Available online at http://sanad.iau.ir/journal/ijim/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol.15, No. 4, 2023 Article ID IJIM-1629, 13 pages
Research Article

Using Data Envelopment Analysis-Discriminant Analysis for predicting the congestion

S. Navidi[1], M. Rostamy-Malkhalifeh[*] [1], G. Tohidi[2], MH. Behzadi[3]
[1] Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran,
[2] Department of Mathematics, Islamic Azad University, Central Tehran Branch, Tehran, Iran
[3] Department of Statistics, Science and Research Branch, Islamic Azad University, Tehran, Iran.

Submission Date:2020/12/15, Revised Date:2021/06/05, Date of Acceptance:2021/10/17

Abstract
Two of the essential and important topics of scholars' research are congestion and classification in Data Envelopment Analysis. There are lots of papers that researchers represented their methods in these fields separately. Assume that there is a different method that can predict the congestion of Decision-Making Units. In this paper, we represented our method that predicts the congestion of DMUs instead of calculating their congestion. The advantage of this method is for the time that measured the congestion of DMUs but we need to add new DMUs and we do not want to calculate the congestion of all DMUs again. For this reason, we define available DMUs into three groups such as DMUs with strong congestion, DMUs with weak congestion, and DMUs with no congestion; then predict the congestion of new DMU. In the last section, we represent the numerical example of our purpose method. The result shows that the prediction of congestion is so correct.

Keywords: Data Envelopment Analysis, Discriminant Analysis, Congestion, Classification, Decision Making Unit

---

[*] Corresponding author: Email: mohsen_rostamy@yahoo.com

1. Introduction

Discriminant Analysis (DA) is one of the powerful tools for classification and predicting group membership of new Decision-Making Units (DMUs). Data Envelopment Analysis (DEA) is used to classifying the DMUs into efficient and inefficient groups by measuring their relative efficiency. DEA has lots of applied models such as BCC (Banker et al. [1]), CCR (Charnes et al. [2]), Additive, and so on.

In using DEA models, must defined inputs and outputs for DMUs. When an increase in one/more inputs causes reduce in one/more outputs, without improving any other inputs or outputs, congestion happens (Cooper et al. [3], Cooper et al. [4]). Färe and Grosskopf [5], [6] represented a method to recognize the input factors responsible for the congestion. Brockett et al. [7] believed that congestion of DMU depends on its inputs. Some of the researchers try to present the method by solving fewer models to measure the congestion (Cooper et al. [8], Noura et al. [9], Navidi et al. [10]). Congestion has usage in different cases, such as the economy, industry, energy, and so on. Sueyoshi et al. [11], [12] have perused in these cases. They expressed their method by explaining desirable and undesirable congestion. Khoveyni et al. [13] present their method for identifying congestion with negative data. Wang et al. [14] present a new definition of congestion. Sole-Ribalta et al. [15] researched on congestion of multiplex networks. The other interesting subject for the researcher in the congestion topic is energy efficiency (Zhou et al. [16], Zhou et al. [17], Hu et al. [18]).

In (1999) the additive model of DEA compared with the represented GP approach for DA. Sueyoshi believed that combining DA and DEA in the framework of GP is so useful and helps us to specify the group membership of new observation, more accurately. So, he presented his DEA-DA method by using GP (Sueyoshi [19]) then he completed his model (Sueyoshi [20]).

In the real situation, sometimes we have not accessed to exact data, there should be used imprecise data. Jahanshahloo et al. [21], Duarte Silva et al. [22, 23] and Angulo et al. [24] represented their method for interval data and Hosseinzadeh Lotfi et al. [25, 26], Khalili-Damghani et al. [27], Omrani et al. [28], Ji et al. [29], Ghasemi et al. [30] and Dotoli et al. [31] represented their method for fuzzy data.

Measuring the congestion of DMUs needs a complex method for calculateing it and this is time-consuming. In the case that we measure the congestion of all DMUs but in the middle of the process we have to add new DMUs, what should we do? Should we calculate the congestion of all DMUs again?

