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Research Article

Simulating Exchange Rate Volatility in Iran Using Stochastic Differential Equations

P. Fakhraiepour *, P. Nabati †‡, R. Taghizadeh §

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Abstract

The main purpose of this paper is to analyze the exchange rate volatility in Iran in the time period between 2011/11/27 and 2017/02/25 on a daily basis. As a tradable asset and as an important and effective economic variable, exchange rate plays a decisive role in the economy of a country. In a successful economic management, the modeling and prediction of the exchange rate volatility is essential for economic policies. Therefore, modeling and forecasting the changes in exchange rates for economic policies is vital. Foreign currency has the particular property of stochastic volatility, which can be modeled as a stochastic differential equation. In order to provide the best model, first, we studied the effectiveness of different stochastic models, drew upon the daily price of the exchange rate, and investigated the performance of these models. Finally, the best model was achieved by taking into account the numerical simulation and the mean square error, Akaikes (AIC), Schwarzs Bayesian (SBIC), and the Hannan-Quinn (HQIC) criteria.

Keywords : Stochastic differential equation; Geometric Brownian motion; Volatility of exchange rate; White noise.

1 Introduction

Foreign exchange is a tradable asset and as a very important and influential macroeconomic variable in different domestic sectors of the country, exchange rate plays a decisive role in the economic management policy in a successful management and planning. Forecasting and simulating the volatility are the key elements in financial engineering. Therefore, modeling and forecasting exchange rate volatility to implement economic policies has considerable importance. Financial science is one of the branches of humanities which mathematical models have a significant impact on. Since many financial quantities are random variables, their variations cannot be fully explained by nonrandom variables. Given this feature, stochastic differential equations are considered a branch of mathematics in the field of finance for modeling and forecasting the financial volatility. These models were first considered by Black-Scholes and Merton (1973) with the issue of stock price modeling. Their model appeared in the form of a stochastic differential equation called geometric Brownian motion in economic discussions [3], [7].The geometric Brownian motion model has a weakness in modeling such issues [12], because several empirical studies

*Department of Science, Urmia University of Technology, Urmia, Iran.
†Corresponding author. p.nabati@uut.ac.ir, Tel: +98(44)31980317.
‡Department of Science, Urmia University of Technology, Urmia, Iran.
§Department of Industrial Engineering, Urmia University of Technology, Urmia, Iran.

have shown that the prices of some commodities are not exactly log-normally distributed. Therefore, different models of stochastic differential equations such as nonlinear stochastic differential equations, mean reversion stochastic differential equations, differential equation with stochastic volatility, etc., were created which can precisely simulate the exchange rate volatility. Garman and Kohlhagen (1983) were the first to derive the FX European option formula by using the Black-Scholes approach and taking into account the domestic foreign interest rate. They developed an exact model based on the assumption on the log-normality of the underlying asset with constant volatility [7].This article presents different models of stochastic differential equations for modeling exchange rate volatility in Iran and the comparison between these models is based on the mean square error criteria. Finally, using information criteria are Akaikes information criterion (AIC), Schwarzs Bayesian information criterion (SBIC), and the Hannan-Quinn criterion (HQIC) are selected. As a result, this article is categorized as follows. The second part deals with the definition of differential equations in general and in financial markets, especially the foreign exchange market. In the third section, we examine various models of the available stochastic differential equations for the exchange rate. The fourth section is dedicated to data analysis. Finally, the last section compares the proposed models and presents the best model according to the proposed criteria.

2 Mathematical modeling

In general, the definition of the stochastic differential equation is as follows:

$$dx_t = f(x_t, t)dt + g(x_t, t)dB_t \qquad (2.1)$$

where $f(x_t, t)$ is the drift term, $g(x_t, t)$ is the diffusion term according to time t and the random process B_t are a standard wiener process [13]. Equation (2.1) is the basic form of stochastic differential equation. This equation is used in the field of economics, individually in modeling exchange rate which would be explained in more details.

