.54 & 0.66 & 0.82 & 0.50 & 1 & 1.19 & 0.73 \\ 0.46 & 0.60 & 0.69 & 0.39 & 0.84 & 1 & 0.61 \\ 1 & 0.85 & 1.12 & 0.50 & 1.37 & 1.64 & 1 \end{pmatrix}$$

The vector w is obtained by least squares method and its results are explained in Table 2. As it is seen here, efficient units are ranked in higher level than inefficient units. The proposed method presents an overall ranking for efficient decision making units.

The rank of this decision making units by the proposed method are explained in Table 3. The rank of unit A, D, E and$G$ has been one, because these units in relation to the other individual units are efficient in pair-wise comparison. Therefore, it is impossible to rank them in this special case, that it is one of the problems of proposed method in(*3.2*). On the one hand, unit F despite has the most optimal overall priority and unit B despite has the least overall priority. So in the ranking, unit F will be located in the higher level that B. The other problem of proposed method in (*3.2*), is that by change εs, different ranking are obtained.

In Table 4 we compare the results of the proposed model with AP andMAJ models. Efficient DMUs have same ranking in all three methods and Due to the weight vector that was resulted from paired comparison matrix, other units have also been ranked.

6 Application

In this section, an application is used to compare these models. Finally, we analyzed the results. Consider 23 universities. each universities has five inputs: the amount of Educational environment , The number of school classes ,the number of Employee ,the number of teacher, The total budget and two outputs: the number of students, amount of Quality of education as output. All inputs and outputs are shown in Table 5.
Input 2: The number of school classes.
Entry 3 the number of Employee.
Entry 4: the number of teacher.
Entry 5: The total budget.
Output 1: The number of students.
Output 2: amount of Quality of education.

These units evaluated by CCR model, $DMU3, 4, 7, 9, 10, 11, 12, 15, 18, 22$ are efficient. Efficient DMUs are ranked. The results of ranking the efficient DMUs by the different ranking Methods are showed in Table 6.

Sign (*) means DMU is inefficient. However, the weight of decision maker are used in proposed model, the results of ranking with proposed model are shown in Table 7.

As it was considered, in CSW models, some DMUs are inefficient, despite Other models were evaluated efficient then they are not ranked. DMU_{12} has best rank in proposed model and in most the ranking's models. Other DMUs have same rank with all of the models. As you can see, all DMUs was ranked with the weight vector that was obtained from paired comparison matrix. This method is the incorporation of AHP and DEA and rank all of efficient and inefficient DMUs.

7 Conclusion

We have presented a two-stage method for ranking of decision making units with incorporating DEA and AHP. In the first stage All DMUs are evaluated and compared Pairwise and in the second stage by using analytic hierarchy process, we have presented a full ranking for all DMUs. In contrast to DEA and AHP models, the advantage of the AHP/DEA approach is that it has not limitations of both DEA and AHP because of using the incorporation of AHP/DEA model . The another advantage of this method is that, the pairwise comparisons matrixes of AHP are obtained mathematically from the input/output data. Thus, the evaluation is not based on subjective judgments of a decision maker. on the one hand, since we are using of given inputs and outputs of DMUs, the utility theory non-axiomatic limitations of AHP are irrelevant. The Difficulty of Sinuany-Sterns method, was that by different s, different ranking are got; but, by considering, two evaluated DMUs are removed of production possibility set in the propose model, we can not be used the proposed model for ranking the two decision making units. For future researches Future research can be study on finding a relationship between obtaining weights of DEA and AHP, and using Topsis method in ranking.

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