In this paper, we represent our method for the time when we have the congestion of some DMUs and we need to know the congestion of new DMU without calculating its congestion, so we predict its congestion.

One of the important usages of the DEA-DA method is classification. In this paper, we use DEA-DA methods to classify the DMUs according to their congestion into three groups. At first, we measure the congestion. Then by using discriminant analysis can divide DMUs into defined groups such as DMUs with strong congestion, DMUs with weak congestion, and DMUs with no congestion; then by using this information we predict the group membership of new DMUs.

The remainder of this paper is organized as follows: In section 2, we reviewed some related previous works. Our proposed method is presented in section 3. The empirical example of our purpose method is represented in section 4. The conclusion is represented in section 5.

2. Background

In this section, we reviewed some related previous works.

2.1. Congestion

Assume that, the number of DMUs, inputs, and outputs are n, m, and s. The vectors $x_j=(x_{1j},\dots,x_{mj})^T$ and $y_j=(y_{1j},\dots,y_{mj})^T$ are the input and output values of DMU_j, $j=1,\dots,n$, respectively.

Noura et al. [9] represented their method as follow:

First, solve the output-oriented BCC model (Banker et al. [1]):

$$Max\ \ \varphi+\varepsilon(\sum_{r=1}^{s}s_r^{+}+\sum_{i=1}^{m}s_i^{-}) \qquad (1)$$

$$s.t.$$

$$\sum_{j=1}^{n}\lambda_j x_{ij}+s_{io}^{-}=x_{io} \qquad ,\ i=1,2,\dots,m$$

$$\sum_{j=1}^{n}\lambda_j y_{rj}-s_{ro}^{+}=\varphi_o y_{ro} \quad ,\ r=1,2,\dots,s$$

$$\sum_{j=1}^{n}\lambda_j=1$$

$$0\le\lambda_j, s_{ro}^{+}, s_{io}^{-} \qquad , j=1,2,\dots,n,\ i=1,2,\dots,m,\ r=1,2,\dots,s$$

The optimal solution of (1) is $(\varphi^*,\ \lambda^*,\ s^{+*},\ s^{-*})$.

Then define set E as follow:

$$E=\left\{j\,|\,\varphi_j^*=1\right\} \qquad (2)$$

$(\varphi_j^*\ is\ \varphi^*\ \ for\ DMU_j)$

A DMU in set E has the highest amounts of ith input component compared with other DMUs is selected.

$$\exists\left(t\in E\right);\ \forall\left(j\in E\right)\Rightarrow x_{it}\ge x_{ij} \qquad (3)$$

$$x_{it}=x_i^* \quad ,\quad i=1,\dots,m$$

We have congestion if and only if, at least one of the two following conditions is satisfied:

I. $\varphi^*>1$ and there is at least one $x_{io}>x_i^*$ $\ (i=1,2,\dots,m)$

II. There is at least one $s_r^{+*}>0$ $\ (r=1,2,\dots,s)$ and there is at least one $x_{io}>x_i^*$ $\ (i=1,2,\dots,m)$

2.2 DEA-DA

Sueyoshi [20] represented his extended DEA-DA method for two groups.

The first stage that is classification and overlap identification is formulated as follows:

$$
\begin{aligned}
& min \sum_{j\in G_1} S_{1j}^{+} + \sum_{j\in G_2} S_{2j}^{-} \\
& s.t. \ \sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} + S_{1j}^{+} - S_{1j}^{-} = d+1, \qquad j \in G_1 \\
& \quad \sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} + S_{2j}^{+} - S_{2j}^{-} = d, \qquad j \in G_2 \\
& \quad \sum_{i=1}^{k} (\lambda_i^{+} + \lambda_i^{-}) = 1 \\
& \quad all\ slacks \geq 0 \ , \ \lambda_i^{+} \geq 0 \ , \ \lambda_i^{-} \geq 0 \ , \ d: unrestricted
\end{aligned}
\tag{4}
$$

Where S_{1j}^{+}, S_{1j}^{-} $(j \in G_1)$ are the positive and negative aberration of a piecewise linear discriminant function $\sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij}$ from a discriminant score d of G_1, respectively. The positive aberration, specify an event of group misclassification on the jth observation in G_1 and the negative aberration, specify an event of group correct classification. The above description for G_1 expand to G_2 (S_{2j}^{+}, S_{2j}^{-} $(j \in G_2)$).