2.1 Stochastic differential equations in financial markets

We investigate the definition of the stochastic differential equations in the financial markets. As a tradable asset, exchange rate volatility has risk-free and risky changes. Therefore, the modeling of the price behavior of this asset is started as risk free asset, and by adding a risk element to the model, a stochastic differential equation is obtained. Suppose the exchange rate volatility relative to the time is a certain function of the price value. So if $S(t)$ is the exchange price at time t then

$$\frac{dS(t)}{dt} = a(t)S(t) \qquad S(0) = S_0 \qquad (2.2)$$

In which $a(t)$ is a nonrandom coefficient at time t .
By adding the noise term to the $a(t)$ we have

$$a(t) = r(t) + noise$$

Where $a(t)$ is divided into two parts that are constant and random. Therefore

$$dS(t) = r(t)S(t)dt + noise \times S(t)dt \qquad (2.3)$$

Let us consider $noise \times dt = dB_t$, where B_t is a standard Brownian motion. So

$$dS_t = f(S_t, t)dt + g(S_t, t)dB_t \qquad (2.4)$$

The term $f(S_t, t)$ is the drift and $g(S_t, t)$ is the diffusion coefficient. Two steps must be taken to simulate the random differential equations [9] ,[2].
(i)Estimation of the unknown parameters of the equation.
(ii)Find the solution path of the equation.
Each of these cases is explained in the next sections. Since most of the stochastic differential equations do not have the analytical solution, the numerical methods have been considered by many researchers. The simplest time discrete approximation of an Ito process is the Euler Maruyama approximation. We shall consider an Ito process $X = \{X_t, t_0 \le t \le T\}$ satisfying the scalar stochastic differential equation

$$dX = f(t, X_t)dt + g(t, X_t)dB_t \ \ ; X_{t_0} = x_0 \quad (2.5)$$

For a given discretization $t_0 < t_1 < ... < t_N = T$ of the time interval $[to, T]$, an Euler approximation is a continuous time stochastic process

$Y = \{Y_t, t_0 \leq T\}$ satisfying the iterative scheme which is considered as an approximation of X.

$$Y_{n+1} = Y_n + f(t_n, Y_n)(t_{n+1} - t_n) + g(t_n, Y_n)(B_{t_{n+1}} - B_{t_n}) \qquad (2.6)$$

for $n = 0, 1, 2, ..., N-1$ with initial value $Y_0 = x_0$ where we have written $Y_n = Y(t_n)$ for the value of the approximation at the discretization time t_n. We shall also write $\triangle_n = t_{n+1} - t_n$ and $\delta = max$ n $\triangle_n$ Of course, $\triangle$ is generally considered the same. that's mean

$$\delta = \triangle_n \equiv \frac{T - t_0}{N} \quad , t_n = t_0 + n\delta$$

for some integer N large enough so that $\delta \in (0, 1)$. So we will have

$$Y_{n+1} = Y_n + a(t_n, Y_n) \triangle_n + b(t_n, Y_n) \triangle B_n$$

where

$$\triangle B_n \sim N(0, \triangle_n)$$

3 Stochastic models

Table 1 presents some different stochastic differential equations. Using these models, we describe the volatility of exchange rate. Model 1 is a geometric Brownian motion with expected growth rate μ and standard deviation σ, one of the simplest models in financial markets, this model assumes that the percentage of expected changes in prices and the percentage of fluctuations in prices is constant [4]. Models 2 and 3 are considered by Zhong et al for modeling the exchange rate of China [14] and model 4 presented by Farnoosh et al in the modeling of the OPEC oil price [6]. Models 5, 6 and 7 are used to model oil prices and future prices [1]. Model 9 is considered for the dynamics of oil price volatility.

4 Data analysis method

In this section, we set out to examine the proposed models and compare them with each other. It should be noted that the selected financial variable is the foreign exchange market of Iran for the time period 2011/11/27 to 2017/02/25, which is obtained from the Central Bank of Iran. These historical observations are on a daily basis. The unknown parameters were estimated based on the given data. Each of the steps is as follows.

4.1 Parameter estimation

In econometrics and statistics, the generalized method of moments (GMM) is a generic method for estimating parameters in statistical models. It is commonly applied in the context of semi-parametric models, where the parameter of interest is finite-dimensional, whereas the full shape of the distribution function of the data may not be known, and therefore maximum likelihood estimation is not applicable. The method requires that a certain number of moment conditions be specified for the model. These moment conditions are functions of the model parameters and the data, such that their expectation is zero at the true values of the parameters. The GMM method then minimizes a certain norm of the sample averages of the moment conditions [8]. The GMM estimators are known to be consistent, asymptotically normal, and efficient in the class of all estimators that do not use any extra information aside from that contained in the moment conditions. The GMM of Hansen (1982) is used in this article to estimate the parameters of the continuous-time models in Table 3 by using the corresponding discrete-time econometric specification:

$$P_{n+1} - P_n = f(t_n, P_n) + \varepsilon_{n+1}, \qquad (4.7)$$

$$E[\varepsilon_{n+1}] = 0, \qquad (4.8)$$

$$E[{\varepsilon_{n+1}}^2] = g(t_n, P_n)^2 \triangle t \qquad (4.9)$$

We let θ be the parameter vector with elements μ , κ and σ .
We used 1266 normalize data in order to estimate the parameters in proposed models. Operation estimating parameters μ , σ and κ has been done by the generalized method of moment (GMM). The estimated results are shown in Table 4, which is used to perform the estimation of the Eviews software [8].

4.2 Path simulation

In general, the solution of stochastic differential equation is obtained from two analytical and numerical methods. We used the Euler-Maruyama (EM) numerical method [13]. Using the EM scheme and MATLAB programming with mean

Table 1: Stochastic models

Models	
model 1	$dS_t = \mu S_t dt + \sigma S_t dB_t$
model 2	$d\ln S_t = \mu dt + \sigma dB_t$
model 3	$d\ln S_t = \kappa(\mu - \ln S_t)dt + \sigma dB_t$
model 4	$dS_t = \kappa S_t(\mu - S_t)dt + \sigma\sqrt{S_t}dB_t$
model 5	$dS_t = \mu S_t dt + \sigma S_t^{\frac{3}{4}} dB_t$
model 6	$dS_t = \mu\sqrt{S_t}dt + \sigma S_t^{\frac{3}{4}} dB_t$
model 7	$dS_t = (\mu\sqrt{S_t} + \kappa S_t)dt + \sigma S_t^{\frac{3}{4}} dB_t$
model 8	$dS_t = \kappa(\mu - S_t)dt + \sigma dB_t$
model 9	$dS_t = \kappa S_t(\mu - S_t)dt + \sigma S_t^{\frac{3}{2}} dB_t$

Table 2: Estimated parameters for models in Table 1

Models	μ	κ	σ
model 1	0.000545	-	-0.000169
model 2	3429414.007	0.004983	5.3570
model 3	10.4390974	0.005673	0.000153
model 4	-38.140547	-0.0005024	0.019430
model 5	0.000451	-	0.004146
model 6	0.081272	-	-0.003267
model 7	1.821448	-0.009860	0.000188
model 8	0.000628	-	0.000178
model 9	34128.63	2.41e-07	3.79e-07

Table 3: The amount of MSE values for models

Models	MSE
model 1	0.02329297
model 2	0.00488627
model 3	0.00449501
model 4	0.01882939
model 5	0.01420544
model 6	0.01974035
model 7	0.00454764
model 8	0.03417456
model 9	0.00453337

Table 4: Information criteria

Models	AIC	SBIC	HQIC
model 1	−3.7571	−3.7504	−3.7546
model 2	−5.3176	−5.3075	−5.3139
model 3	−5.4010	−5.3910	−5.3973
model 4	−3.9686	−3.9585	−3.9649
model 5	−4.2516	−4.2449	−4.2491
model 6	−3.9226	−3.9159	−3.9201
model 7	−5.3894	−5.3794	−5.3857
model 8	−3.3738	−3.3671	−3.3713
model 9	−5.3926	−5.3825	−5.3888

of 1000 iteration, we simulate and plot the path of proposed models. The comparisons between the exact and simulated values are presented in figure 1.