All the observed factors Z_{ij} are connected by $\sum_{i=1}^{k} \lambda_i Z_{ij}$ where λ_i is a weight for the ith factor.

These weights are limited in the way that the sum of total values of $\lambda_i = (\lambda_i^{+} - \lambda_i^{-})$ for all $i = 1, ..., k$ is unity.

The new sample that is mth observation, whose value is defined by Z_{im}, can be classified by the following principle:

I. $If \ \sum_{i=1}^{k} \lambda_i^{*} Z_{im} \geq d^{*} + 1 \ then \ m \in G_1$

II. $If \ d^{*} + 1 > \sum_{i=1}^{k} \lambda_i^{*} Z_{im} > d^{*} \ then \ m \in G_1 \cap G_2$

III. $If \ d^{*} \geq \sum_{i=1}^{k} \lambda_i^{*} Z_{im} \ then \ m \in G_2$

($\lambda_i^{*} = (\lambda_i^{+^{*}} - \lambda_i^{-^{*}})$, d^{*} are the optimal solutions of (4))

For using these principles, the whole set G has divided into the following subset:

$$C_1 = \{ j \in G_1 \mid \sum_{i=1}^{k} \lambda_i^{*} Z_{im} \geq d^{*} + 1 \}$$

$$C_2 = \{ j \in G_2 \mid \sum_{i=1}^{k} \lambda_i^{*} Z_{im} \leq d^{*} \}$$

$$D_1 = G_1 - C_1$$

$D_2 = G_2 - C_2$

When the overlap is identified, the second stage that is handling overlap has used for two subgroups $(D_1 \cup D_2)$. The handling overlap is formulated as follows:

$$
\begin{aligned}
& min \sum_{j\in D_1} S_{1j}^{+} + \sum_{j\in D_2} S_{2j}^{-} \\
& s.t. \ \sum_{i=1}^{k}(\lambda_i^{+}-\lambda_i^{-})Z_{ij} \ge d+1, && j\in C_1 \\
& \sum_{i=1}^{k}(\lambda_i^{+}-\lambda_i^{-})Z_{ij} + S_{1j}^{+} - S_{1j}^{-} = c, && j\in D_1 \\
& \sum_{i=1}^{k}(\lambda_i^{+}-\lambda_i^{-})Z_{ij} + S_{2j}^{+} - S_{2j}^{-} = c, && j\in D_2 \\
& \sum_{i=1}^{k}(\lambda_i^{+}-\lambda_i^{-})Z_{ij} \le d, && j\in C_2 \\
& \sum_{i=1}^{k}(\lambda_i^{+}+\lambda_i^{-}) = 1 \\
& d \le c \le d+1 \\
& all\ slacks \ge 0 \ , \ \lambda_i^{+} \ge 0 \ , \ \lambda_i^{-} \ge 0 \ , \ d: unrestricted \ , \ c: unrestricted
\end{aligned}
\tag{5}
$$

All correct classified observations have limited by constraints numbers 1 and 4 in the model (5).

$$(\sum_{i=1}^{k}(\lambda_i^{+}-\lambda_i^{-})Z_{ij} \ge d+1, \quad j\in C_1 \quad ; \quad \sum_{i=1}^{k}(\lambda_i^{+}-\lambda_i^{-})Z_{ij} \le d, \quad j\in C_2)$$

The new discriminant score (c) is specified by minimizing the total aberration of observation in the overlap. The new sample that is rth observation which is identified as an overlap in the first stage can be classified by the following principle:

I. $If \ \sum_{i=1}^{k}\lambda_i^{*}Z_{ir} \ge c^{*} \ then \ r\in G_1$

II. $If \ \sum_{i=1}^{k}\lambda_i^{*}Z_{ir} < c^{*} \ then \ r\in G_2$

($\lambda_i^{*} = (\lambda_i^{+^{*}} - \lambda_i^{-^{*}})$, c^{*} are the optimal solutions of (5))