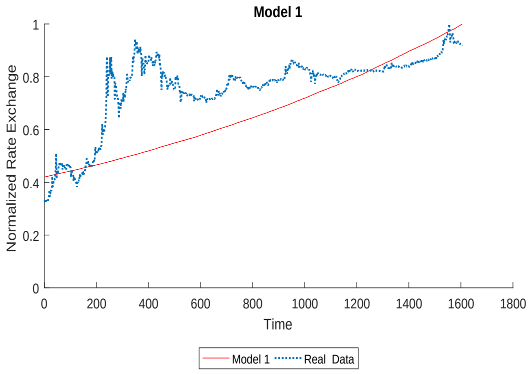

Figure 1: Compare numerical simulation of model 1 with real data

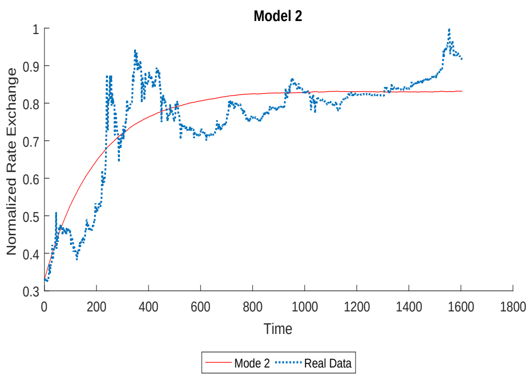

Figure 2: Compare numerical simulation of model 2 with real data

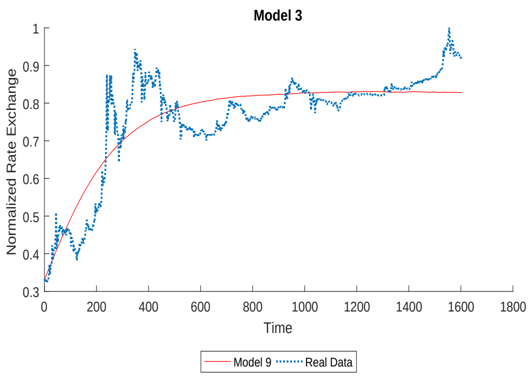

Figure 3: Compare numerical simulation of model 9 with real data

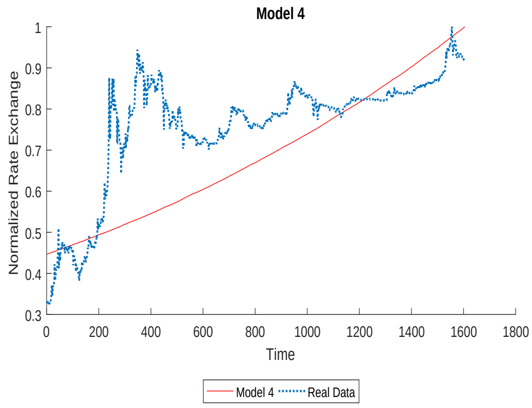

Figure 4: Compare numerical simulation of model 4 with real data

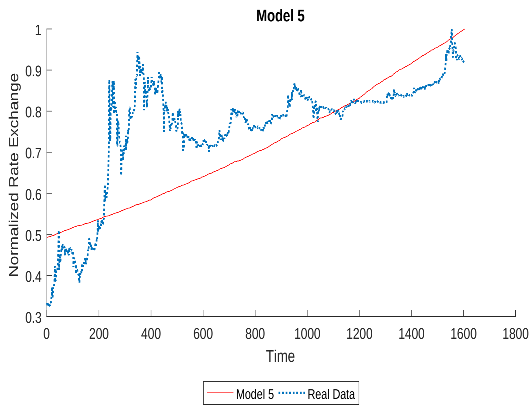

Figure 5: Compare numerical simulation of model 5 with real data

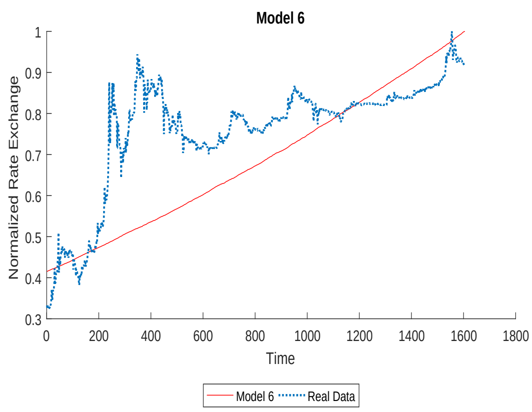

Figure 6: Compare numerical simulation of model 6 with real data

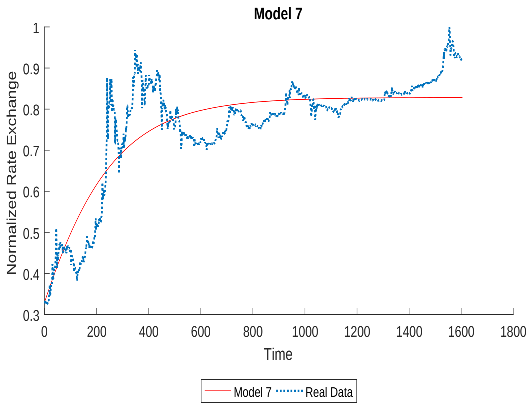

Figure 7: Compare numerical simulation of model 7 with real data

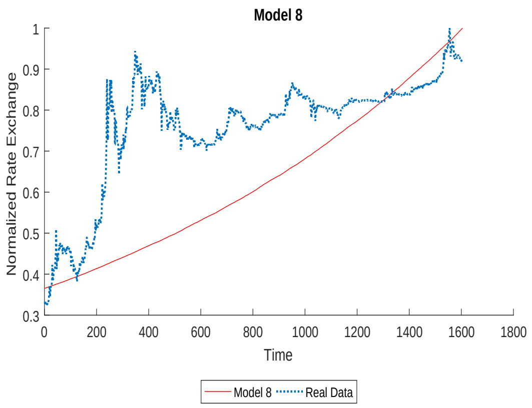

Figure 8: Compare numerical simulation of model 8 with real data

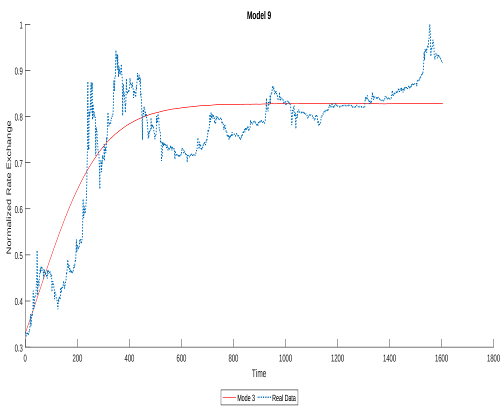

Figure 9: Compare numerical simulation of model 3 with real data

5 Comparision the models

After simulating the 9 models as presented in Table 1, we compared them with each other. The method of comparing the models with each other is considered to be the criterion MSE , and the results obtained for this criterion are presented in Table 4. Finally, the top model should be introduced. To select the best model, there are various criteria that are presented in the following three criteria of information.

In statistical modelling, the MSE can represent the difference between the actual observations and the observation values predicted by the model. In this context, it is used to determine the extent to which the model fits the data as well as whether removing some explanatory variables is possible without significantly harming the models predictive ability[10].

If $\hat{Y}$ is a vector of n predictions, and Y is the vector of observed values corresponding to the inputs to the function which generated the predictions, then the MSE of the predictor can be estimated by

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y)^2 \qquad (5.10)$$

Also, we used the metrics to choose the best model. Information criteria embody two factors: a term which is a function of the residual sum of squares (RSS or MSE), and some penalty for the loss of degrees of freedom as a result of adding extra parameters. Hence, adding a new variable or an additional lag to a model will have two competing effects on the information criteria: the residual sum of squares will fall but the value of the penalty term will increase.

The objective is to choose a number of parameters which minimise the value of the information criteria. Consequently, adding an extra term will reduce the value of the criteria only if the fall in the residual sum of squares is sufficient to more than outweigh the increased value of the penalty term. There are several different criteria, which vary according to how stiff the penalty term is. The three most popular information criteria are Akaikes (1974) information criterion (AIC), Schwarzs (1978) Bayesian information criterion (SBIC), and the HannanQuinn criterion (HQIC[5].

Algebraically, these are expressed, respectively, as

$$AIC = \ln(\delta^2) + \frac{2K}{T} \tag{5.11}$$

$$SBIC = \ln(\delta^2) + \frac{K}{T}\ln(T) \tag{5.12}$$

$$HQIC = \ln(\delta^2) + \frac{2K}{T}\ln\ln(T) \tag{5.13}$$

where δ^2 is he residual variance (also equivalent to the residual sum of squares divided by the number of observations, T), K is the total number of parameters estimated and T is the sample size. A model with lower information criterion is a superior model [5].

6 Conclusion

In this paper, various models for the simulation of exchange rate fluctuations in Iran are investigated and the EM approximation for numerical solution of these SDEs are presented. The parameters of models are estimated using the GMM estimator with the EM scheme and MATLAB programing with mean of 1000 iteration, we simulate and plot the path of proposed models. These simulations are compared to the proposed criteria. According to the results of the criteria as shown in Table 5, the third model that has a logarithmic form could be selected as the best model because of the lowest amount of criteria. In the future work we plan to find the option price for this model and simulate the exchange rate volatility with jump noise.

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Pouya Fakhraiepour. He received a B.Sc in Applied mathematics from University of Guilan, Rasht, Iran, (2015) and M.Sc from Urmia University of Technology, Urmia, Iran, (2017). Now he is working on financial mathematics.

Parisa Nabati is Assistant professor of Applied Mathematics in probability and statistics at Urmia University of Technology. Her research interests are in the areas of Applied Mathematics, Partial Differential Equations, Stochastic Differential Equations, Stochastic Calculus in Finance and Stochastic Processes. She has published research articles in international journals of Mathematics.

Rahim Taghizadeh is Assistant professor of Labor Economics and Industrial Relations at Urmia University of Technology. His research interests are in the areas of Labor Economics, SME Supports policies, Social Policy, Knowledge-Based Economy and Applied Econometrics. He has published research articles in international journals of Industrial Engineering.