For more comprehension, we bring Fig.1

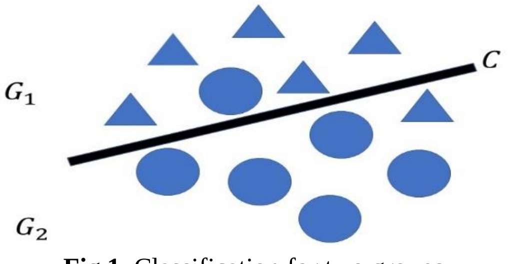

Fig 1. Classification for two groups

3. Proposed method

3.1. Measuring congestion

In this section, we used the Navidi et al. [10] method for measuring the congestion for verification of the DEA-DA method that represents in the model (5) to see its prediction of DMUs congestion is correct or not.

Assume that, the number of DMUs, inputs, and outputs are n, m, and s. The vectors $x_j = (x_{1j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{rj})^T$ are the input and output values of DMU_j, $j = 1, \dots, n$, respectively.

we will consider the maximum value in each component of the output. For DMU_1 we have:

$$Max\ y_{11} = y_{11}^* \ ,\ Max\ y_{21} = y_{21}^* \ ,\ \dots,\ Max\ y_{s1} = y_{s1}^* \qquad (6)$$

.

.

.

For DMU_n we have:

$$Max\ y_{1n} = y_{1n}^* \ ,\ Max\ y_{2n} = y_{2n}^* \ ,\ \dots,\ Max\ y_{sn} = y_{sn}^*$$

Afterward, we define the set F as follow:

$$F = \left\{ y_{rj}^* \ , r = 1, \dots, s \ , \ j = 1, \dots, n \right\} \qquad (7)$$

A DMU in set F has the highest amounts of ith input component compared with other DMUs is selected.

$$\exists (o \in F); \forall (j \in F) \Rightarrow x_{io} \geq x_{ij} \qquad (8)$$

$$x_{io} = x_i^* \quad , \quad i = 1, \dots, m$$

For example, in set F:

DMU_o has the highest amounts of first input component compared with the first input component of the other DMUs:

$$\exists (p \in F); \forall (j \in F) \Rightarrow x_{1p} \geq x_{1j} \quad ; \quad x_{1p} = x_1^*$$

and so on.

We have congestion if at least there is one $x_{ip} > x_i^*$ $(i = 1, 2, \dots, m)$

The congestion in the ith input of DMU_P is

$$s_i^c = x_{ip} - x_i^* \quad , \quad (x_{ip} > x_i^*) \qquad (9)$$

3.2. Classification of observations

In this section, we used our represented Discriminant Analysis method for the classification of observations.

The proposed is the classification of the congestion with the DEA-DA model. Therefore, assume that there are n observations $j = (1, ..., n)$ that are belong to 3 groups, each observation defines by k independent factors $i = (1, ..., k)$ indicated by Z_{ij}.

The DEA-DA model for more than two groups is formulated as follows:

$$
\begin{aligned}
& min \sum_{j\in G_1} t_{1j} + \sum_{j\in G_2} t_{2j} + \sum_{j\in G_2} t_{3j} + \sum_{j\in G_3} t_{4j} \\
& s.t. \ \sum_{i=1}^{k} \lambda_i Z_{ij} - c_1 \geq -t_{1j}, && j \in G_1 \\
& \sum_{i=1}^{k} \lambda_i Z_{ij} - c_1 \leq t_{2j}, && j \in G_2 \\
& \sum_{i=1}^{k} \lambda_i Z_{ij} - c_2 \geq -t_{3j}, && j \in G_2 \\
& \sum_{i=1}^{k} \lambda_i Z_{ij} - c_2 \leq t_{4j}, && j \in G_3 \\
& \sum_{i=1}^{k} |\lambda_i| = 1 \\
& c_1, c_2 : unrestricted \ , \ \lambda_i : unrestricted \ , \ t_{1j} \geq 0 \ , \ t_{2j} \geq 0 \ , \ t_{3j} \geq 0 \ , \ t_{4j} \geq 0
\end{aligned}
\tag{10}
$$

All the observed factors Z_{ij} are connected by $\sum_{i=1}^{k} \lambda_i Z_{ij}$ where λ_i is a weight for the ith factor. The different 3 groups separate with discriminant scores c_g. The variables $t_{1j}, t_{2j}, t_{3j}, t_{4j}$, are the aberration of discriminant function $\sum_{i=1}^{k} \lambda_i Z_{ij}$ from a discriminant score c_g to minimize an event of group misclassification.
The new sample that is lth observation, whose value is defined by Z_{il}, can be classified by the following principle:

I. $If \ \sum_{i=1}^{k} \lambda_i^* Z_{il} \geq c_1^* \quad then \ \ l \in G_1$

II. $If \ c_1^* \geq \sum_{i=1}^{k} \lambda_i^* Z_{il} \geq c_2^* \quad then \ \ l \in G_2$

III. $If \ c_2^* \geq \sum_{i=1}^{k} \lambda_i^* Z_{il} \geq c_3^* \quad then \ \ l \in G_3$

(c_g^* and λ_i^* are the optimal solutions of (5))

4. Numerical example

4.1. Measuring congestion

In this section, at first, we calculate the congestion for verification of the DEA-DA method to see its prediction of DMUs congestion is correct or not.
Assume that we have 15 DMUs as shown in Table 1.

Table 1. Source: Khoveyni et al. [13]

DMU	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Input1	-1	-3	0	-2	-2	2	4	-2	-2	4	3	-2	3	2	3
Input2	-3	-1	-2	0	2	-2	-2	4	2	4	-2	3	3	1	3
Output	-1	-1	1	1	1	1	0	0	1	0	0.5	0.5	0.5	-2	-3

By using Navidi et al. [10] method we have:

$$F=\{y_C=y_D=y_E=y_F=y_I=1\} \quad , \quad x_1^*=x_{1F}=2 \quad , \quad x_2^*=x_{2E}=x_{2I}=2$$

$$x_G^C=x_{1G}-x_1^*=4-2=2$$

$$x_H^C=x_{2H}-x_2^*=4-2=2$$

$$x_J^C=\begin{cases} x_{1J}-x_1^*=4-2=2 \\ x_{2J}-x_2^*=4-2=2 \end{cases}$$

$$x_K^C=x_{1K}-x_1^*=3-2=1$$

$$x_L^C=x_{2L}-x_2^*=3-2=1$$

$$x_M^C=\begin{cases} x_{1M}-x_1^*=3-2=1 \\ x_{2M}-x_2^*=3-2=1 \end{cases}$$

$$x_O^C=\begin{cases} x_{1O}-x_1^*=3-2=1 \\ x_{2O}-x_2^*=3-2=1 \end{cases}$$

As you see here, we measure the congestion in each input and discern weakly or strongly congestion of all DMUs so easy. The results are shown in Table 2.

Table 2. Results of congestion

DMU	Congestion
G	Weak congestion
H	Weak congestion
J	Strong congestion
K	Weak congestion
L	Weak congestion
M	Strong congestion
O	Strong congestion

For more comprehension, we bring Fig.2.

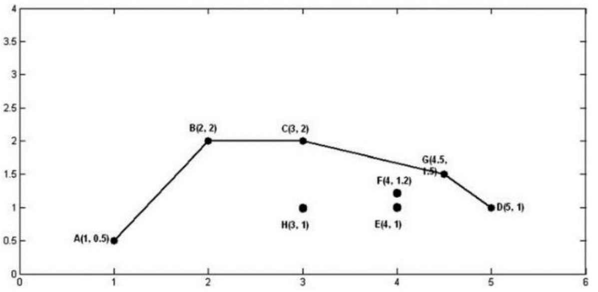

Fig. 2. Source: Brockett et al. [7] and Noura et al. [9]

4.2. Classification

In this section, we apply model (5) to our data. Table 3 present the λ_i^* $(i = 1, 2, 3)$ and c_g^* $(g = 1, 2)$.

Table 3. Weight estimates and discriminant scores

c_1^*	0.69
c_2^*	0.37
λ_1^*	0.32
λ_2^*	0.32
λ_3^*	0.36

In this research we used 3 indexes (2 inputs and 1 output that has shown in Table 1) to discern the 15 DMUs to 3 following groups:

G_1= {Strong congestion}

G_2= {Weak congestion}

G_3= {No congestion}

Table 4 present the group membership and prediction of the group membership of DMUs that were achieved from using the model (5).

Table 4. Classification

Obs	Group	Prediction
J	G_1	G_1
M	G_1	G_1
O	G_1	G_1
G	G_2	G_2
H	G_2	G_2
K	G_2	G_2
L	G_2	G_2
A	G_3	G_3
B	G_3	G_3
C	G_3	G_3
D	G_3	G_3
E	G_3	G_3
F	G_3	G_3
I	G_3	G_3
N	G_3	G_3

As you see in Table 4, all of the 15 DMUs are classified as 100% correct. Model (5) is a simple and convenient model that can correctly predict group membership easily.

5. Conclusion

As we know, two of the significant topics in DEA are measuring the congestion and conjecturing the correct classification of a new sample. There are lots of models and methods represented in these fields. The represented method for measuring the congestion is tried to present their method that needs to solve the fewer model. Also, most of the represented methods for classification are just useful for classifying observation into two groups.
In this paper, at first, we used a convenient method to measure the congestion of DMUs that includes simple calculations to verify the DEA-DA method. Then we used the DEA-DA method that can be classified the congestion of DMUs into three groups such as DMUs with strong congestion, DMUs with weak congestion, and DMUs with no congestion. As shown in Table 4, the method predicted all of the congestion of DMUs 100% correct.
Future work can expand our framework to other alterations of the DEA methods. Also, the represented DEA-DA method can use for imprecise data, so it seems good to measuring the congestion with imprecise data and then predict the congestion of new DMUs with these methods.

References

[1] Banker RD, Charnes A, Coopper WW, (1984), Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis, Journal of Management Science, 30:1078-1092.

[2] Charnes A, Cooper W.W, Rhodes E, (1978), Measuring the efficiency of decision-making units, European Journal of Operational Research, 2:429–444.

[3] Cooper WW, Seiford LM, Zhu J, (2000), A unified additive model approach for evaluating inefficiency and congestion whit associated measures in DEA, Socio-Economic Planning Sciences, 34:1_25.

[4] Cooper WW, Deng H, Seiford LM, Zhu J, (2004), Congestion: its identification and management with DEA, In: Cooper WW, Seiford LM, Zhu J, editors, Handbook of data envelopment analysis, Boston: Kluwer; p. 177e201.

[5] Färe R, Grosskopf S, (1983), Measuring congestion in production, Journal of Economics, 43:257e71.

[6] Färe R, Grosskopf S, (2001), When can slacks be used to identify congestion. An answer to W. W. Cooper, L. Seiford, J. Zhu, Socio-Economic Planning Sciences, 35:1e10.

[7] Brockett PL, Cooper WW, Shin HC, Wang Y, (1998), Inefficiency and congestion in Chinese production before and after the 1978 economic reforms, Socio-Economic Planning Sciences, 32:1-20.

[8] Cooper WW, Deng H, Huang ZM, Li SX, (2002), A one-model approach to congestion in data envelopment analysis, Socio-Economic Planning Sciences, 36:231_8.

[9] Noura AA, Hosseinzadeh Lotfi F, Jahanshahloo GR, Fanati Rashidi S, Barnett R. Parker, (2010), A new method for measuring congestion in data envelopment analysis, Socio-Economic Planning Sciences, 44: 240_246.

[10] Navidi S, Rostamy-Malkhalifeh M, Hosseinzadeh Lotfi F, (2019), Measuring congestion in data envelopment analysis without solving any models, Scientia Iranica.

[11] Sueyoshi T, Goto M, (2016), Undesirable congestion under natural disposability and desirable congestion under managerial disposability in U.S. electric power industry measured by DEA environmental assessment, Energy Economics, 55: 173_188.

[12] Sueyoshi T, Goto M, (2016), Returns to damage under undesirable congestion and damages to return under desirable congestion measured by DEA environmental assessment with multiplier restriction: Economic and energy planning for social sustainability in China, Energy Economics, 56: 288_309.

[13] [Khoveynia M, Eslamib R, Yang GL, (2017), Negative data in DEA: Recognizing congestion and specifying the least and the most congested decision-making units, Computers & Operations Research, 79:39_48.

[14] Wang Q, Wan J, YuanY, (2018), Locality constraint distance metric learning for traffic congestion detection, Pattern Recognition, 75:272_281.

[15] Sole-Ribalta A, Arenas A, Gomez S, (2019), Effect of shortest path multiplicity on congestion of multiplex networks, new journal of physics, 21: Open access.

[16] Zhou DQ, Meng FY, Bai Y, Cai SQ, (2017), Energy efficiency and congestion assessment with energy mix effect: The case of APEC countries, Journal of Cleaner Production, 142:819_828.

[17] Zhou P, Wu F, Zhou DQ, (2017), Total-factor energy efficiency with congestion, Annals of Operations Research, 255:241_256.

[18] Hu JL, Chang MC, Tsay HW, (2017), The congestion total-factor energy efficiency of regions in Taiwan, Energy Policy, 110: 710_718.

[19] Sueyoshi T, (1999), DEA–discriminant analysis in the view of goal programming, European Journal of Operational Research, 115:564–582.

[20] Sueyoshi T, (2001), Extended DEA–discriminant analysis, European Journal of Operational Research, 131:324–351.

[21] Jahanshahloo G.R, Hosseinzadeh Lotfi F, Rezai Balf F, Zhiani Rezai H, (2007), Discriminant analysis of interval data using monte carlo method in assessment of overlap, Applied Mathematics and Computation, 191:521–532.

[22] Duarte Silva A.P, Brito P, (2006), Linear discriminant analysis for interval data, Computational Statistics, 21:289–308.

[23] Duarte Silva A.P, Brito P, (2015), Discriminant analysis of interval data: an assessment of parametric and distance-based approaches, Journal of Classification, 32:516–541.

[24] Angulo C, Anguita D, Gonzalez-Abril L, Ortega J.A, (2008), Support vector machines for interval discriminant analysis, Neurocomputing, 71:1220–1229.

[25] Hosseinzadeh Lotfi F, Jahanshahloo G.R, Rezai Balf F, Zhiani Rezai H, (2007), Discriminant Analysis of Imprecise Data, Applied Mathematical Sciences, 1:723 – 737.

[26] Hosseinzadeh Lotfi F, Mansouri B, (2008), The Extended Data Envelopment Analysis/Discriminant Analysis Approach of Fuzzy Models, Applied Mathematical Sciences, 2:1465 – 1477.

[27] Khalili-Damghani K, Tavana M, J. Santos-Arteaga F, (2016), A comprehensive fuzzy DEA model for emerging market assessmentand selection decisions, Applied Soft Computing, 38:676–702.

[28] Omrani H, Shafaat Kh, Emrouznejad A, (2018), An Integrated Fuzzy Clustering Cooperative Game Data Envelopment Analysis Model with application in Hospital efficiency, Expert Systems with Applications, 114:615-628.

[29] Ji A, Qiao Y, Liu C, (2019), Fuzzy DEA-based classifier and its applications in healthcare management, Health Care Manag Sci 22:560–568.

[30] Ghasemi M-R, Ignatius J, Lozano S, Emrouznejad A, Hatami-Marbini A, (2015), A fuzzy expected value approach under generalized data envelopment analysis, Knowl-Based Syst 89:148–159.

[31] Dotoli M, Epicoco N, Falagario M, Sciancalepore F, (2015), Acrossefficiency fuzzy Data Envelopment Analysis technique for performance evaluation of Decision-Making Units under uncertainty, Comput Ind Eng 79:103–114.